(i) $f \in \mathcal{S}(\mathbb{R}), g \in \mathcal{S}(\mathbb{R}) \Rightarrow f * g \in C^\infty(\mathbb{R})$ ($f \in \mathcal{S}(\mathbb{R}) \subseteq L_1(\mathbb{R})$ and for all $k$, $g \in C^k(\mathbb{R})$, $g^{(k)} \in L^\infty(\mathbb{R})$).

(ii) $$x^p (f*g)^{(q)}(x) = x^p \int f(t)\, g^{(q)}(x-t)\, dt$$

$$= \int f(t)\, (t+x-t)^p\, g^{(q)}(x-t)\, dt$$

$$= \sum_{j=0}^{p} \binom{p}{j} \int t^j f(t)\, (x-t)^{p-j}\, g^{(q)}(x-t)\, dt.$$

Since $\sup_u |u|^{p-j} g^{(q)}(u) < \infty$ $\quad |t^j f(t)(x-t)^{p-j} g^{(q)}(x-t)| \leq C_{p,j} |t^j f(t)|$

$\hookrightarrow$ dominated convergence theorem.

$$\lim_{|x|\to\infty} x^p (f*g)^{(q)}(x) = \sum_{j=0}^{p} \binom{p}{j} \int t^j f(t) \lim_{|x|\to\infty} (x-t)^{p-j} g^{(q)}(x-t)\, dt = 0.$$