Density: $\forall \varepsilon > 0$, $\exists f_\varepsilon \in C_c^0(\mathbb{R})$ such that $\|f - f_\varepsilon\|_p \leq \varepsilon$.

Assume that $\mathrm{supp}(f_\varepsilon) \subseteq [a,b]$. Denote by $g_n = f_\varepsilon * \rho_n$, where $(\rho_n)_{n \geq 0}$ is a sequence of regularizer.

(1) $\mathrm{supp}(g_n) \subseteq [a-1, b+1]$ for all sufficently large n. In addition, $g_n \in C_\infty$.

(2) $\|f_\varepsilon - g_n\|_p = \left\{\int |f_\varepsilon(x) - g_n(x)|^p dx\right\}^{1/p} \leq (b-a+2) \sup\limits_{x \in [a-1, b+1]} |f_\varepsilon(x) - g_n(x)|$.

(3) $f_\varepsilon(x) - g_n(x) = \int \{f_\varepsilon(x) - f_\varepsilon(x-t)\} \rho_n(t)\, dt$ (since $\int \rho_n(t) dt = 1$).

(4) $|f_\varepsilon(x) - g_n(x)| \leq \sup\limits_{|t| \leq \varepsilon_n} |f_\varepsilon(x) - f_\varepsilon(x-t)|$

(5) f_ε is continuous and is compactly supported; it is uniformly continuous and $\|f_\varepsilon - g_n\|_\infty \xrightarrow[n\to\infty]{} 0$

Choose n sufficently large. $\|f_\varepsilon - g_n\|_p \leq \varepsilon$.

We get $\|f - g_n\|_1 \leq \|f - f_\varepsilon\|_1 + \|f_\varepsilon - g_n\|_1 \leq 2\varepsilon$ ✓.