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Radio spectral ageing in a random magnetic field

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ABSTRACT

I study the ageing of radio synchrotron emission from relativistic electrons in a random magnetic field. There is a range of break frequencies corresponding to the range in field strength, so that the sharp break in the spectrum given by the Jaffe–Perola model is not always seen, and the spectra can be very similar to that of the Kardashev–Pacholczyk model. This explains the good fit of the Kardashev–Pacholczyk model to observations, even though the Jaffe–Perola model is more physically reasonable. The lack of an exponential break in the spectrum does not mean that pitch angle scattering does not occur. In the presence of inverse Compton losses, the spectral shape is strongly dependent on rms field strength, allowing equipartition estimates to be checked.

Key words: magnetic fields – radiation mechanisms: miscellaneous – radio continuum: galaxies.

1 INTRODUCTION

The radio emission from powerful extragalactic radio galaxies and quasars is synchrotron radiation from relativistic electrons spiralling in a magnetic field. The most energetic electrons lose energy fastest, so that the high-frequency emission falls. This phenomenon, known as spectral ageing, has been studied by many authors (Kardashev 1962; Pacholczyk 1970; Jaffe & Perola 1973; Myers & Spangler 1985; Alexander 1987).

Two models for the evolution of an initially pure power-law electron energy spectrum can be distinguished (Myers & Spangler 1985). In the Kardashev–Pacholczyk (KP) model, the pitch angle of an individual electron is constant, and as the electrons whose motion is more perpendicular to the field lose energy faster than electrons travelling nearly parallel to the field, there are always some high-energy electrons left to radiate at high frequencies. In the Jaffe–Perola (JP) model, the pitch-angle distribution is rapidly isotropized, with individual electrons sampling all pitch angles. The energy losses are independent of an electron's initial pitch angle, leading to a sharp break in the energy spectrum.

In the conventional approach, one averages over all orientations of a constant field (Myers & Spangler 1985). This is not strictly correct. Random orientations also imply random field strengths, and the field strength is normally assumed to be constant. In addition, the large observed polarizations rule out a truly random field, although large polarizations can be given by a sheared random field (Laing 1980).

It should be emphasized that, of these two models, the JP model has a far more secure physical basis, and would be expected to give the best description of reality. An anisotropic pitch-angle distribution excites Alfvén waves which scatter the electrons in pitch angle (Wentzel 1969; Melrose 1980). Even if this does not occur, the pitch angle changes as an electron moves between regions of differing field strength. It is surprising, therefore, that detailed observations of Cygnus A (Carilli et al. 1991) are well fitted by the KP model and strongly rule out the JP model spectrum.

In this paper I examine the radio spectrum allowing for the random nature of the magnetic field. In Section 2 I first consider the spectra from a volume containing a random magnetic field with the KP and JP prescriptions for the evolution of the electron energy spectrum. In Section 3 I go on to consider a realistic non-scattering model in which the pitch angles change as the relativistic electrons move along the varying field. A JP-type model allowing for diffusion between regions of different field strength and the effects of inverse Compton losses is presented in Section 4. I consider some effects relevant to real sources in Section 5, and give my conclusions in Section 6.

2 A SIMPLE MODEL

In the presence of synchrotron and inverse Compton losses, an initial power-law distribution of electron energies $N(E)=N_0E^{-\gamma}$ changes to (Pacholczyk 1970)

$$N(E,\phi,t)=N_0E^{-\gamma}[1-C_2Et(B_{\rm IC}^2+B^2\sin^2\phi)]^{\gamma-2}, \qquad (1)$$

where ϕ is the pitch angle, C_2 a constant and

$B_{\rm IC} = 3.18(1+z)^2 \mu$G is the magnitude of the magnetic field equivalent to the microwave background. In the JP model the pitch angles are continually randomized and the above expression is changed by the replacement $\sin^2\phi \rightarrow \langle \sin^2\phi \rangle = 2/3$. The exponent γ is related to the radio spectral index n by $\gamma = 2n+1$. The depletion of high-energy electrons is seen as a fall in the high-frequency radio emission below the extrapolation of the low-frequency emission.

Electron energy losses are highest in the strong field regions, so the spectrum of the radio emission from these regions steepens most rapidly. Thus at high frequencies the emission is biased towards regions of low field strength. There is a difference between the KP and JP models, as in the former the emission from regions where the magnetic field is perpendicular to the line of sight fades faster.

The total emission is given by

$$I(\nu, t) = C \int N(E, t) F(x) \, dE, \tag{2}$$

where $x = \nu/\nu_{\rm T}$, with

$$F(x) = x \int_x^{\infty} K_{5/3}(z) dz, \tag{3}$$

and

$$\nu_{\rm T} = c_1 B E_{\rm T}^2 \sin\phi, \tag{4}$$

where the energy $E_{\rm T}$ is defined by

$$E_{\rm T} = 1/[C_2 t (B_{\rm IC}^2 + B^2 \sin^2\phi)]. \tag{5}$$

The initial radio spectrum is a power law $I \propto \nu^{-n}$. The total aged spectrum can be characterized by a function $A(\gamma, x)$, which simply multiplies the initial spectrum,

$$I(\nu, t) = A(\gamma, x) I(\nu, 0). \tag{6}$$

Here A is a fixed function, so that the shape of the spectrum is the same at all times everywhere, and only the single parameter x is a function of position, while γ is a constant for any source (assuming no reacceleration) but can vary between sources.

To obtain the total spectrum for a source, one must then sum over the entire volume, or equivalently sum over all angles and field strengths. The initial intensity $I(\nu, 0) \propto (B \sin\phi)^{n+1}$ determines the relative contributions to the spectrum from different points.

I assume that the magnetic field can be described by a Gaussian random field. Then the angles are distributed isotropically and the field is drawn from a Maxwellian distribution. In Fig. 1 I show the resulting spectra for both KP and JP models compared with the spectra that would result if the field strength were uniform. Especially in the JP case, the spectra are flatter at high frequencies than if the field were constant. The spectrum initially falls below the constant-field case but remains fairly flat. The initial fall is due to increased losses from the regions of highest field strength; the remaining emission then decays more slowly.

The reason for the smoothing out of the sharp spectral decline is simple. The resultant spectrum is the superposition of many spectra, all with different break frequencies $\nu_{\rm T}$. In the standard KP model, the break frequency depends on pitch angle. In the models in this paper, the break frequency depends in addition on the local field strength. The resultant smearing out of the sharp spectral break depends simply on the presence of a range of field strengths and is not specific to a Gaussian field.

In Fig. 2, I show how the spectral index varies with time. The spectral index is that between the two frequencies ν_1 and ν_2, plotted against $\sqrt{\nu_1/\nu_{\rm T}}$ which is proportional to time. This figure is then directly comparable to figs 2 and 3 of Myers & Spangler (1985), although I have used consistent definitions of $\nu_{\rm T}$ in the KP and JP cases (Leahy, Muxlow & Stephens 1989). When plotted in this way, considerable differences between the models are apparent. When the amount of spectral ageing is small, the two-point spectral index tends to overestimate the age slightly. When the spectral ageing is large, the two-point spectral index substantially underestimates the age.

3 A FREE-STREAMING MODEL

The previous section showed that emission from a volume in which the field strength was not constant gave a spectrum without an exponential decline at high frequencies. The relativistic electrons are expected to move between regions

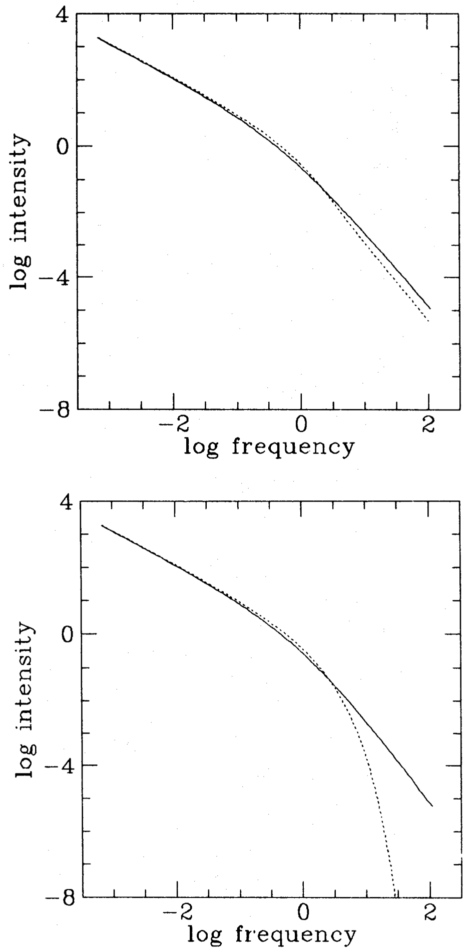

Figure 1. Aged spectra as a function of frequency for the KP model (top) and JP model (bottom). The aged spectrum assuming a random field strength is shown as a solid line, and that from a constant field shown as a dotted line for comparison.

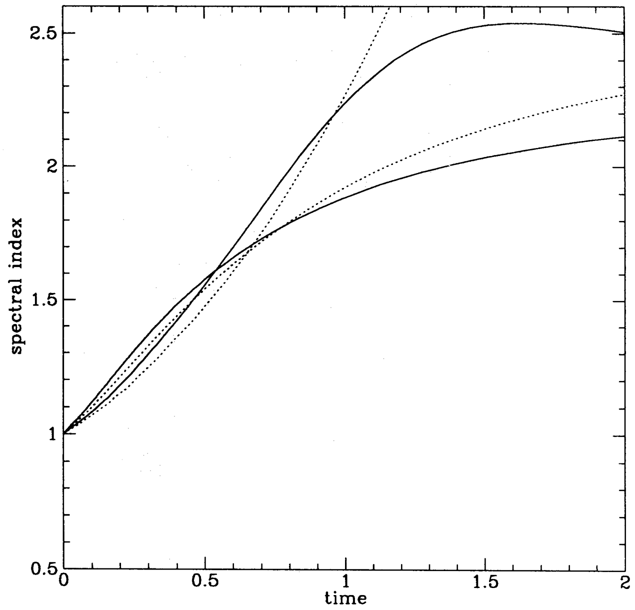

Figure 2. Two-frequency spectral index as a function of time. Solid lines are KP models, while JP models are dashed lines. The higher two curves are for the unmodified (uniform field strength) models, while the lower curves are for the random field strength case.

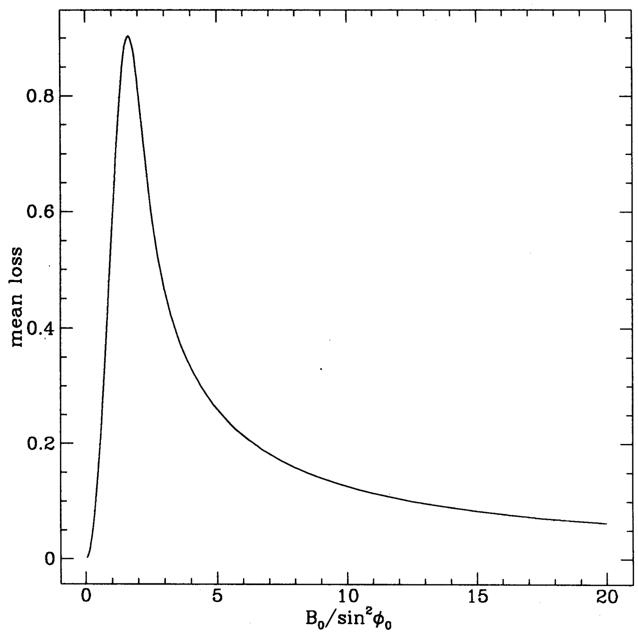

Figure 3. The mean losses of an electron as a function of the initial value of $B_0/\sin^2\phi_0$.

of different field strength, so in this section I allow electrons to stream freely along the field. This is an extension of the KP model. In Section 4, I consider diffusion along the field in the JP model.

As an electron travels along a field line, any change of magnetic field strength causes the pitch angle of the electron to change so as to keep the magnetic flux linked by the orbit constant. If the electron is initially at pitch angle ϕ_0 to a field of strength B_0 and travelling at speed v, then

$$B/\sin^2\phi = B_0/\sin^2\phi_0. \tag{7}$$

The velocity components perpendicular and parallel to the field are

$$v_\perp = v\sin\phi_0\sqrt{B/B_0}; \qquad v_\parallel = v\sqrt{1-B\sin^2\phi_0/B_0}, \tag{8}$$

respectively.

The losses are proportional to $B^2\sin^2\phi$. This needs to be averaged along the orbit, along which $B/\sin^2\phi$ is a constant, so that the average losses are proportional to

$$\int B^3 W(s)\,\mathrm{d}s. \tag{9}$$

The integral is over all the path s that the electron can reach, with $B<B_{\max}$ where $B_{\max}=B_0/\sin^2\phi_0$ is the maximum field strength into which the electron can travel (it is mirrored out of stronger fields) and $W(s)$ is proportional to the length of time the electron spends in a region ds of a particular field strength. I will assume that the probability of a field strength B is the same for all electrons, and that the time spent in a region of field strength B is inversely proportional to the velocity $v_\parallel$ corresponding to that field strength. The average losses for an electron are then

$$\langle B^2\sin^2\phi\rangle_{\text{orbits}} = \frac{\sin^2\phi_0}{B_0} \times \frac{\int_0^{B_0/\sin^2\phi_0} B^3 P(B)(1-B\sin^2\phi_0/B_0)^{-1/2}\,\mathrm{d}B}{\int_0^{B_0/\sin^2\phi_0} P(B)(1-B\sin^2\phi_0/B_0)^{-1/2}\,\mathrm{d}B} = L(B_0/\sin^2\phi_0). \tag{10}$$

The mean losses are shown as a function of $B_0/\sin^2\phi_0$ in Fig. 3. Note that average electrons have the largest losses. An electron with a large $B_0/\sin^2\phi_0$ spends the vast majority of its time in regions of low field strength travelling nearly parallel to the field, and consequently has low average losses. The mean losses in this case are only about 80 per cent of what they were before.

The electron energy spectrum in this case becomes

$$N(E,\phi,t) = N_0E^{-\gamma}\{1-C_2Et[B_{\mathrm{IC}}^2+L(B/\sin^2\phi)]\}^{\gamma-2} \tag{11}$$

The total spectrum is again obtained by using equation (11) in equation (2) and averaging over all angles and field strengths. The result is shown in Fig. 4. In this case, spectral ageing is systematically slower than in the standard KP model.

4 DIFFUSION AND INVERSE COMPTON MODELS

Section 2 considered the effect on the spectrum of electrons being trapped in place in a variable magnetic field. If the electrons are free to move along the field and not scattered in pitch angle, then the free-streaming KP model discussed in Section 3 is appropriate. In reality, the situation will be intermediate between these two extremes, and pitch-angle scattering will occur along with diffusion along the field. The

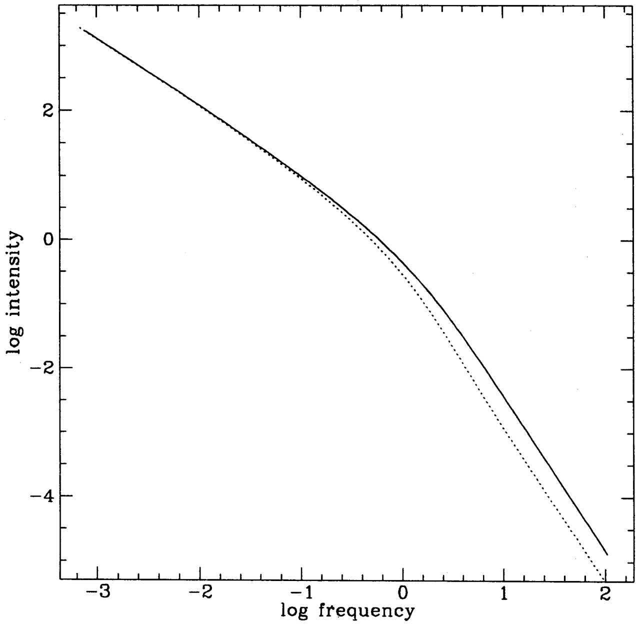

Figure 4. The synchrotron spectrum from electrons in a random magnetic field with no pitch-angle scattering (solid line), with the standard KP model spectrum as a dotted line for comparison.

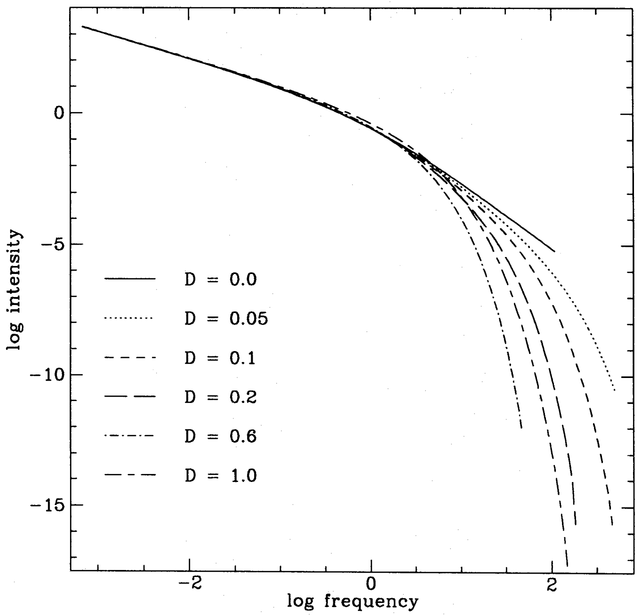

Figure 5. The synchrotron spectrum from electrons in a random magnetic field with pitch-angle scattering for various values of the diffusion coefficient *D*.

pitch-angle scattering makes the electron energy distribution independent of pitch angle, and the diffusion between regions of different field strength tends to make the electron energy spectrum independent of position. How much this is true depends on the efficiency of pitch-angle scattering in preventing diffusion.

If diffusion between regions of different field strengths is only partial, then we might consider a model in which the energy losses of an electron are proportional to

$$\langle B^2 \rangle_{\text{electron}} = D \langle B^2 \rangle + (1-D) B^2, \tag{12}$$

which is a combination of the local field B and the average field seen throughout the electron's lifetime. If $D=0$ then we recover the spectrum described in Section 2, whereas for larger D the spectrum declines more sharply at high frequencies. Strictly, D should be a function of energy as it depends on both the diffusion time and the energy loss time.

Exactly the same formalism can be applied to a source with a weak magnetic field, so that energy losses are dominated by inverse Compton scattering off the microwave background [this might also be true for radio haloes in galaxy clusters (Kim, Tribble & Kronberg 1991), or for sources at high redshift where the energy density of the microwave background is much greater]. In the extreme case the electron energy spectrum is the same everywhere. The radio spectrum differs from place to place because the break frequency $\nu_T \propto B$ varies. This is equivalent to a case where the electrons sample not only all pitch angles but all field strengths. Models with both synchrotron and inverse Compton losses can be described by equation (12) if we make the identification

$$D = B_{\text{IC}}^2 / (B_{\text{IC}}^2 + \langle B^2 \rangle). \tag{13}$$

I have calculated the spectra for various values of D and show the results in Fig. 5. For small values of D the spectra have no sharp break, but this break is apparent for $D \gtrsim 0.2$. For $D=1$ a sharp break is seen, although not quite so sharp as in the standard JP model. Interestingly, for large values of D the spectrum steepens more rapidly than for $D=1$. This is because a small variation in the energy losses compensates for the factor B in the numerator of ν_T, so that ν_T varies very little.

If we just consider the effect of inverse Compton losses, then powerful sources with very high field strengths should have radio spectra similar to the KP model. This is indeed the case for Cygnus A (Carilli et al. 1991), where the averaging is mainly along the line of sight as the observations have good resolution. Other sources have equipartition field strengths that are at most a few B_{IC} (e.g. 3C 234, Alexander 1987), and these sources would be expected to have spectra that are much closer to the JP model. Careful fitting of high-frequency spectra could be used to check the reliability of the estimates of magnetic fields in radio sources.

5 EXTRA EFFECTS IN REAL SOURCES

In addition to the effects considered so far, there are several factors that must be taken into account when studying real sources. Most of these effects are small, and will be hard to measure.

The age increases back from the hotspot (Myers & Spangler 1985; Alexander & Leahy 1987), so that a line of sight through a source not exactly in the plane of the sky will intercept regions of different ages. Also, the derived lobe-hotspot separation velocities are substantial, particularly in powerful FRII sources (Liu, Pooley & Riley 1992), so that the back of a lobe will be younger than the front by the time photons take to traverse the lobe. This effect can be estimated from the data, as the difference between the spectra from the front and back of the source can be related to the difference between spectra at different points along the source axis, with due allowance for projection effects. In addition, the age at a given distance from the hotspot might depend on distance from the source axis, giving a spread of ages along the line of sight even for sources in the plane of the sky. Also, the ages might be influenced by changing field strength along the source axis (Wiita & Gopal-Krishna 1990). All these effects will smear out any break in the spectrum even more.

Although it would be difficult to do in practice, there should be differences between the spectra at the edges of a source and in the centre of the lobes (Carilli et al. 1991 looked for this effect in Cygnus A and failed to find it). These differences depend on source orientation, and so should be more pronounced for quasars if the standard unification scheme is correct (Orr & Browne 1982; Barthel 1989). In addition, in an inclined source the spectral break in the back lobe should be more smeared out, because in the front lobe the time delay from front to back of the source partially compensates for the different distances back from the hotspot.

5.1 Filling factors and filamentation

Detailed observations of Cygnus A (Perley, Dreher & Cowan 1984) have shown that the emission is filamentary, and Perley et al. gave a volume filling factor in the range 0.03–0.3. However, the emission from a volume containing a random magnetic field will have structure (for an example, see Tribble 1991). Half of the low-frequency (unaged) emission from a three-dimensional random magnetic field comes from only 23 per cent of the volume. In two and one dimensions the fraction of the volume required for half the emission is 0.2 and 0.13 respectively.

Filamentary structure (Perley et al. 1984; Fomalont et al. 1989; Hines, Owen & Eilek 1990) is observed at high frequencies (for high resolution) where ageing is important. As the electron population ages and the radio spectrum steepens, the emissivity contrast between regions of different field strength increases. The observed emission variations might simply be due to a random magnetic field combined with some spectral ageing. If this is the case, one should probably take a value for the filling factor close to unity.

5.2 Emission in non-isotropic fields

I have also calculated aged spectra for models in which the magnetic field configuration is sheared into a plane (Laing 1980). There are two differences between these and the preceding models. The probability distribution of field strengths is assumed to be given in this case by a two-dimensional Maxwellian. Also, the distribution of field-line orientations is given by

$$P(\theta)=\frac{\sin\theta\,\mathrm{d}\theta}{2\pi\sqrt{\cos^2\theta_s-\cos^2\theta}}, \tag{14}$$

where θ_s is the angle between the line of sight and the plane in which the field is sheared. The range of angles is from $\theta=\theta_s$ to $\theta=\pi-\theta_s$. One can also consider the case where the field is one-dimensional: we then see electrons at one pitch angle only and the probability distribution of field strengths is a simple Maxwellian.

In all cases the resultant spectra are very similar to those of the three-dimensional isotropic models considered in previous sections.

6 CONCLUSIONS

The aged synchrotron spectra from electrons in a strong random magnetic field are flatter than the spectrum from the JP model, and are similar to the KP model spectrum. This agrees well with the recent observations of Cygnus A by Carilli et al. (1991), and explains why the observed spectra are well fitted by the KP model, even though the JP model is more physically plausible.

If losses due to inverse Compton scattering are neglected, two physically distinct models can be distinguished. If the electron positions are localized then the break energy and frequency depend only on the local field strength. The resultant spectra are qualitatively similar to that of the KP model, independent of whether pitch-angle scattering is effective. In particular, the sharp break characteristic of the JP model does not occur. Failure to observe an exponential fall of the spectrum at high frequencies does not rule out the existence of pitch-angle scattering.

The second class of models allows for electrons to move between regions of different field strength. Without pitch-angle scattering, the pitch angle of an electron changes so as to keep the magnetic flux linked by the orbit constant. Such models give KP-type spectra but age somewhat more slowly. Strong pitch-angle scattering tends to prevent diffusion between regions of different field strength, so I have considered models where the energy losses are due to some combination of the global rms field and the instantaneous local field. In this case, a wide variety of spectra are possible, covering the entire range from the basic KP model to the JP model, depending on the diffusion efficiency.

The inclusion of inverse Compton losses gives models that are formally identical to the diffusion models. For sources in which the field is strong and the losses are dominated by synchrotron emission, the spectra should be close to the KP model. This is indeed seen in Cygnus A. For less powerful sources with weaker fields, and sources at high redshift where inverse Compton losses are greater, the spectral decline is much sharper. An interesting point is that the variation of spectral shape with field strength is not monotonic. The steepening is sharpest when synchrotron and inverse Compton losses are about equally important, as then the dependence of break frequency on field strength is weakest.

I have also considered spectral ageing in sheared field configurations as suggested by Laing (1980) to explain the polarization properties of edge-brightened double sources. Again, spectra similar to those of the three-dimensional models result.

In summary, realistic magnetic field configurations can give a spectrum without the sharp break of the JP model. The resultant spectra are very similar to the KP spectrum. The observation of such a spectrum does not imply that the electron pitch angles remain constant. I conclude that the JP model is probably correct, as would be expected, and that the sharp break in the JP model is smoothed out because there is a range of break frequency corresponding to the range of field strength. The sharp break should become more pronounced for sources with lower field strengths when inverse Compton losses are more important.

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