A Note on Interferometric Calibration
**** DRAFT ****

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We derive the UW definition of reference truth convolved with responsivity. Note that this does not prove the definition is "correct" in any particular way, just that it follows from the assumption that responsivity does not commute with resampling for earth-scene looks, that is, that $F\rho\, r_{\text{ES}} \neq \rho\, F\, r_{\text{ES}}$. where r_{ES} is a high resolution approximation to earth-scene radiance, ρ is instrument responsivity, and F is resampling from the high resolution to the sensor grid. The derivation may suggest why the UW definition works better as a specification for some forms of the calibration equation.

Calibration of the on-axis optical path of a Michaelson interferometer can be represented as

$$r_{\text{cal}}^{\text{LR}} = r_{\text{ICT}}^{\text{LR}} \frac{ES - SP}{IT - SP} \tag{1}$$

where $r_{\text{cal}}^{\text{LR}}$ is calibrated radiances, $r_{\text{ICT}}^{\text{LR}}$ is expected radiance from the internal calibration target, and ES, IT, and SP are uncalibrated spectra for earth scene, internal calibration target, and space looks, respectively. All values in equation 1 are at the "sensor grid" $dv = 1/(2\,\text{OPD})$, where OPD is the optical path difference.

Let $ES \approx F\rho(r_{\text{ES}} + r_{\text{SP}})$, $IT \approx F\rho(r_{\text{ICT}} + r_{\text{SP}})$, and $SP \approx F\rho\, r_{\text{SP}}$, where r_{ES}, r_{ICT}, and r_{SP} are high resolution approximations to the true radiances, ρ is responsivity, and F is resampling from the high resolution to the sensor grid dv. Let $r_{\text{ICT}}^{\text{LR}} = F(r_{\text{ICT}})$. Substituting this into (1) gives

$$
\begin{aligned}
r_{\text{cal}}^{\text{LR}} &\approx r_{\text{ICT}}^{\text{LR}} \frac{F\rho(r_{\text{ES}} + r_{\text{SP}}) - F\rho\, r_{\text{SP}}}{F\rho(r_{\text{ICT}} + r_{\text{SP}}) - F\rho\, r_{\text{SP}}} \\
&= r_{\text{ICT}}^{\text{LR}} \frac{F\rho\, r_{\text{ES}}}{F\rho\, r_{\text{ICT}}} && (2) \\
&= r_{\text{ICT}}^{\text{LR}} \frac{F\rho\, r_{\text{ES}}}{\rho\, F\, r_{\text{ICT}}} && (3) \\
&= \frac{F\rho\, r_{\text{ES}}}{\rho} = r_{\text{resp}} && (4)
\end{aligned}
$$

We go from equation 2 to 3 because responsivity commutes with resampling for the ICT look, that is, $F\rho\, r_{\text{ICT}} = \rho\, F\, r_{\text{ICT}}$. The key assumption underlying the UW definition is that responsivity does *not* commute with resampling for the ES look, that is, that $F\rho\, r_{\text{ES}} \neq \rho\, F\, r_{\text{ES}}$. Equation 4 is the UW definition of "reference truth with responsivity".

A more conventional and user-friendly definition of reference truth is

$$r_{\text{flat}} = F r_{\text{ES}} \tag{5}$$

In practice we find the "ratio first" UMBC CCAST reference calibration equation has smaller residuals when compared with r_{flat}, while the "SA^{-1} first" NOAA 4 algorithm has smaller residuals with r_{resp}. It seems to us the proper focus for calibration algorithm development should be minimizing residuals in comparison with r_{flat}.