3 Resampling

We often use double Fourier interpolation to move from one frequency grid to another [4]. For interpolation from a relatively fine grid (for example a step of 0.0025 cm^{-1}) to a typical instrument grid this gives an $O(n \log n)$ algorithm. But for two relatively close grids we may get better runtimes with a resampling matrix. Perhaps the simplest way to represent such a matrix is with an explicit sinc basis,

$$R(i,j) = \frac{dv_s}{dv_u} \cdot \operatorname{sinc}(\frac{v_s(j) - v_u(i)}{dv_u}) \tag{12}$$

Here v_s is an n–vector of sensor-grid frequencies, v_u an m–vector of user-grid frequencies, dv_s the sensor-grid spacing, and dv_u the user-grid spacing. R is an $m \times n$ matrix whose columns correspond to sensor-grid channels and rows to user-grid channels. R is applied as $r_u = R\, r_s$, where r_s is an n–vector of sensor-grid radiances and r_u an m–vector of user-grid radiances. One way to see this works is to note that an impulse at a sensor grid channel gives a sinc function sampled at the user grid, and that a linear transform is uniquely determined by its action on an orthogonal basis.

R defined this was is a close cousin to the CrIS SA matrix. The key difference is that for the SA matrix we have an ILS rather than pure sinc basis. The limit as $dv_u \to dv_s$ gives the identity matrix. In that case we still have a sinc basis, but values for $i \neq j$ are sampled at the zero crossings. In practice equation (12) gives good agreement with double Fourier interpolation. For the NOAA 18-20 Jan 2016 test the mean difference was less than 0.01K in the SW and less than 0.002K for the MW and LW CrIS bands.

The NOAA/MIT CrIS resampling function is

$$R(i,j) = \frac{dv_s}{dv_u} \cdot \frac{\sin(\pi \frac{v_s(j) - v_u(i)}{dv_u})}{N \cdot \sin(\pi \frac{v_s(j) - v_u(i)}{N \cdot dv_u})} \tag{13}$$

For CrIS the convention seems to be to take $N = n \times d$ for a band-dependent decimation factor d of roughly 20. This also gives a good approximation to double Fourier interpolation. Note that $N \cdot \sin(c/N) \approx N \cdot c/N$ for large N, so the limit as $N \to \inf$ of $N \cdot \sin(c/N)$ is c. Applying this to equation (13) we see the limit as $N \to \inf$ gives equation 12, and in practice the resulting matrices are very close for CrIS with $N = n \times d$.