$$\hat{x}_k = \hat{x}_k^- + K(z_k - H\hat{x}_k^-)$$

最终估计值　　模型　　测量　　模型（算出来的）

same.

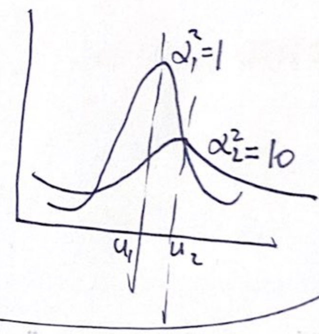
目标：寻找K，使 $\hat{x}_k \to x_k$

$e_k = x_k - \hat{x}_k$　　$p(e_k) \sim (0, P)$

$$P = E[ee^T] = \begin{bmatrix} \sigma_{e_1}^2 & \sigma_{e_1}\sigma_{e_2} \\ \sigma_{e_2}\sigma_{e_1} & \sigma_{e_2}^2 \end{bmatrix}$$

因此，e_k要越小，在$(p(e_k))$最小

方差越小，越靠近0 ⇒ 误差最小



合适的K，$tr(P)$ 最小

$$tr(P) = \sigma_{e_1}^2 + \sigma_{e_2}^2 \quad 最小$$

$$P = E[ee^T] = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T]$$

$z_k = Hx_k + v_k$

$$\Rightarrow x_k - \hat{x}_k = x_k - \hat{x}_k^- - Kz_k + K_k H \hat{x}_k^-$$

$$= x_k - \hat{x}_k^- - K_k H x_k - K_k v_k + K_k H \hat{x}_k^-$$

$$= x_k - \hat{x}_k^- - K_k H (x_k - \hat{x}_k^-) - K_k v_k$$

$$= (I - K_k H) \underbrace{(x_k - \hat{x}_k^-)}_{= e_k^-} - K_k v_k$$

$$\Rightarrow E\left[(x-\hat{x}^-)(x-\hat{x}^-)^T\right]$$

$$= E\left(\left((I-K_kH)e_k^- - K_kv_k\right)\left((I-K_kH)e_k^- - K_kv_k\right)\right]^T\Big]$$

$$= E\left[\left[(I-K_kH)e_k^- - K_kv_k\right]\left[e_k^{-T}(I-K_kH)^T - v_k^TK_k^T\right]\right]$$

$$= E\Big[(I-K_kH)e_k^-e_k^{-T}(I-K_kH)^T - \underbrace{(I-K_kH)e_k^-v_k^TK_k^T}_{=0} - \underbrace{K_kv_ke_k^{-T}(I-K_kH)^T}_{=0}$$

$$+ K_kv_kv_k^TK_k^T\Big)\Big]$$

$(I-K_kH)\,E[e_k^-v_k^T]\,K_k^T$

$E(AB)=E(A)E(B)$

e_k^- 和 v_k^T 相互独立

$E(e_k^-] = 0 \quad E[v_k] = 0$

$\Rightarrow 0$

$$= E\Big[(I-K_kH)\underbrace{E(e_k^-e_k^{-T})}_{\Downarrow\ P_k^-}(I-K_kH)^T + K_k\underbrace{E(v_kv_k^T)}_{\Downarrow\ R}K_k^T$$

P_k^- 因为 $E(e_ke_k^T)=P_k$

R（测量噪声）

$E(v_kv_k^T)=R$

$$= (I-K_kH)P_k^{-1}(I-K_kH)^T + K_kRK_k^T$$

3.

$\Rightarrow \quad P_k = (I - K_k H) P_k^- (I - K_k H_k)^T + K_k R_k K_k^T$

$\Rightarrow (P_k^- - K_k H P_k^-)(I^T - H_k^T K_k^T) + K_k R_k K_k^T$

$\Rightarrow P_k^- - \underline{K_k H P_k^-} - \underline{P_k^- H_k^T K_k^T} + K_k H P_k^- H_k^T K_k^T + K_k R_k K_k^T$

$$\boxed{\begin{aligned} & K_k H P_k^- = ((P_k^- H^T) K_k^T)^T \\ & \Rightarrow tr(K_k H P_k^-) = tr(P_k^- H_k^T K_k^T) \end{aligned}}$$

$\Rightarrow$ s.t. $\min_{K} tr(P_k)$

$\Rightarrow tr(P_k) = tr(P_k^-) - 2\,tr(K_k H P_k^-) + tr(K_k H P_k^- H_k^T K_k^T) + tr(K_k R K_k^T)$

tip: $\boxed{\dfrac{d\,tr(AB)}{dA} = B^T}$ $\quad$ $\boxed{\dfrac{d\,tr(ABA^T)}{dA} = 2AB}$

$\downarrow$

proof: $\Rightarrow tr\left[\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\right] = tr\begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & \\ & a_{22}b_{12} + a_{2} \end{bmatrix}$

$tr(AB) = a_{11}b_{11} + a_{12}b_{21} + a_{22}b_{12} + a_{22}b_{22}$

$$\frac{d\,tr(AB)}{dA} = \begin{bmatrix} \dfrac{d\,tr(AB)}{\partial a_{11}} & \dfrac{\partial tr(AB)}{\partial a_{12}} \\ \dfrac{d\,tr(AB)}{\partial (a_{21})} & \dfrac{d\,tr(AB)}{\partial a_{22}} \end{bmatrix}$$

$$= \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix}$$

$\Rightarrow \dfrac{d\,tr(P_k)}{dK_k} = 0 \quad -2(H P_k^-)^T + 2K_k H P_k^- H^T + 2K_k R_k = 0$

P_k^-

$\Updownarrow$

$\Rightarrow -P_k^{-T} H^T + K_k (H P_k^- H^T + R_k) = 0$

$\Rightarrow \quad K_k (H P_k^- H^T + R_k) = P_k^- H^T$

$\Rightarrow \quad K_k = P_k^- H_k^T [H_k P_k^- H^T + R_k]^{-1}$