A Note on Interferometric Calibration
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We consider the definition of reference truth for measurements made with a Michelson interferometer, taking into account relatively small (or possibly non-existant) effects such as filter position and potential mathematical artifacts. The immediate application is defining reference truth and a corresponding calibration equation for the for the CrIS instrument.

1 Michelson Interferometer

Figure 1 shows a basic Michelson interferometer. Let $r_{\text{in}}(\nu)$ be incoming radiance as a function of frequency ν, δ mirror displacement, and $r_{\text{out}}(\nu, \delta)$ radiance on the path to the detector. In practice the signal from the detector is the product of incoming radiance, beam-splitter efficiency, and detector responsivity. But suppose for the moment that we have a perfect beam splitter and mirrors, and that there are no filters. Then radiance r_{out} on the path to the detector can be represented as

$$r_{\text{out}}(\nu, \delta) = r_{\text{in}}(\nu)(1 + \cos 2\pi\nu\delta)/2 \tag{1}$$

including a term $r_{\text{in}}(\nu)/2$ that depends only on ν. Integrating over frequency, we have

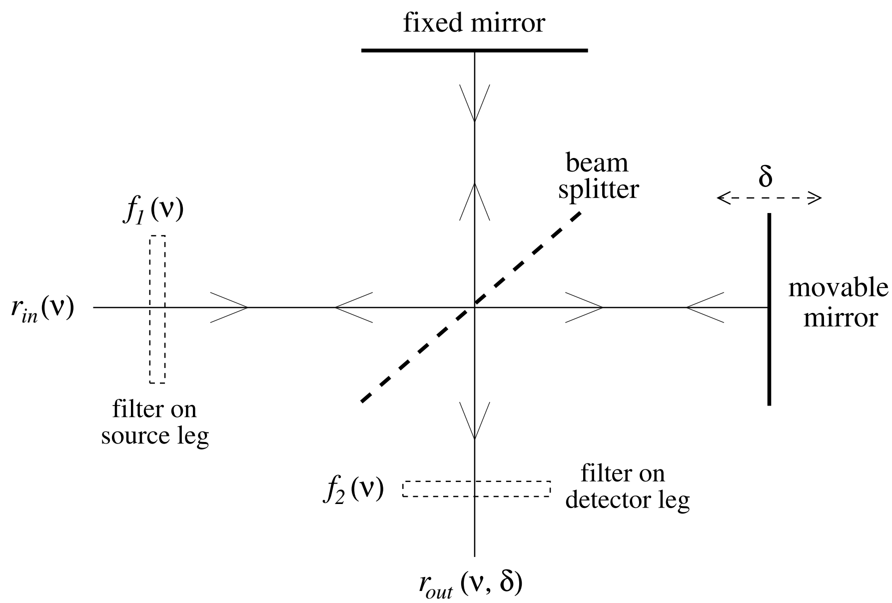

Figure 1: Michelson interferometer with filters

$$r_{\text{out}}(\delta) = \frac{1}{2}\int_{\nu=0}^{\text{inf}} r_{\text{in}}(\nu) d\nu + \frac{1}{2}\int_{\nu=0}^{\text{inf}} r_{\text{in}}(\nu)\cos(2\pi\nu\delta) d\nu \tag{2}$$

This is the continuous interferogram as a function of displacement δ.

We are interested in the following question: if we have a single filter $f(\nu)$, does it make a difference if f is on the input or detector leg of the interferometer? We present two related arguments that it does not. Consider figure 1 again, and suppose we have $f = f_1(\nu)$ on the input leg and nothing on the output leg. This can be represented as

$$r_{\text{out}}(\nu, \delta) = f(\nu) r_{\text{in}}(\nu)(1 + \cos 2\pi\nu\delta)/2 \tag{3}$$

Now suppose we move f to the detector leg. The assumption in the literature seems to be that equation (3) stll holds. Note that filter position does not matter for the $r_{\text{in}}(\nu)/2$ term, or for the case when we remove one of the mirrors.

Consider two lines at frequencies ν_1 and ν_2, where ν_1 is in the passband of f and ν_2 is not. If f is on the detector leg then both ν_1 and ν_2 will participate in interference, with radiance before the filter as in equation (1). After the filter we have $r_{\text{out}}(\nu_2, \delta) = 0$ for all δ, with $r_{\text{out}}(\nu_1, \delta)$ unchanged. If f is on the input leg then ν_1 will be as before but ν_2 will not be present anywhere downstream, so again we have $r_{\text{out}}(\nu_2, \delta) = 0$ for all δ. So the cases are the same.

Note that in this hypothetical situation there is no nonlinearity or intermodulation because we are assuming a perfect beamsplitter and mirrors, and no Nyquest limit, truncation, or discretization errors because we are considering the case for arbitraly real-valued δ. But the argument may not apply as $\nu_1 \to \nu_2$, because at some point f can not separate them.

The case we are interested in practice is where ν_1 and ν_2 are close and fall on a slope or shoulder of f. But for the following argument we don't need to assume that. So consider the case of arbitrary ν_1 and ν_2. The AC component of equation (2) with f on the input leg is

$$r_{\text{out}}(\delta) = \int_{\nu=0}^{\text{inf}} f(\nu) \cos(2\pi\nu\delta) d\nu \tag{4}$$

For the case where ν takes on only the two values ν_1 and ν_2 this simplifies to

$$r_{\text{out}}(\delta) = f(\nu_1) \cos(2\pi\nu_1\delta) + f(\nu_2) \cos(2\pi\nu_2\delta) \tag{5}$$

This is interesting because of the recognizable beat pattern in the interferogram. Now consider f on the output leg. On the path before f we have the case of no filter,

$$r_{\text{out}}(\delta) = \cos(2\pi\nu_1\delta) + \cos(2\pi\nu_2\delta) \tag{6}$$

And after f we have equation (5) again. Since ν_1 and ν_2 were chosen arbitrarily we conclude that the property holds for all ν and δ, and that the filter position does not matter.

In practice, f_1 might be beam splitter efficiency, f_2 detector responsivity, and $g(r_{\text{out}}(\delta))$ a function taking radiance to voltage. In the case of the CrIS instrument, f_2 could be detector responsivity plus optical filter effects. The particular question for CrIS we were interested in was if it was correct to model the effects of a filter on the detector leg with a filter on the input. The answer seems to be yes.