INDU 323 – Operations Research I

Term Project

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In this question, we aim to find two things:

i) Which cities should install lockboxes?
ii) Which region should send the payment to which city?

Hence, our decision variables will be as follows:

$$x_i(i = 1{,}2{,}3{,}4) = \{1\ if\ Region\ i\ sends\ payments\ to\ city\ j \mid 0\ otherwise\}$$

$$y_j(j = 1{,}2{,}3{,}4) = \{1\ if\ a\ lockbox\ operated\ in\ city\ j \mid 0\ otherwise\}$$

[1]

Nickle has to minimize the total annual cost. It is calculated as:
$Total\ annual\ cost\ =\ Annual\ cost\ of\ operating\ lockboxes\ +\ Annual\ lost\ interest$

The daily value of payments from each region is shown below:

Region	Daily Value of Payments ($)
West	70000
Midwest	50000
East	60000
South	40000

The number of days it takes from a time a payment is sent from a region to city and until a check is cleared is shown in the table below:

From/To	L.A.	Chicago	N.Y.	Atlanta
West	2	6	8	8
Midwest	6	2	5	5
East	8	5	2	5
South	8	5	5	2

The Annual lost interest is calculated as follows:
$interest\ rate \times daily\ value\ of\ payments \times number\ of\ days$

for e.g. for West to L.A.:

$0.20 \times 70000 \times 2\ =\ 28000$

For all assignments (routes) the annual lost interest is shown in the table below:

Assignment	Annual Lost Interest (Lij) ($)
West to L.A.	28000

West to Chicago	84000
West to N.Y.	112000
West to Atlanta	112000
Midwest to L.A.	60000
Midwest to Chicago	20000
Midwest to N.Y.	50000
Midwest to Atlanta	50000
East to L.A.	96000
East to Chicago	60000
East to N.Y.	24000
East to Atlanta	60000
South to L.A.	64000
South to Chicago	40000
South to N.Y.	40000
South to Atlanta	16000

Hence, the objective function will be:

Let I represent the regions and J represent the cities.

$$minimize\ Z = \sum_{j \in J} 50000 \times y_j + \sum_{j \in J} \sum_{i \in I} x_{ij} \times Loss_{ij}$$

The constraints are as follows:

$$\sum_{j \in J} x_{1j} = 1, \sum_{j \in J} x_{2j} = 1, \sum_{j \in J} x_{3j} = 1, \sum_{j \in J} x_{4j} = 1$$

$$x_{ij} \leq y_j \ \forall i \in I \,\&\, j \in J$$

First constraint is to ensure all money from each region can go to a single city only. The above formulation is solved using IBM ILOG CPLEX Optimization Studio and the data is saved in an excel file. For simplicity, the values are taken as multiples of 1000 (for e.g. 96 instead of 96000).

Solutions Tab

```
// solution (optimal) with objective 242
// Quality Incumbent solution:
// MILP objective                              2.4200000000e+02
// MILP solution norm |x| (Total, Max)         6.00000e+00  1.00000e+00
// MILP solution error (Ax=b) (Total, Max)     0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)             0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)       0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)         0.00000e+00  0.00000e+00
//

y = [1
         0 1 0];
x = [[1 0 0 0]
             [0 0 1 0]
             [0 0 1 0]
             [0 0 1 0]];
```

The first line tells us that the solution is optimal and the amount of the objective function (minimum cost) is $242,000.

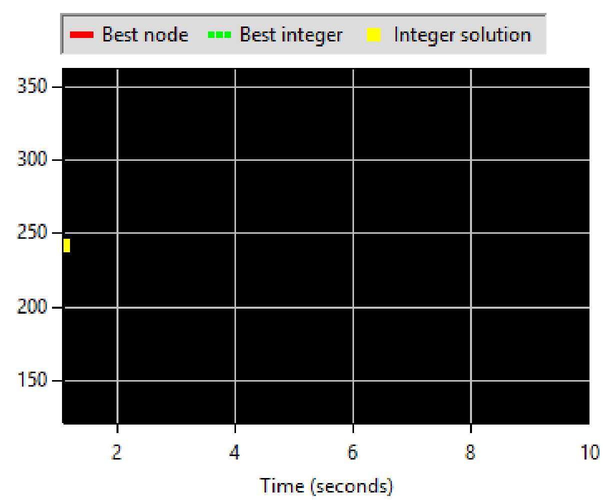
At the bottom, we see two arrays for x and y.

The 1D array y = [1 0 1 0] shows that the lockbox will be installed in city 1 (L.A.) and city 3 (New York).

The 2D array x has four 1D arrays inside. The first array [1 0 0 0] shows that from region 1 (West) should be sent to city 1 (L.A.). For Midwest, East, and South, the payment will be sent to city 3 (New York).

Statistics Tab

Statistic	Value
Cplex	solution (optimal) with objectiv...
Constraints	20
Variables	20
Binary	20
Non-zero coefficients	48
MIP	
Objective	242
Incumbent	242
Nodes	0
Remaining nodes	0
Iterations	9
Solution pool	
Count	3
Mean objective	322.666667



The statistics tab shows us the number of constraints (20), number of variables (20), and non-zero coefficients (48), among other data. The optimal solution is shown in the graph on the right. It is indicated by a yellow dot which corresponds to the value of 242 which is our optimal solution.

Engine log

```
        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt     Gap

*     0+    0                         438.0000        0.0000           100.00%
*     0+    0                         288.0000        0.0000           100.00%
*     0     0      integral     0      242.0000      242.0000        9    0.00%
Elapsed time = 0.00 sec. (0.15 ticks, tree = 0.00 MB, solutions = 3)

Root node processing (before b&c):
  Real time             =    0.00 sec. (0.15 ticks)
Parallel b&c, 8 threads:
  Real time             =    0.00 sec. (0.00 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =    0.00 sec. (0.15 ticks)
```

The Engine log provides us important information on how the optimal solution is reached. The node number can be seen in the second column. The interval is set to 10 in the settings so it shows 0 in our case, but it is a number between 0 and 10. On its right, the objective column displays the optimal amount in each node. The "Best Integer" and "Best Bound" show the non-integer and optimal integer solutions. Lastly, the Gap column on the right displays the difference between best integer and best bound which is 0 in our case.

References

[1] N. Shirzadi, "INDU 323 Lab Manual", Concordia University, Montreal, 2022.