$(x,t) \mapsto f(t)\, g(x-t) = h(x,t)$. is p- times differentiable.

$\frac{\partial}{\partial x^k} h(x,t) = f(t)\, g^{(k)}(x-t)$ for all $k \in [\![0, p]\!]$.

$\left| \frac{\partial}{\partial x^k} h(x,t) \right| \leqslant \|g^{(k)}\|_\infty \cdot |f(t)|$ for all $x \in \mathbb{R}$

Therefore: $x \mapsto \int h(x,t)\, dt$ is p - times continuously differentiable and:

$$\frac{\partial}{\partial x^k} \int h(x,t)\, dt = \int \frac{\partial}{\partial x^k} h(x,t)\, dt = \int f(t)\, g^{(k)}(x-t)\, dt.$$