Let $(\phi_n)_{n\geq 0}$ a sequence of functions in $\mathcal{S}(\mathbb{R})$ $\quad \|f-\phi_n\|_2 \to 0 \quad n\to\infty$.

$$\|f-\mathcal{F}\bar{\mathcal{F}}f\|_2 \leq \|f-\phi_n\|_2 + \|\phi_n-\bar{\mathcal{F}}\tilde{\mathcal{F}}\phi_n\|_2 + \|\mathcal{F}\bar{\mathcal{F}}\phi_n-\mathcal{F}\bar{\mathcal{F}}f\|_2$$

We have: $\phi_n = \mathcal{F}\bar{\mathcal{F}}\phi_n$. for all $n\in\mathbb{N}$

Since $\mathcal{F}$ is an isometry $\|\mathcal{F}\bar{\mathcal{F}}\phi_n-\mathcal{F}\bar{\mathcal{F}}f\|_2 = \|\phi_n-f\|_2$

Therefore: $\|f-\mathcal{F}\bar{\mathcal{F}}f\|_2 = 0 \Rightarrow f = \mathcal{F}\bar{\mathcal{F}}f$ a.e.