ASSIGNMENT - 8

Q1) Given,

$G1 = (V_1, T_1, P_1, S_1)$

$G2 = (V2, T2, P2, S2)$

T's ≡ Terminals   P's ≡ Productions

V's ≡ Variables   S's ≡ Start symbols

i) Prove that CFLs are CLOSED UNDER UNION

Construct a new grammar G3 such that

$L(G3) = L(G1) \cup L(G2)$

Sol:- For this, we need to prove that $L_1 \cup L_2$ is CONTEXT-FREE for CFLs L_1 and L_2.

Lets suppose, grammar G_3 generates a language $L(G3)$ as below:-   S: Being the START SYMBOL

$G3 = (V_1 \cup V_2 \cup \{S\}, T1 \cup T_2, P_1 \cup P2 \cup \{S \to S_1 / S_2\}, S)$

Now, a string "s" such that $s \in (T_1 \cup T_2)^*$ is considered.

If $S_1 \Rightarrow^* s$ (or) $S_2 \Rightarrow^* s$

From the grammar,

$S \to S_1$ (and)

$S \to S_2$

Hence, we can conclude that S leads to s.

$L_1 \cup L_2$ will be the language of the Grammar G_3, as L_1 and L_2 are context-free languages.

Now,

We can conclude that $L_1 \cup L_2$ is CONTEXT-FREE as G_3 is context-free.

$\therefore$ $L(G3) = L(G1) \cup L(G2)$ //PROVED

(ii) Concatenation of languages :- $L(G3) = L(G1)\,L(G2)$

Sol:- For this, we need to prove that L_1L_2 is a CONTEXT FREE LANGUAGE for the languages L_1 and L_2.

Lets suppose, grammar G_3 generates a language $L(G3)$ as below, with S being the new start symbol.

$$G3 = (V_1 \cup V_2 \cup \{S\},\ T_1 \cup T_2,\ P_1 \cup P_2 \cup \{S \to S_1S_2\},\ S)$$

Like we did before, let's consider a string "s_1" such that $s_1 \in L_1$ (and) a string "s_2" such that $s_2 \in L_2$

As, $S_1 \Rightarrow^* s_1$ (or) $S_2 \Rightarrow^* s_2$,

From the grammar above :- $S \Rightarrow S_1S_2$

$\therefore$ S leads to s_1s_2

As, L_1 and L_2 are context free languages, the language of the grammar G_3 is going to be L_1L_2.

Since, G is a context free Grammar, L_1L_2 is a context free language.

$$\therefore \; L(G_3) = L(G_1)\,L(G_2) \quad // \text{PROVED}$$

(iii) Kleen Closure :- $L(G') = L(G)^*$

Sol:- For this, we need to prove that L^* is a CONTEXT FREE LANGUAGE for the language L.

Lets suppose, grammar G_3 generates a language $L(G')$ as below, with S being the new start symbol.

$$G' = (V_1 \cup \{S\}, T_1, P_1 \cup \{S \to \epsilon, S \to SS_1\}, S)$$

Lets suppose, $S_1 \to aS_1b$ and

$S_1 \to \epsilon$

Since, $S \to \epsilon$

Hence, $S \to SS_1$

$\therefore L(G') = \{a^n b^n ; n \geq 0\}^*$ CONTEXT FREE LANGUAGE.

//PROVED

Q2) Prove that CFLs are NOT CLOSED under

(a) INTERSECTION

(b) COMPLEMENT

Sol:- (a)

We already know that L_1 and L_2 below are CONTEXT FREE LANGUAGES.

$$L_1 = \{0^m 1^m 0^n : m, n > 0\}$$

$$L_2 = \{0^m 1^n 0^n : m, n > 0\}$$

Now:- $L_1 \cap L_2 = \{0^m 1^m 0^m : m > 0\}$

(NOT CONTEXT FREE LANGUAGE)

Hence Proved that CFLs are NOT CLOSED under INTERSECTION

(b)

We already know that CFLs are closed under UNION, for L_1 and L_2 context free languages,

$\therefore$ $L_1 \cup L_2$ is a CONTEXT FREE LANGUAGE.

Lets suppose that,

CFL is closed under COMPLEMENT

$(L_1)^c$, $(L_2)^c$, $(L_1)^c \cup (L_2)^c$ are CONTEXT FREE LANGUAGES

Therefore, $((L_1)^c \cup (L_2)^c)^c$ is a CONTEXT FREE LANGUAGE.

$$L_1 \cap L_2 = ((L_1)^c \cup (L_2)^c)^c$$

As, we already know that CONTEXT FREE LANGUAGES are NOT CLOSED UNDER INTERSECTION, $L_1 \cap L_2$ is not a CONTEXT FREE LANGUAGE.

This is a CONTRADICTION of our assumption.

$\therefore$ CONTEXT FREE LANGUAGE is NOT CLOSED UNDER COMPLEMENT.

Q3) $L = \{a^n b^m c^n \mid m \leq n\}$

Sol:- Using PUMPING LEMMA we can prove this by the RULE OF CONTRADICTION.

Let $m = 3$, $n = 4$ as it is given that $m \leq n$.

Now we can split "S" into $u\,v\,x\,y\,z$

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$S = a^4 b^3 c^4$ (As $m=3$ and $n=4$)

$\Rightarrow S = \underbrace{aa}_{u}\ \underbrace{aa}_{v}\ \underbrace{bbb}_{x}\ \underbrace{c}_{y}\ \underbrace{ccc}_{z}$

this shortens down to

$\Rightarrow S = u v^k x y^k z$ where $k=2$

$\therefore u v^2 x y^2 z$

$\Rightarrow S = aaaaaa\ bbb\ cc\ ccc$

$= a^6 b^3 c^5$

NOW as $a^6 \neq c^5$ ($\because a^n b^m c^n$)

$\therefore$ Our assumption that L is a CONTEXT FREE LANGUAGE is FALSE.

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(b) $L = \{xx \mid x \in \Sigma^*\}$ for any alphabet Σ with atleast two symbols.

Sol:- Lets suppose,

L is a CONTEXT FREE LANGUAGE.

and "p" be the pumping length.

Lets consider:-

$z = 0^p 1^p 0^p 1^p \in L$

As, $|z| > p$, there exists u, v, w, x, y such that

$z = uvwxy$, $|vwx| \le p$, $|vx| > 0$ and

$uv^iwx^iy \in L \quad \forall\ i \ge 0$

vwx must straddle the MIDPOINT of z.

> Let suppose vwx is only in the first half.

Then in, uv^2wx^2y the second half starts with 1.

$\therefore$ This is not of the form ww.

> When vwx is only in the second half.

Then in, uv^2wx^2y the first half ends in a 0.

$\therefore$ This is not of the form ww.

> Lets suppose vwx straddles the middle.

Then uv^0wx^0y must be of the form $0^p 1^i 0^j 1^p$ where either i or j, is not p.

$\therefore uv^0wx^0y \notin L$

Q4) $f(x) = 3x$

Sol:-

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     IF x != 0 GOTO C
     GOTO E
[C]  Z2 ← 3
[A]  IF Z2 != 0 GOTO B
     GOTO E
[B]  Z2 ← Z2 - 1
     Z1 ← x + y
     y ← Z1
     GOTO A
```