

A Note on Interferometric Calibration

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We consider the definition of reference truth for measurements made with a Michelson interferometer, taking into account relatively small (or possibly non-existent) effects such as filter position and potential mathematical artifacts. The immediate application is defining reference truth and a corresponding calibration equation for the CrIS instrument.

1 Michelson Interferometer

Figure 1 shows a basic Michelson interferometer. Let $r_{\text{in}}(\nu)$ be incoming radiance as a function of frequency ν , δ mirror displacement, and $r_{\text{out}}(\nu, \delta)$ radiance on the path to the detector. In practice the signal from the detector is the product of incoming radiance, beam-splitter efficiency, and detector responsivity. But suppose for the moment that we have a perfect beam splitter and mirrors, and that there are no filters. Then radiance r_{out} on the path to the detector can be represented as

$$r_{\text{out}}(\nu, \delta) = r_{\text{in}}(\nu)(1 + \cos 2\pi\nu\delta)/2 \quad (1)$$

including a term $r_{\text{in}}(\nu)/2$ that depends only on ν . Integrating over frequency, we have

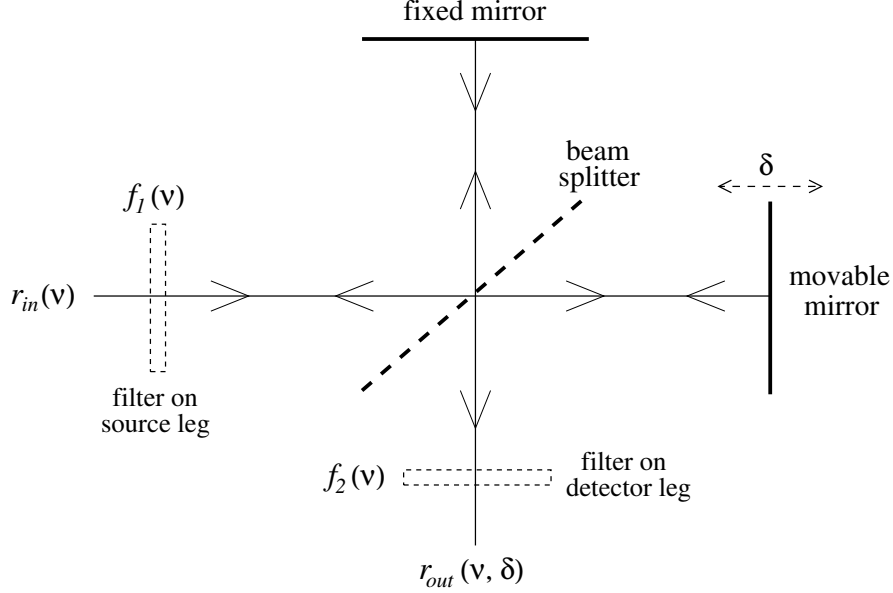


Figure 1: Michelson interferometer with filters

$$r_{\text{out}}(\delta) = \frac{1}{2} \int_{\nu=0}^{\text{inf}} r_{\text{in}}(\nu) d\nu + \frac{1}{2} \int_{\nu=0}^{\text{inf}} r_{\text{in}}(\nu) \cos(2\pi\nu\delta) d\nu \quad (2)$$

This is the continuous interferogram as a function of displacement δ .

We are interested in the following question: if we have a single filter $f(\nu)$, does r_{out} (output radiance, in our diagram) change when f is moved from the input to the detector leg of the interferometer? We present two related arguments that it does not. Consider figure 1 again, and suppose we have $f = f_1(\nu)$ on the input leg and nothing on the output leg. This can be represented as

$$r_{\text{out}}(\nu, \delta) = f(\nu) r_{\text{in}}(\nu) (1 + \cos 2\pi\nu\delta) / 2 \quad (3)$$

Now suppose we move f to the detector leg. The assumption in the literature seems to be that equation (3) still holds. Note that filter position does not matter for the $r_{\text{in}}(\nu)/2$ term, or for the case when we remove one of the mirrors.

Consider two lines at frequencies ν_1 and ν_2 , where ν_1 is in the passband of f and ν_2 is not. If f is on the detector leg then both ν_1 and ν_2 will participate in interference, with radiance before the filter as in equation (1). After the

filter we have $r_{\text{out}}(\nu_2, \delta) = 0$ for all δ , with $r_{\text{out}}(\nu_1, \delta)$ unchanged. If f is on the input leg then ν_1 will be as before but ν_2 will not be present anywhere downstream, so again we have $r_{\text{out}}(\nu_2, \delta) = 0$ for all δ . So the cases are the same.

Note that in this hypothetical situation there is no nonlinearity or inter-modulation because we are assuming a perfect beamsplitter and mirrors, and no Nyquist limit, truncation, or discretization errors because we are considering the case for arbitrarily real-valued δ . But the argument may not apply as $\nu_1 \rightarrow \nu_2$, because at some point f can not separate them.

The case we are interested in practice is where ν_1 and ν_2 are close and fall on a slope or shoulder of f . But for the following argument we don't need to assume that. So consider the case of arbitrary ν_1 and ν_2 . The AC component of equation (2) with f on the input leg is

$$r_{\text{out}}(\delta) = \int_{\nu=0}^{\text{inf}} f(\nu) \cos(2\pi\nu\delta) d\nu \quad (4)$$

For the case where ν takes on only the two values ν_1 and ν_2 this simplifies to

$$r_{\text{out}}(\delta) = f(\nu_1) \cos(2\pi\nu_1\delta) + f(\nu_2) \cos(2\pi\nu_2\delta) \quad (5)$$

This is interesting because of the recognizable beat pattern in the interferogram. Now consider f on the output leg. On the path before f we have the case of no filter,

$$r_{\text{out}}(\delta) = \cos(2\pi\nu_1\delta) + \cos(2\pi\nu_2\delta) \quad (6)$$

And after f we have equation (5) again. Since ν_1 and ν_2 were chosen arbitrarily we conclude that the property holds for all ν and δ , and that the filter position does not matter.

In practice, f_1 might be beam splitter efficiency, f_2 detector responsivity, and $g(r_{\text{out}}(\delta))$ a function taking radiance to voltage. In the case of the CrIS instrument, f_2 could be detector responsivity plus optical filter effects. The particular question for CrIS we were interested in was if it was correct to model the effects of a filter on the detector leg with a filter on the input. The answer seems to be yes.

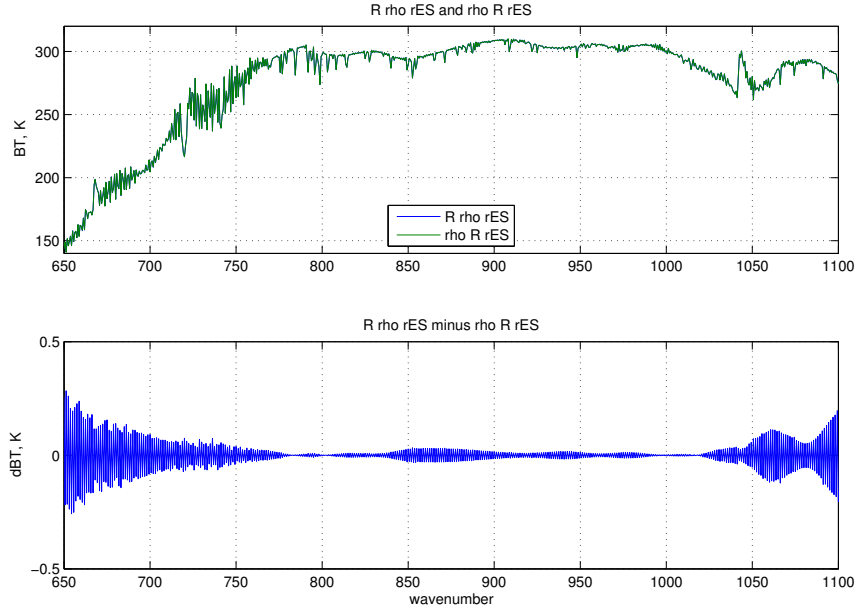


Figure 2: $R\rho r_{\text{ES}}$ and $\rho R r_{\text{ES}}$

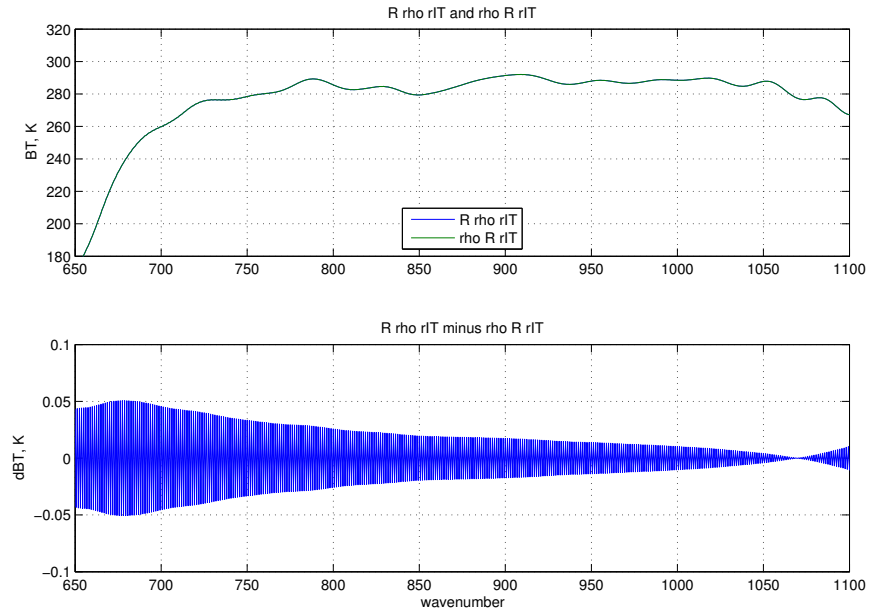


Figure 3: $R\rho r_{\text{ICT}}$ and $\rho R r_{\text{ICT}}$

2 CrIS reference truth

We can derive reference truth from a basic form of the CrIS calibration equation, with details depending on assumptions about instrument behavior. Suppose r_{es} is high resolution earth scene radiance, r_{ict} high resolution black-body radiance from the internal calibration target, ρ instrument responsivity, and R resampling from high resolution to the instrument sensor grid.

If r is some arbitrary high resolution radiance, we might expect that $R\rho r \approx \rho Rr$. Figure 2 shows the difference for a typical earth scene, and figure 3 the difference for an ICT (black body) look. Suppose we interpret this as $R\rho r_{\text{ict}} \approx \rho Rr_{\text{ict}}$ for ICT looks, but $R\rho r_{\text{es}} \neq \rho Rr_{\text{es}}$ for earth scenes. Calibration of the on-axis optical path of a Michaelson interferometer can be represented as

$$r_{\text{cal}}^{\text{LR}} = r_{\text{ict}}^{\text{LR}} \frac{ES - SP}{IT - SP} \quad (7)$$

where $r_{\text{cal}}^{\text{LR}}$ is calibrated radiances, $r_{\text{ict}}^{\text{LR}}$ is expected radiance from the internal calibration target, and ES , IT , and SP are uncalibrated spectra for earth scene, internal calibration target, and space looks, respectively. Note that all values in equation 7 are at the “sensor grid” $dv = 1/(2 \text{ OPD})$.

Let $ES \approx R\rho(r_{\text{es}} + r_{\text{sp}})$, $IT \approx R\rho(r_{\text{ict}} + r_{\text{sp}})$, and $SP \approx R\rho r_{\text{sp}}$, where r_{es} , r_{ict} , and r_{sp} are high resolution approximations to the true radiances, ρ is responsivity, and R is resampling from the high resolution to the sensor grid dv . Let $r_{\text{ict}}^{\text{LR}} = R(r_{\text{ict}})$. Substituting this into (7) gives

$$\begin{aligned} r_{\text{cal}}^{\text{LR}} &\approx r_{\text{ict}}^{\text{LR}} \frac{R\rho(r_{\text{es}} + r_{\text{sp}}) - R\rho r_{\text{sp}}}{R\rho(r_{\text{ict}} + r_{\text{sp}}) - R\rho r_{\text{sp}}} \\ &= r_{\text{ict}}^{\text{LR}} \frac{R\rho r_{\text{es}}}{R\rho r_{\text{ict}}} \end{aligned} \quad (8)$$

$$\approx r_{\text{ict}}^{\text{LR}} \frac{R\rho r_{\text{es}}}{\rho R r_{\text{ict}}} \quad (9)$$

$$= \frac{R\rho r_{\text{es}}}{\rho} = r_{\text{resp}} \quad (10)$$

We go from equation 8 to 9 because we have assumed responsivity commutes at least approximately with resampling for the ICT look. Equation 10 is the UW definition of “reference truth with responsivity”.

A more conventional and user-friendly definition of reference truth is

$$r_{\text{flat}} = R r_{\text{ES}} \quad (11)$$

If we assume $R\rho r_{\text{ES}} \approx \rho R r_{\text{ES}}$ then we get “flat” reference truth. In practice we find the “ratio first” UMBC CCAST reference calibration equation has smaller residuals when compared with r_{flat} , while the “SA⁻¹ first” NOAA 4 algorithm has smaller residuals with r_{resp} . It seems to us the proper focus for calibration algorithm development should be minimizing residuals in comparison with r_{flat} .

3 Resampling

We often use double Fourier interpolation to move from one frequency grid to another [4]. For interpolation from a relatively fine grid (for example a step of 0.0025 cm^{-1}) to a typical instrument grid this gives an $O(n \log n)$ algorithm. But for two relatively close grids we may get better runtimes with a resampling matrix. Perhaps the simplest way to represent such a matrix is with an explicit sinc basis,

$$R(i, j) = \frac{dv_s}{dv_u} \cdot \text{sinc}\left(\frac{v_s(j) - v_u(i)}{dv_u}\right) \quad (12)$$

Here v_s is an n -vector of sensor-grid frequencies, v_u an m -vector of user-grid frequencies, dv_s the sensor-grid spacing, and dv_u the user-grid spacing. R is an $m \times n$ matrix whose columns correspond to sensor-grid channels and rows to user-grid channels. R is applied as $r_u = R r_s$, where r_s is an n -vector of sensor-grid radiances and r_u an m -vector of user-grid radiances. One way to see this works is to note that an impulse at a sensor grid channel gives a sinc function sampled at the user grid, and that a linear transform is uniquely determined by its action on an orthogonal basis.

R defined this way is a close cousin to the CrIS SA matrix. The key difference is that for the SA matrix we have an ILS rather than pure sinc basis. The limit as $dv_u \rightarrow dv_s$ gives the identity matrix. In that case we still have a sinc basis, but values for $i \neq j$ are sampled at the zero crossings. In practice equation (12) gives good agreement with double Fourier interpolation. For the NOAA 18-20 Jan 2016 test the mean difference was less than 0.01K in the SW and less than 0.002K for the MW and LW CrIS bands.

The NOAA/MIT CrIS resampling function is

$$R(i, j) = \frac{dv_s}{dv_u} \cdot \frac{\sin\left(\pi \frac{v_s(j) - v_u(i)}{dv_u}\right)}{N \cdot \sin\left(\pi \frac{v_s(j) - v_u(i)}{N \cdot dv_u}\right)} \quad (13)$$

For CrIS the convention seems to be to take $N = n \times d$ for a band-dependent decimation factor d of roughly 20. This also gives a good approximation to double Fourier interpolation. Note that $N \cdot \sin(c/N) \approx N \cdot c/N$ for large N , so the limit as $N \rightarrow \text{inf}$ of $N \cdot \sin(c/N)$ is c . Applying this to equation (13) we see the limit as $N \rightarrow \text{inf}$ gives equation 12, and in practice the resulting matrices are very close for CrIS with $N = n \times d$.