0.1 Mathematical Model

Sets :

W : Set of weeks.
O : Set of orders.
SKU : Set of SKUs.

Decision Variables :

$P_{SKU,W}$: Quantity of SKU SKU produced in week W.
$I_{SKU,W}$: Inventory level of SKU SKU at the end of week W.
A_O : Availability date for order O.
$R_{SKU,W,O}$: Number of units of SKU SKU assigned from week W batch production to order O.
$\delta_{SKU,W}$: Binary variable indicating whether SKU SKU is produced in week W.
$\gamma_{SKU,W,O}$: Binary variable indicating whether production of SKU SKU in week W is assigned to order O.

Parameters :

$D_{O,SKU}$: Quantity requested in Demand for SKU in order O.
C_W : Production capacity available in week W.
SBO_{SKU} : Initial stock of SKU SKU at week W_0.
QRD_{SKU} : Quality release duration for SKU SKU.
LD_O : Loading date for order O.
$Batch_W$: Parameter that gives the batch associated with each week.
N : A large positive number to limit R when γ is 1.

Objective Function :

Minimize the absolute difference between the availability date and the loading date for each order :

$$\sum_{O} |A_O - LD_O|$$

Constraints

1. **Production Capacity Constraint :** Ensure that the total production in each week does not exceed the available capacity :
$$\sum_{SKU} P_{SKU,W} \leq C_W \quad \forall W \in W$$

2. **Inventory Balance Constraint (Modified) :** Ensure that the inventory level at the end of the week for each SKU is equal to the projected inventory from the previous week plus the production minus the demand. For week W_0, the initial inventory is set to SBO_{SKU} :
$$I_{SKU,W} = \begin{cases} SBO_{SKU}, & \text{if } W = W_0 \\ I_{SKU,W-1} + P_{SKU,W} - \sum_O R_{SKU,W,O}, & \text{if } W > W_0 \end{cases} \quad \forall SKU, W]$$

3. **Binary Production Constraint :** Ensure that $P_{SKU,W}$ is less than or equal to $\delta_{SKU,W} \cdot M$ where M is a large positive number :
$$P_{SKU,W} \leq \delta_{SKU,W} \cdot M \quad \forall SKU, W$$

4. **Availability Calculation Constraints (Updated) :** Calculate the availability date for each order based on the maximum availability date among the weeks when production for that SKU occurs, considering the binary variable $\gamma_{SKU,W,O}$:
$$A_O \geq \max_{SKU}(Batch_W + QRD_{SKU}) \cdot \gamma_{SKU,W,O} \quad \forall O \in O$$

5. **Demand Affected by Inventory Constraint (Updated) :** Ensure that the demand for SKU SKU in order O is exactly met by the sum of affected production quantities over all weeks :
$$\sum_{W} R_{SKU,W,O} = D_{O,SKU} \quad \forall O \in O, SKU$$

6. Limit on R with γ Constraint (Updated) : Ensure that $R_{SKU,W,O}$ is limited by a large number N when $\gamma_{SKU,W,O}$ is set to 1 :

$$R_{SKU,W,O} \leq N \cdot \gamma_{SKU,W,O} \quad \forall O \in O, SKU, W$$