

UKF notes

1. Motion models: (CTRV: constant turn rate and velocity model)

此模型也可以用于 **ekf**。这个模型常用于解决在大转弯的情况下跟踪目标。

- **cv** 模型：假设物体沿直线移动

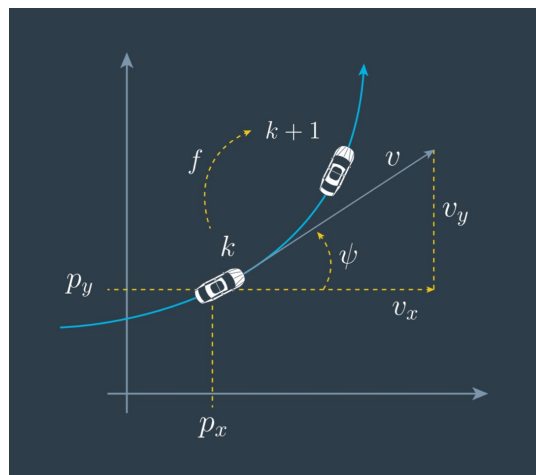
- **cvrt** 模型：物体可以沿直线移动，也可以以 constant turn rate + constant velocity 移动

- **define state vector:**

$x = [p_x \ p_y \ v \ \text{yaw} \ \text{yaw_rate}]$

其中, $p_x \ p_y$ 表示位置, v 表示速度的大小, yaw 为偏航角 (ϕ), yawrate 为 yaw 的角速度。

- **process model**



f : 表示 process model, 表示物体从 k 时刻到 $(k-1)$ 时刻的位置变换的预测, 即 $x_{k+1} = f(x_k, v_k)$ [v_k 为 noise vector]

change rate of state: $\dot{x}' = [\dot{p}_x' \ \dot{p}_y' \ \dot{v}' \ \dot{\psi}' \ \dot{\psi}_{rate}']$

$$= [v_x \ v_y \ 0 \ \dot{\psi}_{rate} \ 0]$$

$$= [\cos(\psi) * v, \sin(\psi) * v, 0, \dot{\psi}_{rate}, 0]$$

define $\Delta t = t_{k+1} - t_k$

We assume the velocity and yaw rate to be constant

=>integration of $\dot{v}' = 0$

integration of $\dot{\psi}_{rate} = \dot{\psi}_{rate} * \Delta t$

integration of $\dot{\psi}_{rate}' = 0$

DIFFERENTIAL EQUATION

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{v} \\ \dot{\psi} \\ \dot{\psi}_{rate} \end{bmatrix} = \begin{bmatrix} v \cdot \cos(\psi) \\ v \cdot \sin(\psi) \\ 0 \\ \dot{\psi} \\ 0 \end{bmatrix}$$

INTEGRAL

$$x_{k+1} = x_k + \int_{t_k}^{t_{k+1}} \begin{bmatrix} \dot{p}_x(t) \\ \dot{p}_y(t) \\ \dot{v}(t) \\ \dot{\psi}(t) \\ \dot{\psi}_{rate}(t) \end{bmatrix} dt$$

INTEGRAL

$$x_{k+1} = x_k + \begin{bmatrix} \int_{t_k}^{t_{k+1}} v(t) \cdot \cos(\psi(t)) dt \\ \int_{t_k}^{t_{k+1}} v(t) \cdot \sin(\psi(t)) dt \\ 0 \\ \dot{\psi}_k \Delta t \\ 0 \end{bmatrix}$$

由于， v_k 为 constant，所以提取出积分外侧

由于， $\dot{\psi}_{rate}$ 为 constant， $\psi(t) = \psi(k) + \dot{\psi} * \Delta t$

INTEGRAL

$$x_{k+1} = x_k + \begin{bmatrix} v_k \int_{t_k}^{t_{k+1}} \cos(\psi_k + \dot{\psi}_k \cdot (t - t_k)) dt \\ v_k \int_{t_k}^{t_{k+1}} \sin(\psi_k + \dot{\psi}_k \cdot (t - t_k)) dt \\ 0 \\ \dot{\psi}_k \Delta t \\ 0 \end{bmatrix}$$

对于 1、2 两行数据，进行积分计算，结果如下：

Question 1: $v_k \int_{t_k}^{t_{k+1}} \cos(\psi_k + \dot{\psi}_k \cdot (t - t_k)) dt = ???$

A. $\frac{\dot{\psi}_k}{v_k} (\cos(\psi_k) - \sin(\psi_k))$

B. $\frac{v_k}{\dot{\psi}_k} (\sin(\psi_k + \dot{\psi}_k \Delta t) - \sin(\psi_k))$ ✓

Question 2: $v_k \int_{t_k}^{t_{k+1}} \sin(\psi_k + \dot{\psi}_k \cdot (t - t_k)) dt = ???$

A. $\frac{v_k}{\dot{\psi}_k} (-\cos(\psi_k + \dot{\psi}_k \Delta t) + \cos(\psi_k))$ ✓

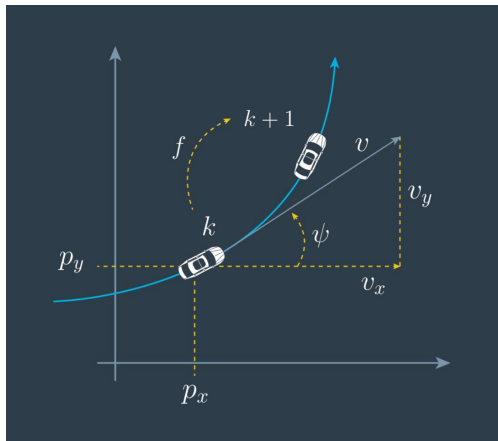
B. $\frac{\dot{\psi}_k}{\Delta t} (\tan(\psi_k) - \sin(\psi_k))$

- special case: yaw rate = 0

problem: 上述公式内 需要有 yaw_rate 做分母, 无法计算

solution:

- 1) 解法 1: 将 yaw-rate = 0, 带入初始式子, 再解积分
- 2) 解法 2: 当 yaw_rate = 0, 目标沿 yaw 角方向做直线运动



因此:

$$p_x' = v_x * \text{deltat} = v_k * \cos(\text{yaw}) * \text{deltat}$$

$$p_y' = v_y * \text{deltat} = v_k * \sin(\text{yaw}) * \text{deltat}$$

其余几项与之前一致

- process noise

PROCESS MODEL	NOISE VECTOR
$x_{k+1} = f(x_k, \nu_k)$	$\nu_k = \begin{bmatrix} \nu_{a,k} \\ \nu_{\ddot{\psi},k} \end{bmatrix}$
LONGITUDINAL ACCELERATION NOISE: STATISTICAL PROPERTIES	
$\nu_{a,k} \sim \mathcal{N}(0, \sigma_a^2)$	
YAW ACCELERATION NOISE: STATISTICAL PROPERTIES	
$\nu_{\ddot{\psi},k} \sim \mathcal{N}(0, \sigma_{\ddot{\psi}}^2)$	

- $v_{a,k}$ 该表示加速度噪声，它影响车辆的速度，假设它的噪声分布为高斯分布

- $v_{\psi'',k}$ 该噪声表示为 yaw 的加速度，也符合高斯分布

- how the process noise influence process

PROCESS MODEL

$$x_{k+1} = x_k + \begin{bmatrix} \frac{v_a}{v_{\psi}} (\sin(\psi_k + \dot{\psi}_k \Delta t) - \sin(\psi_k)) \\ \frac{v_a}{v_{\psi}} (-\cos(\psi_k + \dot{\psi}_k \Delta t) + \cos(\psi_k)) \\ 0 \\ \dot{\psi}_k \Delta t \\ 0 \end{bmatrix} + \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

INFLUENCE YAW ACCELERATION ON YAW RATE

$$e = \Delta t \cdot v_{\psi'',k}$$

process noise 表现了状态转移矩阵的误差。

- influence of $v_{a,k}$ & $v_{\psi'',k}$ on yaw rate: ($v_{a,k}$ 的影响为 0)

对应 e 代表的 process noise，即 yaw_rate 的 process noise。

假设影响因子 yaw acceleration($v_{\psi'',k}$) 在 k 与 $k+1$ 时刻为常量，则经过 Δt 时刻，对 yaw-rate 累计 e 数量的 process noise (如上所示)

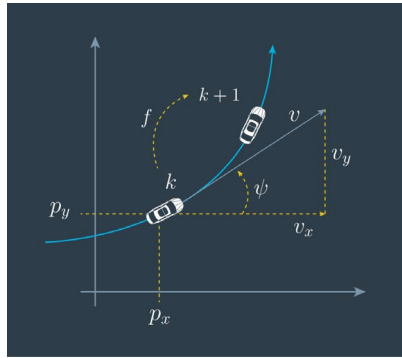
- influence of $v_{a,k}$ and $v_{\psi'',k}$ on velocity

$v_{\psi'',k}$ 对其影响为 0

$v_{a,k}$: 假设加速度 v_a 在时刻间为常量，则它对于 velocity 的影响为 $\Delta t * v_{a,k}$

- influence of two noise on yaw angle: $0.5 * \Delta t^2 * v_{\psi'',k}$

- influence of two noise on position



如果没有两个加速 noise，轨迹如蓝色路径所示。

然而，yaw acceleration noise will change the radius of the circle ->

influence on position

yaw acceleration positive, 往蓝线上方移动 (yaw 变大变得更快了)

yaw acceleration negative, 往蓝线下方移动 (yaw 变大变的越来越慢)

-yaw rate 变化变慢)

注意: effect of yaw acceleration on the position is relatively small

compared to other factor.

- influence of the va,k noise on position

假设 yaw-rate = 0, 即车为直线运动 (即类似 efk, 短时间内虽然有 yaw-rate, 但是可以近似为直线运动)

因此 $a = 0.5 * \text{deltat}^2 \cos(\psi) * va,k$

- influence of va,k on y position

that is, what is y acceleration offset

依旧假设, 物体做直线运动

$0.5 * \text{delta-t}^2 \sin(\psi) va,k$