(i) Write $|f(u)\, g(x-u)| = (|f(u)|\,|g(x-u)|^2)^{1/2} (|f(u)|)^{1/2}$.

Since $f \in L_1(\mathbb{R})$ and $|g|^2 \in L_1(\mathbb{R}) \Rightarrow u \mapsto |f(u)|\,|g(x-u)|^2$ is integrable a.e. (Fubini: $\iint |f(u)|\,|g(x-u)|^2\, du\, dx = \|f\|_1 \|g\|_2^2 < \infty$).

$$\int |f(u)\, g(x-u)|\, du \leq \underbrace{\left(\int |f(u)|\,|g(x-u)|^2\, du\right)^{1/2}}_{<\infty \text{ a.e.}} \left(\int |f(u)|\, du\right)^{1/2}$$ (Cauchy-Schwarz).

$\Rightarrow$ $f * g(x)$ is defined a.e.

(ii)
$$|f * g(x)|^2 \leq \|f\|_1 \int |f(u)|\,|g(x-u)|^2\, du$$

$$\Rightarrow \int |f * g(x)|^2\, dx \leq \|f\|_1 \, \|f\|_1 \, \|g\|_2^2$$

Fubini-Tonelli:
$$\iint |f(u)|\,|g(x-u)|^2\, du\, dx = \int |f(u)|\, du \int |g(x-u)|^2\, dx = \|f\|_1 \|g\|_2^2.$$