interferometric interpolation

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basic equations

The Nyquist equations for Fourier transform interferometry are

$$V_{\mathrm{max}} = n\,dv = 1/(2dx)$$

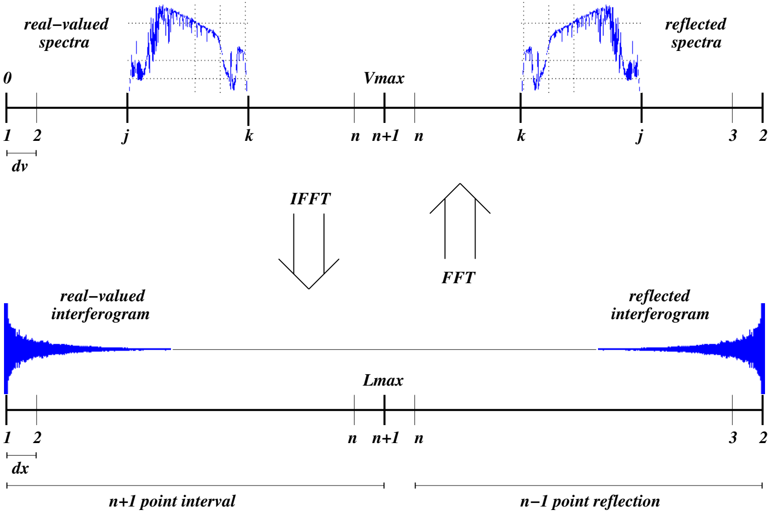
$$L_{\mathrm{max}} = n\,dx = 1/(2dv)$$

where

- L_{max} is maximum OPD
- V_{max} is maximum frequency
- dx is the distance step
- dv is the frequency step
- n is the number of steps

We consider the application to interferometric or "double Fourier" interpolation

example



n+1 point real–valued cosine transform with a 2n point FFT

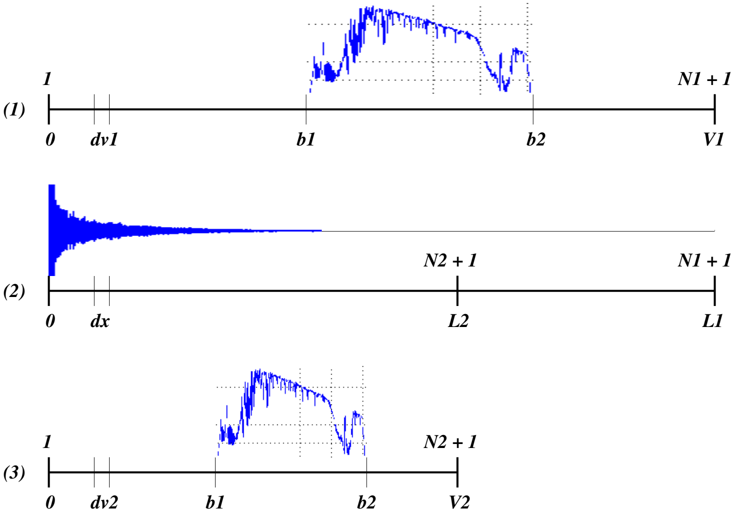
discussion

- $V_{\max} = n\,dv = 1/(2dx)$ and $L_{\max} = n\,dx = 1/(2dv)$
- $L_{\max}$ and $V_{\max}$ fall on index $n+1$ when we start counting from 1
- index 1 is optical displacement 0 and index $n+1$ is optical displacement $L_{\max}$. We need $n+1$ points to represent a full single-sided sweep
- starting from spectra we generate the interferogram by embedding the spectral interval from j to k in the larger zero-filled span from 0 to $V_{\max}$, reflecting this from n down to 2 as shown, and doing a $2n$-point inverse FFT
- starting from spectra at resolution dv, note that choosing one of dx, $V_{\max}$, or n fixes the other values

discussion

- if our original $n+1$ point spectra is real-valued then the interferogram will be as well
- the transforms are invertible. If we start with an $n+1$ point sweep from 0 to L_{max}, reflect it as shown, and do a $2n$ point FFT, we get the spectra and reflection
- in practice we might take a $2n$ point sweep from $-L_{\text{max}}$ to L_{max}, get an interferogram centered near point n, swap halves to get something like the interferogram shown, and do a $2n$ point FFT to get the spectra and reflection
- in that case the resulting spectrum will typically be complex, with the imperfect centering appearing as a phase shift

interpolation



basic equations

The Nyquist equations for (1) and (2) are

$$V_1 = N_1 dv_1 = 1/(2dx)$$

$$L_1 = N_1 dx = 1/(2dv_1)$$

and for (2) and (3) are

$$V_2 = N_2 dv_2 = 1/(2dx)$$

$$L_2 = N_2 dx = 1/(2dv_2)$$

If we are given dv_1 and dv_2 then we need dx, N_1, or N_2 to fill out the values above. N_1 and N_2 must be integers.

constraints

Note that dx is the same for both pairs of equations, so $V_1 = V_2$. Then $N_1 dv_1 = N_2 dv_2$, and we have

$$N_2/N_1 = dv_1/dv_2,$$

a constraint on the transform sizes that can only be satisfied if dv_1/dv_2 is rational. In addition, the transforms must include the band of interest, so we require

$$V_1 = N_1 dv_1 \geq b_2$$

Suppose dv_1/dv_2 is rational. Let m_1 and m_2 be the smallest integers such that $m_1/m_2 = dv_1/dv_2$.

constraints

Let k be the smallest integer such that $m_2 \cdot 2^k \cdot dv_1 \geq b_2$, and let $N_1 = m_2 \cdot 2^k$ and $N_2 = m_1 \cdot 2^k$.

Then the constraints above are satisfied, and in addition if m_1 and m_2 are not too large then N_1 and N_2 will have mostly small prime factors, making the FFT calculation more efficient.

If dv_1/dv_2 is not rational or m_1 or m_2 are very large we may want to proceed differently, for example by finding a value for dv_1 close to the desired dv but with a more tractible rational representation. In that case we might want to do a conventional interpolation from dv to dv_1 as a preliminary step.