American University of Technology **Sciences Program**

MAT205- Linear Algebra

Spring 2019

Textbook

Elementary linear algebra, Larson & Falvo, Houghton Mifflin, 6th Edition

A student can buy the 7th Edition of the book.

Course Syllabus

Introduction

The course introduces the elementary linear algebra for students. Students embarking on a linear algebra course should have a thorough knowledge of algebra, and familiarity with analytic geometry and trigonometry. We do not assume that calculus is a prerequisite for this course. Many students will encounter mathematical formalism for the first time in this course. As a result, our primary goal is to present the major concepts of linear algebra clearly and concisely.

The order and coverage of topics were chosen for maximum efficiency, effectiveness, and balance. For example, in Chapter 4 we present the main ideas of vector spaces and bases, beginning with a brief look leading into the vector space concept as a natural extension of these familiar examples. This material is often the most difficult for students, but our approach to linear independence, span, basis, and dimension is carefully explained and illustrated by examples. The eigenvalue problem is developed in detail in Chapter 7. Additional online Chapters 8 cover complex vector spaces.

Mathematica package will be used in class with some applications. An introduction to this package will be done in order to do applications. **http://www.wolfram.com/language/elementary-introduction/preface.html**

Other packages known can also be used by the student such as Maple, MathLab or others.

Course content

The first three chapters of this textbook cover linear systems and two other computational areas you may have probably studied before: matrices and determinants. These discussions prepare the way for the central theoretical topic of linear algebra: the concept of a vector space. Vector spaces generalize the familiar properties of vectors in the plane. It is at this point in the text that you will begin to write proofs and learn to verify theoretical properties of vector spaces.

The concept of a vector space permits you to develop an entire theory of its properties. The theorems you prove will apply to all vector spaces. For example, in Chapter 6 you will study linear transformations, which are special functions between vector spaces. The applications of linear transformations appear almost everywhere—computer graphics, differential equations, and satellite data transmission, to name just a few examples.

Another major focus of linear algebra is the so-called eigenvalue problem. Eigenvalues are certain numbers associated with square matrices and are fundamental in applications as diverse as population dynamics, electrical networks, chemical reactions, differential equations, and economics.

The last part of the materials is the complex vector space, which extend many properties of matrices to a new dual space and adjoint matrices as well Hermitian. Complex number and complex functions are also considered.

Linear algebra strikes a wonderful balance between computation and theory. As you proceed, you will become adept at matrix computations and will simultaneously develop abstract reasoning skills. Furthermore, you will see immediately that the applications of linear algebra to other disciplines are plentiful. In fact, you will notice that each chapter of this textbook closes with a section of applications. You might want to peruse some of these sections to see the many diverse areas to which linear algebra can be applied. (An index of these applications is given on the inside front cover.)

Linear algebra has become a central course for mathematics majors as well as students of science, business, and engineering. Its balance of computation, theory, and applications to real life, geometry, and other areas makes linear algebra unique among mathematics courses. For the many people who make use of pure and applied mathematics in their professional careers, an understanding and appreciation of linear algebra is indispensable.

Course Summary and Objectives

Chapter 1; Systems of Linear Equations

CHAPTER 1 OBJECTIVES

- Recognize, graph, and solve a system of linear equations in *n* variables.
- Use back-substitution to solve a system of linear equations.
- Determine whether a system of linear equations is consistent or inconsistent.
- Determine if a matrix is in row-echelon form or reduced row-echelon form.
- Use elementary row operations with back-substitution to solve a system in row-echelon form.
- Use elimination to rewrite a system in row-echelon form.
- Write an augmented or coefficient matrix from a system of linear equations, or translate a matrix into a system of linear equations.
- Solve a system of linear equations using Gaussian elimination and Gaussian elimination with back-substitution.
- Solve a homogeneous system of linear equations.
- Set up and solve a system of equations to fit a polynomial function to a set of data points, as well as to represent a network.

Chapter 2; Matrices

CHAPTER 2 OBJECTIVES

- Write a system of linear equations represented by a matrix, as well as write the matrix form of a system of linear equations.
- Write and solve a system of linear equations in the form
- Use properties of matrix operations to solve matrix equations.
- Find the transpose of a matrix, the inverse of a matrix, and the inverse of a matrix product (If they exist).
- Factor a matrix into a product of elementary matrices, and determine when they are invertible.
- Find and use the -factorization of a matrix to solve a system of linear equations.
- Use a stochastic matrix to measure consumer preference.
- Use matrix multiplication to encode and decode messages.
- Use matrix algebra to analyze economic systems (Leontief input-output models).
- Use the method of least squares to find the least squares regression line for a set of data.

Chapter 3; Determinants

CHAPTER 3 OBJECTIVES

- Find the determinants of a matrix and a triangular matrix.
- Find the minors and cofactors of a matrix and use expansion by cofactors to find the determinant of a matrix.
- Use elementary row or column operations to evaluate the determinant of a matrix.
- Recognize conditions that yield zero determinants.
- Find the determinant of an elementary matrix.
- Use the determinant and properties of the determinant to decide whether a matrix is singular or nonsingular, and recognize equivalent conditions for a nonsingular matrix.
- Verify and find an eigenvalue and an eigenvector of a matrix.
- Find and use the adjoint of a matrix to find its inverse.
- Use Cramer's Rule to solve a system of linear equations.
- Use determinants to find the area of a triangle defined by three distinct points, to find an

equation of a line passing through two distinct points, to find the volume of a tetrahedron defined by four distinct points, and to find an equation of a plane passing through three distinct points.

<u>Chapter 4; Real Vector Space</u>

CHAPTER 4 OBJECTIVES

- Perform, recognize, and utilize vector operations on vectors in
- Determine whether a set of vectors with two operations is a vector space and recognize standard examples of vector spaces, such as:
- Determine whether a subset of a vector space is a subspace.
- Write a linear combination of a finite set of vectors in
- Determine whether a set of vectors in a vector space is a spanning set of.
- Determine whether a finite set of vectors in a vector space is linearly independent.
- Recognize standard bases in the vector spaces and
- Determine if a vector space is finite dimensional or infinite dimensional.
- Find the dimension of a subspace of and
- Find a basis and dimension for the column or row space and a basis for the nullspace (nullity) of a matrix.
- Find a general solution of a consistent system in the form
- Find in and
- Find the transition matrix from the basis to the basis in
- Find for a vector in
- Determine whether a function is a solution of a differential equation and find the general solution of a given differential equation.
- Find the Wronskian for a set of functions and test a set of solutions for linear independence.
- Identify and sketch the graph of a conic or degenerate conic section and perform a rotation of axes.

<u>Chapter 5; Inner Product Spaces</u>

CHAPTER 5 OBJECTIVES

- Find the length of **v**, a vector **u** with the same length in the same direction as **v**, and a unit vector in the same or opposite direction as **v**
- Find the distance between two vectors, the dot product, and the angle θ between **u** and **v.**
- Verify the Cauchy-Schwarz Inequality, the Triangle Inequality, and the Pythagorean Theorem.
- Determine whether two vectors are orthogonal, parallel, or neither.
- Determine whether a function defines an inner product on R^n, $M_{m,n}$ or P_n, and find the inner product as defined for two vectors $\langle \mathbf{u}, \mathbf{v} \rangle$ in and R^n, $M_{m,n}$ and P_n
- Find the projection of a vector onto a vector or subspace.
- Determine whether a set of vectors in R^n is orthogonal, orthonormal, or neither.
- Find the coordinates of **x** relative to the orthonormal basis R^n
- Use the Gram-Schmidt orthonormalization process.
- Find an orthonormal basis for the solution space of a homogeneous system.
- Determine whether subspaces are orthogonal and, if so, find the orthogonal complement of a subspace.
- Find the least squares solution of a system $A\mathbf{x} = \mathbf{b}$
- Find the cross product of two vectors **u** and **v.**
- Find the linear or quadratic least squares approximating function for a known function.
- Find the *n*th-order Fourier approximation for a known function.

<u>Chapter 6; Linear Transformations</u>

CHAPTER 6 OBJECTIVES

- Find the image and preimage of a function.
- Determine whether a function from one vector space to another is a linear transformation.

- Find the kernel, the range, and the bases for the kernel and range of a linear transformation T and determine the nullity and rank of T
- Determine whether a linear transformation is one-to-one or onto.
- Verify that a matrix defines a linear function that is one-to-one and onto.
- Determine whether two vector spaces are isomorphic.
- Find the standard matrix for a linear transformation and use this matrix to find the image of a vector and sketch the graph of the vector and its image.
- Find the standard matrix of the composition of a linear transformation.
- Determine whether a linear transformation is invertible and find its inverse, if it exists.
- Find the matrix of a linear transformation relative to a nonstandard basis.
- Know and use the definition and properties of similar matrices.
- Identify linear transformations defined by reflections, expansions, contractions, shears, and/or rotations.

Chapter 7; Eigenvalues and Eigenvectors

CHAPTER 7 OBJECTIVES

- Find the eigenvalues and corresponding eigenvectors of a linear transformation, as well as the characteristic equation and the eigenvalues and corresponding eigenvectors of a matrix A
- Demonstrate the Cayley-Hamilton Theorem for a matrix A
- Find the eigenvalues of both an idempotent matrix and a nilpotent matrix.
- Determine whether a matrix is triangular, diagonalizable, symmetric, and/or orthogonal.
- Find (if possible) a nonsingular matrix for a matrix A such that $P^{-1}AP$ is diagonal.
- Find a basis B (if possible) for the domain of a linear transformation T such that the matrix of T relative to B is diagonal.
- Find the eigenvalues of a symmetric matrix and determine the dimension of the corresponding eigenspace.
- Find an orthogonal matrix P that diagonalizes A
- Find and use an age transition matrix and an age distribution vector to form a population model and find a stable age distribution for the population.
- Solve a system of first-order linear differential equations.
- Find a matrix of the quadratic form associated with a quadratic equation.
- Use the Principal Axes Theorem to perform a rotation of axes and eliminate the xy, xz, and *yz*-terms, and find the equation of the rotated quadratic surface.

Chapter 8; Complex Vector Space

CHAPTER 8 OBJECTIVES

- Use Complex Numbers
- Understand Conjugates and Division of Complex Numbers
- Knows Polar Form and De Moivre's Theorem
- Properties Complex Vector Spaces and Inner Products
- Introducing Unitary and Hermitian Matrices

A good introduction to Mathematica can be used from

During the lectures many examples will be solved using this Package.

Attendance:

Students are expected to attend all lectures and all recitations. Absence does not excuse a student from responsibility for class material during his/her absence. The course is presented by a series of lectures. A number of exercises are reserved for ***homework*** to be presented every Monday after the end of each chapter or sections of the chapter.

Exams and Final exam:

Students will sit for 3 quizzes after the end of 3 chapters. A final exam will be done at the end of the course.

Grading Policy:

Exam 1, 20%; Exam 2, 20% , Exam 3, 20% , Homework and Drop quizzes 10% and Final Exam, 30% .

Office hours

When any student needs helps from the instructor, he has to take appointment with him.

Lectures: TTh: 12: 45 – 13.4500

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