Since f and g are in $L^1(\mathbb{R})$, Fubini's theorem shows that $(y,z) \mapsto f(y,z) \in L^1(\mathbb{R}^2)$. By making a change of variable

$$\int_{\mathbb{R}}\int_{\mathbb{R}} f(y)\, g(z)\, dy\, dz = \int_{\mathbb{R}}\int_{\mathbb{R}} f(x-t)\, g(t)\, dx\, dt.$$

Fubini : $x \mapsto \int_{\mathbb{R}} f(x-t)\, g(t)\, dt$ is defined a.e.

② $$\int_{\mathbb{R}}\int_{\mathbb{R}} |f(x-t)|\,|g(t)|\, dx\, dt = \int_{\mathbb{R}} |g(t)| \left\{ \int_{\mathbb{R}} |f(x-t)|\, dx \right\} dt$$

$$\leqslant \|f\|_1 \cdot \|g\|_1.$$