In [1]:
```
import urllib
from urllib.request import urlopen
from bs4 import BeautifulSoup
import requests
import re
from transformers import BartTokenizer, BartForConditionalGeneration
from transformers import T5Tokenizer, T5ForConditionalGeneration, T5Config
import torch
import os, sys
import subprocess
from pptx import Presentation
from math import floor
import time
import numpy as np
import pandas as pd
from PIL import Image
import matplotlib.pyplot as plt
import cv2
from pptx.util import Inches

def geturl(var):
    url_source = urlopen(var).read()
    soup = BeautifulSoup(url_source,'lxml')

    var1= soup.find_all("a",class_="image")
    i=""
    for var in var1:
      i=i+str(var)

    img=[]

    for j in range(len(i)):
      if(j<(len(i)-3)):
        if(i[j]=='c' and i[j+1]=='=' and i[j+2]=='"'):
          h=""
          for k in range(j+3,len(i)):
            if(i[k]=='"'):
                break
            h=h+i[k]
          # if 'gif' not in h:
          img.append(h)
    des=[]
    mydivs = soup.find_all("div", {"class": "thumbinner"})
    for i in mydivs:
      des.append(i.text)
    return img,des

def getnarray(var):

    with urllib.request.urlopen(var) as url:
        s = url.read()

    arr = np.asarray(bytearray(s), dtype=np.uint8)
    img = cv2.imdecode(arr, -1)

    return img


def joinimg(img,text):
    cv2.imwrite('i1.png', img)
    n = len(text)
#     img = cv2.imread('/Users/mayank.bumb@zomato.com/Downloads/add.png')
```

```
    print(img.shape)
    (width, height, dim) = img.shape
    flag = height // 10
    s = text.split()
    down = 0
    count = 0
    li = []
    temp = ''
    for i in s:
        count += len(i)
        if count > flag:
            count = len(i)
            down += 1
            li.append(temp)
            temp = ''
        temp = temp + i + ' '
        count += 1
    down += 1
    li.append(temp)
    print(li)
    down = n // flag + 1
    print(width, height)
    temp = np.ones((10 + down * 20, height, 3)) * 255
    print(temp.shape)

    if dim == 4:
        b,g,r, a = cv2.split(img)
#         print(img.shape)

        new_img  = cv2.merge((b, g, r))
        not_a = cv2.bitwise_not(a)
        not_a = cv2.cvtColor(not_a, cv2.COLOR_GRAY2BGR)
#         plt.imshow(not_a)
#         plt.show()
        new_img = cv2.bitwise_and(new_img,new_img,mask = a)
        img = cv2.add(new_img, not_a)

#         cv2.imwrite(output_dir, new_img)
#         plt.imshow(new_img)
#         print(new_img.shape)
#         img = cv2.cvtColor(img, cv2.COLOR_RGBA2RGB)
#         img = new_img
#         temp = np.zeros((10 + down * 20, height, 4))
#         img = np.resize(img, (width, height, 3))
    img1 = np.concatenate((img, temp), axis = 0)
    cv2.imwrite('i2.jpg', img1)
#     img1 = np.c_[img,temp]
    print(img1.shape)
    font = cv2.FONT_HERSHEY_SIMPLEX

    # org
    org = (0, width + 20)

    # fontScale
    fontScale = 0.6

    # Blue color in BGR
    color = (0, 0, 0)

    # Line thickness of 2 px
    thickness = 2
    count = 1
    for i in li:
        org = (0, width + count * 20)
```

```
    # Using cv2.putText() method
        image = cv2.putText(img1, i, org, font,
                       fontScale, color, thickness, cv2.LINE_AA)
        count += 1


    cv2.imwrite('i.png', image)
    return(image)




print("Input:-")
#Taking the input from the string
#t=input()

# p=input()
p = input()
INPUT = p
t=""
for i in range(len(p)):
  if(p[i]==' '):
    t=t+"_"
  else:
    t=t+p[i]

var='https://en.wikipedia.org/wiki/'+t
# Specifying the url of the web page
url_source = urlopen(var).read()

imlist,urlist = geturl(var)
count = 0
imlist1 = []
urlist1 = []
print(len(imlist), len(urlist))
for i in imlist:
  if 'gif' not in i:
    imlist1.append(imlist[count])
    try:
      urlist1.append(urlist[count])
    except:
      urlist1.append('')
  count += 1
urlist = urlist1[0:8]
imlist = imlist1[0:8]
print(urlist)
num_img = len(imlist)
print(num_img)
# Using beautifulsoup library to get the html file from the url
soup = BeautifulSoup(url_source,'lxml')
soup

# Extracting the text content from the paragraphs by finding the p tag
'''
spans=soup.find_all('span',attrs={'mw-headline'})
raj=[]
raj.append("Introduction")
for span in spans:
  raj.append(span.text)
['p','span.mw-headline']
'''
'''
para = []
for paragraph in soup.find_all(['span',attrs={'mw-headline'},'p']):
```

```
    para.append(str(paragraph.text))
'''
'''
for header in soup.find_all(['h1', 'h2', 'h3']):
  print(header.get_text())
  for elem in header.next_siblings:
    if elem.name and elem.name.startswith('h'):
      break
    if elem.name == 'p':
      print(elem.get_text())
'''


#print(paras)
para=[]
para2=[]
for header in soup.find_all(['h1', 'h2','p']):
  para.append(header.get_text())
for header in soup.find_all('p'):
  para2.append(header.get_text())
l=[]
for i in soup.find_all(['h1','h2']):
  l.append(i.get_text())
print(l)
para.pop(0)


for i in range(len(l)):
  r=l[i]
  value2=""
  for x in range(len(r)):
    if(r[x]=='['):
      break
    else:
      value2=value2+r[x]
  l[i]=value2
l.pop(0)
l.pop(0)
print(l)
# Adding all pragraph in a single sring for further use

final="Introduction"
final2="Introduction"
for val in para:
  final=final+" "+str(val)
final = re.sub(r"\[.*?\]+", '',final)
final = final.replace('\n', '')
print("Output:-")
print(final)
for val in para2:
  final2=final2+" "+str(val)
final2 = re.sub(r"\[.*?\]+", '',final2)
final2 = final2.replace('\n', '')
print(final2)
fun=[]
st=""
j=0
odd=0
print(len(final))
print(len(final2))
for i in range(len(final)):
  if(j>=len(final2)):
    break
  if(final[i]==final2[j]):
```

```
    st=st+final[i]
    j=j+1
    odd=0
  else:
    if(odd==0):
      fun.append(st)
      st=""
      st=st+final[i]
      odd=1
    else:
      st=st+final[i]
fun.append(st)
# print(len(fun))
for i in fun:
  print('-'*50)
  print(i)
fun2=[]
last=0
l.insert(0,'Introduction')
fun3=[]
for i in fun:
  z1=i
  y1=len(l[last])
  last=last+1
  y1=y1+1
  if(last==2):
    y1=y1+8
  z2=z1[y1:]
  fun3.append(z2)
last=0
for i in fun3:
  fun2.append(l[last])
  last=last+1
  fun2.append(i)
it=iter(fun2)
result = dict(zip(it, it))

model = BartForConditionalGeneration.from_pretrained("facebook/bart-large-c
tokenizer = BartTokenizer.from_pretrained("facebook/bart-large-cnn")
# model = BartForConditionalGeneration.from_pretrained("facebook/bart-large-
# tokenizer = BartTokenizer.from_pretrained("facebook/bart-large-cnn")

count = 0
final_output = {}
for i in result:
  # i = 'Addition of numbers'
  ARTICLE_TO_SUMMARIZE = result[i][0:min(len(result[i]), 3500)]
  # s = ARTICLE_TO_SUMMARIZE.split()
  # print(len(s))
  print(i, len(ARTICLE_TO_SUMMARIZE))
  # print(len(ARTICLE_TO_SUMMARIZE))
  inputs = tokenizer([ARTICLE_TO_SUMMARIZE], max_length=len(ARTICLE_TO_SUMM

  # Generate Summary
  summary_ids = model.generate(inputs["input_ids"], num_beams=4, max_length=
  ans = tokenizer.batch_decode(summary_ids, skip_special_tokens=True, clean_
  final_output[i] = ans[0]
  count += 1
  if count > 6:
    break

length = 8
copy1 = final_output
design = int(input("Choose a template "))
```

```
if(design == 1):
    prs = Presentation(r"C:\Users\amaji\Desktop\FYPtemplates\1.pptx")
elif(design == 2):
    prs = Presentation(r"C:\Users\amaji\Desktop\FYPtemplates\2.pptx")
elif(design == 3):
    prs = Presentation(r"C:\Users\amaji\Desktop\FYPtemplates\3.pptx")
elif(design == 4):
    prs = Presentation(r"C:\Users\amaji\Desktop\FYPtemplates\4.pptx")
elif(design == 5):
    prs = Presentation(r"C:\Users\amaji\Desktop\FYPtemplates\5.pptx")
elif(design == 6):
    prs = Presentation(r"C:\Users\amaji\Desktop\FYPtemplates\6.pptx")
elif(design == 7):
    prs = Presentation(r"C:\Users\amaji\Desktop\FYPtemplates\7.pptx")
elif(design == 8):
    prs = Presentation(r"C:\Users\amaji\Desktop\FYPtemplates\8.pptx")

num = 3

#Title Page Design
slide_layout = prs.slide_layouts[0]
slide1 = prs.slides.add_slide(slide_layout)
title = slide1.shapes.title
title.text = p

titles_for_slide = []
for x in copy1:
    titles_for_slide.append(x)

nch = min(8, num_img, len(titles_for_slide))
for i in range(nch-1):
    imarray = getnarray("http:"+imlist[i])
    im = Image.fromarray(imarray)
    newsize = (700,700)
    im = im.resize(newsize)
#     im = im.convert('RGB')
    im.save(r"C:\Users\amaji\Desktop\im_file.png")
#     htmlim = joinimg(imarray,urlist[i])
    slide = prs.slides.add_slide(prs.slide_layouts[3])
    shapes = slide.shapes

    title_shape = shapes.title
    body_shape = shapes.placeholders[1]

    title_shape.text = titles_for_slide[i]

    img_path = r'C:\Users\amaji\Desktop\im_file.png'

    left = Inches(8)
    top = Inches(2)
    height = Inches(3)
    width = Inches(4)
    pic = slide.shapes.add_picture(img_path, left, top, height=height)

    left = Inches(8)
    top = Inches(5)
    width = Inches(4)
    height = Inches(2)
    txBox = slide.shapes.add_textbox(left, top, width, height)
    tf = txBox.text_frame
    tf.word_wrap = True

    p = tf.add_paragraph()
```

```
    p.text = urlist[i]

    collect = copy1[titles_for_slide[i]].split('.')
    collect = list(filter(None, collect))
    count_of_sentences = len(collect)
    count = 0

    text = urlist[i]

    for y in range(num):
        if(count == count_of_sentences):
            flag = 0
            break
        else:
            print("here")
            tf = body_shape.text_frame
            p = tf.add_paragraph()
            p.text = collect[count]
#             tf.text = collect[count]
            print(collect[count])
            count = count + 1


slide = prs.slides.add_slide(prs.slide_layouts[3])
shapes = slide.shapes
title_shape = shapes.title
body_shape = shapes.placeholders[1]
title_shape.text = "Sources"
tf = body_shape.text_frame
p = tf.add_paragraph()
p.text = str(var)
xml_slides = prs.slides._sldIdLst
slides = list(xml_slides)
xml_slides.remove(slides[0])
prs.save(r"C:\Users\amaji\Desktop\majid5.pptx")
print(r"PPT Saved at C:\Users\amaji\Desktop")
```

```
Input:-
Addition
18 18
[' 3 + 2 = 5 with apples, a popular choice in textbooks[1]', ' The plus sig
n', ' Columnar addition – the numbers in the column are to be added, with t
he sum written below the underlined number.', ' Redrawn illustration from T
he Art of Nombryng, one of the first English arithmetic texts, in the 15th
century.[12]', ' ', ' A number-line visualization of the algebraic addition
2 + 4 = 6. A translation by 2 followed by a translation by 4 is the same as
a translation by 6.', ' A number-line visualization of the unary addition 2
+ 4 = 6. A translation by 4 is equivalent to four translations by 1.', ' 4
+ 2 = 2 + 4 with blocks']
8
['Addition', 'Contents', 'Notation and terminology[edit]', 'Interpretations
[edit]', 'Properties[edit]', 'Performing addition[edit]', 'Addition of numb
ers[edit]', 'Generalizations[edit]', 'Related operations[edit]', 'In music
[edit]', 'See also[edit]', 'Notes[edit]', 'Footnotes[edit]', 'References[ed
it]', 'Further reading[edit]', 'Navigation menu']
['Notation and terminology', 'Interpretations', 'Properties', 'Performing a
ddition', 'Addition of numbers', 'Generalizations', 'Related operations',
'In music', 'See also', 'Notes', 'Footnotes', 'References', 'Further readin
g', 'Navigation menu']
Output:-
Introduction  Addition (usually signified by the plus symbol +) is one of t
he four basic operations of arithmetic, the other three being subtraction,
multiplication and division. The addition of two whole numbers results in t
he total amount or sum of those values combined. The example in the adjacen
t image shows a combination of three apples and two apples, making a total
of five apples. This observation is equivalent to the mathematical expressi
on "3 + 2 = 5" (that is, "3 plus 2 is equal to 5"). Besides counting items,
addition can also be defined and executed without referring to concrete obj
ects, using abstractions called numbers instead, such as integers, real num
bers and complex numbers. Addition belongs to arithmetic, a branch of mathe
matics. In algebra, another area of mathematics, addition can also be perfo
rmed on abstract objects such as vectors, matrices, subspaces and subgroup
s. Addition has several important properties. It is commutative, meaning th
at order does not matter, and it is associative, meaning that when one adds
more than two numbers, the order in which addition is performed does not ma
tter (see Summation). Repeated addition of 1 is the same as counting. Addit
ion of 0 does not change a number. Addition also obeys predictable rules co
ncerning related operations such as subtraction and multiplication. Perform
ing addition is one of the simplest numerical tasks. Addition of very small
numbers is accessible to toddlers; the most basic task, 1 + 1, can be perfo
rmed by infants as young as five months, and even some members of other ani
mal species. In primary education, students are taught to add numbers in th
e decimal system, starting with single digits and progressively tackling mo
re difficult problems. Mechanical aids range from the ancient abacus to the
modern computer, where research on the most efficient implementations of ad
dition continues to this day. Contents Notation and terminology Addition is
written using the plus sign "+" between the terms; that is, in infix notati
on. The result is expressed with an equals sign. For example, There are als
o situations where addition is "understood", even though no symbol appears:
The sum of a series of related numbers can be expressed through capital sig
ma notation, which compactly denotes iteration. For example, The numbers or
the objects to be added in general addition are collectively referred to as
the terms, the addends or the summands;this terminology carries over to the
summation of multiple terms.This is to be distinguished from factors, which
are multiplied.Some authors call the first addend the augend. In fact, duri
ng the Renaissance, many authors did not consider the first addend an "adde
nd" at all. Today, due to the commutative property of addition, "augend" is
rarely used, and both terms are generally called addends. All of the above
terminology derives from Latin. "Addition" and "add" are English words deri
ved from the Latin verb addere, which is in turn a compound of ad "to" and
dare "to give", from the Proto-Indo-European root *deh₃- "to give"; thus to
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add is to give to. Using the gerundive suffix -nd results in "addend", "thi
ng to be added". Likewise from augere "to increase", one gets "augend", "th
ing to be increased". "Sum" and "summand" derive from the Latin noun summa
"the highest, the top" and associated verb summare. This is appropriate not
only because the sum of two positive numbers is greater than either, but be
cause it was common for the ancient Greeks and Romans to add upward, contra
ry to the modern practice of adding downward, so that a sum was literally h
igher than the addends.Addere and summare date back at least to Boethius, i
f not to earlier Roman writers such as Vitruvius and Frontinus; Boethius al
so used several other terms for the addition operation. The later Middle En
glish terms "adden" and "adding" were popularized by Chaucer. The plus sign
"+" (Unicode:U+002B; ASCII: +) is an abbreviation of the Latin word et,
meaning "and". It appears in mathematical works dating back to at least 148
9. Interpretations Addition is used to model many physical processes. Even
for the simple case of adding natural numbers, there are many possible inte
rpretations and even more visual representations. Possibly the most fundame
ntal interpretation of addition lies in combining sets: This interpretation
is easy to visualize, with little danger of ambiguity. It is also useful in
higher mathematics (for the rigorous definition it inspires, see § Natural
numbers below). However, it is not obvious how one should extend this versi
on of addition to include fractional numbers or negative numbers. One possi
ble fix is to consider collections of objects that can be easily divided, s
uch as pies or, still better, segmented rods. Rather than solely combining
collections of segments, rods can be joined end-to-end, which illustrates a
nother conception of addition: adding not the rods but the lengths of the r
ods. A second interpretation of addition comes from extending an initial le
ngth by a given length: The sum a + b can be interpreted as a binary operat
ion that combines a and b, in an algebraic sense, or it can be interpreted
as the addition of b more units to a. Under the latter interpretation, the
parts of a sum a + b play asymmetric roles, and the operation a + b is view
ed as applying the unary operation +b to a. Instead of calling both a and b
addends, it is more appropriate to call a the augend in this case, since a
plays a passive role. The unary view is also useful when discussing subtrac
tion, because each unary addition operation has an inverse unary subtractio
n operation, and vice versa. Properties Addition is commutative, meaning th
at one can change the order of the terms in a sum, but still get the same r
esult. Symbolically, if a and b are any two numbers, then The fact that add
ition is commutative is known as the "commutative law of addition" or "comm
utative property of addition". Some other binary operations are commutativ
e, such as multiplication, but many others are not, such as subtraction and
division. Addition is associative, which means that when three or more numb
ers are added together, the order of operations does not change the result.
As an example, should the expression a + b + c be defined to mean (a + b) +
c or a + (b + c)? Given that addition is associative, the choice of definit
ion is irrelevant. For any three numbers a, b, and c, it is true that (a +
b) + c = a + (b + c). For example, (1 + 2) + 3 = 3 + 3 = 6 = 1 + 5 = 1 + (2
+ 3). When addition is used together with other operations, the order of op
erations becomes important. In the standard order of operations, addition i
s a lower priority than exponentiation, nth roots, multiplication and divis
ion, but is given equal priority to subtraction. Adding zero to any number,
does not change the number; this means that zero is the identity element fo
r addition, and is also known as the additive identity. In symbols, for eve
ry a, one has This law was first identified in Brahmagupta's Brahmasphutasi
ddhanta in 628 AD, although he wrote it as three separate laws, depending o
n whether a is negative, positive, or zero itself, and he used words rather
than algebraic symbols. Later Indian mathematicians refined the concept; ar
ound the year 830, Mahavira wrote, "zero becomes the same as what is added
to it", corresponding to the unary statement 0 + a = a. In the 12th centur
y, Bhaskara wrote, "In the addition of cipher, or subtraction of it, the qu
antity, positive or negative, remains the same", corresponding to the unary
statement a + 0 = a. Within the context of integers, addition of one also p
lays a special role: for any integer a, the integer (a + 1) is the least in
teger greater than a, also known as the successor of a. For instance, 3 is
the successor of 2 and 7 is the successor of 6. Because of this succession,

the value of a + b can also be seen as the bth successor of a, making addit ion iterated succession. For example, 6 + 2 is 8, because 8 is the successo r of 7, which is the successor of 6, making 8 the 2nd successor of 6. To nu merically add physical quantities with units, they must be expressed with c ommon units. For example, adding 50 milliliters to 150 milliliters gives 20 0 milliliters. However, if a measure of 5 feet is extended by 2 inches, the sum is 62 inches, since 60 inches is synonymous with 5 feet. On the other h and, it is usually meaningless to try to add 3 meters and 4 square meters, since those units are incomparable; this sort of consideration is fundament al in dimensional analysis. Performing addition Studies on mathematical dev elopment starting around the 1980s have exploited the phenomenon of habitua tion: infants look longer at situations that are unexpected. A seminal expe riment by Karen Wynn in 1992 involving Mickey Mouse dolls manipulated behin d a screen demonstrated that five-month-old infants expect 1 + 1 to be 2, a nd they are comparatively surprised when a physical situation seems to impl y that 1 + 1 is either 1 or 3. This finding has since been affirmed by a va riety of laboratories using different methodologies. Another 1992 experimen t with older toddlers, between 18 and 35 months, exploited their developmen t of motor control by allowing them to retrieve ping-pong balls from a box; the youngest responded well for small numbers, while older subjects were ab le to compute sums up to 5. Even some nonhuman animals show a limited abili ty to add, particularly primates. In a 1995 experiment imitating Wynn's 199 2 result (but using eggplants instead of dolls), rhesus macaque and cottont op tamarin monkeys performed similarly to human infants. More dramatically, after being taught the meanings of the Arabic numerals 0 through 4, one chi mpanzee was able to compute the sum of two numerals without further trainin g. More recently, Asian elephants have demonstrated an ability to perform b asic arithmetic. Typically, children first master counting. When given a pr oblem that requires that two items and three items be combined, young child ren model the situation with physical objects, often fingers or a drawing, and then count the total. As they gain experience, they learn or discover t he strategy of "counting-on": asked to find two plus three, children count three past two, saying "three, four, five" (usually ticking off fingers), a nd arriving at five. This strategy seems almost universal; children can eas ily pick it up from peers or teachers. Most discover it independently. With additional experience, children learn to add more quickly by exploiting the commutativity of addition by counting up from the larger number, in this ca se, starting with three and counting "four, five." Eventually children begi n to recall certain addition facts ("number bonds"), either through experie nce or rote memorization. Once some facts are committed to memory, children begin to derive unknown facts from known ones. For example, a child asked t o add six and seven may know that 6 + 6 = 12 and then reason that 6 + 7 is one more, or 13. Such derived facts can be found very quickly and most elem entary school students eventually rely on a mixture of memorized and derive d facts to add fluently. Different nations introduce whole numbers and arit hmetic at different ages, with many countries teaching addition in pre-scho ol. However, throughout the world, addition is taught by the end of the fir st year of elementary school. Children are often presented with the additio n table of pairs of numbers from 0 to 9 to memorize. Knowing this, children can perform any addition. The prerequisite to addition in the decimal syste m is the fluent recall or derivation of the 100 single-digit "addition fact s". One could memorize all the facts by rote, but pattern-based strategies are more enlightening and, for most people, more efficient: As students gro w older, they commit more facts to memory, and learn to derive other facts rapidly and fluently. Many students never commit all the facts to memory, b ut can still find any basic fact quickly. The standard algorithm for adding multidigit numbers is to align the addends vertically and add the columns, starting from the ones column on the right. If a column exceeds nine, the e xtra digit is "carried" into the next column. For example, in the addition 27 + 59 7 + 9 = 16, and the digit 1 is the carry. An alternate strategy sta rts adding from the most significant digit on the left; this route makes ca rrying a little clumsier, but it is faster at getting a rough estimate of t he sum. There are many alternative methods. Since the end of the XXth centu ry, some US programs, including TERC, decided to remove the traditional tra nsfer method from their curriculum. This decision was criticized that is why some states and counties didn’t support this experiment. Decimal fractions can be added by a simple modification of the above process. One aligns two decimal fractions above each other, with the decimal point in the same location. If necessary, one can add trailing zeros to a shorter decimal to make it the same length as the longer decimal. Finally, one performs the same addition process as above, except the decimal point is placed in the answer, exactly where it was placed in the summands. As an example, 45.1 + 4.34 can be solved as follows: In scientific notation, numbers are written in the form x=a×10b{\displaystyle x=a\times 10^{b}}, where a{\displaystyle a} is the significand and 10b{\displaystyle 10^{b}} is the exponential part. Addition requires two numbers in scientific notation to be represented using the same exponential part, so that the two significands can simply be added. For example: Addition in other bases is very similar to decimal addition. As an example, one can consider addition in binary. Adding two single-digit binary numbers is relatively simple, using a form of carrying: Adding two "1" digits produces a digit "0", while 1 must be added to the next column. This is similar to what happens in decimal when certain single-digit numbers are added together; if the result equals or exceeds the value of the radix (10), the digit to the left is incremented: This is known as carrying. When the result of an addition exceeds the value of a digit, the procedure is to "carry" the excess amount divided by the radix (that is, 10/10) to the left, adding it to the next positional value. This is correct since the next position has a weight that is higher by a factor equal to the radix. Carrying works the same way in binary: In this example, two numerals are being added together: 011012 (1310) and 101112 (2310). The top row shows the carry bits used. Starting in the rightmost column, 1 + 1 = 102. The 1 is carried to the left, and the 0 is written at the bottom of the rightmost column. The second column from the right is added: 1 + 0 + 1 = 102 again; the 1 is carried, and 0 is written at the bottom. The third column: 1 + 1 + 1 = 112. This time, a 1 is carried, and a 1 is written in the bottom row. Proceeding like this gives the final answer 1001002 (3610). Analog computers work directly with physical quantities, so their addition mechanisms depend on the form of the addends. A mechanical adder might represent two addends as the positions of sliding blocks, in which case they can be added with an averaging lever. If the addends are the rotation speeds of two shafts, they can be added with a differential. A hydraulic adder can add the pressures in two chambers by exploiting Newton's second law to balance forces on an assembly of pistons. The most common situation for a general-purpose analog computer is to add two voltages (referenced to ground); this can be accomplished roughly with a resistor network, but a better design exploits an operational amplifier. Addition is also fundamental to the operation of digital computers, where the efficiency of addition, in particular the carry mechanism, is an important limitation to overall performance. The abacus, also called a counting frame, is a calculating tool that was in use centuries before the adoption of the written modern numeral system and is still widely used by merchants, traders and clerks in Asia, Africa, and elsewhere; it dates back to at least 2700–2300 BC, when it was used in Sumer. Blaise Pascal invented the mechanical calculator in 1642; it was the first operational adding machine. It made use of a gravity-assisted carry mechanism. It was the only operational mechanical calculator in the 17th century and the earliest automatic, digital computer. Pascal's calculator was limited by its carry mechanism, which forced its wheels to only turn one way so it could add. To subtract, the operator had to use the Pascal's calculator's complement, which required as many steps as an addition. Giovanni Poleni followed Pascal, building the second functional mechanical calculator in 1709, a calculating clock made of wood that, once setup, could multiply two numbers automatically. Adders execute integer addition in electronic digital computers, usually using binary arithmetic. The simplest architecture is the ripple carry adder, which follows the standard multi-digit algorithm. One slight improvement is the carry skip design, again following human intuition; one does not perform all the carries in computing 999 + 1, but one bypasses the group of 9s and skips to the answer. In practice, computational addition may be achieved via XOR and AND bitwise logical operations in conjunction with bitshift ope rations as shown in the pseudocode below. Both XOR and AND gates are straightforward to realize in digital logic allowing the realization of full adder circuits which in turn may be combined into more complex logical operations. In modern digital computers, integer addition is typically the fastest arithmetic instruction, yet it has the largest impact on performance, since it underlies all floating-point operations as well as such basic tasks as address generation during memory access and fetching instructions during branching. To increase speed, modern designs calculate digits in parallel; these schemes go by such names as carry select, carry lookahead, and the Ling pseudocarry. Many implementations are, in fact, hybrids of these last three designs. Unlike addition on paper, addition on a computer often changes the addends. On the ancient abacus and adding board, both addends are destroyed, leaving only the sum. The influence of the abacus on mathematical thinking was strong enough that early Latin texts often claimed that in the process of adding "a number to a number", both numbers vanish. In modern times, the ADD instruction of a microprocessor often replaces the augend with the sum but preserves the addend. In a high-level programming language, evaluating a + b does not change either a or b; if the goal is to replace a with the sum this must be explicitly requested, typically with the statement a = a + b. Some languages such as C or C++ allow this to be abbreviated as a += b. On a computer, if the result of an addition is too large to store, an arithmetic overflow occurs, resulting in an incorrect answer. Unanticipated arithmetic overflow is a fairly common cause of program errors. Such overflow bugs may be hard to discover and diagnose because they may manifest themselves only for very large input data sets, which are less likely to be used in validation tests. The Year 2000 problem was a series of bugs where overflow errors occurred due to use of a 2-digit format for years. Addition of numbers To prove the usual properties of addition, one must first define addition for the context in question. Addition is first defined on the natural numbers. In set theory, addition is then extended to progressively larger sets that include the natural numbers: the integers, the rational numbers, and the real numbers. (In mathematics education, positive fractions are added before negative numbers are even considered; this is also the historical route.) There are two popular ways to define the sum of two natural numbers a and b. If one defines natural numbers to be the cardinalities of finite sets, (the cardinality of a set is the number of elements in the set), then it is appropriate to define their sum as follows: Here, A ∪ B is the union of A and B. An alternate version of this definition allows A and B to possibly overlap and then takes their disjoint union, a mechanism that allows common elements to be separated out and therefore counted twice. The other popular definition is recursive: Again, there are minor variations upon this definition in the literature. Taken literally, the above definition is an application of the recursion theorem on the partially ordered set N2. On the other hand, some sources prefer to use a restricted recursion theorem that applies only to the set of natural numbers. One then considers a to be temporarily "fixed", applies recursion on b to define a function "a +", and pastes these unary operations for all a together to form the full binary operation. This recursive formulation of addition was developed by Dedekind as early as 1854, and he would expand upon it in the following decades. He proved the associative and commutative properties, among others, through mathematical induction. The simplest conception of an integer is that it consists of an absolute value (which is a natural number) and a sign (generally either positive or negative). The integer zero is a special third case, being neither positive nor negative. The corresponding definition of addition must proceed by cases: Although this definition can be useful for concrete problems, the number of cases to consider complicates proofs unnecessarily. So the following method is commonly used for defining integers. It is based on the remark that every integer is the difference of two natural integers and that two such differences, a – b and c – d are equal if and only if a + d = b + c.So, one can define formally the integers as the equivalence classes of ordered pairs of natural numbers under the equivalence relation The equivalence class of (a, b) contains either (a – b, 0) if a ≥ b, or (0, b – a) otherwise. If n is a natural number, one can denote +n the equivalence class of (n, 0), and by –n the equivalence class of (0, n). This allows identifyi

ng the natural number n with the equivalence class +n. Addition of ordered
pairs is done component-wise: A straightforward computation shows that the
equivalence class of the result depends only on the equivalences classes of
the summands, and thus that this defines an addition of equivalence classe
s, that is integers. Another straightforward computation shows that this ad
dition is the same as the above case definition. This way of defining integ
ers as equivalence classes of pairs of natural numbers, can be used to embe
d into a group any commutative semigroup with cancellation property. Here,
the semigroup is formed by the natural numbers and the group is the additiv
e group of integers. The rational numbers are constructed similarly, by tak
ing as semigroup the nonzero integers with multiplication. This constructio
n has been also generalized under the name of Grothendieck group to the cas
e of any commutative semigroup. Without the cancellation property the semig
roup homomorphism from the semigroup into the group may be non-injective. O
riginally, the Grothendieck group was, more specifically, the result of th
is construction applied to the equivalences classes under isomorphisms of t
he objects of an abelian category, with the direct sum as semigroup operati
on. Addition of rational numbers can be computed using the least common den
ominator, but a conceptually simpler definition involves only integer addit
ion and multiplication: As an example, the sum 34+18=3×8+4×14×8=24+432=2832
=78{\displaystyle {\frac {3}{4}}+{\frac {1}{8}}={\frac {3\times 8+4\times
1}{4\times 8}}={\frac {24+4}{32}}={\frac {28}{32}}={\frac {7}{8}}}. Additio
n of fractions is much simpler when the denominators are the same; in this
case, one can simply add the numerators while leaving the denominator the s
ame: ac+bc=a+bc{\displaystyle {\frac {a}{c}}+{\frac {b}{c}}={\frac {a+b}
{c}}}, so 14+24=1+24=34{\displaystyle {\frac {1}{4}}+{\frac {2}{4}}={\frac
{1+2}{4}}={\frac {3}{4}}}. The commutativity and associativity of rational
addition is an easy consequence of the laws of integer arithmetic. For a mo
re rigorous and general discussion, see field of fractions. A common constr
uction of the set of real numbers is the Dedekind completion of the set of
rational numbers. A real number is defined to be a Dedekind cut of rational
s: a non-empty set of rationals that is closed downward and has no greatest
element. The sum of real numbers a and b is defined element by element: Thi
s definition was first published, in a slightly modified form, by Richard D
edekind in 1872.The commutativity and associativity of real addition are im
mediate; defining the real number 0 to be the set of negative rationals, it
is easily seen to be the additive identity. Probably the trickiest part of
this construction pertaining to addition is the definition of additive inve
rses. Unfortunately, dealing with multiplication of Dedekind cuts is a time
-consuming case-by-case process similar to the addition of signed integers.
Another approach is the metric completion of the rational numbers. A real n
umber is essentially defined to be the limit of a Cauchy sequence of ration
als, lim an. Addition is defined term by term: This definition was first pu
blished by Georg Cantor, also in 1872, although his formalism was slightly
different.One must prove that this operation is well-defined, dealing with
co-Cauchy sequences. Once that task is done, all the properties of real add
ition follow immediately from the properties of rational numbers. Furthermo
re, the other arithmetic operations, including multiplication, have straigh
tforward, analogous definitions. Complex numbers are added by adding the re
al and imaginary parts of the summands. That is to say: Using the visualiza
tion of complex numbers in the complex plane, the addition has the followin
g geometric interpretation: the sum of two complex numbers A and B, interpr
eted as points of the complex plane, is the point X obtained by building a
parallelogram three of whose vertices are O, A and B. Equivalently, X is th
e point such that the triangles with vertices O, A, B, and X, B, A, are con
gruent. Generalizations There are many binary operations that can be viewed
as generalizations of the addition operation on the real numbers. The field
of abstract algebra is centrally concerned with such generalized operation
s, and they also appear in set theory and category theory. In linear algebr
a, a vector space is an algebraic structure that allows for adding any two
vectors and for scaling vectors. A familiar vector space is the set of all
ordered pairs of real numbers; the ordered pair (a,b) is interpreted as a v
ector from the origin in the Euclidean plane to the point (a,b) in the plan
e. The sum of two vectors is obtained by adding their individual coordinate

s: This addition operation is central to classical mechanics, in which velocities, accelerations and forces are all represented by vectors. Matrix addition is defined for two matrices of the same dimensions. The sum of two m × n (pronounced "m by n") matrices A and B, denoted by A + B, is again an m × n matrix computed by adding corresponding elements: For example: In modular arithmetic, the set of available numbers is restricted to a finite subset of the integers, and addition "wraps around" when reaching a certain value, called the modulus. For example, the set of integers modulo 12 has twelve elements; it inherits an addition operation from the integers that is central to musical set theory. The set of integers modulo 2 has just two elements; the addition operation it inherits is known in Boolean logic as the "exclusive or" function. A similar "wrap around" operation arises in geometry, where the sum of two angle measures is often taken to be their sum as real numbers modulo 2π. This amounts to an addition operation on the circle, which in turn generalizes to addition operations on many-dimensional tori. The general theory of abstract algebra allows an "addition" operation to be any associative and commutative operation on a set. Basic algebraic structures with such an addition operation include commutative monoids and abelian groups. A far-reaching generalization of addition of natural numbers is the addition of ordinal numbers and cardinal numbers in set theory. These give two different generalizations of addition of natural numbers to the transfinite. Unlike most addition operations, addition of ordinal numbers is not commutative. Addition of cardinal numbers, however, is a commutative operation closely related to the disjoint union operation. In category theory, disjoint union is seen as a particular case of the coproduct operation, and general coproducts are perhaps the most abstract of all the generalizations of addition. Some coproducts, such as direct sum and wedge sum, are named to evoke their connection with addition. Related operations Addition, along with subtraction, multiplication and division, is considered one of the basic operations and is used in elementary arithmetic. Subtraction can be thought of as a kind of addition—that is, the addition of an additive inverse. Subtraction is itself a sort of inverse to addition, in that adding x and subtracting x are inverse functions. Given a set with an addition operation, one cannot always define a corresponding subtraction operation on that set; the set of natural numbers is a simple example. On the other hand, a subtraction operation uniquely determines an addition operation, an additive inverse operation, and an additive identity; for this reason, an additive group can be described as a set that is closed under subtraction. Multiplication can be thought of as repeated addition. If a single term x appears in a sum n times, then the sum is the product of n and x. If n is not a natural number, the product may still make sense; for example, multiplication by −1 yields the additive inverse of a number. In the real and complex numbers, addition and multiplication can be interchanged by the exponential function: This identity allows multiplication to be carried out by consulting a table of logarithms and computing addition by hand; it also enables multiplication on a slide rule. The formula is still a good first-order approximation in the broad context of Lie groups, where it relates multiplication of infinitesimal group elements with addition of vectors in the associated Lie algebra. There are even more generalizations of multiplication than addition. In general, multiplication operations always distribute over addition; this requirement is formalized in the definition of a ring. In some contexts, such as the integers, distributivity over addition and the existence of a multiplicative identity is enough to uniquely determine the multiplication operation. The distributive property also provides information about addition; by expanding the product (1 + 1)(a + b) in both ways, one concludes that addition is forced to be commutative. For this reason, ring addition is commutative in general. Division is an arithmetic operation remotely related to addition. Since a/b = a(b−1), division is right distributive over addition: (a + b) / c = a/c + b/c. However, division is not left distributive over addition; 1 / (2 + 2) is not the same as 1/2 + 1/2. The maximum operation "max (a, b)" is a binary operation similar to addition. In fact, if two nonnegative numbers a and b are of different orders of magnitude, then their sum is approximately equal to their maximum. This approximation is extremely useful in the applications of mathematics, for example in truncating Taylor series.

However, it presents a perpetual difficulty in numerical analysis, essentia lly since "max" is not invertible. If b is much greater than a, then a stra ightforward calculation of (a + b) − b can accumulate an unacceptable round -off error, perhaps even returning zero. See also Loss of significance. The approximation becomes exact in a kind of infinite limit; if either a or b i s an infinite cardinal number, their cardinal sum is exactly equal to the g reater of the two. Accordingly, there is no subtraction operation for infin ite cardinals. Maximization is commutative and associative, like addition. Furthermore, since addition preserves the ordering of real numbers, additio n distributes over "max" in the same way that multiplication distributes ov er addition: For these reasons, in tropical geometry one replaces multiplic ation with addition and addition with maximization. In this context, additi on is called "tropical multiplication", maximization is called "tropical ad dition", and the tropical "additive identity" is negative infinity. Some au thors prefer to replace addition with minimization; then the additive ident ity is positive infinity. Tying these observations together, tropical addit ion is approximately related to regular addition through the logarithm: whi ch becomes more accurate as the base of the logarithm increases. The approx imation can be made exact by extracting a constant h, named by analogy with Planck's constant from quantum mechanics, and taking the "classical limit" as h tends to zero: In this sense, the maximum operation is a dequantized v ersion of addition. Incrementation, also known as the successor operation, is the addition of 1 to a number. Summation describes the addition of arbit rarily many numbers, usually more than just two. It includes the idea of th e sum of a single number, which is itself, and the empty sum, which is zer o. An infinite summation is a delicate procedure known as a series. Countin g a finite set is equivalent to summing 1 over the set. Integration is a ki nd of "summation" over a continuum, or more precisely and generally, over a differentiable manifold. Integration over a zero-dimensional manifold reduc es to summation. Linear combinations combine multiplication and summation; they are sums in which each term has a multiplier, usually a real or comple x number. Linear combinations are especially useful in contexts where strai ghtforward addition would violate some normalization rule, such as mixing o f strategies in game theory or superposition of states in quantum mechanic s. Convolution is used to add two independent random variables defined by d istribution functions. Its usual definition combines integration, subtracti on, and multiplication. In general, convolution is useful as a kind of doma in-side addition; by contrast, vector addition is a kind of range-side addi tion. In music Addition is also used in the musical set theory. George Perl e gives the following example: “do-mi, re-fa♯, mi♭-sol are different requir ements of one interval... or other type of equality... and are connected wi th the axis of symmetry. Do-mi belongs to the family of symmetrically conne cted dyads as it is shown further:” re re♯ mi fa fa♯ sol sol ♯ re do♯ do si la♯ la sol♯ Axis of the pitches are italiciz ed, the axis is defined with the pitch category. Thus, do-mi is a part of a n interval family-4 and a part of sum family -2 (at G♯ = 0). A tonal range of Alban Berg’s Lyric Suite {0,11,7,4,2,9,3,8,10,1,5,6} is a series of six dyads with their total number being 11. If the line is turned and inverte d, then it is {0,6,5,1,...} with all dyads in total being 6. The total numb er of successive dyads of a tonal range in Lyric Suite is 11 do sol re re♯ la♯ mi♯ si mi la sol♯ do♯ fa♯ Axis of the pitches are italicized, the axis is defined by the dyads (interval 1). See also Notes F ootnotes References History Elementary mathematics Education Cognitive scie nce Mathematical exposition Advanced mathematics Mathematical research Comp uting Further reading +Addition(+) −Subtraction(−) ×Multiplication(× or ·) ÷Division(÷ or /) Navigation menu
Introduction Addition (usually signified by the plus symbol +) is one of t he four basic operations of arithmetic, the other three being subtraction, multiplication and division. The addition of two whole numbers results in t he total amount or sum of those values combined. The example in the adjacen t image shows a combination of three apples and two apples, making a total of five apples. This observation is equivalent to the mathematical expressi on "3 + 2 = 5" (that is, "3 plus 2 is equal to 5"). Besides counting items, addition can also be defined and executed without referring to concrete obj

ects, using abstractions called numbers instead, such as integers, real numbers and complex numbers. Addition belongs to arithmetic, a branch of mathematics. In algebra, another area of mathematics, addition can also be performed on abstract objects such as vectors, matrices, subspaces and subgroups. Addition has several important properties. It is commutative, meaning that order does not matter, and it is associative, meaning that when one adds more than two numbers, the order in which addition is performed does not matter (see Summation). Repeated addition of 1 is the same as counting. Addition of 0 does not change a number. Addition also obeys predictable rules concerning related operations such as subtraction and multiplication. Performing addition is one of the simplest numerical tasks. Addition of very small numbers is accessible to toddlers; the most basic task, 1 + 1, can be performed by infants as young as five months, and even some members of other animal species. In primary education, students are taught to add numbers in the decimal system, starting with single digits and progressively tackling more difficult problems. Mechanical aids range from the ancient abacus to the modern computer, where research on the most efficient implementations of addition continues to this day. Addition is written using the plus sign "+" between the terms; that is, in infix notation. The result is expressed with an equals sign. For example, There are also situations where addition is "understood", even though no symbol appears: The sum of a series of related numbers can be expressed through capital sigma notation, which compactly denotes iteration. For example, The numbers or the objects to be added in general addition are collectively referred to as the terms, the addends or the summands;this terminology carries over to the summation of multiple terms.This is to be distinguished from factors, which are multiplied.Some authors call the first addend the augend. In fact, during the Renaissance, many authors did not consider the first addend an "addend" at all. Today, due to the commutative property of addition, "augend" is rarely used, and both terms are generally called addends. All of the above terminology derives from Latin. "Addition" and "add" are English words derived from the Latin verb addere, which is in turn a compound of ad "to" and dare "to give", from the Proto-Indo-European root *deh$_3$- "to give"; thus to add is to give to. Using the gerundive suffix -nd results in "addend", "thing to be added". Likewise from augere "to increase", one gets "augend", "thing to be increased". "Sum" and "summand" derive from the Latin noun summa "the highest, the top" and associated verb summare. This is appropriate not only because the sum of two positive numbers is greater than either, but because it was common for the ancient Greeks and Romans to add upward, contrary to the modern practice of adding downward, so that a sum was literally higher than the addends.Addere and summare date back at least to Boethius, if not to earlier Roman writers such as Vitruvius and Frontinus; Boethius also used several other terms for the addition operation. The later Middle English terms "adden" and "adding" were popularized by Chaucer. The plus sign "+" (Unicode:U+002B; ASCII: +) is an abbreviation of the Latin word et, meaning "and". It appears in mathematical works dating back to at least 1489. Addition is used to model many physical processes. Even for the simple case of adding natural numbers, there are many possible interpretations and even more visual representations. Possibly the most fundamental interpretation of addition lies in combining sets: This interpretation is easy to visualize, with little danger of ambiguity. It is also useful in higher mathematics (for the rigorous definition it inspires, see § Natural numbers below). However, it is not obvious how one should extend this version of addition to include fractional numbers or negative numbers. One possible fix is to consider collections of objects that can be easily divided, such as pies or, still better, segmented rods. Rather than solely combining collections of segments, rods can be joined end-to-end, which illustrates another conception of addition: adding not the rods but the lengths of the rods. A second interpretation of addition comes from extending an initial length by a given length: The sum a + b can be interpreted as a binary operation that combines a and b, in an algebraic sense, or it can be interpreted as the addition of b more units to a. Under the latter interpretation, the parts of a sum a + b play asymmetric roles, and the operation a + b is viewed as applying the unary operation +b to a. Instead of calling both a and b addends, it is more appropriate to call a the a ugend in this case, since a plays a passive role. The unary view is also useful when discussing subtraction, because each unary addition operation has an inverse unary subtraction operation, and vice versa. Addition is commutative, meaning that one can change the order of the terms in a sum, but still get the same result. Symbolically, if a and b are any two numbers, then The fact that addition is commutative is known as the "commutative law of addition" or "commutative property of addition". Some other binary operations are commutative, such as multiplication, but many others are not, such as subtraction and division. Addition is associative, which means that when three or more numbers are added together, the order of operations does not change the result. As an example, should the expression a + b + c be defined to mean (a + b) + c or a + (b + c)? Given that addition is associative, the choice of definition is irrelevant. For any three numbers a, b, and c, it is true that (a + b) + c = a + (b + c). For example, (1 + 2) + 3 = 3 + 3 = 6 = 1 + 5 = 1 + (2 + 3). When addition is used together with other operations, the order of operations becomes important. In the standard order of operations, addition is a lower priority than exponentiation, nth roots, multiplication and division, but is given equal priority to subtraction. Adding zero to any number, does not change the number; this means that zero is the identity element for addition, and is also known as the additive identity. In symbols, for every a, one has This law was first identified in Brahmagupta's Brahmasphutasiddhanta in 628 AD, although he wrote it as three separate laws, depending on whether a is negative, positive, or zero itself, and he used words rather than algebraic symbols. Later Indian mathematicians refined the concept; around the year 830, Mahavira wrote, "zero becomes the same as what is added to it", corresponding to the unary statement 0 + a = a. In the 12th century, Bhaskara wrote, "In the addition of cipher, or subtraction of it, the quantity, positive or negative, remains the same", corresponding to the unary statement a + 0 = a. Within the context of integers, addition of one also plays a special role: for any integer a, the integer (a + 1) is the least integer greater than a, also known as the successor of a. For instance, 3 is the successor of 2 and 7 is the successor of 6. Because of this succession, the value of a + b can also be seen as the bth successor of a, making addition iterated succession. For example, 6 + 2 is 8, because 8 is the successor of 7, which is the successor of 6, making 8 the 2nd successor of 6. To numerically add physical quantities with units, they must be expressed with common units. For example, adding 50 milliliters to 150 milliliters gives 200 milliliters. However, if a measure of 5 feet is extended by 2 inches, the sum is 62 inches, since 60 inches is synonymous with 5 feet. On the other hand, it is usually meaningless to try to add 3 meters and 4 square meters, since those units are incomparable; this sort of consideration is fundamental in dimensional analysis. Studies on mathematical development starting around the 1980s have exploited the phenomenon of habituation: infants look longer at situations that are unexpected. A seminal experiment by Karen Wynn in 1992 involving Mickey Mouse dolls manipulated behind a screen demonstrated that five-month-old infants expect 1 + 1 to be 2, and they are comparatively surprised when a physical situation seems to imply that 1 + 1 is either 1 or 3. This finding has since been affirmed by a variety of laboratories using different methodologies. Another 1992 experiment with older toddlers, between 18 and 35 months, exploited their development of motor control by allowing them to retrieve ping-pong balls from a box; the youngest responded well for small numbers, while older subjects were able to compute sums up to 5. Even some nonhuman animals show a limited ability to add, particularly primates. In a 1995 experiment imitating Wynn's 1992 result (but using eggplants instead of dolls), rhesus macaque and cottontop tamarin monkeys performed similarly to human infants. More dramatically, after being taught the meanings of the Arabic numerals 0 through 4, one chimpanzee was able to compute the sum of two numerals without further training. More recently, Asian elephants have demonstrated an ability to perform basic arithmetic. Typically, children first master counting. When given a problem that requires that two items and three items be combined, young children model the situation with physical objects, often fingers or a drawing, and then count the total. As they gain experience, they learn or discover the strategy of "counting-on": asked to find two plus three, children count thre e past two, saying "three, four, five" (usually ticking off fingers), and arriving at five. This strategy seems almost universal; children can easily pick it up from peers or teachers. Most discover it independently. With additional experience, children learn to add more quickly by exploiting the commutativity of addition by counting up from the larger number, in this case, starting with three and counting "four, five." Eventually children begin to recall certain addition facts ("number bonds"), either through experience or rote memorization. Once some facts are committed to memory, children begin to derive unknown facts from known ones. For example, a child asked to add six and seven may know that 6 + 6 = 12 and then reason that 6 + 7 is one more, or 13. Such derived facts can be found very quickly and most elementary school students eventually rely on a mixture of memorized and derived facts to add fluently. Different nations introduce whole numbers and arithmetic at different ages, with many countries teaching addition in pre-school. However, throughout the world, addition is taught by the end of the first year of elementary school. Children are often presented with the addition table of pairs of numbers from 0 to 9 to memorize. Knowing this, children can perform any addition. The prerequisite to addition in the decimal system is the fluent recall or derivation of the 100 single-digit "addition facts". One could memorize all the facts by rote, but pattern-based strategies are more enlightening and, for most people, more efficient: As students grow older, they commit more facts to memory, and learn to derive other facts rapidly and fluently. Many students never commit all the facts to memory, but can still find any basic fact quickly. The standard algorithm for adding multidigit numbers is to align the addends vertically and add the columns, starting from the ones column on the right. If a column exceeds nine, the extra digit is "carried" into the next column. For example, in the addition 27 + 59 7 + 9 = 16, and the digit 1 is the carry. An alternate strategy starts adding from the most significant digit on the left; this route makes carrying a little clumsier, but it is faster at getting a rough estimate of the sum. There are many alternative methods. Since the end of the XXth century, some US programs, including TERC, decided to remove the traditional transfer method from their curriculum. This decision was criticized that is why some states and counties didn’t support this experiment. Decimal fractions can be added by a simple modification of the above process. One aligns two decimal fractions above each other, with the decimal point in the same location. If necessary, one can add trailing zeros to a shorter decimal to make it the same length as the longer decimal. Finally, one performs the same addition process as above, except the decimal point is placed in the answer, exactly where it was placed in the summands. As an example, 45.1 + 4.34 can be solved as follows: In scientific notation, numbers are written in the form x=a×10b{\displaystyle x=a\times 10^{b}}, where a{\displaystyle a} is the significand and 10b{\displaystyle 10^{b}} is the exponential part. Addition requires two numbers in scientific notation to be represented using the same exponential part, so that the two significands can simply be added. For example: Addition in other bases is very similar to decimal addition. As an example, one can consider addition in binary. Adding two single-digit binary numbers is relatively simple, using a form of carrying: Adding two "1" digits produces a digit "0", while 1 must be added to the next column. This is similar to what happens in decimal when certain single-digit numbers are added together; if the result equals or exceeds the value of the radix (10), the digit to the left is incremented: This is known as carrying. When the result of an addition exceeds the value of a digit, the procedure is to "carry" the excess amount divided by the radix (that is, 10/10) to the left, adding it to the next positional value. This is correct since the next position has a weight that is higher by a factor equal to the radix. Carrying works the same way in binary: In this example, two numerals are being added together: 011012 (1310) and 101112 (2310). The top row shows the carry bits used. Starting in the rightmost column, 1 + 1 = 102. The 1 is carried to the left, and the 0 is written at the bottom of the rightmost column. The second column from the right is added: 1 + 0 + 1 = 102 again; the 1 is carried, and 0 is written at the bottom. The third column: 1 + 1 + 1 = 112. This time, a 1 is carried, and a 1 is written in the bottom row. Proceeding like this gives the final answer 1001002 (3610). Analog computers work dire ctly with physical quantities, so their addition mechanisms depend on the form of the addends. A mechanical adder might represent two addends as the positions of sliding blocks, in which case they can be added with an averaging lever. If the addends are the rotation speeds of two shafts, they can be added with a differential. A hydraulic adder can add the pressures in two chambers by exploiting Newton's second law to balance forces on an assembly of pistons. The most common situation for a general-purpose analog computer is to add two voltages (referenced to ground); this can be accomplished roughly with a resistor network, but a better design exploits an operational amplifier. Addition is also fundamental to the operation of digital computers, where the efficiency of addition, in particular the carry mechanism, is an important limitation to overall performance. The abacus, also called a counting frame, is a calculating tool that was in use centuries before the adoption of the written modern numeral system and is still widely used by merchants, traders and clerks in Asia, Africa, and elsewhere; it dates back to at least 2700–2300 BC, when it was used in Sumer. Blaise Pascal invented the mechanical calculator in 1642; it was the first operational adding machine. It made use of a gravity-assisted carry mechanism. It was the only operational mechanical calculator in the 17th century and the earliest automatic, digital computer. Pascal's calculator was limited by its carry mechanism, which forced its wheels to only turn one way so it could add. To subtract, the operator had to use the Pascal's calculator's complement, which required as many steps as an addition. Giovanni Poleni followed Pascal, building the second functional mechanical calculator in 1709, a calculating clock made of wood that, once setup, could multiply two numbers automatically. Adders execute integer addition in electronic digital computers, usually using binary arithmetic. The simplest architecture is the ripple carry adder, which follows the standard multi-digit algorithm. One slight improvement is the carry skip design, again following human intuition; one does not perform all the carries in computing 999 + 1, but one bypasses the group of 9s and skips to the answer. In practice, computational addition may be achieved via XOR and AND bitwise logical operations in conjunction with bitshift operations as shown in the pseudocode below. Both XOR and AND gates are straightforward to realize in digital logic allowing the realization of full adder circuits which in turn may be combined into more complex logical operations. In modern digital computers, integer addition is typically the fastest arithmetic instruction, yet it has the largest impact on performance, since it underlies all floating-point operations as well as such basic tasks as address generation during memory access and fetching instructions during branching. To increase speed, modern designs calculate digits in parallel; these schemes go by such names as carry select, carry lookahead, and the Ling pseudocarry. Many implementations are, in fact, hybrids of these last three designs. Unlike addition on paper, addition on a computer often changes the addends. On the ancient abacus and adding board, both addends are destroyed, leaving only the sum. The influence of the abacus on mathematical thinking was strong enough that early Latin texts often claimed that in the process of adding "a number to a number", both numbers vanish. In modern times, the ADD instruction of a microprocessor often replaces the augend with the sum but preserves the addend. In a high-level programming language, evaluating a + b does not change either a or b; if the goal is to replace a with the sum this must be explicitly requested, typically with the statement a = a + b. Some languages such as C or C++ allow this to be abbreviated as a += b. On a computer, if the result of an addition is too large to store, an arithmetic overflow occurs, resulting in an incorrect answer. Unanticipated arithmetic overflow is a fairly common cause of program errors. Such overflow bugs may be hard to discover and diagnose because they may manifest themselves only for very large input data sets, which are less likely to be used in validation tests. The Year 2000 problem was a series of bugs where overflow errors occurred due to use of a 2-digit format for years. To prove the usual properties of addition, one must first define addition for the context in question. Addition is first defined on the natural numbers. In set theory, addition is then extended to progressively larger sets that include the natural numbers: the integers, the rational numbers, and the real numbers. (In mathematics education, positive fractions are added before negative numbers are even considered; this is also the historical route.) There are two popular ways to define the sum of two natural numbers a and b. If one defines natural numbers to be the cardinalities of finite sets, (the cardinality of a set is the number of elements in the set), then it is appropriate to define their sum as follows: Here, A ∪ B is the union of A and B. An alternate version of this definition allows A and B to possibly overlap and then takes their disjoint union, a mechanism that allows common elements to be separated out and therefore counted twice. The other popular definition is recursive: Again, there are minor variations upon this definition in the literature. Taken literally, the above definition is an application of the recursion theorem on the partially ordered set N2. On the other hand, some sources prefer to use a restricted recursion theorem that applies only to the set of natural numbers. One then considers a to be temporarily "fixed", applies recursion on b to define a function "a +", and pastes these unary operations for all a together to form the full binary operation. This recursive formulation of addition was developed by Dedekind as early as 1854, and he would expand upon it in the following decades. He proved the associative and commutative properties, among others, through mathematical induction. The simplest conception of an integer is that it consists of an absolute value (which is a natural number) and a sign (generally either positive or negative). The integer zero is a special third case, being neither positive nor negative. The corresponding definition of addition must proceed by cases: Although this definition can be useful for concrete problems, the number of cases to consider complicates proofs unnecessarily. So the following method is commonly used for defining integers. It is based on the remark that every integer is the difference of two natural integers and that two such differences, a – b and c – d are equal if and only if a + d = b + c.So, one can define formally the integers as the equivalence classes of ordered pairs of natural numbers under the equivalence relation The equivalence class of (a, b) contains either (a – b, 0) if a ≥ b, or (0, b – a) otherwise. If n is a natural number, one can denote +n the equivalence class of (n, 0), and by –n the equivalence class of (0, n). This allows identifying the natural number n with the equivalence class +n. Addition of ordered pairs is done component-wise: A straightforward computation shows that the equivalence class of the result depends only on the equivalences classes of the summands, and thus that this defines an addition of equivalence classes, that is integers. Another straightforward computation shows that this addition is the same as the above case definition. This way of defining integers as equivalence classes of pairs of natural numbers, can be used to embed into a group any commutative semigroup with cancellation property. Here, the semigroup is formed by the natural numbers and the group is the additive group of integers. The rational numbers are constructed similarly, by taking as semigroup the nonzero integers with multiplication. This construction has been also generalized under the name of Grothendieck group to the case of any commutative semigroup. Without the cancellation property the semigroup homomorphism from the semigroup into the group may be non-injective. Originally, the Grothendieck group was, more specifically, the result of this construction applied to the equivalences classes under isomorphisms of the objects of an abelian category, with the direct sum as semigroup operation. Addition of rational numbers can be computed using the least common denominator, but a conceptually simpler definition involves only integer addition and multiplication: As an example, the sum 34+18=3×8+4×14×8=24+432=2832=78{\displaystyle {\frac {3}{4}}+{\frac {1}{8}}={\frac {3\times 8+4\times 1}{4\times 8}}={\frac {24+4}{32}}={\frac {28}{32}}={\frac {7}{8}}}. Addition of fractions is much simpler when the denominators are the same; in this case, one can simply add the numerators while leaving the denominator the same: ac+bc=a+bc{\displaystyle {\frac {a}{c}}+{\frac {b}{c}}={\frac {a+b}{c}}}, so 14+24=1+24=34{\displaystyle {\frac {1}{4}}+{\frac {2}{4}}={\frac {1+2}{4}}={\frac {3}{4}}}. The commutativity and associativity of rational addition is an easy consequence of the laws of integer arithmetic. For a more rigorous and general discussion, see field of fractions. A common construction of the set of real numbers is the Dedekind completion of the set of rational numbers. A real number is defined to be a Dedekind cut of rationals: a non-empty set of rationals that is closed downward and has no greatest element. The sum of r eal numbers a and b is defined element by element: This definition was first published, in a slightly modified form, by Richard Dedekind in 1872.The commutativity and associativity of real addition are immediate; defining the real number 0 to be the set of negative rationals, it is easily seen to be the additive identity. Probably the trickiest part of this construction pertaining to addition is the definition of additive inverses. Unfortunately, dealing with multiplication of Dedekind cuts is a time-consuming case-by-case process similar to the addition of signed integers. Another approach is the metric completion of the rational numbers. A real number is essentially defined to be the limit of a Cauchy sequence of rationals, lim an. Addition is defined term by term: This definition was first published by Georg Cantor, also in 1872, although his formalism was slightly different.One must prove that this operation is well-defined, dealing with co-Cauchy sequences. Once that task is done, all the properties of real addition follow immediately from the properties of rational numbers. Furthermore, the other arithmetic operations, including multiplication, have straightforward, analogous definitions. Complex numbers are added by adding the real and imaginary parts of the summands. That is to say: Using the visualization of complex numbers in the complex plane, the addition has the following geometric interpretation: the sum of two complex numbers A and B, interpreted as points of the complex plane, is the point X obtained by building a parallelogram three of whose vertices are O, A and B. Equivalently, X is the point such that the triangles with vertices O, A, B, and X, B, A, are congruent. There are many binary operations that can be viewed as generalizations of the addition operation on the real numbers. The field of abstract algebra is centrally concerned with such generalized operations, and they also appear in set theory and category theory. In linear algebra, a vector space is an algebraic structure that allows for adding any two vectors and for scaling vectors. A familiar vector space is the set of all ordered pairs of real numbers; the ordered pair (a,b) is interpreted as a vector from the origin in the Euclidean plane to the point (a,b) in the plane. The sum of two vectors is obtained by adding their individual coordinates: This addition operation is central to classical mechanics, in which velocities, accelerations and forces are all represented by vectors. Matrix addition is defined for two matrices of the same dimensions. The sum of two m × n (pronounced "m by n") matrices A and B, denoted by A + B, is again an m × n matrix computed by adding corresponding elements: For example: In modular arithmetic, the set of available numbers is restricted to a finite subset of the integers, and addition "wraps around" when reaching a certain value, called the modulus. For example, the set of integers modulo 12 has twelve elements; it inherits an addition operation from the integers that is central to musical set theory. The set of integers modulo 2 has just two elements; the addition operation it inherits is known in Boolean logic as the "exclusive or" function. A similar "wrap around" operation arises in geometry, where the sum of two angle measures is often taken to be their sum as real numbers modulo 2π. This amounts to an addition operation on the circle, which in turn generalizes to addition operations on many-dimensional tori. The general theory of abstract algebra allows an "addition" operation to be any associative and commutative operation on a set. Basic algebraic structures with such an addition operation include commutative monoids and abelian groups. A far-reaching generalization of addition of natural numbers is the addition of ordinal numbers and cardinal numbers in set theory. These give two different generalizations of addition of natural numbers to the transfinite. Unlike most addition operations, addition of ordinal numbers is not commutative. Addition of cardinal numbers, however, is a commutative operation closely related to the disjoint union operation. In category theory, disjoint union is seen as a particular case of the coproduct operation, and general coproducts are perhaps the most abstract of all the generalizations of addition. Some coproducts, such as direct sum and wedge sum, are named to evoke their connection with addition. Addition, along with subtraction, multiplication and division, is considered one of the basic operations and is used in elementary arithmetic. Subtraction can be thought of as a kind of addition—that is, the addition of an additive inverse. Subtraction is itself a sort of inverse to addition, in that adding x and subtracting x are inverse functions. Given a set with an additi on operation, one cannot always define a corresponding subtraction operatio n on that set; the set of natural numbers is a simple example. On the other hand, a subtraction operation uniquely determines an addition operation, an additive inverse operation, and an additive identity; for this reason, an a dditive group can be described as a set that is closed under subtraction. M ultiplication can be thought of as repeated addition. If a single term x ap pears in a sum n times, then the sum is the product of n and x. If n is not a natural number, the product may still make sense; for example, multiplica tion by −1 yields the additive inverse of a number. In the real and complex numbers, addition and multiplication can be interchanged by the exponential function: This identity allows multiplication to be carried out by consulti ng a table of logarithms and computing addition by hand; it also enables mu ltiplication on a slide rule. The formula is still a good first-order appro ximation in the broad context of Lie groups, where it relates multiplicatio n of infinitesimal group elements with addition of vectors in the associate d Lie algebra. There are even more generalizations of multiplication than a ddition. In general, multiplication operations always distribute over addit ion; this requirement is formalized in the definition of a ring. In some co ntexts, such as the integers, distributivity over addition and the existenc e of a multiplicative identity is enough to uniquely determine the multipli cation operation. The distributive property also provides information about addition; by expanding the product (1 + 1)(a + b) in both ways, one conclud es that addition is forced to be commutative. For this reason, ring additio n is commutative in general. Division is an arithmetic operation remotely r elated to addition. Since a/b = a(b−1), division is right distributive over addition: (a + b) / c = a/c + b/c. However, division is not left distributi ve over addition; 1 / (2 + 2) is not the same as 1/2 + 1/2. The maximum ope ration "max (a, b)" is a binary operation similar to addition. In fact, if two nonnegative numbers a and b are of different orders of magnitude, then their sum is approximately equal to their maximum. This approximation is ex tremely useful in the applications of mathematics, for example in truncatin g Taylor series. However, it presents a perpetual difficulty in numerical a nalysis, essentially since "max" is not invertible. If b is much greater th an a, then a straightforward calculation of (a + b) − b can accumulate an u nacceptable round-off error, perhaps even returning zero. See also Loss of significance. The approximation becomes exact in a kind of infinite limit; if either a or b is an infinite cardinal number, their cardinal sum is exac tly equal to the greater of the two. Accordingly, there is no subtraction o peration for infinite cardinals. Maximization is commutative and associativ e, like addition. Furthermore, since addition preserves the ordering of rea l numbers, addition distributes over "max" in the same way that multiplicat ion distributes over addition: For these reasons, in tropical geometry one replaces multiplication with addition and addition with maximization. In th is context, addition is called "tropical multiplication", maximization is c alled "tropical addition", and the tropical "additive identity" is negative infinity. Some authors prefer to replace addition with minimization; then t he additive identity is positive infinity. Tying these observations togethe r, tropical addition is approximately related to regular addition through t he logarithm: which becomes more accurate as the base of the logarithm incr eases. The approximation can be made exact by extracting a constant h, name d by analogy with Planck's constant from quantum mechanics, and taking the "classical limit" as h tends to zero: In this sense, the maximum operation is a dequantized version of addition. Incrementation, also known as the suc cessor operation, is the addition of 1 to a number. Summation describes the addition of arbitrarily many numbers, usually more than just two. It includ es the idea of the sum of a single number, which is itself, and the empty s um, which is zero. An infinite summation is a delicate procedure known as a series. Counting a finite set is equivalent to summing 1 over the set. Inte gration is a kind of "summation" over a continuum, or more precisely and ge nerally, over a differentiable manifold. Integration over a zero-dimensiona l manifold reduces to summation. Linear combinations combine multiplication and summation; they are sums in which each term has a multiplier, usually a real or complex number. Linear combinations are especially useful in contex ts where straightforward addition would violate some normalization rule, su ch as mixing of strategies in game theory or superposition of states in quantum mechanics. Convolution is used to add two independent random variables defined by distribution functions. Its usual definition combines integration, subtraction, and multiplication. In general, convolution is useful as a kind of domain-side addition; by contrast, vector addition is a kind of range-side addition. Addition is also used in the musical set theory. George Perle gives the following example: “do-mi, re-fa♯, mi♭-sol are different requirements of one interval... or other type of equality... and are connected with the axis of symmetry. Do-mi belongs to the family of symmetrically connected dyads as it is shown further:” re re♯ mi fa fa♯ sol sol♯ re do♯ do si la♯ la sol♯ Axis of the pitches are italicized, the axis is defined with the pitch category. Thus, do-mi is a part of an interval family-4 and a part of sum family -2 (at G♯ = 0). A tonal range of Alban Berg’s Lyric Suite {0,11,7,4,2,9,3,8,10,1,5,6} is a series of six dyads with their total number being 11. If the line is turned and inverted, then it is {0,6,5,1,...} with all dyads in total being 6. The total number of successive dyads of a tonal range in Lyric Suite is 11 do sol re re♯ la♯ mi♯ si mi la sol♯ do♯ fa♯ Axis of the pitches are italicized, the axis is defined by the dyads (interval 1). History Elementary mathematics Education Cognitive science Mathematical exposition Advanced mathematics Mathematical research Computing +Addition(+) −Subtraction(−) ×Multiplication(× or ·) ÷Division(÷ or /)

36096

35883

Introduction Addition (usually signified by the plus symbol +) is one of the four basic operations of arithmetic, the other three being subtraction, multiplication and division. The addition of two whole numbers results in the total amount or sum of those values combined. The example in the adjacent image shows a combination of three apples and two apples, making a total of five apples. This observation is equivalent to the mathematical expression "3 + 2 = 5" (that is, "3 plus 2 is equal to 5"). Besides counting items, addition can also be defined and executed without referring to concrete objects, using abstractions called numbers instead, such as integers, real numbers and complex numbers. Addition belongs to arithmetic, a branch of mathematics. In algebra, another area of mathematics, addition can also be performed on abstract objects such as vectors, matrices, subspaces and subgroups. Addition has several important properties. It is commutative, meaning that order does not matter, and it is associative, meaning that when one adds more than two numbers, the order in which addition is performed does not matter (see Summation). Repeated addition of 1 is the same as counting. Addition of 0 does not change a number. Addition also obeys predictable rules concerning related operations such as subtraction and multiplication. Performing addition is one of the simplest numerical tasks. Addition of very small numbers is accessible to toddlers; the most basic task, 1 + 1, can be performed by infants as young as five months, and even some members of other animal species. In primary education, students are taught to add numbers in the decimal system, starting with single digits and progressively tackling more difficult problems. Mechanical aids range from the ancient abacus to the modern computer, where research on the most efficient implementations of addition continues to this day.

Contents Notation and terminology Addition is written using the plus sign "+" between the terms; that is, in infix notation. The result is expressed with an equals sign. For example, There are also situations where addition is "understood", even though no symbol appears: The sum of a series of related numbers can be expressed through capital sigma notation, which compactly denotes iteration. For example, The numbers or the objects to be added in general addition are collectively referred to as the terms, the addends or the summands;this terminology carries over to the summation of multiple terms.This is to be distinguished from factors, which are multiplied.Some authors call the first addend the augend. In fact, during the Renaissance, many authors did not consider the first addend an "addend" at all. Today, due to the commutative property of addition, "augend" is rarely used, and both ter

ms are generally called addends. All of the above terminology derives from
Latin. "Addition" and "add" are English words derived from the Latin verb a
ddere, which is in turn a compound of ad "to" and dare "to give", from the
Proto-Indo-European root *deh$_3$- "to give"; thus to add is to give to. Using
the gerundive suffix -nd results in "addend", "thing to be added". Likewise
from augere "to increase", one gets "augend", "thing to be increased". "Su
m" and "summand" derive from the Latin noun summa "the highest, the top" an
d associated verb summare. This is appropriate not only because the sum of
two positive numbers is greater than either, but because it was common for
the ancient Greeks and Romans to add upward, contrary to the modern practic
e of adding downward, so that a sum was literally higher than the addends.A
ddere and summare date back at least to Boethius, if not to earlier Roman w
riters such as Vitruvius and Frontinus; Boethius also used several other te
rms for the addition operation. The later Middle English terms "adden" and
"adding" were popularized by Chaucer. The plus sign "+" (Unicode:U+002B; AS
CII: +) is an abbreviation of the Latin word et, meaning "and". It appe
ars in mathematical works dating back to at least 1489.

Interpretations Addition is used to model many physical processes. Even for
the simple case of adding natural numbers, there are many possible interpre
tations and even more visual representations. Possibly the most fundamental
interpretation of addition lies in combining sets: This interpretation is e
asy to visualize, with little danger of ambiguity. It is also useful in hig
her mathematics (for the rigorous definition it inspires, see § Natural num
bers below). However, it is not obvious how one should extend this version
of addition to include fractional numbers or negative numbers. One possible
fix is to consider collections of objects that can be easily divided, such
as pies or, still better, segmented rods. Rather than solely combining coll
ections of segments, rods can be joined end-to-end, which illustrates anoth
er conception of addition: adding not the rods but the lengths of the rods.
A second interpretation of addition comes from extending an initial length
by a given length: The sum a + b can be interpreted as a binary operation t
hat combines a and b, in an algebraic sense, or it can be interpreted as th
e addition of b more units to a. Under the latter interpretation, the parts
of a sum a + b play asymmetric roles, and the operation a + b is viewed as
applying the unary operation +b to a. Instead of calling both a and b adden
ds, it is more appropriate to call a the augend in this case, since a plays
a passive role. The unary view is also useful when discussing subtraction,
because each unary addition operation has an inverse unary subtraction oper
ation, and vice versa.

Properties Addition is commutative, meaning that one can change the order o
f the terms in a sum, but still get the same result. Symbolically, if a and
b are any two numbers, then The fact that addition is commutative is known
as the "commutative law of addition" or "commutative property of addition".
Some other binary operations are commutative, such as multiplication, but m
any others are not, such as subtraction and division. Addition is associati
ve, which means that when three or more numbers are added together, the ord
er of operations does not change the result. As an example, should the expr
ession a + b + c be defined to mean (a + b) + c or a + (b + c)? Given that
addition is associative, the choice of definition is irrelevant. For any th
ree numbers a, b, and c, it is true that (a + b) + c = a + (b + c). For exa
mple, (1 + 2) + 3 = 3 + 3 = 6 = 1 + 5 = 1 + (2 + 3). When addition is used
together with other operations, the order of operations becomes important.
In the standard order of operations, addition is a lower priority than expo
nentiation, nth roots, multiplication and division, but is given equal prio
rity to subtraction. Adding zero to any number, does not change the number;
this means that zero is the identity element for addition, and is also know
n as the additive identity. In symbols, for every a, one has This law was f
irst identified in Brahmagupta's Brahmasphutasiddhanta in 628 AD, although
he wrote it as three separate laws, depending on whether a is negative, pos
itive, or zero itself, and he used words rather than algebraic symbols. Lat
er Indian mathematicians refined the concept; around the year 830, Mahavira
wrote, "zero becomes the same as what is added to it", corresponding to the

unary statement 0 + a = a. In the 12th century, Bhaskara wrote, "In the add ition of cipher, or subtraction of it, the quantity, positive or negative, remains the same", corresponding to the unary statement a + 0 = a. Within t he context of integers, addition of one also plays a special role: for any integer a, the integer (a + 1) is the least integer greater than a, also kn own as the successor of a. For instance, 3 is the successor of 2 and 7 is t he successor of 6. Because of this succession, the value of a + b can also be seen as the bth successor of a, making addition iterated succession. For example, 6 + 2 is 8, because 8 is the successor of 7, which is the successo r of 6, making 8 the 2nd successor of 6. To numerically add physical quanti ties with units, they must be expressed with common units. For example, add ing 50 milliliters to 150 milliliters gives 200 milliliters. However, if a measure of 5 feet is extended by 2 inches, the sum is 62 inches, since 60 i nches is synonymous with 5 feet. On the other hand, it is usually meaningle ss to try to add 3 meters and 4 square meters, since those units are incomp arable; this sort of consideration is fundamental in dimensional analysis.

Performing addition Studies on mathematical development starting around the 1980s have exploited the phenomenon of habituation: infants look longer at situations that are unexpected. A seminal experiment by Karen Wynn in 1992 involving Mickey Mouse dolls manipulated behind a screen demonstrated that five-month-old infants expect 1 + 1 to be 2, and they are comparatively sur prised when a physical situation seems to imply that 1 + 1 is either 1 or 3. This finding has since been affirmed by a variety of laboratories using different methodologies. Another 1992 experiment with older toddlers, betwe en 18 and 35 months, exploited their development of motor control by allowi ng them to retrieve ping-pong balls from a box; the youngest responded well for small numbers, while older subjects were able to compute sums up to 5. Even some nonhuman animals show a limited ability to add, particularly prim ates. In a 1995 experiment imitating Wynn's 1992 result (but using eggplant s instead of dolls), rhesus macaque and cottontop tamarin monkeys performed similarly to human infants. More dramatically, after being taught the meani ngs of the Arabic numerals 0 through 4, one chimpanzee was able to compute the sum of two numerals without further training. More recently, Asian elep hants have demonstrated an ability to perform basic arithmetic. Typically, children first master counting. When given a problem that requires that two items and three items be combined, young children model the situation with physical objects, often fingers or a drawing, and then count the total. As they gain experience, they learn or discover the strategy of "counting-on": asked to find two plus three, children count three past two, saying "three, four, five" (usually ticking off fingers), and arriving at five. This strat egy seems almost universal; children can easily pick it up from peers or te achers. Most discover it independently. With additional experience, childre n learn to add more quickly by exploiting the commutativity of addition by counting up from the larger number, in this case, starting with three and c ounting "four, five." Eventually children begin to recall certain addition facts ("number bonds"), either through experience or rote memorization. Onc e some facts are committed to memory, children begin to derive unknown fact s from known ones. For example, a child asked to add six and seven may know that 6 + 6 = 12 and then reason that 6 + 7 is one more, or 13. Such derived facts can be found very quickly and most elementary school students eventua lly rely on a mixture of memorized and derived facts to add fluently. Diffe rent nations introduce whole numbers and arithmetic at different ages, with many countries teaching addition in pre-school. However, throughout the wor ld, addition is taught by the end of the first year of elementary school. C hildren are often presented with the addition table of pairs of numbers fro m 0 to 9 to memorize. Knowing this, children can perform any addition. The prerequisite to addition in the decimal system is the fluent recall or deri vation of the 100 single-digit "addition facts". One could memorize all the facts by rote, but pattern-based strategies are more enlightening and, for most people, more efficient: As students grow older, they commit more facts to memory, and learn to derive other facts rapidly and fluently. Many stude nts never commit all the facts to memory, but can still find any basic fact quickly. The standard algorithm for adding multidigit numbers is to align t he addends vertically and add the columns, starting from the ones column on the right. If a column exceeds nine, the extra digit is "carried" into the next column. For example, in the addition 27 + 59 7 + 9 = 16, and the digit 1 is the carry. An alternate strategy starts adding from the most significa nt digit on the left; this route makes carrying a little clumsier, but it i s faster at getting a rough estimate of the sum. There are many alternative methods. Since the end of the XXth century, some US programs, including TER C, decided to remove the traditional transfer method from their curriculum. This decision was criticized that is why some states and counties didn’t su pport this experiment. Decimal fractions can be added by a simple modificat ion of the above process. One aligns two decimal fractions above each othe r, with the decimal point in the same location. If necessary, one can add t railing zeros to a shorter decimal to make it the same length as the longer decimal. Finally, one performs the same addition process as above, except t he decimal point is placed in the answer, exactly where it was placed in th e summands. As an example, 45.1 + 4.34 can be solved as follows: In scienti fic notation, numbers are written in the form x=a×10b{\displaystyle x=a\tim es 10^{b}}, where a{\displaystyle a} is the significand and 10b{\displaysty le 10^{b}} is the exponential part. Addition requires two numbers in scient ific notation to be represented using the same exponential part, so that th e two significands can simply be added. For example: Addition in other base s is very similar to decimal addition. As an example, one can consider addi tion in binary. Adding two single-digit binary numbers is relatively simpl e, using a form of carrying: Adding two "1" digits produces a digit "0", wh ile 1 must be added to the next column. This is similar to what happens in decimal when certain single-digit numbers are added together; if the result equals or exceeds the value of the radix (10), the digit to the left is inc remented: This is known as carrying. When the result of an addition exceeds the value of a digit, the procedure is to "carry" the excess amount divided by the radix (that is, 10/10) to the left, adding it to the next positional value. This is correct since the next position has a weight that is higher by a factor equal to the radix. Carrying works the same way in binary: In t his example, two numerals are being added together: 011012 (1310) and 10111 2 (2310). The top row shows the carry bits used. Starting in the rightmost column, 1 + 1 = 102. The 1 is carried to the left, and the 0 is written at the bottom of the rightmost column. The second column from the right is add ed: 1 + 0 + 1 = 102 again; the 1 is carried, and 0 is written at the botto m. The third column: 1 + 1 + 1 = 112. This time, a 1 is carried, and a 1 is written in the bottom row. Proceeding like this gives the final answer 1001 002 (3610). Analog computers work directly with physical quantities, so the ir addition mechanisms depend on the form of the addends. A mechanical adde r might represent two addends as the positions of sliding blocks, in which case they can be added with an averaging lever. If the addends are the rota tion speeds of two shafts, they can be added with a differential. A hydraul ic adder can add the pressures in two chambers by exploiting Newton's secon d law to balance forces on an assembly of pistons. The most common situatio n for a general-purpose analog computer is to add two voltages (referenced to ground); this can be accomplished roughly with a resistor network, but a better design exploits an operational amplifier. Addition is also fundament al to the operation of digital computers, where the efficiency of addition, in particular the carry mechanism, is an important limitation to overall pe rformance. The abacus, also called a counting frame, is a calculating tool that was in use centuries before the adoption of the written modern numeral system and is still widely used by merchants, traders and clerks in Asia, A frica, and elsewhere; it dates back to at least 2700–2300 BC, when it was u sed in Sumer. Blaise Pascal invented the mechanical calculator in 1642; it was the first operational adding machine. It made use of a gravity-assisted carry mechanism. It was the only operational mechanical calculator in the 1 7th century and the earliest automatic, digital computer. Pascal's calculat or was limited by its carry mechanism, which forced its wheels to only turn one way so it could add. To subtract, the operator had to use the Pascal's calculator's complement, which required as many steps as an addition. Giova nni Poleni followed Pascal, building the second functional mechanical calcu lator in 1709, a calculating clock made of wood that, once setup, could mul tiply two numbers automatically. Adders execute integer addition in electro nic digital computers, usually using binary arithmetic. The simplest archit ecture is the ripple carry adder, which follows the standard multi-digit al gorithm. One slight improvement is the carry skip design, again following h uman intuition; one does not perform all the carries in computing 999 + 1, but one bypasses the group of 9s and skips to the answer. In practice, comp utational addition may be achieved via XOR and AND bitwise logical operatio ns in conjunction with bitshift operations as shown in the pseudocode belo w. Both XOR and AND gates are straightforward to realize in digital logic a llowing the realization of full adder circuits which in turn may be combine d into more complex logical operations. In modern digital computers, intege r addition is typically the fastest arithmetic instruction, yet it has the largest impact on performance, since it underlies all floating-point operat ions as well as such basic tasks as address generation during memory access and fetching instructions during branching. To increase speed, modern desig ns calculate digits in parallel; these schemes go by such names as carry se lect, carry lookahead, and the Ling pseudocarry. Many implementations are, in fact, hybrids of these last three designs. Unlike addition on paper, add ition on a computer often changes the addends. On the ancient abacus and ad ding board, both addends are destroyed, leaving only the sum. The influence of the abacus on mathematical thinking was strong enough that early Latin t exts often claimed that in the process of adding "a number to a number", bo th numbers vanish. In modern times, the ADD instruction of a microprocessor often replaces the augend with the sum but preserves the addend. In a high-level programming language, evaluating a + b does not change either a or b; if the goal is to replace a with the sum this must be explicitly requested, typically with the statement a = a + b. Some languages such as C or C++ all ow this to be abbreviated as a += b. On a computer, if the result of an add ition is too large to store, an arithmetic overflow occurs, resulting in an incorrect answer. Unanticipated arithmetic overflow is a fairly common caus e of program errors. Such overflow bugs may be hard to discover and diagnos e because they may manifest themselves only for very large input data sets, which are less likely to be used in validation tests. The Year 2000 problem was a series of bugs where overflow errors occurred due to use of a 2-digit format for years.

Addition of numbers To prove the usual properties of addition, one must fir st define addition for the context in question. Addition is first defined o n the natural numbers. In set theory, addition is then extended to progress ively larger sets that include the natural numbers: the integers, the ratio nal numbers, and the real numbers. (In mathematics education, positive frac tions are added before negative numbers are even considered; this is also t he historical route.) There are two popular ways to define the sum of two n atural numbers a and b. If one defines natural numbers to be the cardinalit ies of finite sets, (the cardinality of a set is the number of elements in the set), then it is appropriate to define their sum as follows: Here, A ∪ B is the union of A and B. An alternate version of this definition allows A and B to possibly overlap and then takes their disjoint union, a mechanism that allows common elements to be separated out and therefore counted twic e. The other popular definition is recursive: Again, there are minor variat ions upon this definition in the literature. Taken literally, the above def inition is an application of the recursion theorem on the partially ordered set N2. On the other hand, some sources prefer to use a restricted recursio n theorem that applies only to the set of natural numbers. One then conside rs a to be temporarily "fixed", applies recursion on b to define a function "a +", and pastes these unary operations for all a together to form the ful l binary operation. This recursive formulation of addition was developed by Dedekind as early as 1854, and he would expand upon it in the following dec ades. He proved the associative and commutative properties, among others, t hrough mathematical induction. The simplest conception of an integer is tha t it consists of an absolute value (which is a natural number) and a sign (generally either positive or negative). The integer zero is a special thir d case, being neither positive nor negative. The corresponding definition o f addition must proceed by cases: Although this definition can be useful fo r concrete problems, the number of cases to consider complicates proofs unnecessarily. So the following method is commonly used for defining integers. It is based on the remark that every integer is the difference of two natural integers and that two such differences, a − b and c − d are equal if and only if a + d = b + c.So, one can define formally the integers as the equivalence classes of ordered pairs of natural numbers under the equivalence relation The equivalence class of (a, b) contains either (a − b, 0) if a ≥ b, or (0, b − a) otherwise. If n is a natural number, one can denote +n the equivalence class of (n, 0), and by −n the equivalence class of (0, n). This allows identifying the natural number n with the equivalence class +n. Addition of ordered pairs is done component-wise: A straightforward computation shows that the equivalence class of the result depends only on the equivalences classes of the summands, and thus that this defines an addition of equivalence classes, that is integers. Another straightforward computation shows that this addition is the same as the above case definition. This way of defining integers as equivalence classes of pairs of natural numbers, can be used to embed into a group any commutative semigroup with cancellation property. Here, the semigroup is formed by the natural numbers and the group is the additive group of integers. The rational numbers are constructed similarly, by taking as semigroup the nonzero integers with multiplication. This construction has been also generalized under the name of Grothendieck group to the case of any commutative semigroup. Without the cancellation property the semigroup homomorphism from the semigroup into the group may be non-injective. Originally, the Grothendieck group was, more specifically, the result of this construction applied to the equivalences classes under isomorphisms of the objects of an abelian category, with the direct sum as semigroup operation. Addition of rational numbers can be computed using the least common denominator, but a conceptually simpler definition involves only integer addition and multiplication: As an example, the sum 34+18=3×8+4×14×8=24+432=2832=78{\displaystyle {\frac {3}{4}}+{\frac {1}{8}}={\frac {3\times 8+4\times 1}{4\times 8}}={\frac {24+4}{32}}={\frac {28}{32}}={\frac {7}{8}}}. Addition of fractions is much simpler when the denominators are the same; in this case, one can simply add the numerators while leaving the denominator the same: ac+bc=a+bc{\displaystyle {\frac {a}{c}}+{\frac {b}{c}}={\frac {a+b}{c}}}, so 14+24=1+24=34{\displaystyle {\frac {1}{4}}+{\frac {2}{4}}={\frac {1+2}{4}}={\frac {3}{4}}}. The commutativity and associativity of rational addition is an easy consequence of the laws of integer arithmetic. For a more rigorous and general discussion, see field of fractions. A common construction of the set of real numbers is the Dedekind completion of the set of rational numbers. A real number is defined to be a Dedekind cut of rationals: a non-empty set of rationals that is closed downward and has no greatest element. The sum of real numbers a and b is defined element by element: This definition was first published, in a slightly modified form, by Richard Dedekind in 1872.The commutativity and associativity of real addition are immediate; defining the real number 0 to be the set of negative rationals, it is easily seen to be the additive identity. Probably the trickiest part of this construction pertaining to addition is the definition of additive inverses. Unfortunately, dealing with multiplication of Dedekind cuts is a time-consuming case-by-case process similar to the addition of signed integers. Another approach is the metric completion of the rational numbers. A real number is essentially defined to be the limit of a Cauchy sequence of rationals, lim an. Addition is defined term by term: This definition was first published by Georg Cantor, also in 1872, although his formalism was slightly different.One must prove that this operation is well-defined, dealing with co-Cauchy sequences. Once that task is done, all the properties of real addition follow immediately from the properties of rational numbers. Furthermore, the other arithmetic operations, including multiplication, have straightforward, analogous definitions. Complex numbers are added by adding the real and imaginary parts of the summands. That is to say: Using the visualization of complex numbers in the complex plane, the addition has the following geometric interpretation: the sum of two complex numbers A and B, interpreted as points of the complex plane, is the point X obtained by building a parallelogram three of whose vertices are O, A and B. Equivalently, X is the point such that the triangles with vertices O, A, B, and X, B, A, are congruent.

Generalizations There are many binary operations that can be viewed as gene ralizations of the addition operation on the real numbers. The field of abs tract algebra is centrally concerned with such generalized operations, and they also appear in set theory and category theory. In linear algebra, a ve ctor space is an algebraic structure that allows for adding any two vectors and for scaling vectors. A familiar vector space is the set of all ordered pairs of real numbers; the ordered pair (a,b) is interpreted as a vector fr om the origin in the Euclidean plane to the point (a,b) in the plane. The s um of two vectors is obtained by adding their individual coordinates: This addition operation is central to classical mechanics, in which velocities, accelerations and forces are all represented by vectors. Matrix addition is defined for two matrices of the same dimensions. The sum of two m × n (pron ounced "m by n") matrices A and B, denoted by A + B, is again an m × n matr ix computed by adding corresponding elements: For example: In modular arith metic, the set of available numbers is restricted to a finite subset of the integers, and addition "wraps around" when reaching a certain value, called the modulus. For example, the set of integers modulo 12 has twelve element s; it inherits an addition operation from the integers that is central to m usical set theory. The set of integers modulo 2 has just two elements; the addition operation it inherits is known in Boolean logic as the "exclusive or" function. A similar "wrap around" operation arises in geometry, where t he sum of two angle measures is often taken to be their sum as real numbers modulo 2π. This amounts to an addition operation on the circle, which in tu rn generalizes to addition operations on many-dimensional tori. The general theory of abstract algebra allows an "addition" operation to be any associa tive and commutative operation on a set. Basic algebraic structures with su ch an addition operation include commutative monoids and abelian groups. A far-reaching generalization of addition of natural numbers is the addition of ordinal numbers and cardinal numbers in set theory. These give two diffe rent generalizations of addition of natural numbers to the transfinite. Unl ike most addition operations, addition of ordinal numbers is not commutativ e. Addition of cardinal numbers, however, is a commutative operation closel y related to the disjoint union operation. In category theory, disjoint uni on is seen as a particular case of the coproduct operation, and general cop roducts are perhaps the most abstract of all the generalizations of additio n. Some coproducts, such as direct sum and wedge sum, are named to evoke th eir connection with addition.

Related operations Addition, along with subtraction, multiplication and div ision, is considered one of the basic operations and is used in elementary arithmetic. Subtraction can be thought of as a kind of addition—that is, th e addition of an additive inverse. Subtraction is itself a sort of inverse to addition, in that adding x and subtracting x are inverse functions. Give n a set with an addition operation, one cannot always define a correspondin g subtraction operation on that set; the set of natural numbers is a simple example. On the other hand, a subtraction operation uniquely determines an addition operation, an additive inverse operation, and an additive identit y; for this reason, an additive group can be described as a set that is clo sed under subtraction. Multiplication can be thought of as repeated additio n. If a single term x appears in a sum n times, then the sum is the product of n and x. If n is not a natural number, the product may still make sense; for example, multiplication by −1 yields the additive inverse of a number. In the real and complex numbers, addition and multiplication can be interch anged by the exponential function: This identity allows multiplication to b e carried out by consulting a table of logarithms and computing addition by hand; it also enables multiplication on a slide rule. The formula is still a good first-order approximation in the broad context of Lie groups, where it relates multiplication of infinitesimal group elements with addition of vectors in the associated Lie algebra. There are even more generalizations of multiplication than addition. In general, multiplication operations alwa ys distribute over addition; this requirement is formalized in the definiti on of a ring. In some contexts, such as the integers, distributivity over a ddition and the existence of a multiplicative identity is enough to uniquel y determine the multiplication operation. The distributive property also pr ovides information about addition; by expanding the product (1 + 1)(a + b) in both ways, one concludes that addition is forced to be commutative. For this reason, ring addition is commutative in general. Division is an arithm etic operation remotely related to addition. Since a/b = a(b−1), division i s right distributive over addition: (a + b) / c = a/c + b/c. However, divis ion is not left distributive over addition; 1 / (2 + 2) is not the same as 1/2 + 1/2. The maximum operation "max (a, b)" is a binary operation similar to addition. In fact, if two nonnegative numbers a and b are of different o rders of magnitude, then their sum is approximately equal to their maximum. This approximation is extremely useful in the applications of mathematics, for example in truncating Taylor series. However, it presents a perpetual d ifficulty in numerical analysis, essentially since "max" is not invertible. If b is much greater than a, then a straightforward calculation of (a + b) − b can accumulate an unacceptable round-off error, perhaps even returning zero. See also Loss of significance. The approximation becomes exact in a k ind of infinite limit; if either a or b is an infinite cardinal number, the ir cardinal sum is exactly equal to the greater of the two. Accordingly, th ere is no subtraction operation for infinite cardinals. Maximization is com mutative and associative, like addition. Furthermore, since addition preser ves the ordering of real numbers, addition distributes over "max" in the sa me way that multiplication distributes over addition: For these reasons, in tropical geometry one replaces multiplication with addition and addition wi th maximization. In this context, addition is called "tropical multiplicati on", maximization is called "tropical addition", and the tropical "additive identity" is negative infinity. Some authors prefer to replace addition wit h minimization; then the additive identity is positive infinity. Tying thes e observations together, tropical addition is approximately related to regu lar addition through the logarithm: which becomes more accurate as the base of the logarithm increases. The approximation can be made exact by extracti ng a constant h, named by analogy with Planck's constant from quantum mecha nics, and taking the "classical limit" as h tends to zero: In this sense, t he maximum operation is a dequantized version of addition. Incrementation, also known as the successor operation, is the addition of 1 to a number. Su mmation describes the addition of arbitrarily many numbers, usually more th an just two. It includes the idea of the sum of a single number, which is i tself, and the empty sum, which is zero. An infinite summation is a delicat e procedure known as a series. Counting a finite set is equivalent to summi ng 1 over the set. Integration is a kind of "summation" over a continuum, o r more precisely and generally, over a differentiable manifold. Integration over a zero-dimensional manifold reduces to summation. Linear combinations combine multiplication and summation; they are sums in which each term has a multiplier, usually a real or complex number. Linear combinations are esp ecially useful in contexts where straightforward addition would violate som e normalization rule, such as mixing of strategies in game theory or superp osition of states in quantum mechanics. Convolution is used to add two inde pendent random variables defined by distribution functions. Its usual defin ition combines integration, subtraction, and multiplication. In general, co nvolution is useful as a kind of domain-side addition; by contrast, vector addition is a kind of range-side addition.

In music Addition is also used in the musical set theory. George Perle give s the following example: “do-mi, re-fa♯, mi♭-sol are different requirements of one interval... or other type of equality... and are connected with the axis of symmetry. Do-mi belongs to the family of symmetrically connected dy ads as it is shown further:” re re♯ mi fa fa♯ sol sol♯ re do♯ do si la♯ la sol♯ Axis of the pitches are italicized, the axis is defined with the pitch category. Thus, do-mi is a part of an interv al family-4 and a part of sum family -2 (at G♯ = 0). A tonal range of Alban Berg’s Lyric Suite {0,11,7,4,2,9,3,8,10,1,5,6} is a series of six dyads wit h their total number being 11. If the line is turned and inverted, then it is {0,6,5,1,...} with all dyads in total being 6. The total number of succe ssive dyads of a tonal range in Lyric Suite is 11 do sol re re♯ l

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a♯   mi♯ si    mi     la    sol♯  do♯  fa♯ Axis of the pitches are italiciz
ed, the axis is defined by the dyads (interval 1).
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See also Notes Footnotes References History Elementary mathematics Educatio
n Cognitive science Mathematical exposition Advanced mathematics Mathematic
al research Computing
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Further reading +Addition(+) −Subtraction(−) ×Multiplication(× or ·) ÷Divis
ion(÷ or /)
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Truncation was not explicitly activated but `max_length` is provided a spec
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Introduction 1895
Notation and terminology 2126
Interpretations 1588
Properties 2920
Performing addition 3500
Addition of numbers 3500
Generalizations 2719
Choose a template 5
here
The addition of two whole numbers results in the total amount or sum of tho
se values combined
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 Addition belongs to arithmetic, a branch of mathematics
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 Performing addition is one of the simplest numerical tasks
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Addition is written using the plus sign "+" between the terms; that is, in
infix notation
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 The result is expressed with an equals sign
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 There are also situations where addition is "understood", even though no s
ymbol appears
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Addition is used to model many physical processes
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 Even for the simple case of adding natural numbers, there are many possibl
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 Possibly the most fundamental interpretation of addition lies in combining
sets
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Addition is commutative, meaning that one can change the order of the terms
in a sum, but still get the same result
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 For any three numbers a, b, and c, it is true that (a + b) + c = a + (b +
c) In symbols, for every a, one has
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 This law was first identified in Brahmagupta's Brahmasphutasiddhanta in 62
8 AD
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Studies on mathematical development starting around the 1980s have exploite
d the phenomenon of habituation: infants look longer at situations that are
unexpected
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 Even some nonhuman animals show a limited ability to add, particularly pri
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 Different nations introduce whole numbers and arithmetic at different age
s, with many countries teaching addition in pre-school
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Addition is first defined on the natural numbers
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 In set theory, addition is then extended to progressively larger sets that
include the integers, the rational numbers, and the real numbers
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