

# chap\_10m 频率响应 知识点

- 一、波特图绘制
- 二、放大器频率响应估算
- 三、米勒定理
- 四、器件特征频率 $f_t$

# 波特图绘制

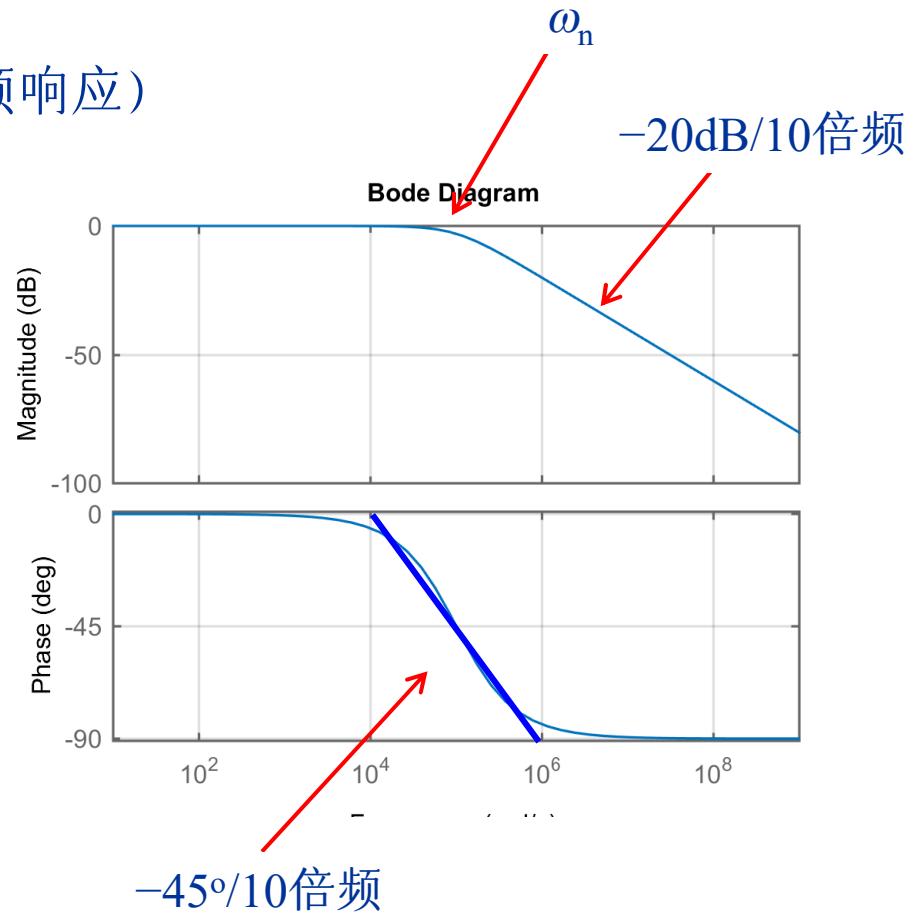
- 一阶RC低通电路（侧重描述高频响应）

$$H(s) = \frac{1}{1+sRC} = \frac{1}{1+\frac{s}{\omega_n}}, \omega_n = \frac{1}{RC}$$

$$H(j\omega) = \frac{1}{1 + \frac{j\omega}{\omega_n}}$$

- 幅度  $20 \log |H(j\omega)|$

- 相位  $\angle H(j\omega)$

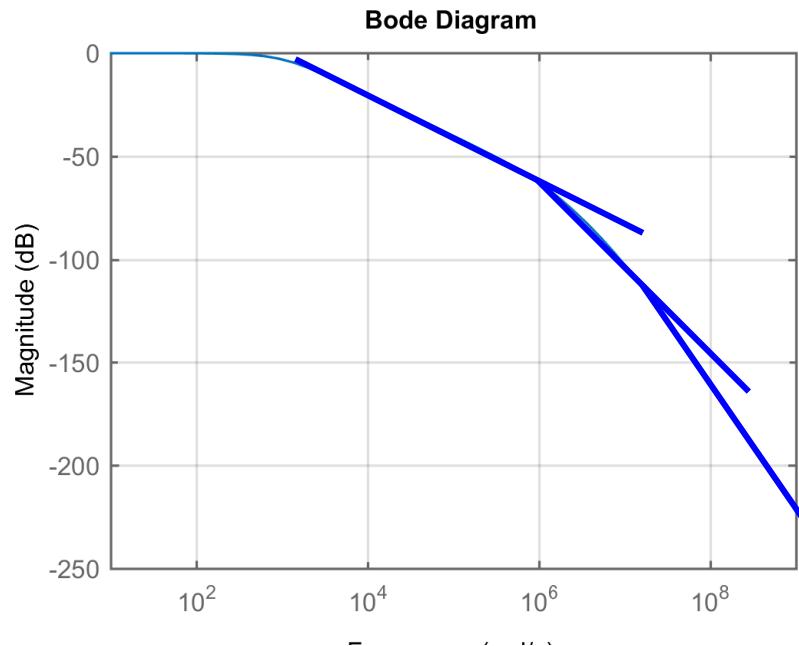


$$\omega_n = 1e5$$

# 波特图绘制

## ■ 多阶RC低通电路

$$H(s) = H_1(s)H_2(s)H_3(s)$$
$$= \frac{1}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)\left(1 + \frac{s}{\omega_3}\right)}$$



## ■ 幅度 $20\log|H(j\omega)| =$

$$20\log|H_1(j\omega)| + 20\log_2|H(j\omega)| + 20\log_3|H(j\omega)|$$

$$\omega_1 = 1e3, \omega_2 = 1e6, \omega_3 = 1e7$$

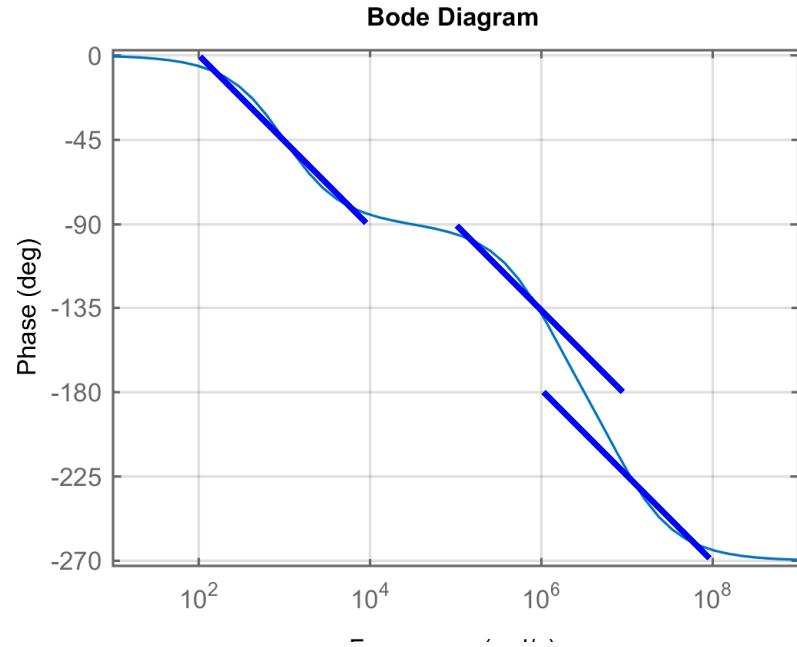
# 波特图绘制

## ■ 多阶RC低通电路

$$H(s) = H_1(s)H_2(s)H_3(s)$$
$$= \frac{1}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)\left(1 + \frac{s}{\omega_3}\right)}$$

## ■ 相位

$$\angle H(j\omega) = \angle H_1(j\omega) + \angle H_2(j\omega) + \angle H_3(j\omega)$$



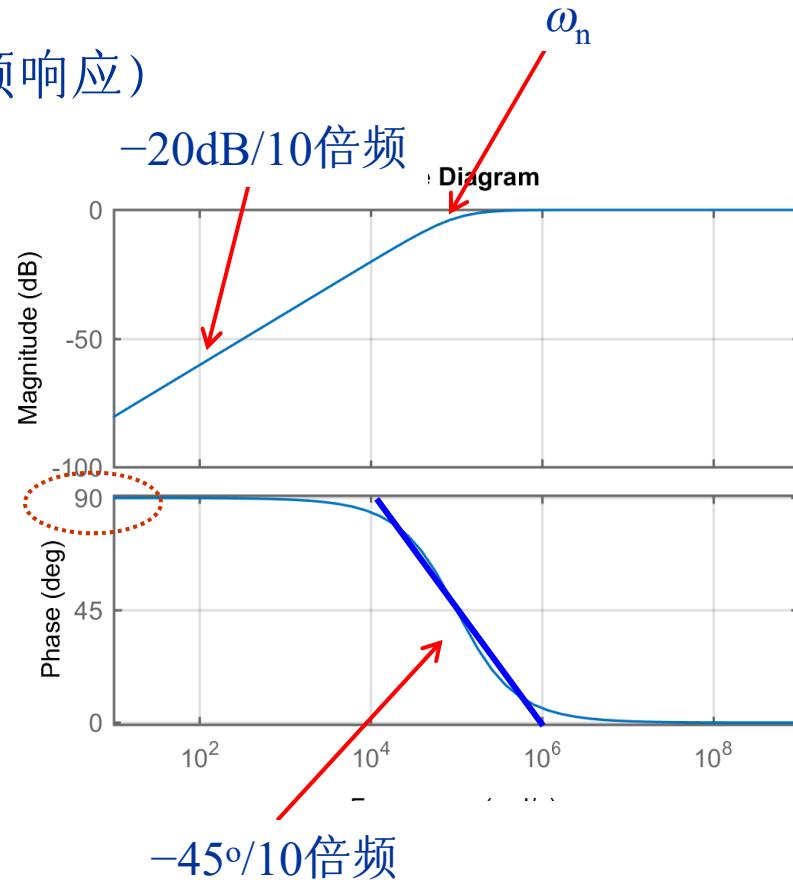
$$\omega_1 = 1e3, \omega_2 = 1e6, \omega_3 = 1e7$$

# 波特图绘制

- 一阶RC高通电路（侧重描述低频响应）

$$H(s) = \frac{sRC}{1+sRC} = \frac{1}{1+\frac{\omega_n}{s}}, \omega_n = \frac{1}{RC}$$

$$H(j\omega) = \frac{1}{1 + \frac{\omega_n}{j\omega}}$$

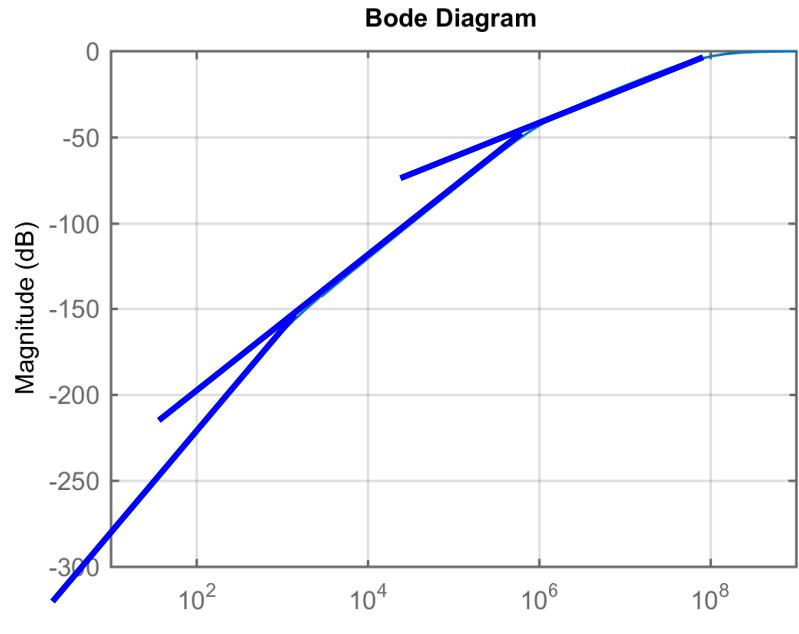


$$\omega_n = 1e5$$

# 波特图绘制

## ■ 多阶RC高通电路

$$H(s) = H_1(s)H_2(s)H_3(s)$$
$$= \frac{1}{\left(1 + \frac{\omega_1}{s}\right)\left(1 + \frac{\omega_2}{s}\right)\left(1 + \frac{\omega_3}{s}\right)}$$



■ 幅度  $20\log|H(j\omega)| =$

$$20\log|H_1(j\omega)| + 20\log_2|H(j\omega)| + 20\log_3|H(j\omega)|$$

$$\omega_1 = 1e8, \omega_2 = 1e6, \omega_3 = 1e3$$

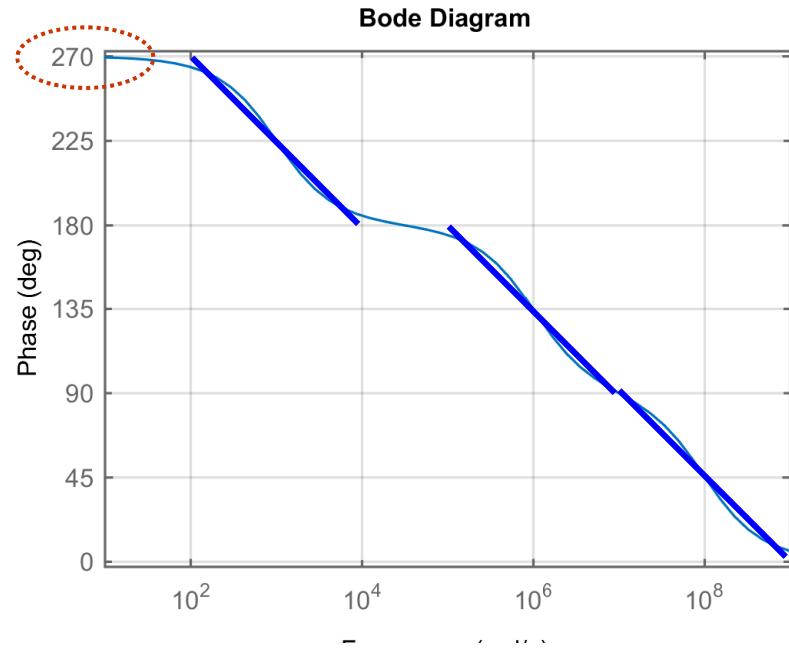
# 波特图绘制

## ■ 多阶RC高通电路

$$H(s) = H_1(s)H_2(s)H_3(s)$$
$$= \frac{1}{\left(1 + \frac{\omega_1}{s}\right)\left(1 + \frac{\omega_2}{s}\right)\left(1 + \frac{\omega_3}{s}\right)}$$

## ■ 相位

$$\angle H(j\omega) = \angle H_1(j\omega) + \angle H_2(j\omega) + \angle H_3(j\omega)$$



$$\omega_1 = 1e8, \omega_2 = 1e6, \omega_3 = 1e3$$

# 波特图绘制

## ■ 零点影响

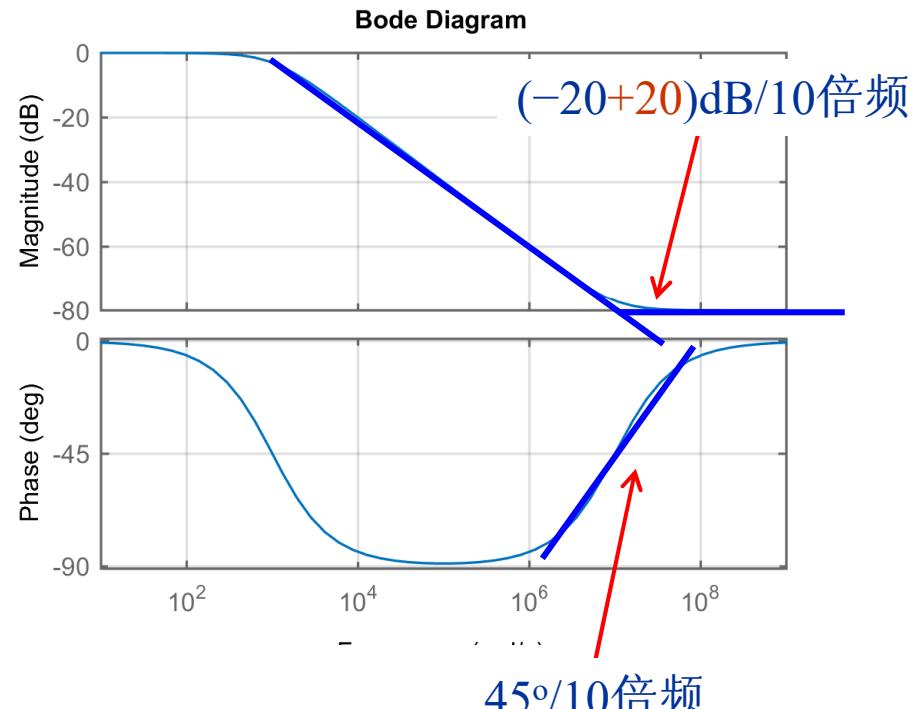
$$H(s) = \frac{\omega_{p1}}{\omega_{z1}} \frac{s + \omega_{z1}}{s + \omega_{p1}}$$

## ■ 对于高频响应

- $\omega_z \gg \omega_p$ , 零点影响可忽略

## ■ 类似地, 对于低频响应

- $\omega_z \ll \omega_p$ , 零点影响可忽略

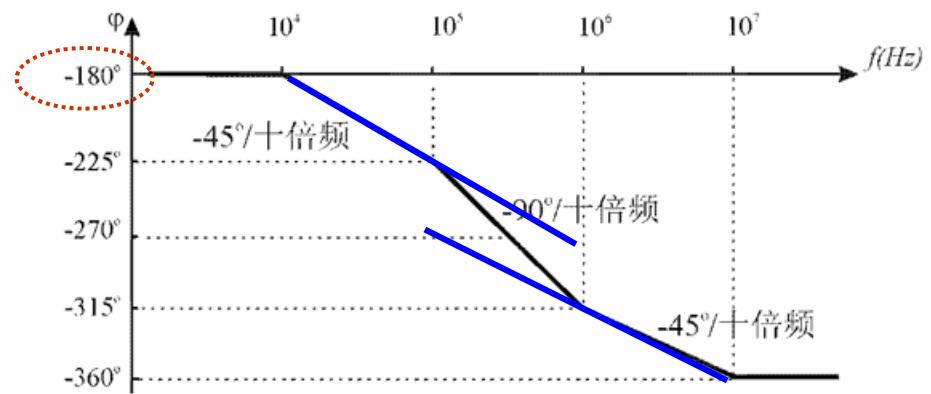


$$\omega_{z1} = 1e7, \omega_{p1} = 1e3$$

## 10.1.2

- 反相
- 二阶低通

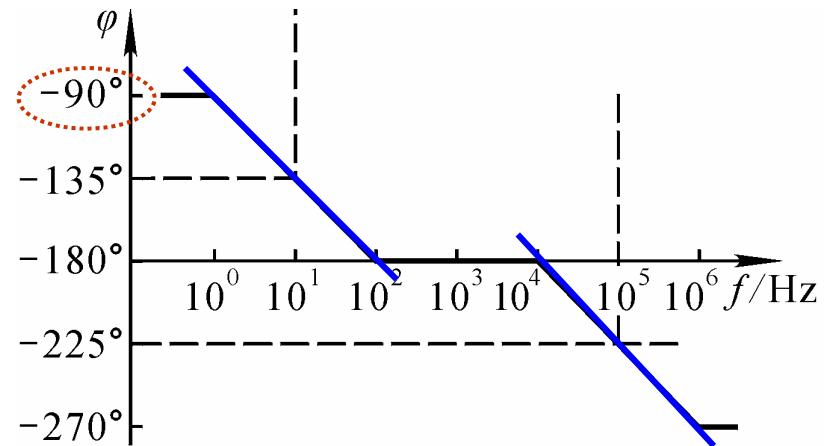
$$f_1 = 1e5, f_2 = 1e6$$



## 10.1.6

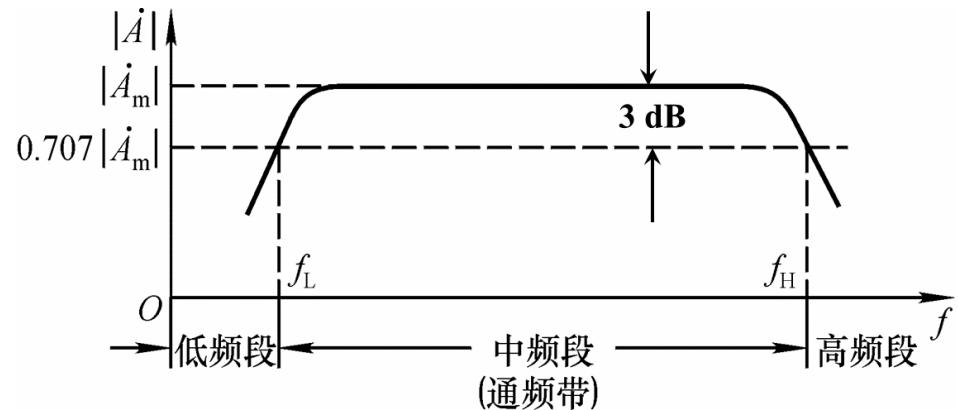
- 反相、一阶高通
- +一阶低通

$$f_1 = 1e1, f_2 = 1e5$$



# 放大器传递函数的一般表示

$$A(s) = A_M F_L(s) F_H(s)$$



■  $A_M$ ——中频增益（与频率无关）

■  $F_L(s)$ ——描述低频响应

$$F_L(s) = F_{L1}(s) F_{L2}(s) \cdots, \quad F_{Ln}(s) = \frac{1}{1 + \frac{\omega_n}{s}} = \frac{s}{s + \omega_n}$$

■  $F_H(s)$ ——描述高频响应

$$F_H(s) = F_{H1}(s) F_{H2}(s) \cdots, \quad F_{Hn}(s) = \frac{1}{1 + \frac{s}{\omega_n}} = \frac{\omega_n}{s + \omega_n}$$

# 放大器 $\omega_L$ 估算

- 如果传递函数已知

$$F_L(s) = \frac{(s + \omega_{z1})(s + \omega_{z2}) \dots (s + \omega_{zn})}{(s + \omega_{p1})(s + \omega_{p2}) \dots (s + \omega_{pn})}$$

- 如果存在主极点 $\omega_{p1}$

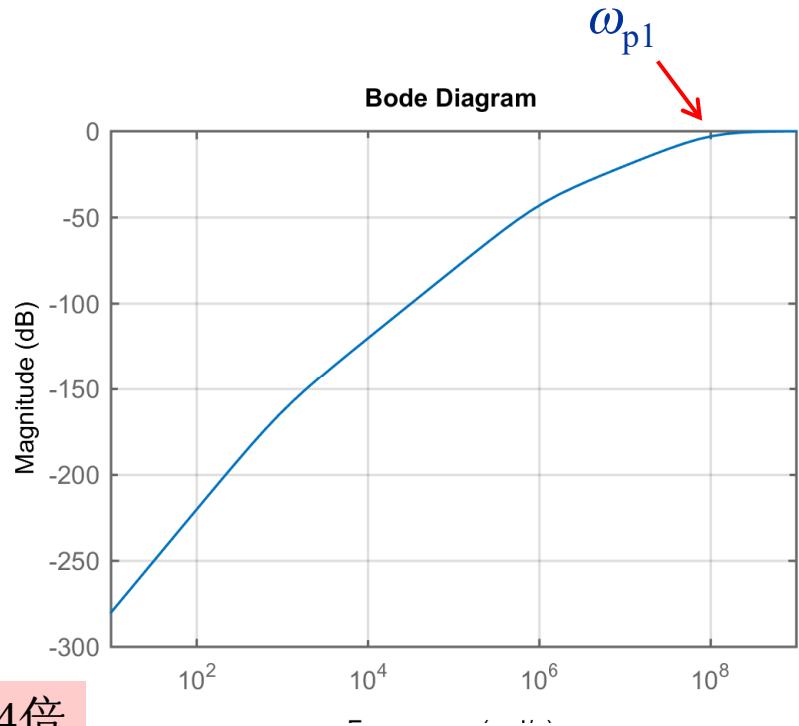
- 假设零点无影响 ( $\omega_{zk} \ll \omega_{p1}$ )

$$F_L(s) \approx \frac{s}{s + \omega_{p1}}, \quad \omega_L \approx \omega_{p1}$$

主极点：较最近的零/极点频率，大至少4倍

- 如果不存在主极点

$$\omega_L \approx \sqrt{(\omega_{p1}^2 + \omega_{p2}^2 + \dots) - 2(\omega_{z1}^2 + \omega_{z2}^2 + \dots)}$$



# 放大器 $\omega_L$ 估算

- 短路时间常数近似法  
Method of Short-Circuit Time Constants
- 估算低频响应 $\Rightarrow$ 低频等效电路 (高频电容视为开路)

- 低次单独考虑每个电容 $C_i$ 
  - 所有其它电容视为短路 (无穷大)

$$\omega = 1e5 \text{ } (f = 16\text{kHz}), \quad C = 5e-12(5\text{pF})$$
$$\left| \frac{1}{j\omega C} \right| = 2e6 \text{ } (2M\Omega)$$

$$\omega_{pi} = \frac{1}{R_i C_i}$$

- 如果存在主极点 $\omega_{p1}$

$$\omega_L \approx \omega_{p1}$$

- 如果不存在主极点

$$\omega_L \approx \sum \omega_{pi}$$

# 放大器 $\omega_H$ 估算

- 如果传递函数已知

$$F_L(s) = \frac{(s + \omega_{z1})(s + \omega_{z2}) \dots (s + \omega_{zn})}{(s + \omega_{p1})(s + \omega_{p2}) \dots (s + \omega_{pn})}$$

- 如果存在主极点 $\omega_{p1}$

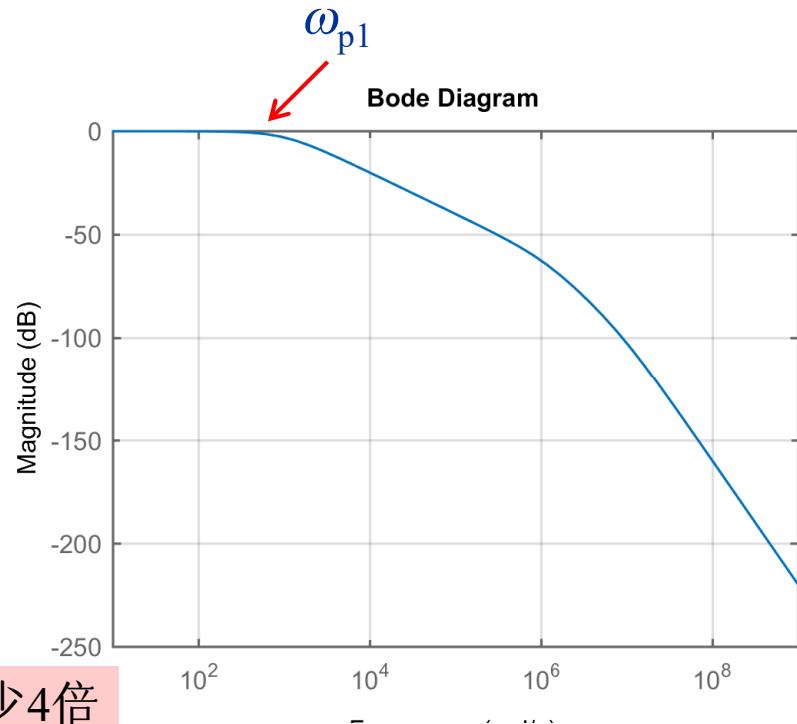
- 假设零点无影响 ( $\omega_{zk} \gg \omega_{p1}$ )

$$F_L(s) \approx \frac{\omega_{p1}}{s + \omega_{p1}}, \quad \omega_H \approx \omega_{p1}$$

主极点：较最近的零/极点频率，小至少4倍

- 如果不存在主极点

$$\frac{1}{\omega_H} \approx \sqrt{\left( \frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \dots \right) - 2 \left( \frac{1}{\omega_{z1}^2} + \frac{1}{\omega_{z2}^2} + \dots \right)}$$



# 放大器 $\omega_H$ 估算

- 开路时间常数近似法  
Method of Open-Circuit Time Constants
- 估算高频响应 $\Rightarrow$ 高频等效电路 (低频电容视为短路)
- 低次单独考虑每个电容 $C_i$ 
  - 所有其它电容视为开路 (零)
- 如果存在主极点 $\omega_{p1}$

$$\omega = 1e5 \text{ } (f = 16\text{kHz}), \quad C = 1e-5(10\mu F)$$

$$\left| \frac{1}{j\omega C} \right| = 1 \Omega$$

$$\omega_{pi} = \frac{1}{R_i C_i}$$

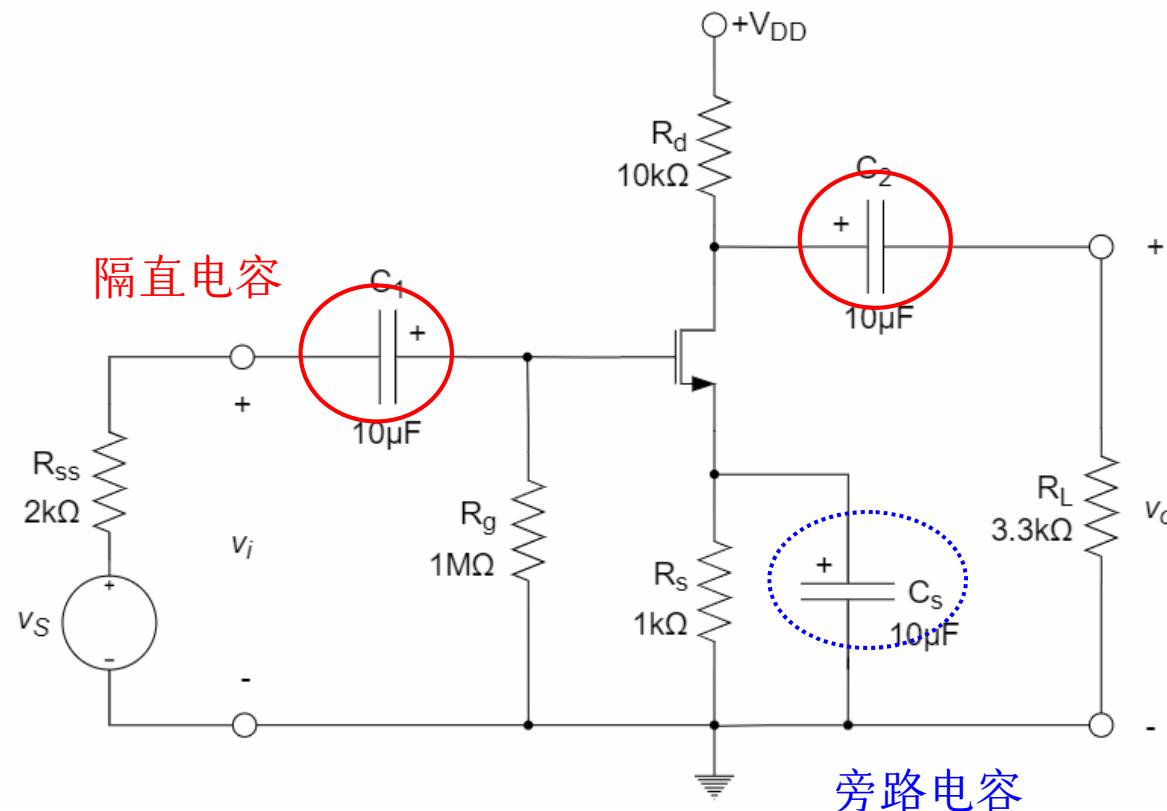
$$\omega_H \approx \omega_{p1}$$

- 如果不存在主极点

$$\frac{1}{\omega_H} \approx \sum \frac{1}{\omega_{pi}}$$

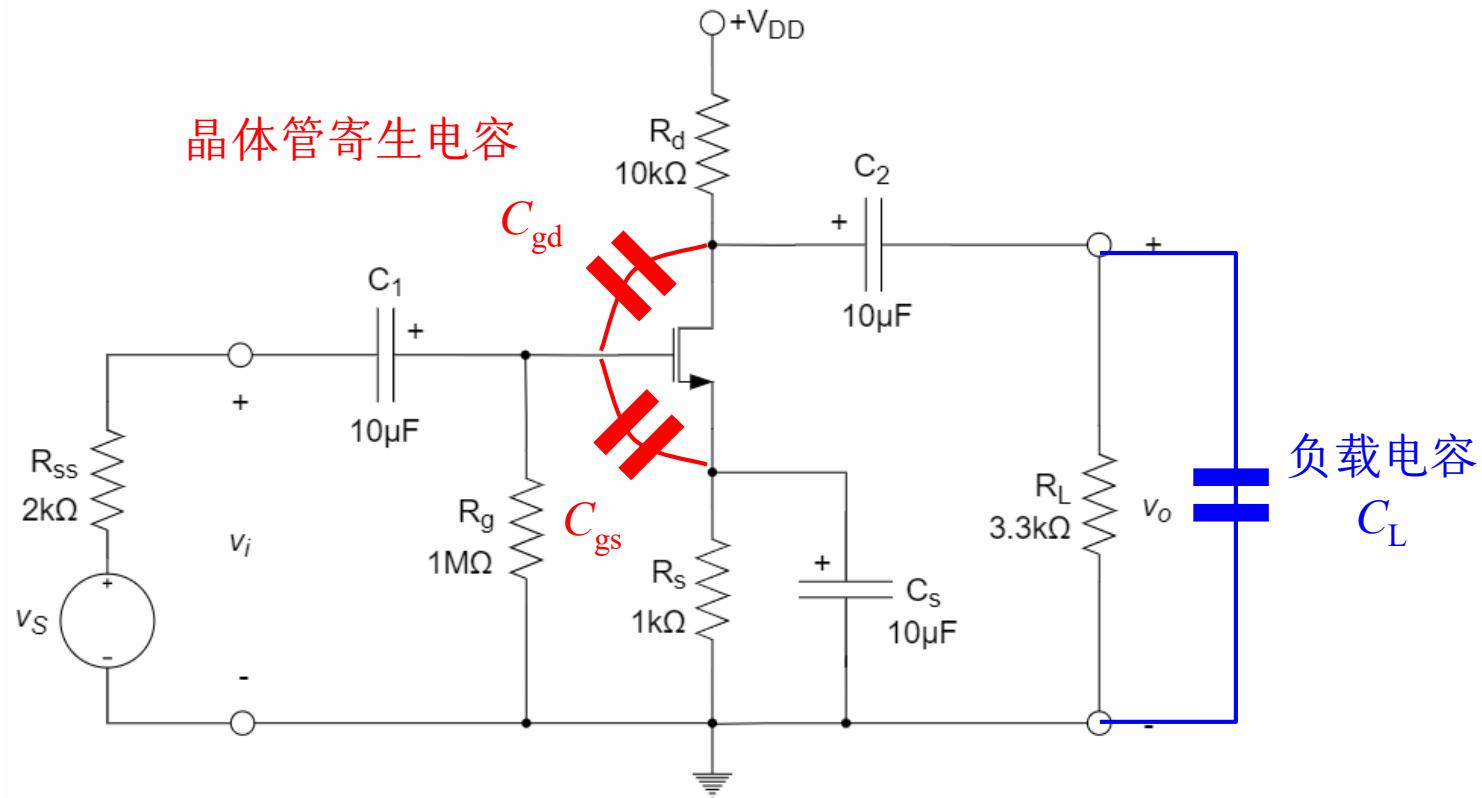
# 电容对极点的贡献

- 隔直电容、旁路电容⇒低频极点



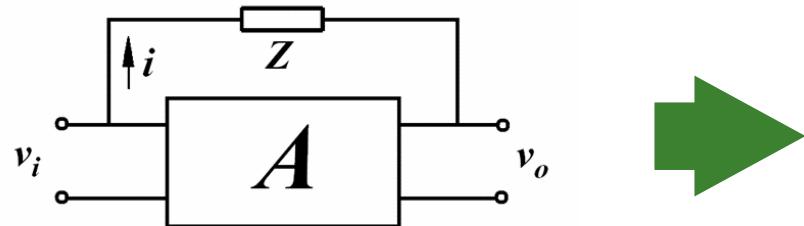
# 电容对极点的贡献

- 晶体管寄生电容、负载电容、以及其他信号通路上并联到地的电容⇒高频极点



# 米勒定理

- 用于将跨接在输出、输入的电容（MOSFET的 $C_{gd}$ 或BJT的 $C_\mu$ ）
  - 等效为两个电容（分别接在输入回路与输出回路）



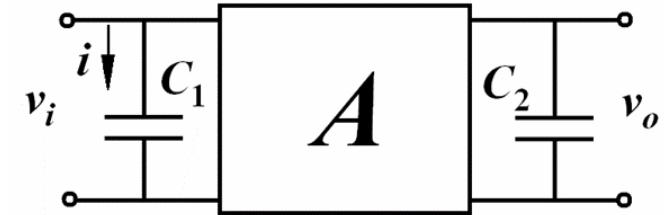
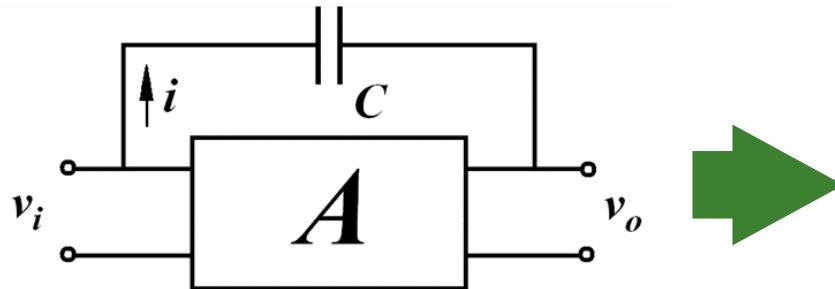
$$i = \frac{v_i - v_o}{Z} = \frac{v_i - Av_i}{Z} = \frac{v_i}{Z/(1-A)} = \frac{v_i}{Z_1}$$

$$Z_1 = \frac{1}{1-A} Z$$

类似地  $Z_2 = \frac{A}{A-1} Z$

# 米勒定理

- 若跨接的元件为电容  $C$



$$Z_1 = \frac{1}{1-A} Z$$

$$C_1 = (1-A)C$$

$$Z_2 = \frac{A}{A-1} Z$$

$$C_2 = \frac{A-1}{A} C \approx C$$

假设增益  $A$  较大

# 器件特征频率 $f_t$

- BJT: 共射, 输出端短路, 电流放大系数降为1的频率
- MOSFET: 共源, 输出端短路, 电流放大系数降为1的频率