

A Note on Interferometric Calibration

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We consider the definition of reference truth for measurements made with a Michelson interferometer, taking into account relatively small (or possibly non-existent) effects such as filter position and potential mathematical artifacts. The immediate application is defining reference truth and a corresponding calibration equation for the CrIS instrument.

1 Michelson Interferometer

Figure 1 shows a basic Michelson interferometer. Let $r_{\text{in}}(\nu)$ be incoming radiance as a function of frequency ν , δ mirror displacement, and $r_{\text{out}}(\nu, \delta)$ radiance on the path to the detector. In practice the signal from the detector is the product of incoming radiance, beam-splitter efficiency, and detector responsivity. But suppose for the moment that we have a perfect beam splitter and mirrors, and that there are no filters. Then radiance r_{out} on the path to the detector can be represented as

$$r_{\text{out}}(\nu, \delta) = r_{\text{in}}(\nu)(1 + \cos 2\pi\nu\delta)/2 \quad (1)$$

including a term $r_{\text{in}}(\nu)/2$ that depends only on ν . Integrating over frequency, we have

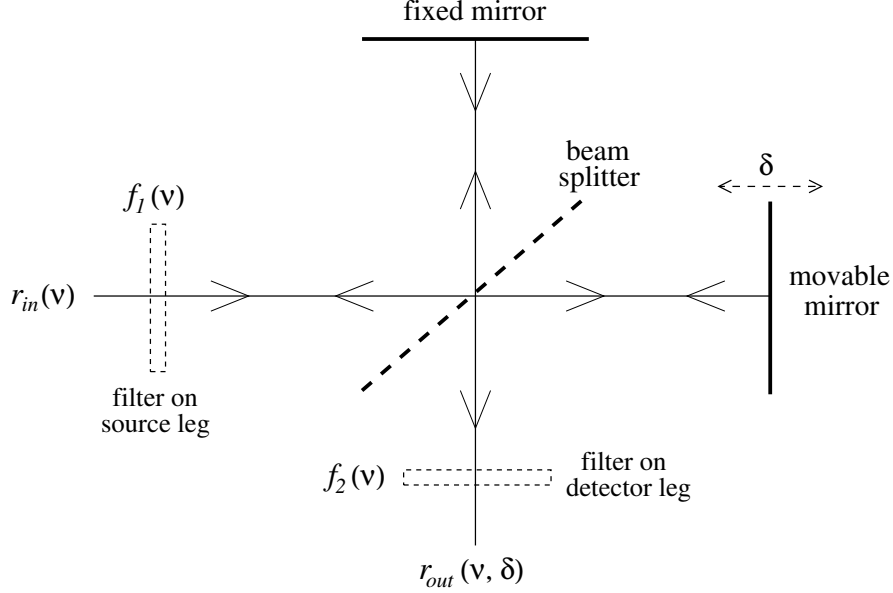


Figure 1: Michelson interferometer with filters

$$r_{\text{out}}(\delta) = \frac{1}{2} \int_{\nu=0}^{\text{inf}} r_{\text{in}}(\nu) d\nu + \frac{1}{2} \int_{\nu=0}^{\text{inf}} r_{\text{in}}(\nu) \cos(2\pi\nu\delta) d\nu \quad (2)$$

This is the continuous interferogram as a function of displacement δ .

We are interested in the following question: if we have a single filter $f(\nu)$, does r_{out} (output radiance, in our diagram) change when f is moved from the input to the detector leg of the interferometer? We present two related arguments that it does not. Consider figure 1 again, and suppose we have $f = f_1(\nu)$ on the input leg and nothing on the output leg. This can be represented as

$$r_{\text{out}}(\nu, \delta) = f(\nu)r_{\text{in}}(\nu)(1 + \cos 2\pi\nu\delta)/2 \quad (3)$$

Now suppose we move f to the detector leg. The assumption in the literature seems to be that equation (3) still holds. Note that filter position does not matter for the $r_{\text{in}}(\nu)/2$ term, or for the case when we remove one of the mirrors.

Consider two lines at frequencies ν_1 and ν_2 , where ν_1 is in the passband of f and ν_2 is not. If f is on the detector leg then both ν_1 and ν_2 will participate

in interference, with radiance before the filter as in equation (1). After the filter we have $r_{\text{out}}(\nu_2, \delta) = 0$ for all δ , with $r_{\text{out}}(\nu_1, \delta)$ unchanged. If f is on the input leg then ν_1 will be as before but ν_2 will not be present anywhere downstream, so again we have $r_{\text{out}}(\nu_2, \delta) = 0$ for all δ . So the cases are the same.

Note that in this hypothetical situation there is no nonlinearity or intermodulation because we are assuming a perfect beamsplitter and mirrors, and no Nyquist limit, truncation, or discretization errors because we are considering the case for arbitrary real-valued δ . But the argument may not apply as $\nu_1 \rightarrow \nu_2$, because at some point f can not separate them.

The case we are interested in practice is where ν_1 and ν_2 are close and fall on a slope or shoulder of f . But for the following argument we don't need to assume that. So consider the case of arbitrary ν_1 and ν_2 . The AC component of equation (2) with f on the input leg is

$$r_{\text{out}}(\delta) = \int_{\nu=0}^{\text{inf}} f(\nu) \cos(2\pi\nu\delta) d\nu \quad (4)$$

For the case where ν takes on only the two values ν_1 and ν_2 this simplifies to

$$r_{\text{out}}(\delta) = f(\nu_1) \cos(2\pi\nu_1\delta) + f(\nu_2) \cos(2\pi\nu_2\delta) \quad (5)$$

This is interesting because of the recognizable beat pattern in the interferogram. Now consider f on the output leg. On the path before f we have the case of no filter,

$$r_{\text{out}}(\delta) = \cos(2\pi\nu_1\delta) + \cos(2\pi\nu_2\delta) \quad (6)$$

And after f we have equation (5) again. Since ν_1 and ν_2 were chosen arbitrarily we conclude that the property holds for all ν and δ , and that the filter position does not matter.

In practice, f_1 might be beam splitter efficiency, f_2 detector responsivity, and $g(r_{\text{out}}(\delta))$ a function taking radiance to voltage. In the case of the CrIS instrument, f_2 could be detector responsivity plus optical filter effects. The particular question for CrIS we were interested in was if it was correct to model the effects of a filter on the detector leg with a filter on the input. The answer seems to be yes.