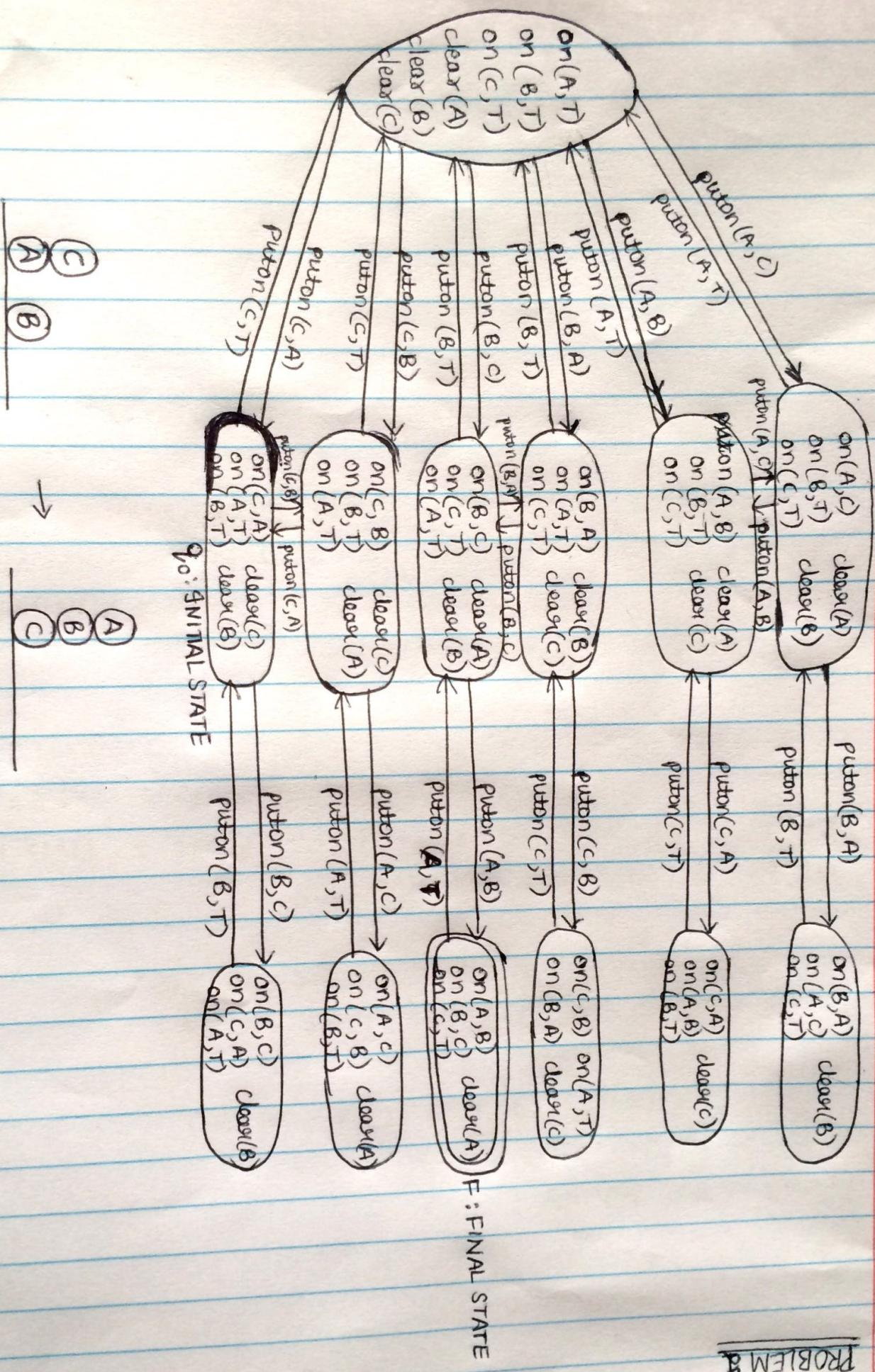
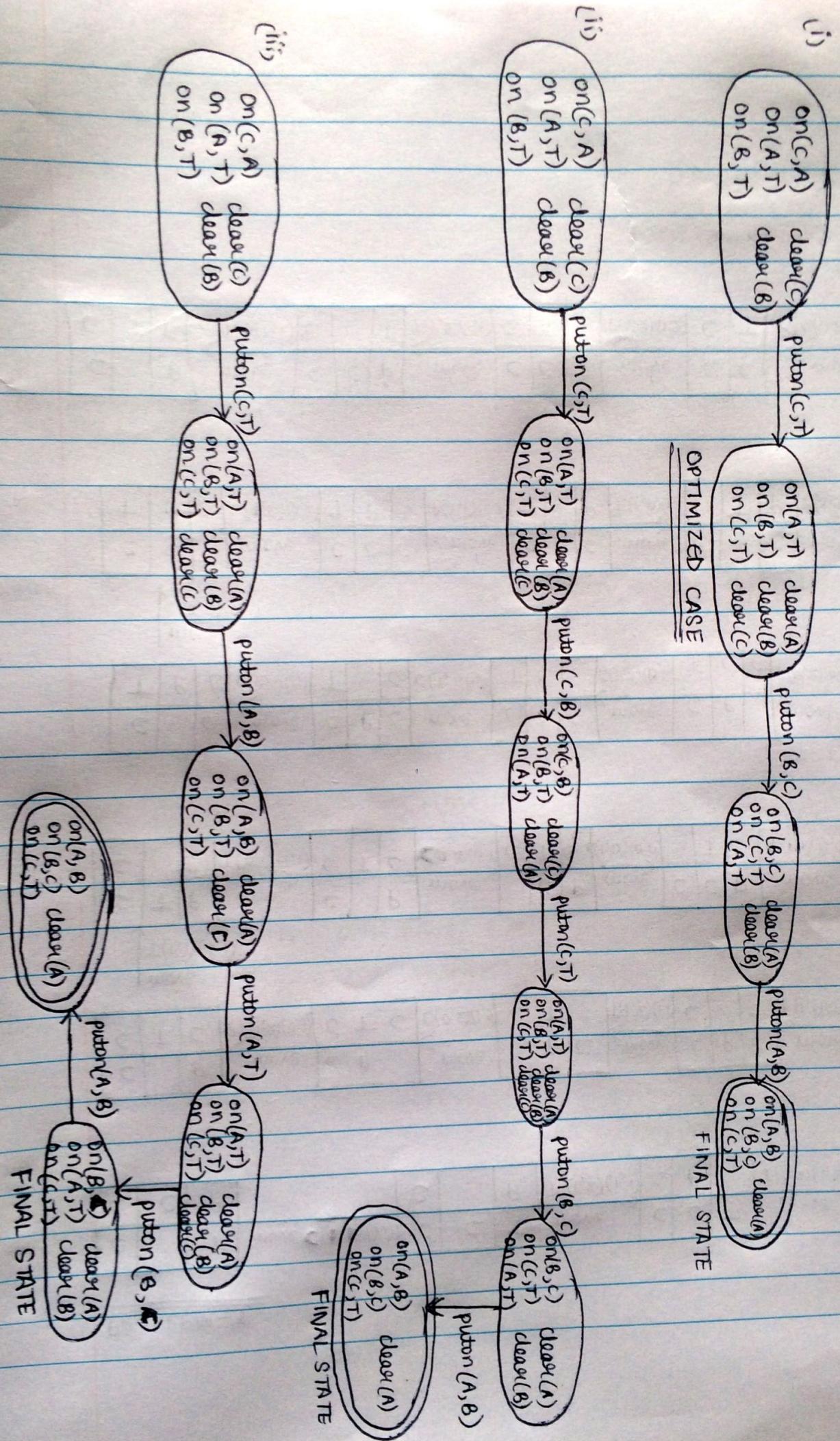


PROBLEM 1

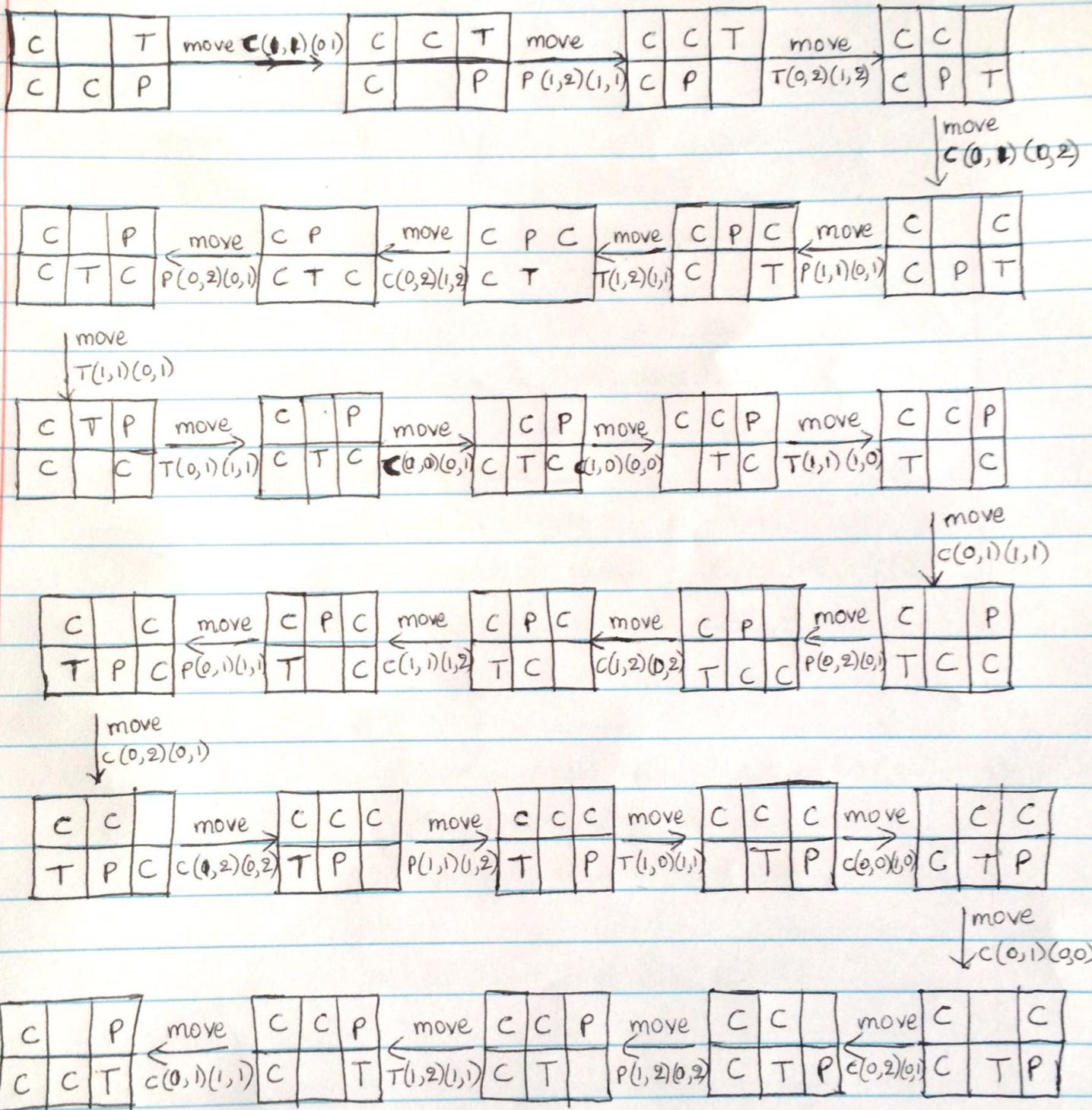


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Testcases



PROBLEM - 3



PROBLEM 4

$$① \sum_{i=1}^n i(i+1) = \frac{n(n+1)(n+2)}{3}$$

By Mathematical Induction:-

$$\text{For } n=1 : \sum_{i=1}^1 i(i+1) = 1(2) = 2 \quad (\text{Left Hand Side})$$

$$\frac{n(n+1)(n+2)}{3} = \frac{1(2)(3)}{3} = 2 \quad (\text{Right Hand Side})$$

$$\text{For } n=2 : \sum_{i=1}^2 i(i+1) = 1(2) + 2(3) = 8 \quad (\text{Left Hand Side})$$

$$\frac{n(n+1)(n+2)}{3} = \frac{2(3)(4)}{3} = 8 \quad (\text{Right Hand Side})$$

Let's assume this to be TRUE for $n=k$

$$\text{For } n=k : \sum_{i=1}^k i(i+1) = 1(2) + 2(3) + 3(4) + \dots + k(k+1) \quad [\text{Left Hand Side}]$$

which should be equal to Right Hand Side

$$\therefore 1(2) + 2(3) + 3(4) + \dots + k(k+1) = \frac{k(k+1)(k+2)}{3} \rightarrow (i)$$

Now, it should be true for $n=k+1$

$$\text{For } n=k+1 : \sum_{i=1}^{k+1} i(i+1) = 1(2) + 2(3) + 3(4) + 4(5) + \dots + k(k+1) + (k+1)(k+2)$$

Substitute equation (i) here

$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2)}{3} + 3(k+1)(k+2)$$

$$= \frac{(k+1)(k+2)[k+3]}{3}$$

Hence Proved

$$\textcircled{2} \quad \sum_{i=0}^n i! = (n+1)! - 1$$

LHS: Left Hand Side

RHS: Right Hand Side

By Mathematical Induction :-

~~$$\text{For } n=0 : \sum_{i=0}^0 i! = 0! = 1 \rightarrow \text{For } n=0 : 0! = 0 \text{ (LHS)}$$~~

$$(0+1)! - 1 = 1 - 1 = 0 \text{ (RHS)}$$

As this is supposed to be true for $n=k$.

$$\text{For } n=k : \sum_{i=0}^k i! = (k+1)! - 1 \rightarrow \text{(i)}$$

Now, let's prove that this is TRUE for $n=k+1$.

$$\text{For } n=k+1 : \sum_{i=0}^{k+1} i! = \sum_{i=0}^k i! + (k+1)! (k+1)$$

Substitute (i) here

$$\begin{aligned} &= [(k+1)! - 1] + [(k+1)! (k+1)] \\ &= (k+1)! [1(k+1)] - 1 \\ &= (k+2)! - 1 \end{aligned}$$

Hence Proved

$$\textcircled{3} \quad n! > 2^n, n \geq 4$$

By Mathematical Induction :-

$$\text{For } n=4 : 4! = 24 \text{ (LHS)}$$

$$2^4 = 16 \text{ (RHS)}$$

$n \geq 4$

Clearly $24 > 16$

\therefore True for n

$$\text{For } n=k : k! > 2^k \rightarrow \text{(i)}, \boxed{k \geq 4}$$

Now, let's prove that this is true for $n=k+1$

$$\text{For } n=k+1 : (k+1)!$$

$$= (k)! (k+1)$$

Sub (i) here

$$> 2^k (k+1)$$

$(k+1) >$
 $\Rightarrow \boxed{k \geq 4}$

Since $k \geq 4 \Rightarrow k+1$ will for sure be greater than 2

$$> 2^k (2)$$

$$> 2^{k+1}$$

Hence Proved

④ A : Set

A^c : Complement of A

$$\text{Show for } n \geq 1, (A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n)^c = A_1^c \cap A_2^c \cap \dots \cap A_n^c$$

For $n=2$:- $(A_1 \cup A_2)^c = A_1^c \cap A_2^c$ (According to De Morgan's Law)

∴ Assume this to be true for $n=k$:-

$$(A_1 \cup A_2 \cup \dots \cup A_k)^c = A_1^c \cap A_2^c \cap \dots \cap A_k^c \rightarrow (i)$$

Now, prove this to be true for $n=k+1$:-

$$\begin{aligned} (A_1 \cup A_2 \cup \dots \cup A_{k+1})^c &= \cancel{(A_1^c \cap A_2^c \cap \dots \cap A_k^c)} \cancel{\cap A_{k+1}^c} \\ &= (A_1^c \cap A_2^c \cap \dots \cap A_k^c \cap A_{k+1}^c) \\ &\quad \text{Sub (i) here} \\ &= (A_1^c \cap A_2^c \cap \dots \cap A_k^c) \cap A_{k+1}^c \\ &= A_1^c \cap A_2^c \cap \dots \cap A_k^c \cap A_{k+1}^c \end{aligned}$$

∴ Hence Proved