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Program Structure & Algorithms Spring 2021

Assignment Number: 4

- **Task:**

We mentioned two alternatives for implementing Union-Find:

1. For weighted quick union, store the depth rather than the size;
2. For weighted quick union with path compression, do two loops, so that all intermediate nodes point to the root, not just the alternates.

For both of these, code the alternative and benchmark it against the implementation in the repository. You have all of that available from a previous assignment.

If you can explain why alternative #1 is unnecessary to be benchmarked, you may skip benchmarking that one.

Usual submission rules apply. 40 points only for this one.

- **Output**

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 100 | Run: 1 | Weighted quick union with path compression: with 10 runs

Run: 1: 0.0 milli-seconds

Mean total pairs generated: 227

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 100 | Run: 1 | Weighted quick union by height without path compression: with 10 runs

Run: 1: 0.0 milli-seconds

Mean total pairs generated: 227

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 100 | Run: 1 | Weighted quick union: with 10 runs

Run: 1: 0.0 milli-seconds

Mean total pairs generated: 227

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 200 | Run: 2 | Weighted quick union with path compression: with 10 runs

Run: 2: 0.0 milli-seconds

Mean total pairs generated: 766

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 200 | Run: 2 | Weighted quick union by height without path compression: with 10 runs

Run: 2: 0.0 milli-seconds

Mean total pairs generated: 766

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 200 | Run: 2 | Weighted quick union: with 10 runs

Run: 2: 0.0 milli-seconds

Mean total pairs generated: 766

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 400 | Run: 3 | Weighted quick union with path compression: with 10 runs

Run: 3: 0.1 milli-seconds

Mean total pairs generated: 1046
2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 400 | Run: 3 | Weighted quick union by height without path compression: with 10 runs
Run: 3: 0.1 milli-seconds
Mean total pairs generated: 1046

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 400 | Run: 3 | Weighted quick union: with 10 runs
Run: 3: 0.0 milli-seconds
Mean total pairs generated: 1046

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 800 | Run: 4 | Weighted quick union with path compression: with 10 runs
Run: 4: 0.1 milli-seconds
Mean total pairs generated: 2500
2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 800 | Run: 4 | Weighted quick union by height without path compression: with 10 runs
Run: 4: 0.1 milli-seconds
Mean total pairs generated: 2500

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 800 | Run: 4 | Weighted quick union: with 10 runs
Run: 4: 0.1 milli-seconds
Mean total pairs generated: 2500

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 1600 | Run: 5 | Weighted quick union with path compression: with 10 runs
Run: 5: 0.3 milli-seconds
Mean total pairs generated: 6567
2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 1600 | Run: 5 | Weighted quick union by height without path compression: with 10 runs
Run: 5: 0.3 milli-seconds
Mean total pairs generated: 6567

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 1600 | Run: 5 | Weighted quick union: with 10 runs
Run: 5: 0.4 milli-seconds
Mean total pairs generated: 6567

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 3200 | Run: 6 | Weighted quick union with path compression: with 10 runs
Run: 6: 0.6 milli-seconds
Mean total pairs generated: 14501
2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 3200 | Run: 6 | Weighted quick union by height without path compression: with 10 runs
Run: 6: 0.5 milli-seconds
Mean total pairs generated: 14501

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 3200 | Run: 6 | Weighted quick union: with 10 runs
Run: 6: 0.4 milli-seconds
Mean total pairs generated: 14501

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 6400 | Run: 7 | Weighted quick union with path compression: with 10 runs
Run: 7: 0.9 milli-seconds
Mean total pairs generated: 30826
2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 6400 | Run: 7 | Weighted quick union by height without path compression: with 10 runs
Run: 7: 1.3 milli-seconds

Mean total pairs generated: 30826

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 6400 | Run: 7 | Weighted quick union: with 10 runs
Run: 7: 0.8 milli-seconds

Mean total pairs generated: 30826

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 12800 | Run: 8 | Weighted quick union with path
compression: with 10 runs

Run: 8: 1.9 milli-seconds

Mean total pairs generated: 60245

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 12800 | Run: 8 | Weighted quick union by height without
path compression: with 10 runs

Run: 8: 3.0 milli-seconds

Mean total pairs generated: 60245

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 12800 | Run: 8 | Weighted quick union: with 10 runs
Run: 8: 1.9 milli-seconds

Mean total pairs generated: 60245

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 25600 | Run: 9 | Weighted quick union with path
compression: with 10 runs

Run: 9: 3.5 milli-seconds

Mean total pairs generated: 121497

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 25600 | Run: 9 | Weighted quick union by height without
path compression: with 10 runs

Run: 9: 5.1 milli-seconds

Mean total pairs generated: 121497

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 25600 | Run: 9 | Weighted quick union: with 10 runs
Run: 9: 4.4 milli-seconds

Mean total pairs generated: 121497

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 51200 | Run: 10 | Weighted quick union with path
compression: with 10 runs

Run: 10: 9.7 milli-seconds

Mean total pairs generated: 298268

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 51200 | Run: 10 | Weighted quick union by height without
path compression: with 10 runs

Run: 10: 14.5 milli-seconds

Mean total pairs generated: 298268

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 51200 | Run: 10 | Weighted quick union: with 10 runs
Run: 10: 9.0 milli-seconds

Mean total pairs generated: 298268

2021-03-02 10:57:34 INFO Benchmark_Timer - Begin run: Sites: 102400 | Run: 11 | Weighted quick union with path
compression: with 10 runs

Run: 11: 17.5 milli-seconds

Mean total pairs generated: 637598

2021-03-02 10:57:35 INFO Benchmark_Timer - Begin run: Sites: 102400 | Run: 11 | Weighted quick union by height
without path compression: with 10 runs

Run: 11: 26.6 milli-seconds

Mean total pairs generated: 637598

2021-03-02 10:57:35 INFO Benchmark_Timer - Begin run: Sites: 102400 | Run: 11 | Weighted quick union: with 10 runs
Run: 11: 17.9 milli-seconds

Mean total pairs generated: 637598

Printing all X axis values (number of sites doubled each iteration):

100,200,400,800,1600,3200,6400,12800,25600,51200,102400

Printing all Y axis values (mean run times for Weighted quick union with path compression):

0.0,0.0,0.1,0.1,0.3,0.6,0.9,1.9,3.5,9.7,17.5

Printing all Y axis values (mean run times for Weighted quick union by height without path compression):

0.0,0.0,0.1,0.1,0.3,0.5,1.3,3.0,5.1,14.5,26.6

Printing all Y axis values (mean run times for Weighted quick union):

0.0,0.0,0.0,0.1,0.4,0.4,0.8,1.9,4.4,9.0,17.9

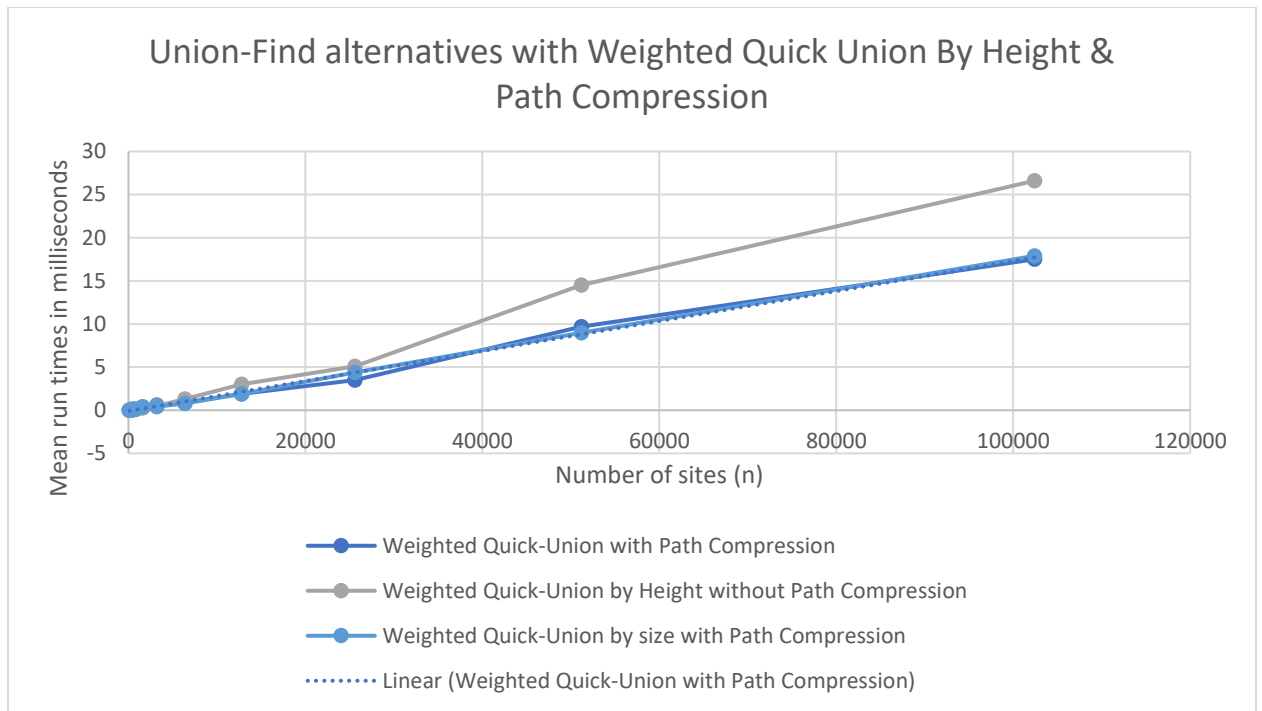
Process finished with exit code 0

- **Relationship conclusion:** I've taken initial number of sites (N) as 100 which is being doubled after each execution for 11 times.

For Weighted Quick Union with Path Compression: We know from the proposition [Hopcroft-Ullman] that any order of M number of union-find operation pairs on N 'sites' makes $\leq c (N + M \lg^* N)$ array accesses where \lg^* is the iterative log function and $\lg^* N$ is basically the number of iterative log functions required to make the log value to 1. And by iterative log formula, $\lg^* N = 1 + \lg^*(\log N)$ if $n > 1$. So, for total number of sites, this $\lg^* N$ would roughly be around the value between 3-4 or less than 5 for all the practical inputs. As when N is 2, $\lg^* N$ is 1; for N = 4, $\lg^* N$ is 2; N = 16, $\lg^* N = 3$; and for N = 65535, $\lg^* N = 4$. For N = 2^{65536} (this is not a practical number in this experiment), $\lg^* N$ is 5.

For, Weighted Quick Union by height: A union operation between elements in different trees either keeps the height unchanged if the two trees have different heights or increase the height by one if the two trees are of same height. The size of the tree would be at least 2^{height} . Therefore, the height can increase at most $\log N$ times.

- **Graphical representation**



N	mean run times (UF_HWQUPC)	mean run times (WQUPC)	mean run times (WeightedQuickUnionByHeight) w/o PC	# of pairs generated (M)	$M \sim (N \ln N)/2$
100	0	0	0	227	230.2
200	0	0	0	766	529.8
400	0.1	0	0.1	1046	1198.2
800	0.1	0.1	0.1	2500	2673.8
1600	0.3	0.4	0.3	6567	5902.2
3200	0.6	0.4	0.5	14501	12913.4
6400	0.9	0.8	1.3	30826	28044.9
12800	1.9	1.9	3	60245	60526.0
25600	3.5	4.4	5.1	121497	129924.4
51200	9.7	9	14.5	298268	277593.4
102400	17.5	17.9	26.6	637598	590676.0

- **Unit tests:**

✓ UF_HWQUPC_Test (edu.neu.coe.info6205.u 0 ms		
✓ testIsConnected01		0 ms
✓ testIsConnected02		0 ms
✓ testIsConnected03		0 ms
✓ testFind0		0 ms
✓ testFind1		0 ms
✓ testFind2		0 ms
✓ testFind3		0 ms
✓ testFind4		0 ms
✓ testFind5		0 ms
✓ testToString		0 ms
✓ testConnect01		0 ms
✓ testConnect02		0 ms
✓ testConnected01		0 ms

✓ WeightedQuickUnionByHeightTest (edu.ne 1 ms		
✓ testIsConnected01		0 ms
✓ testIsConnected02		0 ms
✓ testIsConnected03		0 ms
✓ testFind0		0 ms
✓ testFind1		0 ms
✓ testFind2		0 ms
✓ testFind3		1 ms
✓ testFind4		0 ms
✓ testFind5		0 ms
✓ testToString		0 ms
✓ testConnect01		0 ms
✓ testConnect02		0 ms
✓ testConnected01		0 ms

✓ WQUPCTest (edu.neu.coe.info6205.union_f 8 ms		
✓ testFind0		8 ms
✓ testFind1		0 ms
✓ testFind2		0 ms
✓ testFind3		0 ms
✓ testFind4		0 ms
✓ testConnected01		0 ms

✓ BenchmarkTest (edu.neu.coe.info6205 1 s 473 ms		
✓ testWaitPeriods		1 s 473 ms
✓ getWarmupRuns		0 ms

✓ TimerTest (edu.neu.coe.info6205.util) 2 s 518 ms		
✓ testPauseAndLapResume0		170 ms
✓ testPauseAndLapResume1		325 ms
✓ testLap		218 ms
✓ testPause		223 ms
✓ testStop		100 ms
✓ testMillisecs		107 ms
✓ testRepeat1		204 ms
✓ testRepeat2		286 ms
✓ testRepeat3		777 ms
✓ testPauseAndLap		108 ms