CS 5000: Theory of Computability
Assignment 8

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1 Learning Objectives

1. Closure Properties of Context-Free Languages
2. Programming Language L

Introduction

Before saying our final goodbyes to CFLs, let's investigate some of their closure properties. Problem 4 will give you some exposure to the programming language L, the formalism that we will use to investigate the foundations of computability.

Problem 1 (1 point)

Show that CFLs are closed under union, concatenation, and Kleene closure.

To prove the union closure property, consider two CFGs $G_1 = (V_1, T_1, P_1, S_1)$ and $G_2 = (V_2, T_2, P_2, S_2)$, where V's are variables, T's are terminals, P's are productions, and S's are start symbols.

To prove that CFLs are closed under union, you need to construct a new grammar G_3 such that $L(G_3) = L(G_1) \cup L(G_2)$.

To prove that CFLs are closed under union, you need to construct a new grammar G_3 such that $L(G_3) = L(G_1)L(G_2)$.

To show that CFLs are closed under Kleene closure, you need to show that, given a CFG G, a new grammar $G\prime$ can be constructed such that $L(G\prime) = L(G)^*$.

Problem 2 (2 points)

Show that CFLs are not closed under intersection or complement.

The fact that CFLs are not closed under intersection is quite surprising, given the straightforward proof we did to close regular languages under intersection. But, it is, nonetheless, true. Hint: Try to find two CFL languages whose intersection is not CF.

The fact that CFLs are not closed under complement is also quite surprising, given that regular languages are closed under complement. Hint: This is a one liner once you express the intersection in terms of the complement and the union.

Problem 3 (1 point)

Show that the following two languages are not CF.

1. $L = \{a^n b^m c^n | m \leq n\}$
2. $L = \{xx | x \in \Sigma^*\}$ for any alphabet Σ with at least two symbols.

Problem 4 (1 point)

Write an L program that computes $f(x) = 3x$.

What to Submit?

Save your solutions in hw08.pdf and submit it in Canvas.