

# CS 5000: Theory of Computability

## Assignment 5

### Grammars and Stack Machines

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## 1 Learning Objectives

1. Regular Grammars and Languages
2. Context-Free Grammars
3. Stack Machines

### Problem 1

At the end of my lecture on Thursday, September 28, a student came and ask me if there is a grammar in between linear and context-free. Of course, there is no such grammar, because all linear grammars are context-free, but not all context-free grammars are linear. In other words, once we break the linear constraints, e.g., by adding more variables in the right-hand sides, we get context-free rules. But this was a very interesting question! I am grateful to that student for asking it. So, let's look at it from a slightly different angle. We know that a language is linear if it is generated by a linear grammar, either left-linear or right-linear. But are all languages generated by non-linear grammars necessarily non-linear? Let's do a simple investigation. Let  $L$  be a language over  $\{a, b\}$  generated by the following grammar:

1.  $S \rightarrow aSa$
2.  $S \rightarrow \epsilon$

Is this grammar linear? Is  $L$  linear? If  $L$  is not linear, explain why not. If  $L$  is linear, explain why. So, are all languages generated by non-linear grammars necessarily non-linear?

### Problem 2

Write a context-free grammar  $G$  that generates all strings of balanced left and right parentheses, e.g.,  $()$ ,  $((()))$ ,  $()()$ ,  $((()))$ ,  $((()()))$ , etc. Construct a stack machine  $M$  for it.

### Problem 3

Recall that a *palindrome* is a string that reads the same left to right and right to left, e.g.,  $aba$ ,  $abba$ ,  $abaaba$ , etc. Write a context-free grammar  $G$  for the language of palindromes over  $\{a, b\}$  and construct a stack machine  $M$  for it.

### Problem 4

Consider the context-free grammar below. What language does this grammar generate?

1.  $S \rightarrow 0B|1A$
2.  $A \rightarrow 0|0S|1AA$
3.  $B \rightarrow 1|1S|0BB$

## Problem 5

Consider the context-free grammar below. What language does this grammar generate?

1.  $S \rightarrow \epsilon | 0S | 1T$
2.  $T \rightarrow 0T | 1S$

## What to Submit?

Submit your solutions via Canvas.