# CS 5000: Theory of Computability
# Assignment 11

Vladimir Kulyukin
Department of Computer Science
Utah State University

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## 1 Learning Objectives

1. Primitive recursive functions
2. Computable functions
3. Gödel numbers

## Problem 1 (1 point)

Let $h(x)$ be the integer $n$ such that $n \leq \sqrt{2} < n + 1$. Show that $h(x)$ is primitive recursive.

## Problem 2 (1 point)

Let $gcd(x, y)$ and $lcm(x, y)$ be the greatest common divisor and least common multiple of $x$ and $y$, respectively. Show that both are primitive recursive.

## Problem 3 (1 point)

Is there a computable predicate $P(x_1, ..., x_n, y)$ such that the function $min_y P(x_1, ..., x_n, y)$ is not computable? If not, explain why not. If yes, give such a predicate.

## Problem 4 (1 point)

Let $f(x_1, ..., x_n)$ be a function of $n$ variables. Let $f'(x)$ be defined as follows: $f'([x_1, ..., x_n]) = f(x_1, ..., x_n)$ for all $x_1, ..., x_n$. Show that $f'$ is partially computable if and only if $f$ is partially computable.

## Problem 5 (1 point)

Let $Sort([x_1, ..., x_n]) = [y_1, ..., y_n]$, where $y_1, ..., y_n$ is a permutation of $x_1, ..., x_n$ such that $y_1 \leq y_2 \leq ... \leq y_n$. Show that $Sort(x)$ is primitive recursive.

## What to Submit?

Save your solutions in hw11.pdf and submit it in Canvas.