CS 5000: Theory of Computability
Assignment 10

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1 Learning Objectives

1. Primitive recursive functions
2. Computable functions

Problem 1 (2 points)

Let $f(x)$ be a function of one argument. Let the n-th iteration of f be defined as

$$f^n(x) = f(...f(x)...). \tag{1}$$

For example, $f^0(x) = f(x) = x$, $f^1(x) = f(x)$, $f^2(x) = f(f(x))$, $f^3(x) = f(f(f(x)))$, etc. Let $i_f(n, x) = f^n(x)$. Show that if f is primitive recursive, then so is $i_f(n, x)$.

Problem 2 (2 points)

Let `COMP` be the class of functions obtained from the initial functions by a finite sequence of compositions.

1. Show that for every function $f(x_1, ..., x_n) \in \texttt{COMP}$, either $f(x_1, ..., x_n) = k$, for some constant k, or $f(x_1, ..., x_n) = x_i + k$, for $1 \leq i \leq n$ and some constant k.
2. An n-ary function f is *monotone* if for all n-tuples $(x_1, ..., x_n)$, $(y_1, ..., y_n)$ such that $x_i \leq y_i$, $1 \leq i \leq n$, $f(x_1, ..., x_n) \leq f(y_1, ..., y_n)$. Show that every function in `COMP` is monotone.

Problem 3 (1 point)

Let $\pi(x)$ be the number of primes that are $\leq x$. Show that $\pi(x)$ is primitive recursive.

What to Submit?

Save your solutions in hw10.pdf and submit it in Canvas.