A Note on Interferometric Calibration
**** DRAFT ****

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We consider the definition of reference truth for measurements made with a Michelson interferometer, taking into account relatively small (or possibly non-existant) effects such as filter position and potential mathematical artifacts. The immediate application is defining reference truth and a corresponding calibration equation for the for the CrIS instrument.

1 Michelson Interferometer

Figure 1 shows a basic Michelson interferometer. Let $r_{\rm in}(\nu)$ be incoming radiance as a function of frequency ν, δ mirror displacement, and $r_{\rm out}(\nu,\delta)$ radiance on the path to the detector. In practice the signal from the detector is the product of incoming radiance, beam-splitter efficiency, and detector responsivity. But suppose for the moment that we have a perfect beam splitter and mirrors, and that there are no filters. Then radiance $r_{\rm out}$ on the path to the detector can be represented as

$$r_{\rm out}(\nu,\delta) = r_{\rm in}(\nu)(1+\cos 2\pi\nu\delta)/2 \tag{1}$$

including a term $r_{\rm in}(\nu)/2$ that depends only on ν. Integrating over frequency, we have

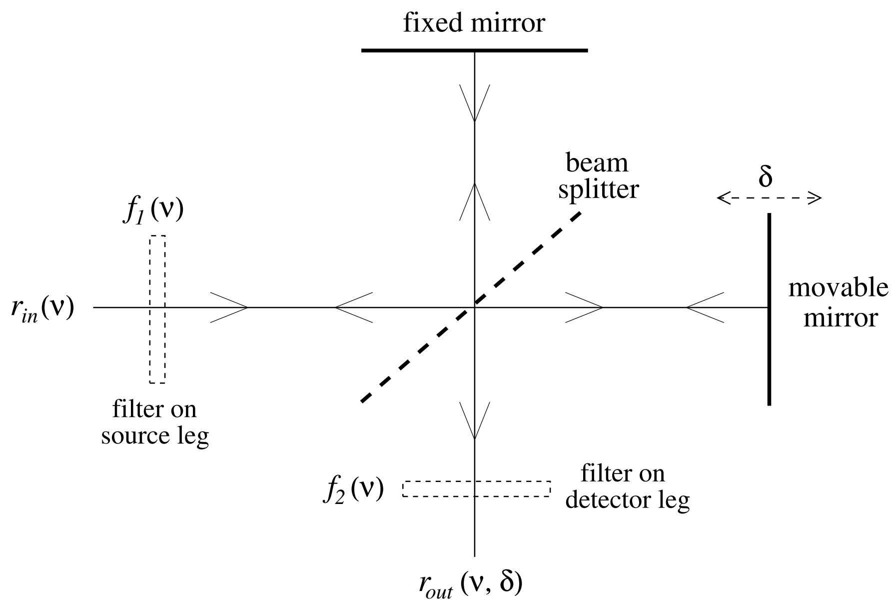

Figure 1: Michelson interferometer with filters

$$r_{\text{out}}(\delta) = \frac{1}{2}\int_{\nu=0}^{\text{inf}} r_{\text{in}}(\nu)d\nu + \frac{1}{2}\int_{\nu=0}^{\text{inf}} r_{\text{in}}(\nu)\cos(2\pi\nu\delta)d\nu \tag{2}$$

This is the continuous interferogram as a function of displacement δ.

We are interested in the following question: if we have a single filter $f(\nu)$, does r_{out} (output radiance, in our diagram) change when f is moved from the input to the detector leg of the interferometer? We present two related arguments that it does not. Consider figure 1 again, and suppose we have $f = f_1(\nu)$ on the input leg and nothing on the output leg. This can be represented as

$$r_{\text{out}}(\nu, \delta) = f(\nu)r_{\text{in}}(\nu)(1 + \cos 2\pi\nu\delta)/2 \tag{3}$$

Now suppose we move f to the detector leg. The assumption in the literature seems to be that equation (3) stll holds. Note that filter position does not matter for the $r_{\text{in}}(\nu)/2$ term, or for the case when we remove one of the mirrors.

Consider two lines at frequencies ν_1 and ν_2, where ν_1 is in the passband of f and ν_2 is not. If f is on the detector leg then both ν_1 and ν_2 will participate in interference, with radiance before the filter as in equation (1). After the filter we have $r_{\text{out}}(\nu_2, \delta) = 0$ for all δ, with $r_{\text{out}}(\nu_1, \delta)$ unchanged. If f is on the input leg then ν_1 will be as before but ν_2 will not be present anywhere downstream, so again we have $r_{\text{out}}(\nu_2, \delta) = 0$ for all δ. So the cases are the same.

Note that in this hypothetical situation there is no nonlinearity or intermodulation because we are assuming a perfect beamsplitter and mirrors, and no Nyquest limit, truncation, or discretization errors because we are considering the case for arbitraly real-valued δ. But the argument may not apply as $\nu_1 \to \nu_2$, because at some point f can not separate them.

The case we are interested in practice is where ν_1 and ν_2 are close and fall on a slope or shoulder of f. But for the following argument we don't need to assume that. So consider the case of arbitrary ν_1 and ν_2. The AC component of equation (2) with f on the input leg is

$$r_{\text{out}}(\delta) = \int_{\nu=0}^{\text{inf}} f(\nu) r_{\text{in}}(\nu) \cos(2\pi\nu\delta) d\nu \tag{4}$$

For the case where ν takes on only the two values ν_1 and ν_2 this simplifies to

$$r_{\text{out}}(\delta) = f(\nu_1)\cos(2\pi\nu_1\delta) + f(\nu_2)\cos(2\pi\nu_2\delta) \tag{5}$$

This is interesting because of the recognizable beat pattern in the interferogram. Now consider f on the output leg. On the path before f we have the case of no filter,

$$r_{\text{out}}(\delta) = \cos(2\pi\nu_1\delta) + \cos(2\pi\nu_2\delta) \tag{6}$$

And after f we have equation (5) again. Since ν_1 and ν_2 were chosen arbitrarily we conclude that the property holds for all ν and δ, and that the filter position does not matter.

In practice, f_1 might be beam splitter efficiency, f_2 detector responsivity, and $g(r_{\text{out}}(\delta))$ a function taking radiance to voltage. In the case of the CrIS instrument, f_2 could be detector responsivity plus optical filter effects. The particular question for CrIS we were interested in was if it was correct to model the effects of a filter on the detector leg with a filter on the input. The answer seems to be yes.

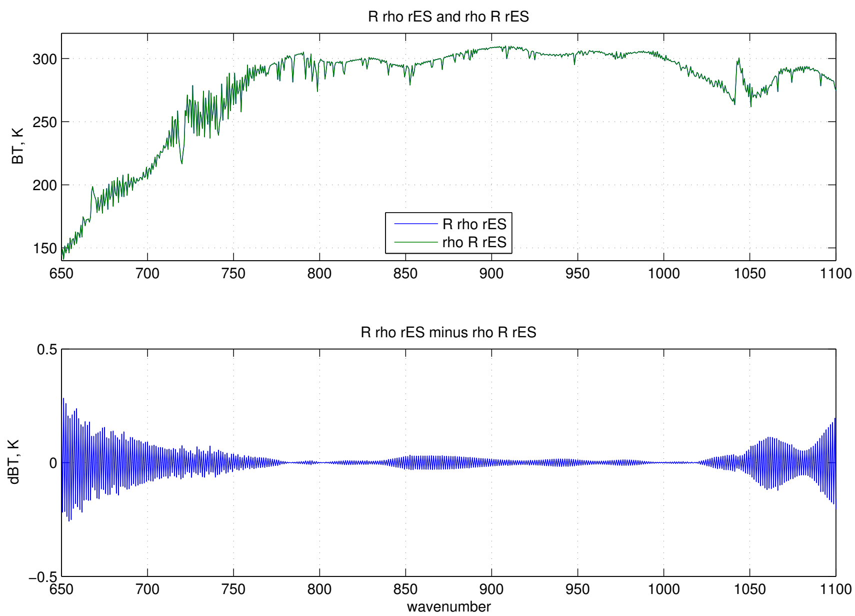

Figure 2: $R\rho\, r_{\rm ES}$ and $\rho\, R\, r_{\rm ES}$

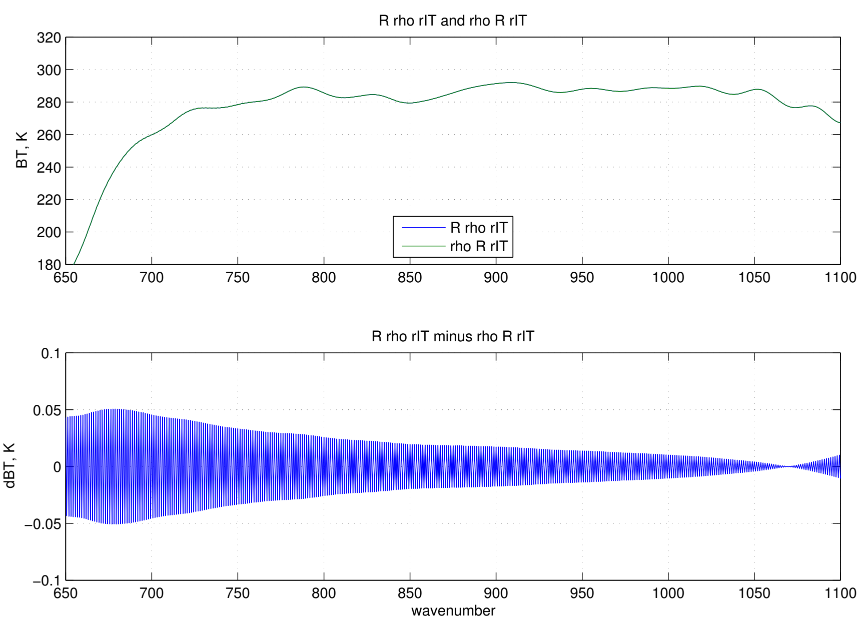

Figure 3: $R\rho\, r_{\rm ICT}$ and $\rho\, R\, r_{\rm ICT}$

2 CrIS reference truth

We can a derive reference truth from a basic form of the CrIS calibration equation, with details depending on assumptions about instrument behavior. Suppose r_{ES} is high resolution earth scene radiance, r_{ICT} high resolution black-body radiance from the internal calibration target, ρ instrument responsivity, and R resampling from high resolution to the instrument sensor grid.

If r is some arbitrary high resolution radiance, we might expect that $R\rho\, r \approx \rho R\, r$. Figure 2 shows the difference for a typical earth scene, and figure 3 the difference for an ICT (black body) look. Suppose we interpret this as $R\rho\, r_{\text{ICT}} \approx \rho\, R\, r_{\text{ICT}}$ for ICT looks, but $R\rho\, r_{\text{ES}} \neq \rho\, R\, r_{\text{ES}}$ for earth scenes. Calibration of the on-axis optical path of a Michaelson interferometer can be represented as

$$r_{\text{cal}}^{\text{LR}} = r_{\text{ICT}}^{\text{LR}} \frac{ES - SP}{IT - SP} \tag{7}$$

where $r_{\text{cal}}^{\text{LR}}$ is calibrated radiances, $r_{\text{ICT}}^{\text{LR}}$ is expected radiance from the internal calibration target, and ES, IT, and SP are uncalibrated spectra for earth scene, internal calibration target, and space looks, respectively. Note that all values in equation 7 are at the "sensor grid" $dv = 1/(2\,\text{OPD})$.

Let $ES \approx R\rho(r_{\text{ES}} + r_{\text{SP}})$, $IT \approx R\rho(r_{\text{ICT}} + r_{\text{SP}})$, and $SP \approx R\rho\, r_{\text{SP}}$, where r_{ES}, r_{ICT}, and r_{SP} are high resolution approximations to the true radiances, ρ is responsivity, and R is resampling from the high resolution to the sensor grid dv. Let $r_{\text{ICT}}^{\text{LR}} = R(r_{\text{ICT}})$. Substituting this into (7) gives

$$\begin{aligned} r_{\text{cal}}^{\text{LR}} &\approx r_{\text{ICT}}^{\text{LR}} \frac{R\rho(r_{\text{ES}} + r_{\text{SP}}) - R\rho\, r_{\text{SP}}}{R\rho(r_{\text{ICT}} + r_{\text{SP}}) - R\rho\, r_{\text{SP}}} \\ &= r_{\text{ICT}}^{\text{LR}} \frac{R\rho\, r_{\text{ES}}}{R\rho\, r_{\text{ICT}}} && (8) \\ &\approx r_{\text{ICT}}^{\text{LR}} \frac{R\rho\, r_{\text{ES}}}{\rho\, R\, r_{\text{ICT}}} && (9) \\ &= \frac{R\rho\, r_{\text{ES}}}{\rho} = r_{\text{resp}} && (10) \end{aligned}$$

We go from equation 8 to 9 because we have assumed responsivity commutes at least approximately with resampling for the ICT look. Equation 10 is the UW definition of "reference truth with responsivity".

A more conventional and user-friendly definition of reference truth is

$$r_{\text{flat}} = R\, r_{\text{ES}} \tag{11}$$

If we assume $R\rho\, r_{\text{ES}} \approx \rho\, R\, r_{\text{ES}}$ then we get "flat" reference truth. In practice we find the "ratio first" UMBC CCAST reference calibration equation has smaller residuals when compared with r_{flat}, while the "SA^{-1} first" NOAA 4 algorithm has smaller residuals with r_{resp}. It seems to us the proper focus for calibration algorithm development should be minimizing residuals in comparison with r_{flat}.

3 Resampling

We often use double Fourier interpolation to move from one frequency grid to another [4]. For interpolation from a relatively fine grid (for example a step of 0.0025 cm^{-1}) to a typical instrument grid this gives an $O(n \log n)$ algorithm. But for two relatively close grids we may get better runtimes with a resampling matrix. Perhaps the simplest way to represent such a matrix is with an explicit sinc basis,

$$R(i,j) = \frac{dv_s}{dv_u} \cdot \mathrm{sinc}(\frac{v_s(j) - v_u(i)}{dv_u}) \tag{12}$$

Here v_s is an n–vector of sensor-grid frequencies, v_u an m–vector of user-grid frequencies, dv_s the sensor-grid spacing, and dv_u the user-grid spacing. R is an $m \times n$ matrix whose columns correspond to sensor-grid channels and rows to user-grid channels. R is applied as $r_u = R\, r_s$, where r_s is an n–vector of sensor-grid radiances and r_u an m–vector of user-grid radiances. One way to see this works is to note that an impulse at a sensor grid channel gives a sinc function sampled at the user grid, and that a linear transform is uniquely determined by its action on an orthogonal basis.

R defined this was is a close cousin to the CrIS SA matrix. The key difference is that for the SA matrix we have an ILS rather than pure sinc basis. The limit as $dv_u \to dv_s$ gives the identity matrix. In that case we still have a sinc basis, but values for $i \neq j$ are sampled at the zero crossings. In practice equation (12) gives good agreement with double Fourier interpolation. For the NOAA 18-20 Jan 2016 test the mean difference was less than 0.01K in the SW and less than 0.002K for the MW and LW CrIS bands.

The NOAA/MIT CrIS resampling function is

$$R(i,j) = \frac{dv_s}{dv_u} \cdot \frac{\sin(\pi \frac{v_s(j) - v_u(i)}{dv_u})}{N \cdot \sin(\pi \frac{v_s(j) - v_u(i)}{N \cdot dv_u})} \tag{13}$$

For CrIS the convention seems to be to take $N = n \times d$ for a band-dependent decimation factor d of roughly 20. This also gives a good approximation to double Fourier interpolation. Note that $N \cdot \sin(c/N) \approx N \cdot c/N$ for large N, so the limit as $N \to \inf$ of $N \cdot \sin(c/N)$ is c. Applying this to equation (13) we see the limit as $N \to \inf$ gives equation 12, and in practice the resulting matrices are very close for CrIS with $N = n \times d$.