If $f \in L_1(\mathbb{R})$, $g \in L_1(\mathbb{R}) \Rightarrow f * g \in L_1(\mathbb{R})$ (see first slide).

$$\int e^{-i2\pi \xi x} f*g(x)\, dx = \int e^{-i2\pi \xi x} \left[\int f(x-t)\, g(t)\, dt \right] dx$$

Since $\iint \left| e^{-i2\pi \xi x} \right| |f(x-t)|\, |g(t)|\, dx\, dt < \infty$, we may apply the Fubini theorem $\rightarrow$

$$\int e^{-i2\pi \xi x} f*g(x)\, dx = \iint e^{-i2\pi \xi (x-t)} f(x-t)\, e^{-i2\pi \xi t} g(t)\, dt =$$

$$= \int e^{-i2\pi \xi s} f(s)\, ds \int e^{-i2\pi \xi t} g(t)\, dt = \hat{f}(\xi)\, \hat{g}(\xi).$$

(ii) if $\hat{f}, \hat{g} \in L_1(\mathbb{R})$, $\quad \hat{f} * \hat{g} \in L_1(\mathbb{R})$

$$\bar{\mathcal{F}}(\hat{f} * \hat{g}) = \bar{\mathcal{F}}(\hat{f})\, \bar{\mathcal{F}}(\hat{g}) = f \cdot g$$

f and g are bounded (since $f = \bar{\mathcal{F}}(\hat{f})$ with $\hat{f} \in L_1(\mathbb{R})$)

$\Rightarrow f \cdot g \in L_1(\mathbb{R}) \quad \int |f \cdot g(x)|\, dx \leq \|f\|_\infty \|g\|_1 < \infty.$