We are going to compute the Fourier transform of $f(x) = e^{-ax^2}$, $a > 0$. A direct method is to evaluate a contour integral in the complex plane, and this is often the best way to proceed when other attempts fail. We present another way to do this evaluation. Observe that $f'(x) = -2axf(x)$ and take the Fourier transform of both sides of this equality. By using equations (17.5) and (17.6) with $k = 1$, we see that

$$2i\pi\xi\widehat{f}(\xi) = \frac{a}{i\pi}[-2i\pi x f(x)]\widehat{\ } = \frac{a}{i\pi}\widehat{f}\,'(\xi).$$

Thus

$$\widehat{f}\,'(\xi) + \frac{2\pi^2}{a}\xi\widehat{f}(\xi) = 0.$$

A particular solution of (17.8) is $e^{-\frac{\pi^2}{a}\xi^2}$. When we look for a general solution of the form $\widehat{f}(\xi) = K(\xi)e^{-\frac{\pi^2}{a}\xi^2}$, we see that $K'(\xi) = 0$, so $K(\xi) = K$, a constant, and $\widehat{f}(0) = K$. But $\widehat{f}(0) = \int_{\mathbb{R}} e^{-ax^2}\,dx = (\pi/a)^{\frac{1}{2}}$. Hence

$$\widehat{f}(\xi) = \sqrt{\frac{\pi}{a}}e^{-\frac{\pi^2}{a}\xi^2}.$$