ASSIGNMENT – 10

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Q1)

We know that :-

$f(x)$ is PRIMITIVE RECURSIVE

And PRIMITIVE RECURSIVE FUNCTIONS are SATISFYING COMPOSITION.

Let $h(x_1, x_2 \dots x_n) = f(g_1(x_1, \dots x_n) \dots g_K(x_1, \dots x_n))$

h : PRIMITIVE RECURSIVE if $f, g_1, g_2 \dots g_K$ are PRIMITIVE RECURSIVE
$\hookrightarrow$ (i)

We'll prove $f^K(x) = f(f(\dots f(x)))$ is PRIMITIVE RECURSIVE by PROOF BY INDUCTION.

Base case:-

$$f^2(x) = f(f(x))$$

$\because$ We know f is primitive recursive.

Hence, by using the composition rule defined in (i),
we can say that $f(f(x))$ is PRIMITIVE RECURSIVE.

$\therefore$ $f^2(x)$ is PRIMITIVE RECURSIVE satisfying the base case.

Hypothesis :-

Let $f^K(x) = f(f(\dots f(x))$ is PRIMITIVE RECURSIVE.

Let's do it for (K+1) :-

We have to prove $f^{K+1}(x)$ is PRIMITIVE RECURSIVE.

$$f^{K+1}(x) = f(f^K(x))$$

We know that f^1 is PRIMITIVE RECURSIVE and $f^K(x)$ is also PRIMITIVE RECURSIVE FROM HYPOTHESIS.

$\therefore$ By composition rule in (i),

$f^{K+1}(x)$ is PRIMITIVE RECURSIVE.

$\therefore$ By MATHEMATICAL INDUCTION,

we can conclude that $f^n(x)$ is PRIMITIVE RECURSIVE.

2)

3. (b)

Let g be a function in COMP.

From 2 (a),

$$g(x_1, \cdots x_n) = K \quad \text{(or)}$$
$$g(x_1, \cdots x_n) = x_i + K$$

CASE (i)

$$g(x_1, \cdots x_n) = K$$

Let's suppose, n-tuples $(x_1, \cdots x_n)$, $(y_1, \cdots y_n)$ such that $x_i \leq y_i$

$$g(x_1, \cdots x_n) = K$$
$$g(y_1, \cdots y_n) = K$$
$$g(x_1, \cdots x_n) = g(y_1, \cdots y_n)$$

$\therefore$ Condition Given is TRUE for case (i).

CASE (ii)

$$g(x_1, \cdots x_n) = x_i + K$$

$$g(x_1, \cdots x_n) = x_i + K$$
$$g(y_1, \cdots y_n) = y_i + K$$

We know, $x_i \leq y_i$,

$$g(x_1, \cdots x_n) \leq g(y_1, \cdots y_n)$$

$\therefore$ Condition Given is TRUE for case (ii)

Now, as it is satisfying both CASE (i) and CASE (ii),

g is MONOTONE.

$\therefore$ $\boxed{\text{Every Function in COMP is MONOTONE}}$

3A:-

Let $\pi(x)$: Number of primes that are less than (or) equal to x.

BASECASE :- $\pi(0) = 0$

The number of primes $\leq x+1$ is given by,

the number of primes less than (or) equal to x,

plus 1 if $x+1$ is a PRIME.

$\therefore$ $\pi(x)$ is given by the recursion equations.

$$\pi(t+1) = \pi(t) + prime(t+1)$$

This is a recursive case.

Now, let's combine both the basecase and recursive case.

So, $\pi(0) = 0$

$$\pi(t+1) = \pi(t) + prime(t+1)$$

Since, the addition (+) and prime are PRIMITIVE RECURSIVE,

$\pi(x)$ is also a PRIMITIVE RECURSIVE.

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2)

(a) Using ZERO, SUCCESOR and PROJECTION RULES.

ZERO:-

$$f(x_1, \cdots x_n) = 0$$

SUCCESSOR:-

$$f(x_1, \cdots x_n) = x_i + 1$$

PROJECTION:-

$$f(x_1, \cdots x_n) = x_i$$

from composition rule,

$$h(x_1, \cdots x_n) = f(g_1(x_1, \cdots x_n) \cdots g_n(x_1 \cdots x_n))$$

$$\therefore h, g_1, g_2, \cdots g_n \in \text{COMP}$$

As $h \in$ COMP, we can write $f(x_1, \cdots x_n) = 0$ (or)

$f(x_1, \cdots x_n) = x_i + 1$ (or)

$f(x_1 \cdots x_n) = x_i$

$\therefore g(x_1, \cdots x_n) = 0$ (or) $x_i + 1$ (or) x_i as $f(x_1, \cdots x_n) = 0$ (or)

$f(x_1, \cdots x_n) = g(x_1 \cdots x_n) + 1$ (or) $f(x_1, \cdots x_n) = g(x_1 \cdots x_n)$

$\therefore$ We can conclude that for "m" compositions,

we can write : $f(x_1, \cdots x_n) = m$ (or)

$f(x_1, x_2 \cdots x_n) = x_i + m$

[If $f(x_1, \cdots x_n) \in$ COMP]