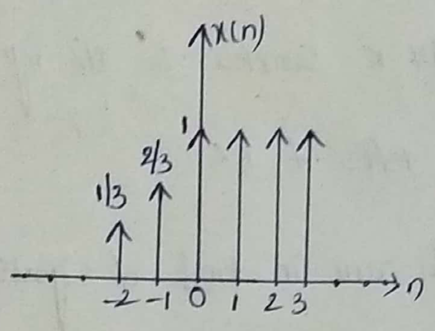
Problems

1) A discrete time signal $x(n)$ is defined as $x(n) = \begin{cases} 1+\frac{n}{3}, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$

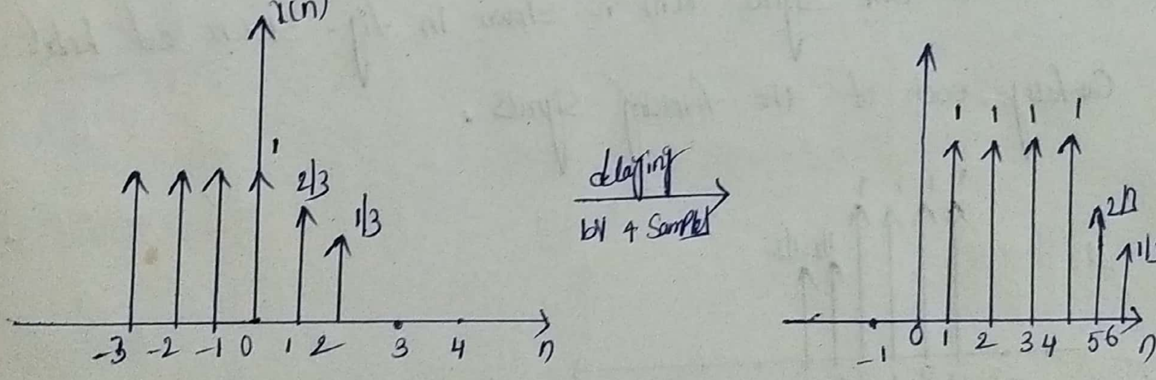
a) Determine its value and sketch the signal $x(n)$.

Sol: $x(n) = \left\{ \ldots 0, \frac{1}{3}, \frac{2}{3}, 1, 1, 1, 1, 0 \ldots \right\}$

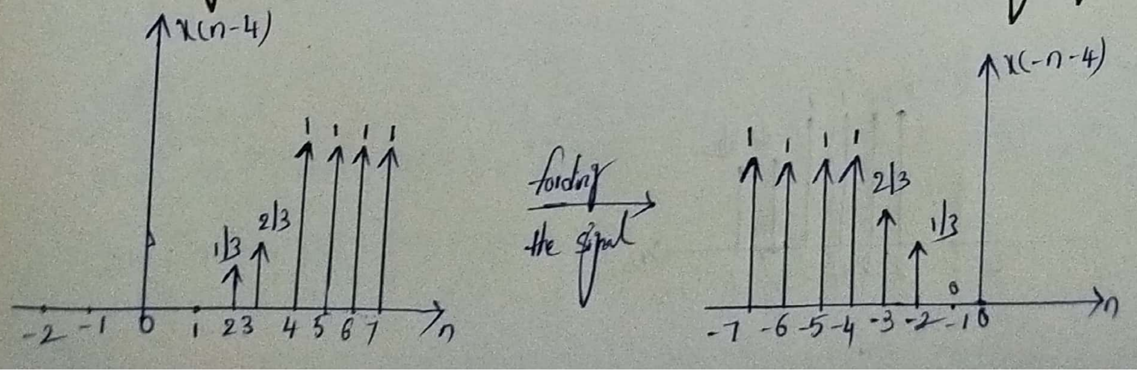


b) Sketch the signals that result if we:

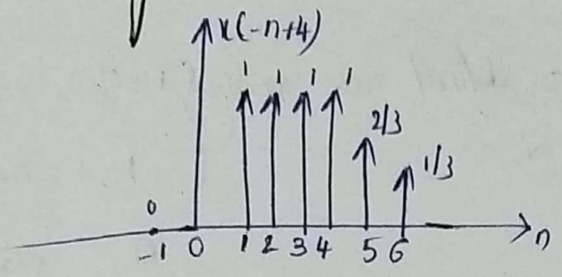
1. first fold $x(n)$ and then delay the resulting signal by four samples.



2. First delay $x(n)$ by four samples and then fold the resulting signal.



c) Sketch the signal $x(-n+4)$



d) Compare the results in Parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$

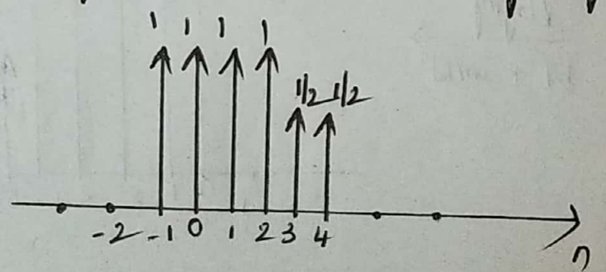
Sol:- To obtain $x(-n+k)$, first we fold $x(n)$. This yields $x(-n)$. Then, we shift $x(-n)$ by k samples to the right if $k>0$, or k samples to the left if $k<0$.

e) Can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$?

Sol:- Yes we can express

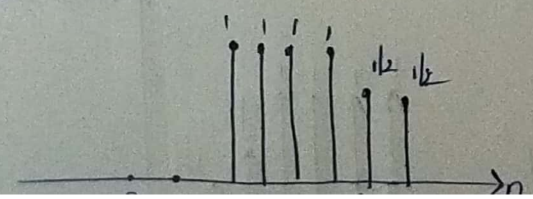
$$x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n+1) + u(n) - u(n-4)$$

2) A discrete time signal $x(n)$ is shown in fig. Sketch and label carefully each of the following signals.



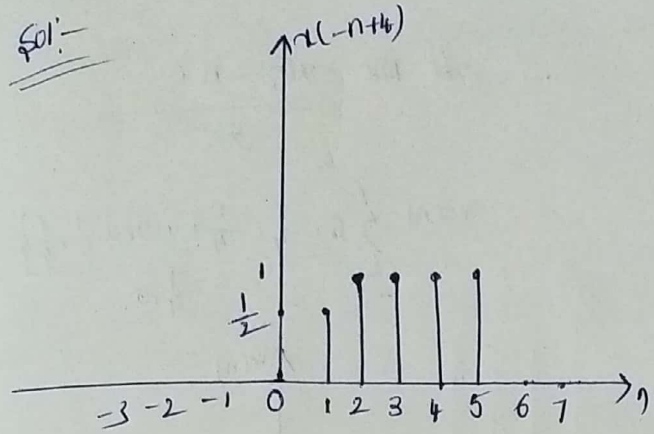
a) $x(n-2)$

Sol:-

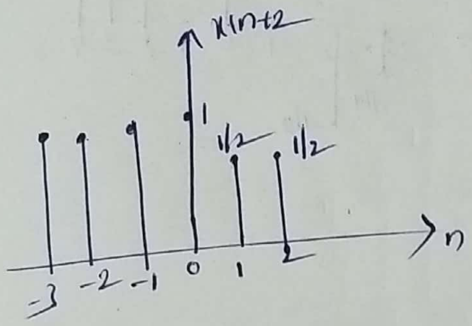


b) $x(4-n)$

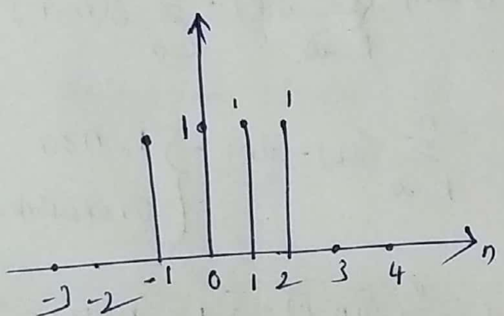
Sol:-



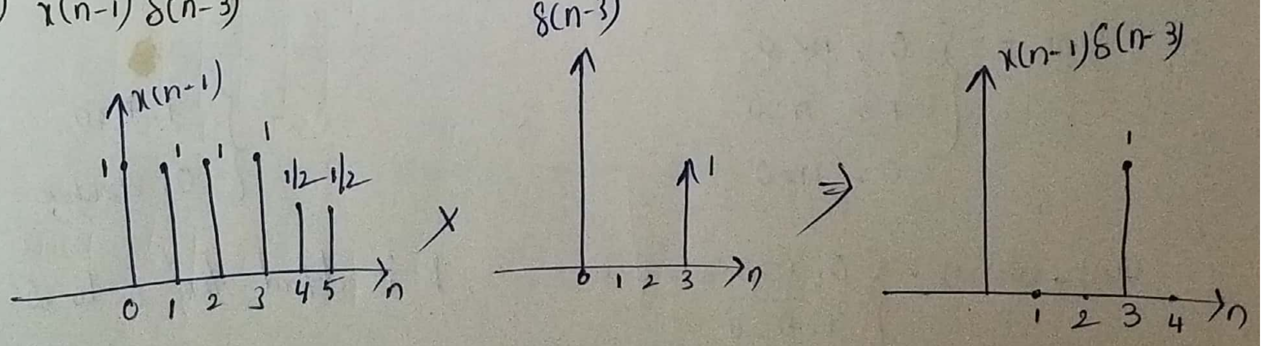
c) $x(n+2)$



d) $x(n)\, u(2-n)$



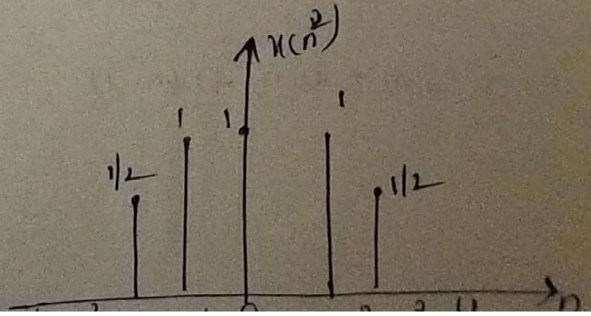
e) $x(n-1)\, \delta(n-3)$



f) $x(n^2)$

$x(-2)^2,\ x(-1)^2,\ x(0^2),\ x(1^2),\ x(2^2),\ x(3)^2$

$x(4),\ x(1),\ x(0),\ x(1),\ x(2),\ x(3)$



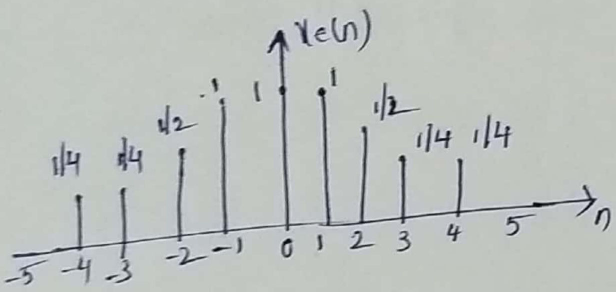
g) even Part of $x(n)$

$x(n) = \{0, 0, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, 1, 1, 0\}$

$x(-n) = \{1/2, 1/2, 1, 1, 1, 1\}$

$$\text{Even Part} = \frac{x(n) + x(-n)}{2}$$

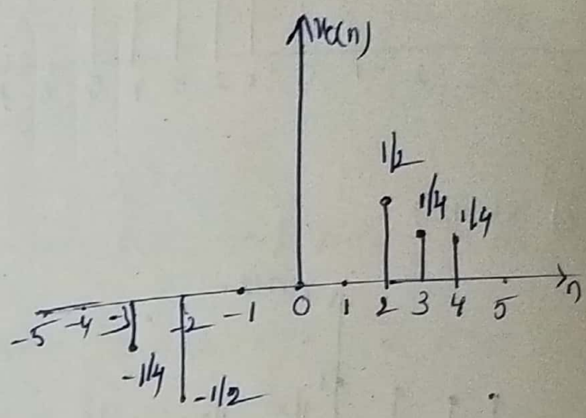
$x_e(n) = \{\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}\}$



h) odd Parts of $x(n)$

$$\text{odd Part} = \frac{x(n) - x(-n)}{2}$$

$x_o(n) = \{0, -\frac{1}{4}, -\frac{1}{4}, 0, 0, 0, 0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0\}$



3. show that

a) $\delta(n) = u(n) - u(n-1)$

Sol: $u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$

$u(n-1) = \begin{cases} 1, & n \geq 1 \\ 0, & \text{else where} \end{cases}$

$\delta(n) = \begin{cases} 0, & n < 0 \\ 1, & n = 0 \\ 0, & n > 0 \end{cases}$

$u(n) - u(n-1) = \begin{cases} 0, & n > 0 \\ 1, & n = 0 \\ 0, & n < 0 \end{cases}$

So $\delta(n) = u(n) - u(n-1)$

b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

Sol: $\sum_{k=-\infty}^{n} \delta(k) = u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$

$\sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 1, & n \geq k \\ 0, & \text{elsewhere} \end{cases}$

$= \begin{cases} 1, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$

k is defined from 0 to ∞.

So $\sum_{k=-\infty}^{\infty} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k) = u(n)$

4) Show that any signal can be decomposed into an even and an odd component. Is the decomposition unique? Illustrate your arguments using the signal.

Sol: Both Even & odd components are unique

Let $x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

$x(-n) = \{6, 5, \underset{\uparrow}{4}, 3, 2\}$

$x_e(n) = \{4, 4, \underset{\uparrow}{4}, 4, 4\}$

$x_o(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

for the above Example we can illustrate even & odd components are unique.

6) Consider the system

$$y(n) = T[x(n)] = x(n^2)$$

a) Determine if the system is time invariant

Sol: $T[x(n)] = x(n^2)$

$y(n) = x(n^2)$

(n at n-K) $y_1(n) = x(n-K) = x((n-K)^2)$

$= x(n^2 + K^2 - 2nK)$

$y_2(n) = x(n^2 - K)$

$y_1(n) \neq y_2(n)$

So, System is Time Variant System.

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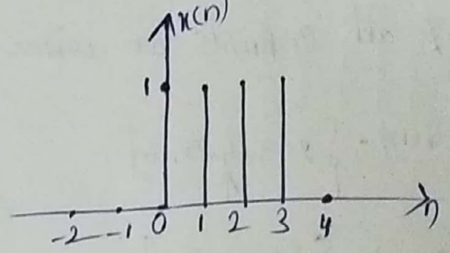
b) To clarify the result in part (a) assume that the signal

$$x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$ is applied into the system.

(1) Sketch the signal $x(n)$

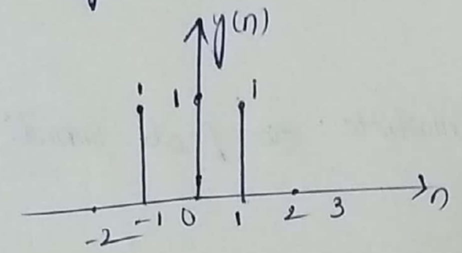
Sol:- $x(n) = \{0, 0, 1, 1, 1, 1, 0\}$



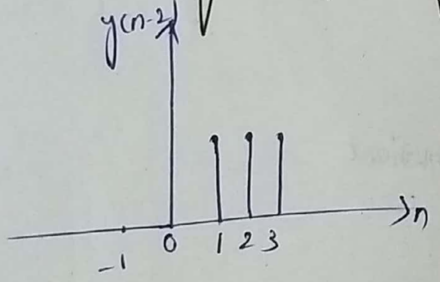
(2) Determine and sketch the signal $y(n) = T[x(n)]$

$y(n) = x(n^2)$

$x(4), x(1), x(0), x(1), x(4)$



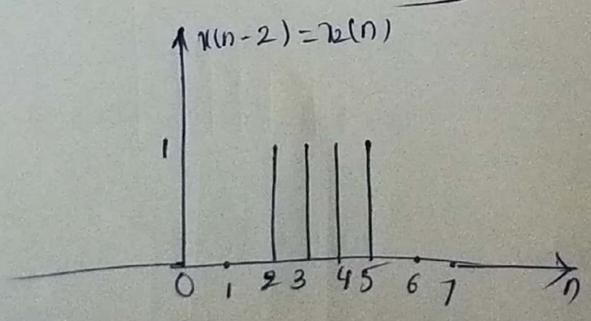
(3) Sketch the signal $y_2'(n) = y(n-2)$



$y(n-2) = \{0, 0, 1, 1, 1, 1\}$

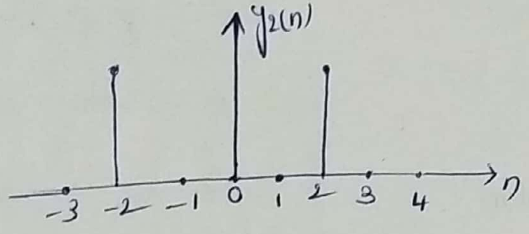
(4) Determine and sketch the signal $x_2(n) = x(n-2)$

Sol:- $x(n-2) = \{0, 0, 1, 1, 1, 1, 0.\}$



(5) Determine and Sketch the Signal $y_2(n) = T[x_2(n)]$

Sol:- $y_2(n) = [x_2^2(n)]$ $x_2(4), x_2(1), x_2(0), x_2(1), x_2(4)$



$y_2(n) = \{0,1,0,\underset{\uparrow}{0},0,1,0\}$

(6) Compare the Signals $y_2(n)$ and $y_2(n-2)$. What is your Conclusion.

Sol:- $y_2(n) = \{0,1,0,0,0,1,0\}$

$y_2(n-2) = \{0,1,0,0,0,1,0\}$

$y_2(n) \neq y_2(n-2)$

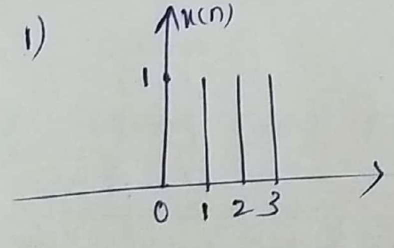
So, System is time Variant

C. Repeat part b for the System

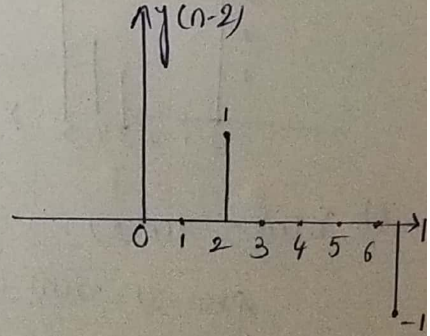
$y(n) = x(n) - x(n-1)$

Can you use this result to make any statement about the time Invariance of this System & why!

1)



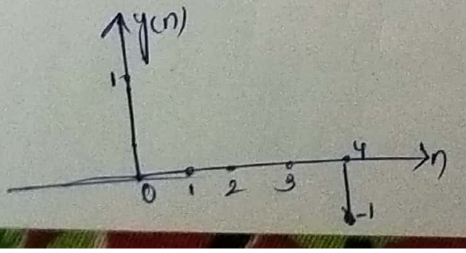
3)



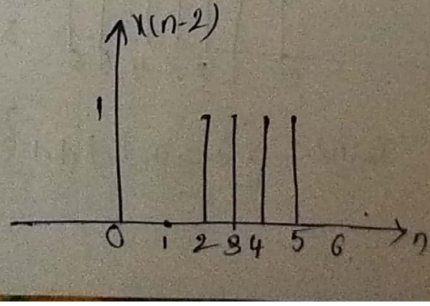
2) $y(n) = x(n) - x(n-1)$

$= \{\underset{\uparrow}{1},1,1,1,0\} - \{\underset{\uparrow}{0},1,1,1,1\}$

$= \{1,0,0,0,-1\}$



4) $x(n-2) = x_2(n)$



5) $y_2(n) = T[x_2(n)]$

$y_2(n) = x_2(n) - x_2(n-1)$

$y_2(n) = \{\underset{\uparrow}{0}, 0, 1, 0, 0, 0, 0, -1\}$

As $y_2(n) = y(n-2)$

So this is time invariant system.

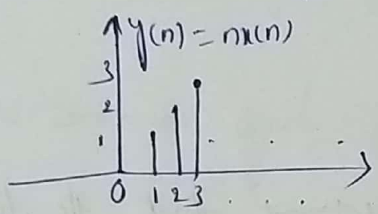
d) Repeat Parts b & c for the system

$$y(n) = T[x(n)] = n x(n)$$

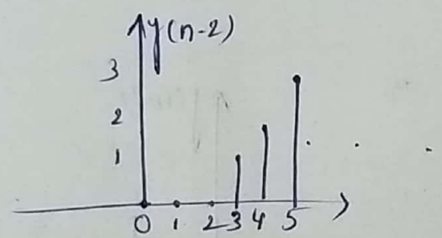
1) $x(n) = \{\ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 1, 0 \ldots\}$

2) $y(n) = n x(n)$

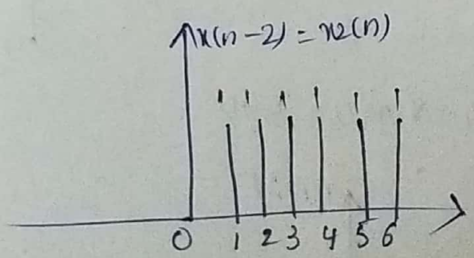
$y(n) = \{\ldots 0, \underset{\uparrow}{0}, 1, 2, 3 \ldots\}$



3) $y(n-2) = \{0, 0, 0, 0, 1, 2, 3\}$



4) $x_2(n) = x(n-2)$



$x_2(n) = \{0, 0, 0, 1, 1, 1, 1, 1\}$

7. A discrete - time System can be

1) Static or dynamic

2) Linear or Non-Linear

3) Time invariant or Varying

4) Causal or Non Causal.

5) Stable or unstable.

Examine the following systems with respect to the properties above.

a) $y(n) = \cos[x(n)]$

Sol: (i) static, (ii) $y_1(n) = \cos[x_1(n)]$

$$y_2(n) = \cos[x_1(n)] + \cos[x_2(n)]$$

$$y'(n) = \cos[x_1(n) + x_2(n)]$$

Non - Linear

(iv) only Present i/p Causal (v) stable.

b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

Sol: dynamic, Linear, Time Invariant, non causal, unstable

[also Depends on future Values]

c) $y(n) = x(n) \cos(w_0 n)$

Sol: Static, Linear, Time Variant, Causal, Stable

$\Downarrow$

$$y(n) = x(n-n_0) \cos(w_0(n-n_0))$$

$$y'(n) = x(n-n_0) \cos(w_0 n)$$

10)

Sol:- As this is a time - invariant system.

$y_2(n)$ should have only 3 Elements and

$y_3(n)$ should have 4 Elements.

So, it is a non-linear.

11)

Sol:- Since $x_1(n)+x_2(n)=\delta(n)$

& s/m is linear, the impulse response of the system is

$$y_1(n)+y_2(n)=\{0,3,-1,2,1\}$$

If s/m was time invariant the response of $x_3(n)$ would be

$$\{3,2,1,3,1\}$$

12) a.

Sol:- Any linear combination of signal in the form of $x_i(n)$;

$i=1,2,3\ldots N$

Because if we take $i=1,3$

$$y_1(n)=x_1(n)$$

$$y_3(n)=x_3(n) \Rightarrow y(n)=y_1(n)+y_3(n)=x_1(n)+x_3(n)$$

$$y_1(n)=x_1(n)+x_3(n)$$

So, it is a linear

b)

Sol:- Any $x_i(n-K)$ where K is any integer, $i=1,2,\ldots N$

1^{st} Replace $n=n-n_0 \Rightarrow x_i(n-n_0-K)$

$x(n)$ by $x(n-n_0) \Rightarrow x_i(n-K-n_0)$

13)

Sol:- A s/m to be BIBO stable only when bounded o/p produce bounded i/p.

$$y(n) = \sum_{K} h(K)\, x(n-K)$$

$$|y(n)| = \sum_{K} |h(K)|\, |x(n-K)|$$

$$= \sum_{K} |x(n-K)| \leq m_n \text{ [Some constant]}$$

$$\text{so } |y(n)| = m_n \sum_{K} |h(K)|$$

$$|y(n)| < \infty \ \forall n \text{, if \& only if } \sum_{K} |h(K)| < \infty$$

$$\text{So } \sum_{n=-\infty}^{\infty} |y(n)|$$

→ A s/m to be BIBO stable only when bounded i/p produce bounded o/p.

$$y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(n-K) \ ; \ n \leq n-K$$

$$|y(n)| = \sum_{K=-\infty}^{\infty} |h(K)|\, |x(n-K)| \ ; \ K \geq 0$$

as $\sum_{K=-\infty}^{\infty} |x(n-K)| \leq m_n$ for some constant.

$$|y(n)| \text{ is } \leq \infty \text{ if \& only if } \sum_{K=-\infty}^{\infty} |h(K)| < \infty$$

$$\text{So } \sum_{n=-\infty}^{\infty} |h(n)| < \infty.$$

14) a)

Sol:- If a s/m is causal, o/p depends only on the present & past i/ps as $x(n) = 0 \ \forall\ n < n_0$ then $y(n)$ also becomes zero $\forall\ n < n_0$.

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b) $y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

for Infinite Impulse Response

$h(n) = 0, \; n<0$ and $n \geq m$

So $y(n)$ reduces to $y(n) = \sum_{k=0}^{n-1} h(k)\, x(n-k)$

15) a)

Sol: For $a=1$, $\sum_{n=M}^{N} a^n = N-M+1$

for $a \neq 1$; $\sum_{n=m}^{N} a^n = \dfrac{a^m - a^{N+1}}{1-a}$

$(1-a) \sum_{n=m}^{N} a^n = a^m + a^{m+1} - a^{m+1} + \cdots - a^N \cdots a^N - a^{N+1}$

$= a^m - a^{N+1}$

b)

Sol: For $M=0$, $|a|<1$ and $N \to \infty$.

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad |a|<1$$

16) a)

Sol: $y(n) = \sum_k h(k)\, x(n-k)$

$\sum_n y(n) = \sum_n \sum_k h(k)\, x(n-k)$

$\sum_n y(n) = \sum_k h(k) . \sum_{n=-\infty}^{\infty} x(n-k)$

$\sum_n y(n) = \left(\sum_k h(k)\right) . \left(\sum_n x(n)\right)$

b.

Sol: $y(n) = \{1, 3, 7, 7, 7, 6, 4\}$

BY Tabular Method

h(n) \ x(n)	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

$\sum_n y(n) = 35$; $\sum_n x(n) = 7$, $\sum_n h(n) = 5$

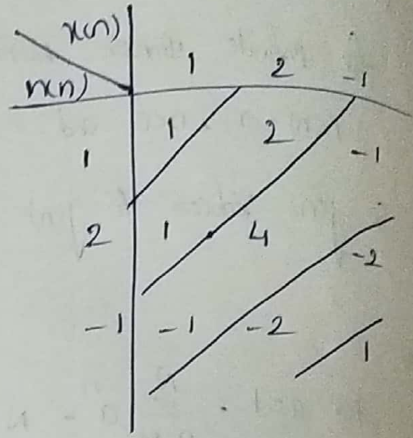
$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$35 = 7 \times 5$

$35 = 35$

ii)

Sol:- $x(n) = \{1, 2, -1\}$, $h(n) = \{1, 2, -1\}$



$y(n) = x(n) * h(n)$

$y(n) = \{1, 4, 2, -4, 1\}$

$\sum_n y(n) = 4$; $\sum_n x(n) = 2$, $\sum_n h(n) = 2$

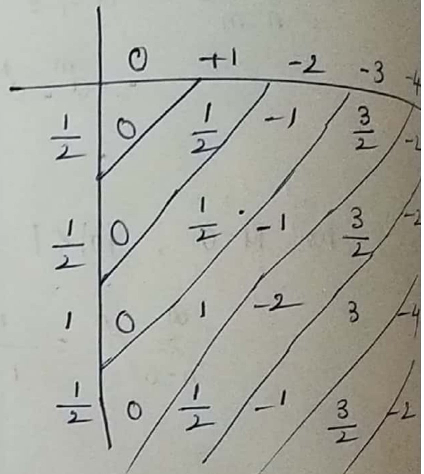
$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$4 = 2 \times 2$

$4 = 4$

iii)

Sol:- $y(n) = \{0, \frac{1}{2}, \frac{-1}{2}, \frac{3}{2}, -2, 0, \frac{-5}{2}, 2\}$



$\sum_n y(n) = 5$, $\sum_n x(n) = -2$, $\sum_n h(n) = \frac{5}{2}$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$-5 = -5$

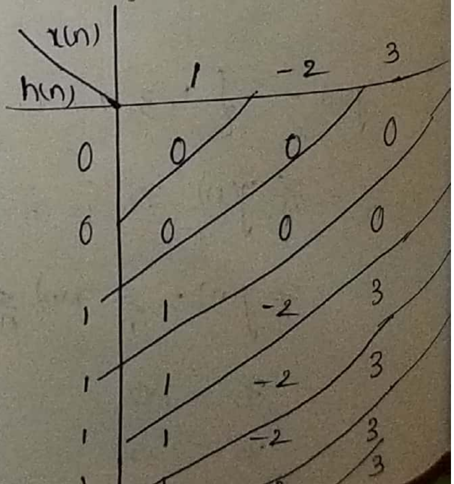
iv)

Sol:- $y(n) = \{1, 2, 3, 4, 5\}$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$15 = 15$

v) $y(n) = \{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$



$\sum_n y(n) = 8$; $\sum_n x(n) = 2$; $\sum_n h(n) = 4$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$8 = 8$

vi)

Sol: $y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n h(n) = 8$; $\sum_n h(n) = 4$, $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$8 = 4 \times 2$

$8 = 8$

h(n) \ x(n)	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

vii)

Sol: $y(n) = \{0, 1, 4, -4, -5, -1, 3\}$

$\sum_n y(n) = -2$; $\sum_n x(n) = -2$; $\sum_n h(n) = 1$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

h(n) \ x(n)	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

viii)

Sol: $y(n) = \{1, 2, 4, 3, 2\}$

$\sum_n y(n) = 12$, $\sum_n x(n) = 4$; $\sum_n h(n) = 3$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$12 = 4 \times 3$

$12 = 12$

h(n) \ x(n)	1	1	2
1	1	1	2
1	1	1	2
1	1	1	2

ix)

Sol: $y(n) = \{1, -1, -5, 2, 3, -5, 1, 4\}$

$\sum_n y(n) = 0$; $\sum_n x(n) = 4$; $\sum_n h(n) = 0$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$0 = 4 \times 0$

$0 = 0.$

h(n) \ x(n)	1	1	$\downarrow$ 0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
-4	-4	-4	0	-4	-4

x)

Sol:- $y(n) = \{1, 4, 4, 10, 4, 4, 4, 1\}$

$\sum_n y(n) = 36$, $\sum_n x(n) = 6$, $\sum_n h(n) = 6$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$36 = 6 \times 6$

$36 = 36$

h(n) \ x(n)	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

xi)

Sol:- $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$

$y(n) = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n u(n)\right]$

$\sum_n y(n) = \frac{8}{3}$, $\sum_n h(n) = \frac{4}{3}$; $\sum_n x(n) = 2$

18) Determine and sketch the convolution $y(n)$ of the signals

$$x(n) = \begin{cases} \frac{1}{3}n, & 0 \le n \le 6 \\ 0, & \text{elsewhere} \end{cases} \qquad h(n) = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

a) Graphically

$x(n) = \{0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\}$

$h(n) = \{1, 1, 1, 1, 1\}$

$y(n) = \{0, \frac{1}{3}, 1, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$

h(n) \ x(n)	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2

b) Analytically

Sol:- $x(n) = \frac{1}{3}n\,[u(n) - u(n-7)]$

$h(n) = u(n+2) - u(n-3)$

$y(n) = \frac{1}{3}n\,[u(n) - u(n-7)] * u(n+2) - u(n-3)$

$= \frac{1}{3}n\,[u(n) * u(n+2)] - \frac{1}{3}n\,[u(n) * u(n-3)] - \frac{1}{3}n\,[u(n-7) * u(n+2)]$
$+ \frac{1}{3}n\,[u(n-7) * u(n-3)]$

20) Consider the following three operations

a) Multiply the Integers ; 131 and 122

Sol:- $131 \times 122 = 15982$

b) Compute the Convolution of Signals $\{1, 3, 1\} *$

Sol: $y(n) = \{1, 5, 9, 8, 2\}$

h(n) \ x(n)	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

c) Multiply the Polynomials :-

$1 + 3z + z^2$ and $1 + 2z + 2z^2$

Sol:- $(z^2 + 3z + 1) \times (2z^2 + 2z + 1) \Rightarrow 2z^4 + 6z^3 + 2z^2 + 2z^3 + 6z^2 + 2z + z^2 + 3z + 1$

$\Rightarrow 2z^4 + 8z^3 + 9z^2 + 5z + 1$

d) Repeat Part (a) for the numbers 1.31 and 12.2

Sol:- $1.31 \times 12.2 = 15.982$

21) Compute the Convolution $y(n) * h(n)$ of the following Pairs of the Signals.

a) $x(n) = a^n u(n)$, $h(n) = b^n u(n)$ when $a \neq b$ & when $a = b$

Sol:- $y(n) = x(n) * h(n)$

$= a^n u(n) * b^n u(n)$

$= [a^n * b^n] u(n)$

$y(n) = \sum_{k=0}^{n} a^k u(k) b^{n-k} u(n-k)$

$= b^n \sum_{k=0}^{n} a^k u(k) b^{-k}$

$= b^n \sum_{k=0}^{n} (a/b)^{-k}$

if $a \neq b$ then $y(n) = \dfrac{b^{n+1} - a^{n+1}}{b-a} u(n)$

if $a = b \Rightarrow b^n (n+1) u(n)$

b) $x(n) = \begin{cases} 1 \; ; & n = -2, 0, 1 \\ 2 \; ; & n = -1 \\ 0 \; ; & \text{elsewhere} \end{cases}$

$h(n) = \delta(n) - \delta(n+1) + \delta(n-4) + \delta(n-5)$

Sol:- $x(n) = \{1, 2, 1, 1\}$ $h(n) = \{1, -1, 0, 0, 1, 1\}$

$y(n) = \{1, 1, -1, 0, 0, 3, 3, 2, 1\}$

c) $x(n) = u(n+1) - u(n-4) - \delta(n-5)$

$h(n) = [u(n+2) - u(n-3)] \, 3(\delta(n))$

Sol:- $x(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$

$h(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$y(n) = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, 1\}$

$h(n)$ \ $x(n)$	1	2	1	1
1	1	2	1	1
−1	−1	−2	−1	−1
0	0	0	0	0
0	0	0	0	0
1	1	2	1	1
1	1	2	1	1

d) $x(n) = u(n) - u(n-5)$

$h(n) = u(n-2) - u(n+8) + u(n-11) - u(n-17)$

Sol:- $x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$h'(n) = \{0, 0, 1, 1, 1, 1, 1, 1, 1\}$

$h(n) = h'(n) + h'(n-9)$, where

$y'(n) = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2\}$

$h(n)$ \ $x(n)$	1	1	1	1	1
0	0	0	0	0	0
0	0	0	0	0	0
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1

22) $x(n) = \{1,4,2,3,5,3,3,4,5,7,6,9\}$

$h_1(n) = \{1,1\}$ $\quad$ $h_4(n) = \{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$

$h_2(n) = \{1,2,1\}$ $\quad$ $h_5(n) = \{\frac{1}{4}, -\frac{1}{2}, \frac{1}{4}\}$

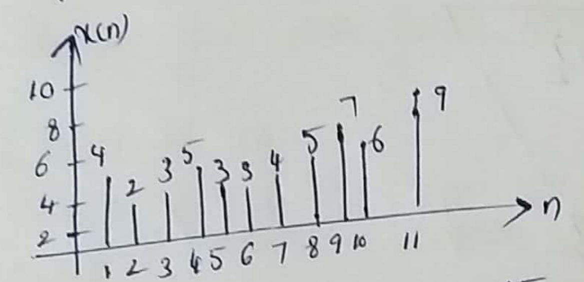
$h_3(n) = \{\frac{1}{2}, \frac{1}{2}\}$

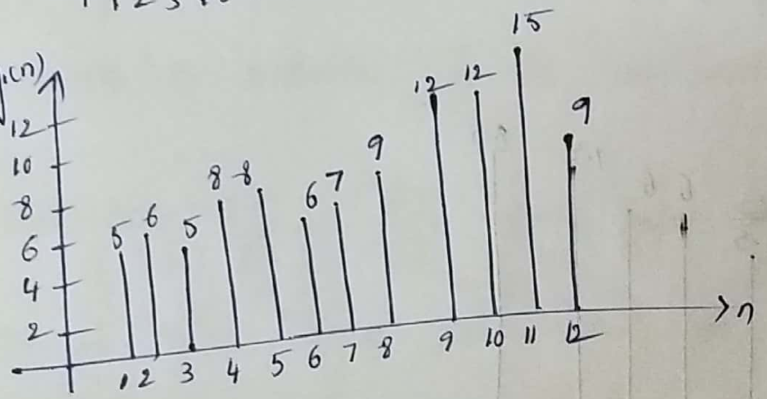
Sketch $x(n)$, $y_1(n)$, $y_2(n)$ one one graph and $x(n)$, $y_3(n)$, $y_4(n)$, $y(5)$ on another graph.

Sol: $y_1(n) = x(n) * h_1(n)$

$y_1(n) = \{1,5,6,5,8,8,6,7,9,12,12,15,9\}$

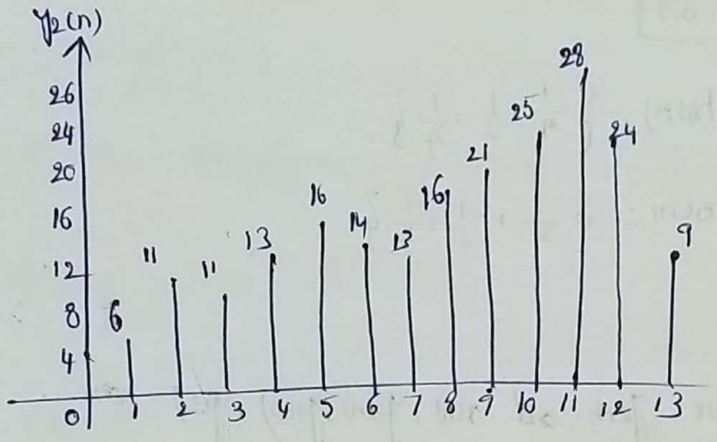
h(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9





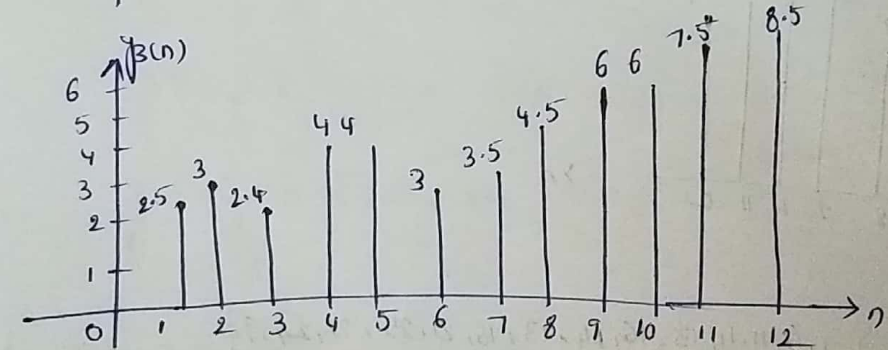
$y_2(n) = x(n) * h_2(n) \Rightarrow \{1,6,11,11,13,16,14,13,16,21,25,28,24,9\}$

h(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
2	2	8	4	6	10	6	6	8	10	14	12	18
1	1	4	2	3	5	1	1	4	5	7	6	9



$\Rightarrow y_3(n) = x(n) * h_3(n) \Rightarrow \{ \frac{1}{2}, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6, 7.5, \frac{9}{2} \}$

$h(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{2}$	$\frac{1}{2}$	2	1	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{4}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	3	$\frac{9}{2}$
$\frac{1}{2}$	$\frac{1}{2}$	2	1	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{4}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	3	$\frac{9}{2}$



$\Rightarrow y_4(n) = x(n) * h_4(n) \Rightarrow \{ 0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 5.25, 6.25, 7, 6, 2.25 \}$

$h(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{6}{4}$	$\frac{9}{4}$
$\frac{1}{2}$	$\frac{1}{2}$	2	1	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$	$\frac{7}{2}$	3	$\frac{9}{2}$
$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{6}{4}$	$\frac{9}{4}$

b) what is the difference b/w $y_1(n)$ & $y_2(n)$ and b/w $y_3(n)$ & $y_4(n)$

Sol:- $y_3(n) = \frac{1}{2} y_1(n)$; $h_3(n) = \frac{1}{2} h_1(n)$.

$y_4(n) = \frac{1}{4} y_2(n)$; $h_4(n) = \frac{1}{4} h_2(n)$

c) Comment on the Smoothness of $y_2(n)$ and $y_4(n)$. which factors affect the Smoothness.

Sol:- $y_2(n)$ and $y_4(n)$ are Smoother than $y_1(n)$ because of Smaller Scalar factor.

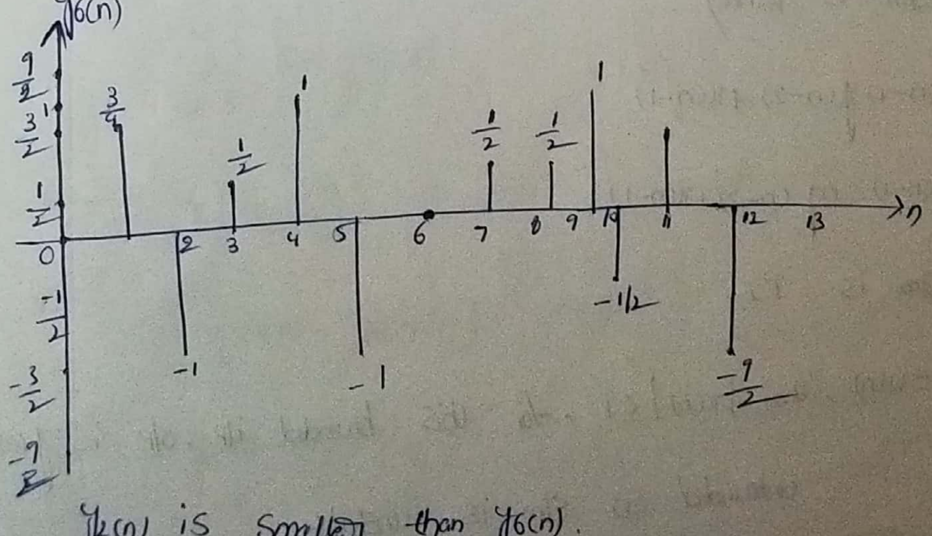
d) Compare $y_4(n)$ with $y_5(n)$. what is the difference Can you Explain it ?

Sol:- $y_4(n)$ results in Smaller o/p, than $y_5(n)$. The negative Value of $h_5(0)$ is responsible for the non-Smooth characteristics of $y_5(n)$.

e) Let $h_6(n) = \left\{ \frac{1}{2}, \frac{-1}{2} \right\}$ Compute $y_6(n)$. Sketch $x(n)$, $y_2(n)$ and $y_6(n)$ on the Same figure and Comment on the results.

Sol:- $y_6(n) = x(n) * h_6(n)$

$y_6(n) = \left\{ \frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, \frac{3}{2}, -\frac{9}{2} \right\}$



$y_2(n)$ is Smaller than $y_6(n)$.

23) $\delta(n) = h(n) * u(n)$ & i/p $x(n)$

Sol:- We can Express $\delta(n) = u(n) - u(n-1)$

$$h(n) = h(n) * \delta(n)$$

$$= h(n) * [u(n) - u(n-1)]$$

$$= h(n) * u(n) - h(n) * u(n-1)$$

$$= \delta(n) - \delta(n-1)$$

then $y(n) = h(n) * x(n)$

$$= [\delta(n) - \delta(n-1)] * x(n)$$

$$= \delta(n) * x(n) - \delta(n-1) * x(n).$$

24) $y(n) = ny(n-1) + x(n)$, $n \geq 0$. Check if system is LTI & Stable.

Sol:- $y(n) = ny(n-1) + x(n)$, $n \geq 0$

$$\left.\begin{aligned} y_1(n) &= ny_1(n-1) + x_1(n) \\ y_2(n) &= ny_2(n-1) + x_2(n) \end{aligned}\right\} \oplus \Rightarrow y(n) = ny_1(n-1) + x_1(n) + ny_2(n-1) + x_2(n)$$

$$y(n) = ny(n-1) + x(n)$$

$$x(n) = ax_1(n) + bx_2(n)$$

$$y(n) = ay_1(n) + by_2(n)$$

Hence the s/m is Linear.

$\Rightarrow y(n-1) = (n-1)y(n-2) + x(n-1)$

delayed $\Rightarrow y(n-1) = ny(n-2) + x(n-1)$

So the s/m is TI.

$\Rightarrow$ If $x(n) = u(n)$, then $|x(n)| \leq 1$, for this bounded i/p, o/p is $y(\infty)$

$y(1) = 5$ . . . . . unbounded. So s/m is unstable.

25) Consider the signal $\delta(n) = a^n u(n)$, $0 < a < 1$

a) Show that any sequence $x(n)$ can be decomposed as

$$x(n) = \sum_{n=-\infty}^{\infty} C_k\, \gamma(n-k)$$ and Express C_k in terms of $x(n)$.

Sol:- $\delta(n) = \gamma(n) - a\gamma(n-1)$

$$\delta(n-k) = \gamma(n-k) - a\gamma(n-k-1)$$

$$x(n) = \sum_{k=\infty}^{\infty} x(k)\, \delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k) \left[\gamma(n-k) - a\gamma(n-k-1)\right]$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \gamma(n-k) - a \sum_{k=-\infty}^{\infty} x(k)\, \gamma(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \gamma(n-k) - a \sum_{k=-\infty}^{\infty} x(k-1)\, \gamma(n-k)$$

$$= \sum_{k=\infty}^{\infty} \left[x(k) - a x(k-1)\right] \gamma(n-k)$$

thus $C_k = x(k) - a x(k-1)$.

b) $y(n) = \mathcal{T}[x(n)]$

$$= \mathcal{T}\left[\sum_{k=-\infty}^{\infty} C_k\, \gamma(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} C_k\, \mathcal{T}\left[\gamma(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} C_k\, g(n-k)$$

c) $h[n] = \mathcal{T}[\delta(n)]$

$$h[n] = \mathcal{T}\left[\gamma(n) - a\gamma(n-1)\right]$$

$$= g(n) - a g(n-1).$$

26) Determine the zero i/p response of the s/m described by the second-order differential equation.

$$x(n) - 3y(n-1) - 4y(n-2) = 0$$

Sol:- With $x(n) = 0$

$$-3y(n-1) - 4y(n-2) = 0 \quad [\div(-3)]$$

$$y(n-1) + \frac{4}{3}y(n-2) = 0$$

at $n=0$

$$y(-1) = -\frac{4}{3}y(-2)$$

at $n=1$

$$y(0) = -\frac{4}{3}y(-1) = \left(-\frac{4}{3}\right)^2 y(-2)$$

$$y(1) = \left(-\frac{4}{3}\right)^3 y(-2)$$

$\vdots$

$$y(k) = \left(-\frac{4}{3}\right)^{k+2} y(-2)$$

$\downarrow$

zero i/p response.

27) $y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$ when $x(n) = 2^n u(n)$

Sol:- $y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$

$$x(n) = y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2)$$

characteristic equation is

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0 \;;\; \lambda = \frac{1}{2}, \frac{1}{3}$$

So $y_h(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{3}\right)^n$

$x(n) = 2^n u(n)$

$y_p(n) = k(2^n)\,u(n)$

So $k(2^n)u(n) - k\left(\frac{5}{6}\right)\left(2^{n-1}\right)u(n-1) + k\left(\frac{1}{6}\right)\left(2^{n-2}\right)u(n-2) = 2^n u(n)$

for $n=2$,

$$4k - \frac{5k}{3} + \frac{k}{6} = 4$$

$$k = \frac{8}{5}$$

Total Solution is

$$y_p(n) + y_h(n) = y(n)$$

$$y(n) = \frac{8}{5}(2^n)u(n) + c_1\left(\frac{1}{2}\right)^n u(n) + c_2\left(\frac{1}{3}\right)^n u(n)$$

Assume $y(-2) = y(-1) = 0$ So $y(0) = 1$

then $y(1) = \frac{5}{6}y(0) + 2 = \frac{17}{6}$

So $\frac{8}{5} + c_1 + c_2 = 1$

$$c_1 + c_2 = -\frac{3}{5} \rightarrow ①$$

$$\frac{16}{5} + \frac{1}{2}c_1 + \frac{1}{3}c_2 = \frac{17}{6}$$

$$3c_1 + 2c_2 = \frac{-11}{5} \rightarrow ②$$

By solving ① & ②

So the Total Solution is

$$y(n) = \left[\frac{8}{5}(2^n) - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right]u(n)$$

28)

Sol:- at $y(-1) = 1$

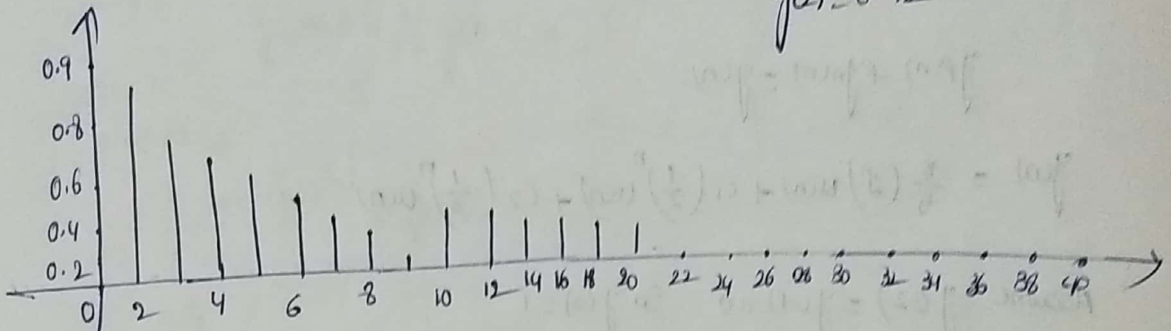
The given equation is $y(n) = (-a)^{n+1} + \frac{(1-(-a)^{n+1})}{1+a}$.

$y(n) = y_{zi}(n) + y_{zs}(n)$ $\Downarrow$ Transient + steady state.

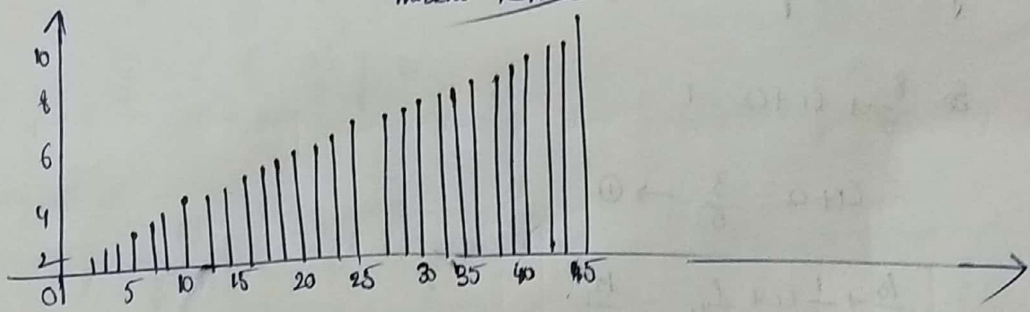
$y(0) = 0.9$

$y(1) = 0.87$

$y(2) = 0.72$



Transient Response.



steady state Response

29) $h_1(n) = a^n[u(n) - u(n-N)]$ & $h_2(n) = [u(n) - u(n-m)]$

Sol:-

$h(n) = h_1(n) * h_2(n)$

$= \sum_{k=-\infty}^{\infty} a^k [u(k)\, u(k-N)][u(n-k) - u(n-k-m)]$

$= \sum_{k=-\infty}^{\infty} a^k u(k)\, u(n-k) - \sum_{k=-\infty}^{\infty} a^k u(k)\, u(n-k-m) - \sum_{k=-\infty}^{\infty} a^k u(k-N)$

$u(n-k) + \sum_{k=-\infty}^{\infty} a^k u(k-N)\, u(n-k-m)$

$= \left(\sum_{k=0}^{n} a^k - \sum_{k=0}^{n-M} a^k\right) - \left(\sum_{k=N}^{n} a^k - \sum_{k=n}^{n-M} a^k\right)$

$h(n) = 0$.

30) $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ to i/p $x(n) = 4^n u(n)$

Sol:- $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

$\lambda^2 - 3\lambda - 4 = 0$

$\lambda = 4, -1$

So $y_h(n) = C_1 4^n + C_2 (-1)^n$

$x(n) = 4^n u(n)$

$y_p(n) = Kn 4^n u(n)$

$Kn 4^n u(n) - 3K(n-1) 4^{n-1} u(n-1) - 4K(n-2) 4^{n-2} u(n-2) = 4^n u(n) + 2(4)^{n-1} u(n-1)$

for $n=2$, $K(32-12) = 4^2 + 8 = 24 \rightarrow K = \frac{6}{5}$

the Total Solution is,

$y(n) = y_p(n) + y_h(n)$

$= \left[\frac{6}{5} n 4^n + C_1 4^n + C_2 (-1)^n\right] u(n)$

to find C_1 and C_2, Let $y(-2)$ & $y(-1) = 0$ then $y(0) = 1$

$y(1) = 3y(0) + 4 + 2 = 9$

$C_2 + C_1 = 1 \rightarrow ①$

$\frac{24}{5} + 4C_1 - C_2 = 9 \Rightarrow 4C_1 - C_2 = \frac{21}{5} \rightarrow ②$

from ① & ②

$C_1 = \frac{26}{25}$ & $C_2 = \frac{-1}{25}$

So $y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25} (-1)^n\right] u(n)$

31) $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

Sol:- characteristic equation :- $\lambda^2 - 3\lambda - 4 = 0$

$\lambda = -4, 1$

$y_h(n) = C_1 4^n + C_2(-1)^n$

$x(n) = \delta(n)$

$y(0) = 1$ and $y(1) = 3y(0) = 2$

$y(1) = 5$

So $C_1 + C_2 = 1 \rightarrow ①$

$4C_1 - C_2 = 5 \rightarrow ②$

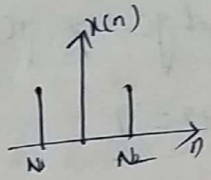
from ① & ② $C_1 = \frac{6}{5}$ & $C_2 = \frac{-1}{5}$

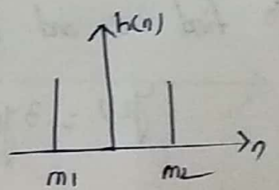
$\therefore h(n) = \left[\frac{6}{5} 4^n - \frac{1}{3} (-1)^n\right] u(n)$

32] Determine the range $L_1 \le n \le L_2$ of their convolution, in terms of N_1, N_2, M_1 and M_2.

$L_1 = N_1 + M_1$

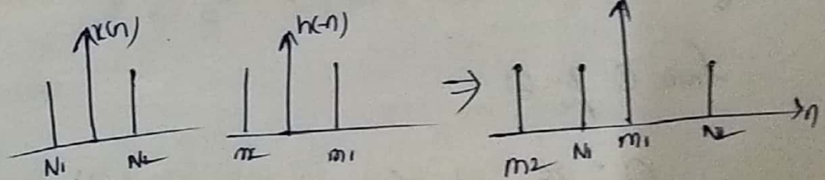
$L_2 = N_2 + M_2$





b) Partial overlap from left

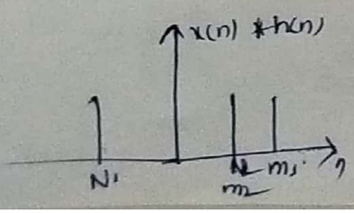
$\rightarrow x(n) * h(n) \Rightarrow$



low $N_1 + m_1$ & high $m_2 + N_1 - 1$

If fully overlap then $N_1 + m_2$ (low) & high $N_2 + m_1$.

Partial overlap from right

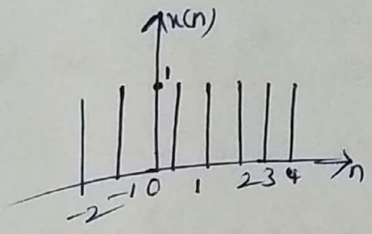


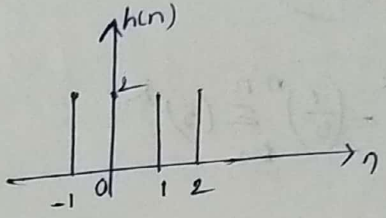
low $\Rightarrow N_2 + m_1 + 1$

high $\Rightarrow N_2 + m_2$

If fully overlaped high N_2+m_1 ; low $= N_1+m_2$.

c) $x(n) = \{1,1,\underset{\uparrow}{1},1,1,1,1\}$ $h(n) = \{2,2,2,2\}$





$m_1 = -1$, $m_2 = 2$

$N_1 = -2$, $N_2 = 4$

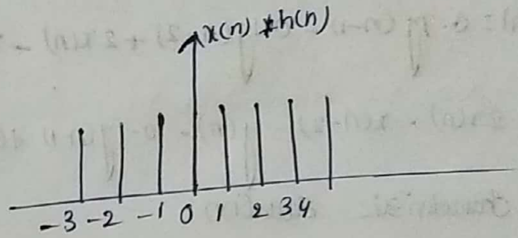
Partial overlap from left

low $N_1+m_1 = -3$

high $m_2+N_1-1 = 2-2-1 = -1$

full overlap $n=0$, $n=3$

Partial right ; $n=4$, $n=6$, $L_2=6$



38)

a) $y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

Sol: $x(n) = y(n) - 0.6y(n-1) - 0.08y(n-2)$

Characteristic Equation

$\lambda^2 - 0.6\lambda + 0.08 = 0$

$\lambda = \frac{1}{2}, \frac{2}{5}$

$y_h(n) = c_1\left(\frac{1}{5}\right)^n + c_2\left(\frac{2}{5}\right)^n$

Impulse response $x(n) = \delta(n)$ with $y(0) = 1$

$y(1) - 0.6\,y(0) = 0 \Rightarrow y(1) = 0.6$

So $c_1 + c_2 = 1 \rightarrow ①$

$\frac{1}{5}c_1 + \frac{2}{5}c_2 = 0.6 \rightarrow ②$

from ① & ② $c_1 = -1$, $c_2 = 3$

$\therefore h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right] u(n)$

Step response $x(n) = u(n)$

$$S(n) = \sum_{k=0}^{n} y\, h(n-k),\ n \geq 0$$

$$= \sum_{k=0}^{n} \left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= 2\left(\frac{2}{5}\right)^{n} \sum_{k=0}^{n} \left(\frac{5}{2}\right)^{k} - \left(\frac{1}{5}\right)^{n} \sum_{k=0}^{n} (5)^{k}$$

$$= \left[2\left(\frac{2}{5}\right)^{n}\left(\frac{5}{2}\right)^{n+1} - 1\right] - \left[\left(\frac{1}{5}\right)^{k}\left(5^{n+1} - 1\right)\right] u(n)$$

b) $y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2)$

Sol:- $2x(n) - x(n-2) = y(n) - 0.7y(n-1) + 0.1y(n-2)$

characteristic Equation

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$$\lambda = \frac{1}{2}, \frac{1}{5}$$

$$y_h(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{5}\right)^n$$

Impulse Response $x(n) = \delta(n)$, $y(0) = 2$

$$y(1) - 0.7y(0) = 0 \Rightarrow y(1) = 1.4$$

$$C_1 + C_2 = 2$$

$$\frac{1}{2}C_1 + \frac{1}{5}C_2 = \frac{7}{5} \rightarrow ①$$

$$C_1 + \frac{2}{5}C_2 = \frac{14}{5} \rightarrow ②$$

Solving ① & ②

$$C_1 = \frac{10}{3},\ C_2 = \frac{-4}{3}$$

So $h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right] u(n)$

Step Response $s(n) = \sum_{k=0}^{n} h(n-k)$

$$= \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left[\frac{1}{2}^n (2^{n+1}-1)\, u(n)\right] - \frac{4}{3}\left[\frac{1}{5}^n (5^{n+1}-1)\right] u(n)$$

34) $h(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \le n \le 4 \\ 0, & \text{elsewhere.} \end{cases}$

$y(n) = \{\underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0\}$

Sol: $h(n) = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \dots\}$

$y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0 \dots\}$

$y(0) = x(0)\, h(0)$

$y(0) = x(0) \cdot 1 \Rightarrow x(0) = 1$

$y(1) = x(1) + h(1)\, x(0)$

$2 = x(1) + \frac{1}{2}(1) \Rightarrow x(1) = \frac{3}{2}$

$y(2) = x(2) + h(2)\, x(1) + h(4)\, x(0)$

$2.5 = x(2) + \frac{1}{4}\left(\frac{3}{2}\right) + \frac{1}{2}(1)$

So $x(n) = \{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2} \dots\}$

35)
a) Express the overall impulse response in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$ & $h_4(n)$

Sol: $h(n) = h_1(n) * [h_2(n) - \{h_3(n) * h_4(n)\}]$

b) Determine $h(n)$ where $h_1(n) = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{2}\}$

$h_2(n) = h_3(n) = (n+1)\, u(n)$

$h_4(n) = \delta(n-2)$

Sol: $h_3(n) * h_4(n) = (n+1)\, u(n) * \delta(n-2)$

$= (n+1)\, u(n-2) = (n+1)\, u(n-2)$

$h_2(n) - [h_3(n) * h_4(n)] = (n+1)u(n) - (n+1)u(n-2)$

$= 2u(n) - \delta(n)$

$h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$

$h(n) = \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right] * \left[2u(n) - \delta(n)\right]$

$= \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$

c) Determine the Response of the sm in Part (b) if

$x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$

Sol:- $x(n) = \{1, 0, 0, 3, 0, -4\}$

36)

Sol:- $s(n) = u(n) * h(n)$

$s(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$

$= \sum_{k=0}^{\infty} h(n-k)$

$= \sum_{k=0}^{\infty} a^{n-k}$

$= \frac{a^{n+k} - 1}{a-1}$; $n \geq 0$

for $x(n) = u(n+5) - u(n-10)$ then

$s(n+5) - s(n-10) = \frac{a^{n+6} - 1}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10)$

from given figure $y(n) = x(n) * h(n) - x(n) * h(n-2)$

$y(n) = \frac{a^{n+6} - 1}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10) - \frac{a^{n+4} - 1}{a-1} u(n+3) + \frac{a^{n-11} - 1}{a-1} u(n-12)$

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31) Compute & sketch step response of the s/m.

$$y(n) = \frac{1}{M}\sum_{k=0}^{m-1} x(n-k)$$

Sol:-

$$h(n) = \left[\frac{u(n)-u(n-m)}{m}\right]$$

$$s(n) = \sum_{k=-\infty}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k) = \begin{cases} \frac{n+1}{m}, & n<m \\ 1, & n \geq m \end{cases}$$

38) $h(n) = \begin{cases} a^n, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$

Sol:-

$$\sum_{k=-\infty}^{\infty} |h(n)| = \sum_{\substack{n=0 \\ n=\text{even}}}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n}$$

$$= \frac{1}{1-|a|^2}$$

stable if $|a|<1$

39) $h(n) = a^n u(n)$ to i/p signal $x(n) = u(n) - u(n-10)$

Sol:- $h(n) = a^n u(n)$

$$y_1(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} a^{n-k}$$

$$= a^m \sum_{k=0}^{n} a^{-k}$$

$$= \frac{1-a^{n+1}}{1-a} u(n)$$

$$y(n) = y_1(n) - y_1(n-10)$$

$$= \frac{1}{1-a}\left[(1-a^{n+1})u(n) - (1-a^{n-9})u(n-10)\right]$$

40] $x(n) = \begin{cases} 1, & 0 \le n \le 10 \\ 0, & \text{otherwise} \end{cases}$

Sol:- from 36^{th} Problem with $a = \frac{1}{2}$

$$y(n) = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right]u(n) - 2\left[1\left(\frac{1}{2}\right)^{n-9}\right]u(n-10)$$

41]

a) $x(n) = 2^n u(n)$

Sol:- $$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k 2^{n-k}$$

$$= 2^n \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k$$

$$= 2^n \left[1 - \left(\frac{1}{4}\right)^{n+1}\right]\left(\frac{4}{3}\right)$$

$$= \frac{2}{3}\left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right] u(n)$$

b) $x(n) = u(-n)$

Sol:- $$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{\infty} h(k)$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = 2, \quad n < 0$$

$$y(n) = \sum_{k=n}^{\infty} h(k)$$

$$= \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)^k$$

$$= 2 - \left(\frac{1 - \left(\frac{1}{2}\right)^n}{\frac{1}{2}}\right)$$

$$= 2\left(\frac{1}{2}\right)^n, \quad n \ge 0$$

42]

a) What is the Impulse Response, $h_e(n)$ of the overall s/m.

Sol:

$$h_e(n) = h_1(n) * h_2(n) * h_3(n)$$

$$= [\delta(n) - \delta(n-1)] * u(n) * h(n)$$

$$= [u(n) - u(n-1)] * h(n)$$

$$= \delta(n) * h(n) = h(n)$$

b) No

43]

a) $x(n)\,\delta(n-n_0) = x(n_0)$. Thus only the value of $x(n)$ at $n=n_0$ is of Interest.

$x(n) * \delta(n-n_0) = x(n-n_0)$. Thus, we obtained shifted version of $x(n)$ Sequence.

b]

Sol:

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= h(n) * x(n)$$

Linearity: $x_1(n) \rightarrow y_1(n) = h(n) * x_1(n)$

$$x_2(n) \rightarrow y_2(n) = h(n) * x_2(n)$$

$$= \alpha\, h(n) * x_1(n) + \beta\, h(n) * x_2(n)$$

$$= \alpha\, y_1(n) + \beta\, y_2(n)$$

Time Variance:

$$x(n) \rightarrow y_1(n) = h(n) * x(n)$$

$$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0)$$

$$= \sum_k h(k)\, x(n-n_0-k)$$

$$= y(n-n_0)$$

c] $h(n) = \delta(n-n_0)$

45]

Sol:- $y(n) = -\frac{1}{2} y(n-1) + x(n) + 2x(n-2)$

at $y(-2) = -\frac{1}{2} y(-3) + x(-2) + 2x(-4) = 1$

$y(-1) = -\frac{1}{2} y(-2) + x(-1) + 2x(-3) = \frac{3}{2}$

$y(0) = -\frac{1}{2} y(-1) + 2x(-2) + x(0) = \frac{17}{4}$

$y(1) = -\frac{1}{2} y(0) + x(1) + 2x(-1) = \frac{47}{8}$

47]

a] $x(n) = \{\underset{\uparrow}{1}, 0, 0 \dots \}$

$y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

$y(0) = x(0) = 1$

$y(1) = \frac{1}{2} y(0) + x(1) + x(0) = \frac{3}{2}$

$y(2) = \frac{1}{2} y(1) + x(2) + x(1) = \frac{3}{4}$. thus we obtain

$y(n) = \{1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, \dots \}$

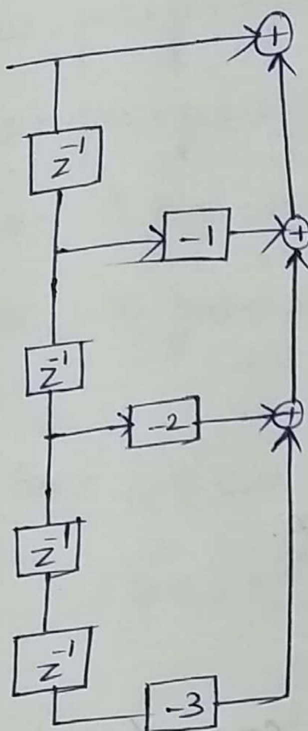
b] $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$.

c] as in Part (a) we obtain

$y(n) = \{1, \frac{5}{2}, \frac{13}{4}, \frac{29}{8}, \frac{61}{16} \dots \}$

d) $y(n) = u(n) * h(n)$

$= \sum_{K} u(K)\, h(n-K)$



$= \sum_{n=0}^{n} h(n-K)$

$y(0) = h(0) = 1$

$y(1) = h(0) + h(1) = \frac{5}{2}$

$y(2) = h(0) + h(1) + h(2) = \frac{13}{4}$ etc.,

e) from Part (a), $h(n) = 0$ for $n<0 \Rightarrow$ The s/m is Causal.

$\sum_{n=0}^{\infty} |h(n)| = 1 + \frac{3}{2}\left(1+\frac{1}{2}+\frac{1}{4}+\cdots\right) = 4 \Rightarrow$ s/m is Stable.

48]

a] $y(n) = ay(n-1) + bx(n)$

$h(n) = ba^n u(n)$

$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1$

$b = 1-a$

b] $s(n) = \sum_{K=0}^{n} h(n-K)$

$= b\left[\frac{1-a^{n+1}}{1-a}\right] u(n)$

$s(\infty) = \frac{b}{1-a} = 1$

$b = 1-a$.

c] $b = 1-a$ in both the cases.

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49]

a] $y(n) = 0.8\,y(n-1) + 2x(n) + 3x(n-1)$

$y(n) - 0.8\,y(n-1) = 2x(n) + 3x(n-1)$

the characteristic Equation is

$\lambda - 0.8 = 0$

$\lambda = 0.8$

$y_h(n) = c(0.8)^n$

Let us first Consider the response of the s/m.

$y(n) - 0.8\,y(n-1) = x(n)$

to $x(n) = \delta(n)$. Since $y(0) = 1$, it follows that $c = 1$. Then, the

Impulse response of the original s/m is

$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$

$= 2\delta(n) + 4.6(0.8)^{n-1} u(n-1)$

b) The Inverse s/m is characterized by the difference Equation.

$x(n) = -1.5\,x(n-1) + \frac{1}{2}y(n) - 0.4\,y(n-1)$

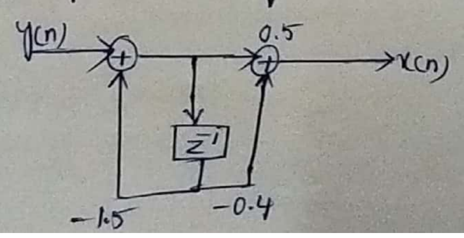
50]

a] $y(n) = 0.9\,y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$

$y(n) - 0.9\,y(n-1) = x(n) + 2x(n-1) + 3x(n-2)$

for $x(n) = \delta(n)$, we have

$y(0) = 1,\ y(1) = 2.9,\ y(2) = 5.61,\ y(3) = 5.049,\ y(4) = 4.544,\ y(5) = 4.090$



b] $S(0) = y(0) = 1$

$S(1) = y(0) + y(1) = 3.91$

$S(2) = y(0) + y(1) + y(2) = 9.51$

$S(3) = y(0) + y(1) + y(2) + y(3) = 14.56$

$S(4) = \sum_{0}^{4} y(n) = 19.16$

$S(5) = \sum_{0}^{5} y(n) = 23.19$

c] $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2)$

$= \delta(n) + 2.9\,\delta(n-1) + 5.61\,(0.9)^{n-2} u(n-2)$

5]

a] $y(n) = \frac{1}{3} x(n) + \frac{1}{3} x(n-3) + y(n-1)$

for $x(n) = \delta(n)$, we have

$h(n) = \left\{ \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3} \ldots \right\}$

b]

$y(n) = \frac{1}{2} y(n-1) + \frac{1}{8} y(n-2) + \frac{1}{2} x(n-2)$

With $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$, we obtain

$h(n) = \left\{ 0, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{16}, \frac{1}{8}, \frac{11}{128}, \frac{15}{256}, \frac{41}{1024} \ldots \right\}$

c] $y(n) = 1.4y(n-1) - 0.48y(n-2) + x(n)$

With $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$, we obtain

$h(n) = \{1, 1.4, 1.48, 1.4, 1.2496, 1.0774, 0.9086 \ldots\}$

d] All three systems are IIR.

e] $y(n) = 1.4y(n-1) - 0.48y(n-2) + x(n)$

The characteristic equation is

$$\lambda^2 - 1.4\lambda + 0.48 = 0 \quad \text{hence}$$

$$\lambda = 0.8, 0.6 \text{ and}$$

$y_h(n) = c_1(0.8)^n + c_2(0.6)^n$ for $x(n) = \delta(n)$. We have,

$$c_1 + c_2 = 1 \text{ and}$$

$$0.8c_1 + 0.6c_2 = 1.4$$

$$c_1 = 4, \quad c_2 = -3$$

$$h(n) = \left[4(0.8)^n - 3(0.6)^n\right] u(n)$$

52]

a] $h_1(n) = c_0\delta(n) + c_1\delta(n-1) + c_2\delta(n-2)$

$h_2(n) = b_2\delta(n) + b_1\delta(n-1) + b_0\delta(n-2)$

$h_3(n) = a_0\delta(n) + (a_1 + a_0 + a_2)\delta(n-1) + a_1a_2\delta(n-2)$

b] The only question is whether

$$h_3(n) \overset{?}{=} h_2(n) = h_1(n)$$

Let $a_0 = c_0$

$$a_1 + a_2c_0 = c_1 \Rightarrow a_1 + a_2c_0 - c_1 = 0$$

$$a_2a_1 = c_2 \Rightarrow \frac{c_2}{a_2} = a_1$$

$$\Rightarrow \frac{c_2}{a_2} \neq a_2c_0 - c_1 = 0$$

$$\Rightarrow c_0a_2^2 - c_1a_2 + c_2 = 0.$$

For $c_0 \neq 0$, the quadratic has a real solution if and only if $c_1^2 - 4c_0c_2 \geq 0$

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53]

a] $y(n) = \frac{1}{2} y(n) + x(n) + x(n-1)$

$x(n) = \delta(n)$

for $y(n) - \frac{1}{2} y(n-1) = x(n) + x(n-1)$

$h(n) = \left(\frac{1}{2}\right)^{n} u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$

b] $h_1(n) * [\delta(n) + \delta(n-1)] = \left(\frac{1}{2}\right)^{n} u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$

54]

a) $x_1(n) = \{1, 2, 4\}$ $\quad h(n) = \{1, 1, 1, 1, 1\}$

Sol: Convolution : $y_1(n) = \{1, 3, 7, 7, 7, 6, 4\}$

Correlation : $r_1(n) = \{1, 3, 7, 7, 7, 6, 4\}$

b) $x_2(n) = \{0, 1, -2, 3, -4\}$ $\quad h_2(n) = \{\frac{1}{2}, 1, 2, 1, \frac{1}{2}\}$

Sol: Convolution : $y_2(n) = \{\frac{1}{2}, 0, \frac{3}{2}, -2, \frac{1}{2}, -6, -\frac{5}{2}, -2\}$

Correlation : $r_2(n) = \{\frac{1}{2}, 0, \frac{3}{2}, -2, \frac{1}{2}, -6, -\frac{5}{2}, -2\}$

Note $y_2(n) = r_2(n)$, $\because h_2(-n) = h_2(n)$ (c)

c) $x_3(n) = \{1, 2, 3, 4\}$ $\quad h_3(n) = \{4, 3, 2, 1\}$

Sol: Convolution : $y_3(n) = \{4, 11, 20, 30, 20, 11, 4\}$

Correlation : $r_3(n) = \{1, 4, 10, 20, 25, 24, 16\}$

d) $x_4(n) = \{1, 2, 3, 4\}$ $\quad h_4(n) = \{1, 2, 3, 4\}$

Sol: Convolution : $y_4(n) = \{1, 4, 10, 20, 25, 24, 16\}$

Correlation : $r_4(n) = \{4, 11, 20, 30, 20, 11, 4\}$

Note that $h_3(-n) = h_4(n+3)$

hence $r_3(n) = y_4(n+3)$

$h_4(-n) = h_3(n+3)$

$y_4(n) = y_3(n+3)$

55]

length of $h(n) = 2$

$h(n) = \{h_0, h_1\}$

$h_0 = 1$

$3h_0 + h_1 = 4 \Rightarrow \boxed{h_0 = 1,\ h_1 = 1}$

56]

Sol:-

$$(2.5.6)\quad y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

$$(2.5.9)\quad w(n) = -\sum_{k=1}^{N} a_k\, w(n-k) + x(n)$$

$$(2.5.10)\quad y(n) = \sum_{k=0}^{M} b_k\, w(n-k)$$

from 2.5.9 we obtain $x(n) = w(n) + \sum_{k=1}^{N} a_k w(n-k) \rightarrow (A)$

by substituting (2.5.10) for $y(n)$ and (A) into (2.5.6)

we obtain L.H.S = R.H.S

57]

Sol:- $y(-1) = y(-2) = 0$

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

The characteristic equation is

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$. Hence

$y_h(n) = c_1 2^n + c_2 n 2^n$

The Partial Solution is

$y_p(n) = K(-1)^n u(n)$

Substituting this solution is into the difference equation, we obtain,

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n)$$

for $n=2$, $k(1+4+4) = 2 \Rightarrow k = \frac{2}{9}$. The Total Solution is

$$y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

from the initial, we obtain $y(0)=1$, $y(1)=2$.

$$c_1 + \frac{2}{9} = 1$$

$$\Rightarrow c_1 = \frac{7}{9}.$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2$$

$$\Rightarrow c_2 = \frac{1}{3}.$$

59]

Sol:

$$u(n-k) = \begin{cases} 1, & n \geq k \\ 0, & \text{otherwise} \end{cases}$$

$$x(n) = x(n) * \delta(n)$$

$$= x(n) * [u(n) - u(n-1)]$$

$$= [x(n) - x(n-1)] * u(n)$$

$$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, u(n-k)$$

60]

Let $h(n)$ be the impulse response of the s/m.

$$s(k) = \sum_{m=-\infty}^{k} h(m)$$

$$h(k) = s(k) - s(k-1)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]\, x(n-k)$$

61] $x(n) = \begin{cases} 1, & n_0 - N \le n \le n_0 + N \\ 0, & \text{otherwise} \end{cases}$

Sol:

$y(n) = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise} \end{cases}$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

The range of non-zero values of $r_{xx}(l)$ is determined by

$n_0 - N \le n \le n_0 + N$

$n_0 - N \le n - l \le n_0 + N$

Which implies

$-2N \le l \le 2N$

for a given shift l, the number of terms in the summation for which both $x(n)$ and non-zero is $2N+1-(l)$ and the value each term is 1. Hence,

$$r_{xx}(l) = \begin{cases} 2N+1-|l|, & -2N \le l \le 2N \\ 0, & \text{otherwise} \end{cases}$$

for $r_{xy}(l)$ we have

$$r_{xy}(l) = \begin{cases} 2N+1-|l-n_0|, & n_0 - 2N \le l \le n_0 + 2N \\ 0, & \text{otherwise} \end{cases}$$

62] a]

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$r_{xx}(-3) = x(0)\, x(3) = 1$

$r_{xx}(-2) = x(0)\, x(2) + x(1) + x(3) = 3$

$r_{xx}(-1) = x(0)\, x(1) + x(1)\, x(2) + x(2)\, x(3) = 5$

$r_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7$

also $r_{xx}(-l) = r_{xx}(l)$

$\therefore r_{xx}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

b] $r_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)\, y(n-l)$

We obtain $r_{yy}(l) = \{1, 3, 5, 7, 5, 3, \underline{1}\}$

We obtain $y(n) = x(-n+3)$ which is equivalent to reversing the sequence

$x(n)$. This has not changed the value.

G3]

Sol: $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$

$$= \begin{cases} 2N+1-|l|, & -2N \le l \le 2N \\ 0, & \text{otherwise} \end{cases}$$

$r_{xx}(0) = 2N+1$

∴ The normalized auto correlation is

$$\rho_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1-|l|), & -2N \le l \le 2N \\ 0, & \text{otherwise} \end{cases}$$

G4] a]

Sol: $r_{xx}(l) = \sum_{n=-\infty}^{\infty} r(n)\, x(n-l)$

$$= \sum_{n=-\infty}^{\infty} \left[\delta(n) + r_1\,\delta(n-k_1) + r_2\,\delta(n-k_2)\right] * \left[\delta(n-l) + r_1\,\delta(n-l-k_1) + r_2\,\delta(n-l-k_2)\right]$$

$$= (1+r_1^2+r_2^2)\, r_{ss}(l) + r_1\left[r_{ss}(l+k_1) + r_{ss}(l-k_1)\right] + r_2\left[r_{ss}(l+k_2) + r_{ss}(l-k_2)\right] + r_1 r_2\left[r_{ss}(l+k_1-k_2) + r_{ss}(l+k_2-k_1)\right]$$

b] $r_{ss}(l)$ has peaks at $l=0, \pm k_1, \pm k_2$ and $\pm(k_1+k_2)$. Suppose that $k_1 < k_2$. Then, we can determine r_1 and k_1. The problem is to determine r_2 and k_2 from the other peaks.

c] If $r_2 = 0$, the peaks occur at $l=0$, and $l = \pm k_1$, that is easy to obtain r_1 and k_1.