vendredi 4 octobre 2019 18:01

Modélisation Système Terre Lune Fixe

$$M_L \frac{d^2}{dt^2} r_L(t) = F_g = \frac{G M_T M_L}{r^2(t)} \left(- \frac{r_L(t)}{r(t)}\right)$$

$$\Rightarrow \frac{d^2}{dt^2} r_L(t) = \frac{G M_T}{r^2(t)} \left(- \frac{r_L(t)}{r(t)}\right) \Rightarrow \frac{d^2}{dt^2} \begin{vmatrix} x_L(t) \\ y_L(t) \end{vmatrix} = \frac{G M_T}{r^2(t)}$$

$$\Rightarrow \begin{cases} \dfrac{d^2}{dt^2} x_L(t) = - \dfrac{G M_T}{r^3(t)} x_L(t) \\[2ex] \dfrac{d^2}{dt^2} y_L(t) = - \dfrac{G M_T}{r^3(t)} y_L(t) \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{d}{dt} V_L(t) = - \dfrac{G M_T}{r^3(t)} x_L(t) = f(t, x(t)) \\[2ex] \dfrac{d}{dt} V_y(t) = - \dfrac{G M_T}{r^3(t)} y_L(t) = f(t, y(t)) \end{cases}$$

on a : $\dfrac{dx}{dt} = v \Rightarrow \begin{cases} \dfrac{d x_L}{dt} = V_x(t) = (f(t), V \\[2ex] \dfrac{d y_L}{dt} = V_y(t) = (f(t), V \end{cases}$

$$\begin{cases} V_x'(t) = f(t, x(t)) \\ x_L(0) = \text{posit° \_x\_Lune} \end{cases} \quad ① \qquad \begin{cases} V_y'(t) = f(t, y(t)) \\ y_L(0) = \text{position\_y\_lune} \end{cases} \quad ②$$

$$\begin{cases} x_i'(t) = f(t, V_x(t)) \\ V_x(t) = \text{Vitesse\_x0\_lune} \end{cases} \quad ③$$

$$\begin{cases} y_i'(t) = f(t, V_y(t)) \\ V_y(t) = \text{Vitesse\_y0\_lune} \end{cases} \quad ④$$

En format numérique → Euler

$$\begin{cases} V_{x_0} \\ V_{i+1} = V_i + f(t_i, x_i) \cdot h \end{cases} \quad ①$$

$$\begin{cases} V_{y_0} \\ V_{y_{i+1}} = V_{y_i} + f(t_i, y_i) \cdot h \end{cases} \quad ②$$

$$\begin{cases} x_0 \\ x_{i+1} = x_i + f(t_i, V_i) \cdot h \end{cases} \quad ③$$

$$\begin{cases} y_0 \\ y_{i+1} = y_i + f(t_i, V_{y_i}) \cdot h \end{cases} \quad ④$$

Méthode Runge-Kutta

$$\begin{cases} y'(t) = f(t, y(t)) \\ y(t_0) = y_0 \end{cases}$$

$$y(t_{n+1}) = y(t_n) + h\,\phi(t_n, y(t_n))$$

$$\phi(t_n, y(t_n)) = c_1 k_1 + c_2 k_2$$

$$\begin{cases} k_1 = f(t_n, y(t_n)) \\ k_2 = f(t_n + \alpha h, y(t_n) + \beta h k_1) \end{cases}$$

Résultat calcul : $\begin{cases} c_1 = c_2 = \frac{1}{2} \\ \alpha = \beta = 1 \end{cases}$

$$\Rightarrow \begin{cases} k_1 = f(t_n, y(t_n)) \\ k_2 = f(t_n + h, y(t_n) + h f(t_n, y(t_n))) \end{cases}$$

$$\Rightarrow \phi(t_n, y(t_n)) = \frac{1}{2} f[t_n, y(t_n)] + \frac{1}{2} f\left[t_n + h,\ y(t_n) + h\, f(t_n + y(t_n))\right]$$

$$y_{t_{n+1}} = y(t_n) + \left[\frac{h}{2} f[t_n, y(t_n)] + \frac{h}{2} f\left[t_n + h,\ y(t_n) + h\, f(t_n, y(t_n))\right]\right]$$

$$\begin{cases} y_{t_{n+1}} = V_{t_{n+1}} \\ y(t_n) = V(t_n) \end{cases}$$

$$V_{t_{n+1}} = V(t_n) + \left[\frac{h}{2} f[t_n, V(t_n)] + \frac{h}{2} f\left[t_n + h,\ V(t_n) + h f(t_n, V(t_n))\right]\right]$$

Remember :

$$\begin{cases} f[t_n, V(t_n)] = -\dfrac{GM_T}{\|r\|^3} \begin{vmatrix} x(t) \\ y(t) \end{vmatrix} \\ f(t_n, x(t_n)) = V_x(t_n) \end{cases}$$

$\Rightarrow$ pour la composante en x :

$$V_x(n+1) = V_x(n) + \left[\frac{h}{2}\left(-\frac{GM_T}{\|r\|^3} x(n)\right) + \frac{h}{2} f\left[t_n + h,\ V(n) + h\left(-\frac{GM_T}{\|r\|^3} x(\right.\right.\right.$$

$$\Rightarrow V_x(n+1) = V_x(n) - \frac{h\, GM_T}{2\|r\|^3} x(n) + \left[\frac{h}{2}\left(-\frac{GM_T}{\|r\|^3} x(n)\right) + \frac{h^2}{2}\left(-\frac{GM_T}{\|r\|^3} V_x(n)\right)\right.$$

$$= V_x(n) - \frac{h^2 GM_T}{2\|r\|^3} V_x(n) - \frac{GM_T}{\|r\|^3} x(n)$$

$$\boxed{V_x(n+1) = V_x(n)\left[1 - \frac{h^2 GM_T}{2\|r\|^3}\right] - \frac{GM_T}{\|r\|^3} x(n)}$$

Interract: Lune - Terre - soleil

$$\sum F_{ext} = M \vec{a}$$

$$M_L \vec{a}_L = F_{\frac{T}{L}} = - \frac{G M_T M_L}{\| r_{TL} \|^3} \times r_{TL}(t)$$

$$\vec{a}_L = - \frac{G M_T}{\| r_{TL} \|^3} \times r_{TL}(t) = - \frac{G M_T}{\| r_{TL} \|^3} \times \left\{ \begin{matrix} x \\ \end{matrix} \right.$$