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Algorithms Lab

Exercise – *GoldenEye*

The Janus syndicate has gained control over the GoldenEye satellite. They can program the satellite to send electromagnetic pulses that wreak havoc anywhere on the planet. As a countermeasure, MI6 has installed jammers that provide umbrellas of protection from such pulses within a certain radius. A directive has been given that all agents must operate under the umbrella of protection at any time. However, it turned out quickly that the directive is problematic: (1) an agent may not be able to reach the target of her/his mission under the umbrella and (2) the jammers need so much energy to operate that the collective energy costs stretch the budget inacceptibly. Research is ongoing so as to (1) estimate how much energy would be needed to allow all missions to be executed and (2) how much energy could be saved without further restricting operational capacities.

The jammers $j_0,\ldots,j_{n-1}\in\mathbb{R}^2$ are given along with an initial power consumption. For a given power consumption $\omega\geq 0$, a position $q\in\mathbb{R}^2$ is safe under the ω -umbrella provided by j_0,\ldots,j_{n-1} , if there is an $i\in\{0,\ldots,n-1\}$ so that $||q-j_i||\leq \sqrt{\omega/4}$.

A *mission* is specified by two positions s and t so that an agent has to move from s to t. During such a movement we do not want the agent to leave the umbrella of protection. More formally, a movement from s to t is a continuous map $\gamma:[0,1]\to\mathbb{R}^2$ with $\gamma(0)=s$ and $\gamma(1)=t$. A movement γ is safe under the ω -umbrella provided by j_0,\ldots,j_{n-1} , if $\gamma(x)$ is safe under the ω -umbrella provided by j_0,\ldots,j_{n-1} , for all $x\in[0,1]$.

Input The first line of the input contains the number $t \le 30$ of test cases. Each of the t test cases is described as follows.

- It starts with a line that contains three integers n m p, separated by a space. They denote
 - n, the number of jammers $(1 \le n \le 3 \cdot 10^4)$;
 - m, the number of missions $(1 \le m \le 3 \cdot 10^4)$;
 - p, the initial power consumption (0 < p < 2^{53}).
- The following n lines define the individual positions j_0, \ldots, j_{n-1} of the jammers. Each position is described by two integer coordinates x y, separated by a space and such that $|x|, |y| < 2^{24}$. You may assume that these positions are pairwise distinct.
- The following m lines define the missions $(s_0,t_0),\ldots,(s_{m-1},t_{m-1})$. Each pair (s_i,t_i) , for $i\in\{0,\ldots,m-1\}$, is described by four integer coordinates $\mathsf{x}_0\ \mathsf{y}_0\ \mathsf{x}_1\ \mathsf{y}_1$, separated by a space and such that $|x_0|,|y_0|,|x_1|,|y_1|<2^{24}$.

mmers positions

Output For each test case output three lines.

- The first line contains a string $c_0c_1 \dots c_{m-1}$ of m characters, where $c_i = y$, if there exists a valid safe movement from s_i to t_i under the p-umbrella provided by j_0, \dots, j_{n-1} , and $c_i = n$, otherwise.
- The second line consists of a single integer a that denotes the smallest power consumption that allows to execute all given missions. More precisely, a is the smallest integer such that there exists a valid safe movement from s_i to t_i under the a-umbrella provided by j_0, \ldots, j_{n-1} , for all $i \in \{0, \ldots, m-1\}$.

the initial power comsumption p allows us to execute some missions M. we want to lower power consumption to b <= p such that we can execute the same missions with the smaller power consumption b The third line consists of a single integer b that denotes the smallest power consumption that allows to execute the same set of missions as possible at power consumption p. More precisely, b is the smallest integer such that if there exists a valid safe movement from s_i to t_i under the p-umbrella provided by j_0, \ldots, j_{n-1} , then there exists a valid safe movement from s_i to t_i under the b-umbrella provided by j_0, \ldots, j_{n-1} .

Points There are five groups of test sets. Each is worth 20 points, which amounts to 100 points in total.

- 1. For the first group of test sets, you may assume that there are no more than $8 \cdot 10^2$ jammers, no more than 80 missions, and that you already know the values of a and b ($n \le 8 \cdot 10^2$, $m \le 80$, b = p, and a = 4p).
- 2. For the second group of test sets, you may assume that there are no more than 10^3 jammers and no more than 10^3 missions ($n \le 10^3$ and $m \le 10^3$).
- 3. For the third group of test sets, you may assume that there are no more than 80 missions $(m \le 80)$.
- 4. For the fourth and fifth group of test sets there are no additional assumptions.
- 5. The fifth group of test sets is hidden.

Corresponding sample test sets are contained in test i. in/out, for $i \in \{1, 2, 3, 4\}$.

Sample Input	Sample Output
2	У
2 1 7	1
0 0	1
1 0	yn
0 0 1 0	64
2 2 12	9
0 0	
3 0	
-1 0 4 0	
-4 0 0 0	