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Radio emission in a random magnetic field: radio haloes and the structure of the magnetic field in the Coma cluster

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SUMMARY

Models of cluster radio haloes have been constructed allowing for a random magnetic field. The resulting halo is far from smooth, both in intensity and polarization. The magnetic field must therefore be tangled on sufficiently small scales to reduce intensity fluctuations to observed levels. The smoothness of the inner regions of the radio halo in the Coma cluster, with fluctuations at about the 5 per cent level at 1-arcmin resolution, indicates that the outer scale of magnetic field fluctuations is less than 15 kpc. Similar considerations indicate that less than 10 per cent of the total halo emission can be due to galactic wakes and demonstrate that the halo emission and the magnetic field are spread throughout the whole volume of the cluster and are not localized with a small filling factor. In the simplest models the fractional polarization should be similar to the fluctuation level, although the polarization can be substantially decreased if the field has a high degree of symmetry.

1 INTRODUCTION

Some of the richest and most luminous X-ray clusters of galaxies contain radio haloes, i.e. diffuse cluster-wide radio emission not associated with individual galaxies. The most prominent and well-studied example is the Coma cluster (Willson 1970; Jaffe, Perola & Valentijn 1976; Valentijn 1978; Hanisch 1980; Cordey 1985; Schlickeiser, Sievers & Thiemann 1987; Henning 1989; Kim *et al.* 1990).

The power-law radio spectrum shows that the emission is synchrotron radiation, so the cluster contains both relativistic electrons and a magnetic field. Lack of associated X-rays from inverse Compton radiation sets a lower limit of approximately 0.1 μG on the field strength (Gursky & Schwarz 1977; Rephaeli & Gruber 1988). The combination of hot gas and magnetic fields leads to an observable Faraday rotation measure (RM) in galaxy clusters which has been detected (Kim, Tribble & Kronberg 1991), although the RMs are smaller than expected if the field known to be present were uniform. The magnetic field is therefore tangled, and resolved RM fluctuations of sources at the centres of cooling flows (Dreher, Carilli & Perley 1987; Taylor *et al.* 1990) indicate that the tangling scale of the field is of order 10 kpc, although it must be emphasized that this value is uncertain. It is also not clear that the magnetic field at cooling flow centres is typical of the general intracluster medium.

The origin of the relativistic electrons and magnetic fields remains unknown. Jaffe (1977) proposed that the relativistic electrons came from radio sources in the cluster and diffused out to fill the cluster volume. Unfortunately the electrons lose energy rather rapidly and have difficulty reaching the outer reaches of the halo if they diffuse at the Alfvén speed, although Holman, Ionson & Scot (1979) showed that the ion sound speed was more appropriate, rescuing the model. Alternatively, Dennison (1980) and Vestrand (1982) proposed that relativistic protons diffused throughout the cluster and produced a secondary population of relativisitic electrons in collisions with thermal protons. In this paper I do not address the formation of the radio halo, but rather consider what the present state of the halo can tell us about the magnetic field structure.

I first consider the properties of radio emission from a volume containing a random magnetic field. I then present simulations of the emission from such a volume and evaluate the variations in intensity. These models are then compared with observations of the radio halo in the Coma cluster and used to constrain the properties of the magnetic field there.

Throughout this paper I assume $H_0 = 50$ km s^{-1} Mpc^{-1}.

2 RADIO EMISSION IN A RANDOM MAGNETIC FIELD

The observed emission at frequency ν from a point with magnetic field B at an angle θ to the line of sight is proportional to

$$\varepsilon = (B \sin \theta)^{n+1} \nu^{-n}, \tag{1}$$

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where the spectral index n corresponds to a power law electron energy distribution with power law index $\gamma = 2n+1$. As the field strength and direction vary from cell to cell of the random field, so will the emitted radio intensity. I assume throughout that the electron pitch angle distribution is isotropic, that the electron energy spectrum is independent of position, and that there is no spectral curvature.

I assume the magnetic field to be turbulent, with the turbulence being homogeneous and isotropic. No mean field is present. I further assume that the magnetic field is a Gaussian random field. This makes it easy to construct examples from a power spectrum. Also, the angles are isotropic and B is drawn from a Maxwellian distribution. The mean emissivity averaging over all angles and field strengths is

$$\langle \varepsilon \rangle = \frac{2(3/2)^{-(n+1)/2}}{n+3} \frac{\Gamma[(n+4)/2]}{\Gamma(3/2)} \nu^{-n} \qquad (2)$$

and the mean square emissivity is

$$\langle \varepsilon^2 \rangle = \frac{(3/2)^{-(n+1)}}{n+2} \frac{\Gamma[(2n+4)/2]}{\Gamma(3/2)} \nu^{-2n}, \qquad (3)$$

where I have set the rms field strength equal to unity. I define the contrast Δ as the dispersion divided by the mean,

$$\Delta = \sqrt{(\langle \varepsilon^2 \rangle / \langle \varepsilon \rangle^2) - 1}. \qquad (4)$$

This is of order unity showing that the possible emissivities of a coherent magnetic cell cover a wide range, the width of the distribution being larger for steeper spectra.

If an emitting region is made up of a number of cells along the line of sight each with weight W_i then the total contrast is easily shown to be

$$\Delta_{\mathrm{T}}^2 = \sum_i W_i^2 \Delta^2 \bigg/ \left(\sum_i W_i \right)^2, \qquad (5)$$

with Δ the contrast for each cell. For N equal cells the contrast is reduced by a factor $1/\sqrt{N}$, as expected. For a Gaussian emissivity profile of equal FWHM the contrast is further reduced by a factor $[(2 \ln 2)/\pi]^{1/4} = 0.81$. If one could measure the contrast in a radio halo then the cell size could be estimated. One complicating factor is that radio haloes are observed with only finite resolution so that, if the cells are small, the contrast is reduced by averaging within the observing beam.

For such a structured magnetic field the emissivity differs from that expected if the field strength is everywhere the same. The ratio is simply the average of the appropriate power of the field,

$$\langle B^{n+1} \rangle = (3/2)^{-(n+1)/2} \Gamma[(n+4)/2] / \Gamma(3/2). \qquad (6)$$

Differences from the uniform field strength case are rather small, so that the mean emissivity gives a good estimate of the rms field strength.

The relativistic electrons lose energy by inverse Compton scattering and synchrotron emission. The inverse Compton scattering is equivalent to synchrotron losses in a 3.2 μG field and will dominate if the magnetic field is weaker than this. This paper considers radio haloes with field strengths of order 1 μG, so that electrons lose energy at a rate approximately independent of the local field strength. I therefore ignore changes in energy losses from cell to cell. If the field is stronger then electrons in the most luminous regions lose energy faster and the emission fades relative to other cells, reducing the intensity contrast. I only consider situations where the field is sufficiently weak that this effect can be neglected, which would not be true for powerful radio sources.

2.1 Halo polarization

The radio emission from a volume element has fractional polarization $p_0 = (3n+3)/(3n+5)$, which is $p_0 = 0.75$ when $n=1$. Also, when $n=1$ it is easy to show that

$$p = p_0 \sqrt{2} \Delta_{\mathrm{T}}, \qquad (7)$$

as long as the correlation length of the field is much smaller than the size of the system. If the halo has many cells along the line of sight then $p \sim \Delta_{\mathrm{T}} \ll 1$.

The observed polarization will be further reduced by two effects: line of sight depolarization and beam averaging. Line of sight depolarization (Burn 1966) only affects the polarization from a single cell – as the polarization position angles from different cells are random, rotating them relative to each other has no effect. In each cell the depolarization is due to half the total RM of the cell,

$$\mathrm{RM_{cell}} \approx 10 \left(\frac{n_{\mathrm{e}}}{2 \times 10^{-3}\ \mathrm{cm}^{-3}} \right) \left(\frac{B}{1\ \mu\mathrm{G}} \right) \left(\frac{r_0}{10\ \mathrm{kpc}} \right) \mathrm{rad\ m}^{-2}, \qquad (8)$$

where n_{e} is the thermal electron density. At a wavelength of 20 cm each cell will suffer very little depolarization, even at the centre of the Coma cluster. There is a slight bias against depolarization in that the strongest emission comes from the cells with small B_z which are depolarized the least. Depolarization by averaging within the beam is expected to have a similar effect to the smoothing of intensity. In these simple models, polarization at about the same level as the intensity contrast is expected.

3 MODELS

I have simulated magnetic fields in a cubical box. The field is constructed by selecting a power spectrum for the vector potential $\boldsymbol{A}$ and choosing components $\tilde{\boldsymbol{A}}(\boldsymbol{k})$ accordingly. The transform $\tilde{\boldsymbol{B}}(\boldsymbol{k}) = i\boldsymbol{k} \wedge \tilde{\boldsymbol{A}}(\boldsymbol{k})$ is then transformed back to the real field using an FFT routine (Press *et al.* 1986). This automatically ensures that the magnetic field is divergence free.

The $\tilde{\boldsymbol{A}}(\boldsymbol{k})$ are chosen according to the prescription

$$P(A_i) = \frac{1}{\sqrt{2\pi}\sigma(k)} \exp\left[\frac{-A_i^2}{2\sigma^2(k)} \right], \qquad (9)$$

where A_i represents either the real or imaginary part of a component of $\tilde{\boldsymbol{A}}$. It is convenient to change to polar coordinates and look at the amplitude and phase of A,

$$P(A, \phi)\, dA d\phi = \frac{A}{\sigma^2(k)} \exp\left[\frac{-A^2}{2\sigma^2(k)} \right] dA \frac{d\phi}{2\pi}. \qquad (10)$$

Therefore ϕ is uniformly distributed between 0 and 2π and A is drawn from a Rayleigh distribution. A Gaussian power spectrum was used,

$$\sigma^2(k) = k^2 \exp(-k^2/k_0^2). \qquad (11)$$

This is equivalent to a Gaussian longitudinal magnetic field autocorrelation function $f = \exp(-r^2/2r_0^2)$, which defines the field scalelength r_0. With this definition the RM autocorrelation function varies as $1 - r^2/r_0^2$ near the origin. The scale k_0 is adjusted to vary the field correlation length relative to that of the grid. Because the grid is of finite size, only a small range of scale sizes can be investigated while keeping a reasonable number of grid points per cell and a reasonable number of cells in the box. The results were found to be similar for a range of power spectra, although none of the power spectra had power on a large range of scales.

I have smoothed the maps with a Gaussian observing beam of FWHM w. A simple analytic approximation to the reduction in contrast is

$$\Delta_S = \Delta_0 / \sqrt{1 + w^2/r_0^2}, \tag{12}$$

where r_0 is the field scalelength, Δ_0 the map's intrinsic contrast and Δ_S the observed contrast after smoothing. For the observations by Kim *et al.* (1990) analysed below, $w = 1$ arcmin $\equiv 40$ kpc.

I present models that have the same overall profile as the Coma radio halo, approximated as a circular Gaussian of FWHM 16 arcmin (Kim *et al.* 1990), and smoothed to 1-arcmin resolution, for a variety of field scale sizes. These models assume a spectral index $n = 1$ which is a compromise between the slightly flatter spectral index of $n \approx 0.7$ at the centre of the halo and the value of $n \approx 1.3$ for the halo as a whole (Kim *et al.* 1987). This approximation is not a serious concern, as Section 2 shows that the variation of contrast with spectral index is reasonably small over the range of interest. Examples are shown in Figs 1, 2 and 3. These maps ae constructed by periodically extending a half-size emission map, shaping to the desired profile and smoothing to 1-arcmin resolution. The emission contrast is then corrected from a rectangular to a Gaussian emission distribution along the line of sight.

4 COMPARISON WITH THE COMA RADIO HALO

The most detailed observations (in the sense of having the greatest resolution) are the 1.4-GHz observations by Kim *et al.* (1990), and these will be considered here. Lower-frequency maps cover a larger area, because the halo is effectively larger at lower frequency, and possibly indicate some structure at large radii (Cordey 1985; Hanisch 1987). This structure might not, however, be indicative of small-scale structure in the magnetic field but could simply be large-scale features related to the spatial distribution of the relativistic electrons, which is not what I am considering in this paper. In addition, the Coma cluster will not have reached equilibrium at large radii so there will still be traces of the cluster formation process.

The noise level of the Coma radio halo has been calculated from the map of Kim *et al.* (1990). Regions near strong radio sources were avoided, and the contrast calculated after fitting to the overall profile of the halo. As the Coma cluster has two strong sources (5C4.81 and 5C4.85) at its centre, the contrast was calculated in regions about 8 arcmin from the cluster centre where the halo flux at 1.4 GHz is approximately 2mJy per 1 arcmin beam. The contrast was found to be similar in all the regions examined, and shows structure to be present at a contrast level of approximately 5 per cent.

The measured contrast does not give the intrinsic fluctuations in the halo, as thermal noise, errors from the discrete source subtraction, and Poisson noise from unremoved cluster and background sources will be present. The residual noise level due to spurious features is quoted as 30 μJy per beam (Kim *et al.* 1990), close to the thermal noise level. The actual noise level on the map (outside the halo and away from strong sources) is reasonably uniform at approximately 70 μJy per beam.The 5 per cent contrast represents a dispersion of 100 μJy on a mean level of 2mJy per beam, so that some of the observed contrast is probably real. Source sub-

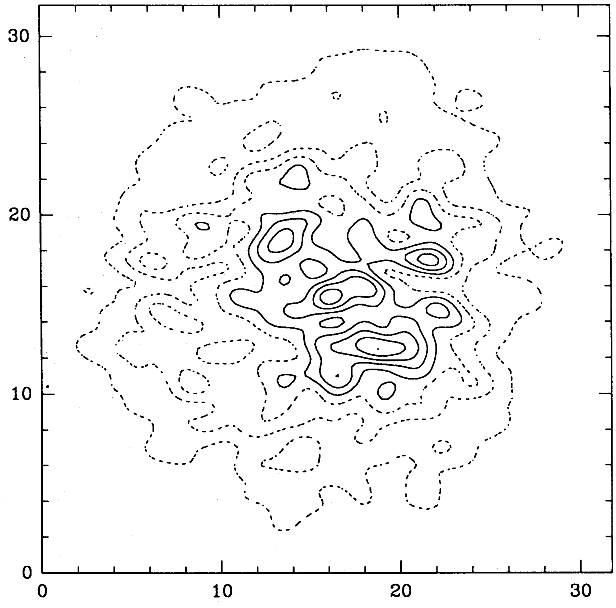

Figure 1. A simulation of the Coma radio halo at 1 arcmin = 40-kpc resolution, for a magnetic field scalelength of 40 kpc. The scale is in arcmin, the solid contours are the higher values and the dashed contours the lower values.

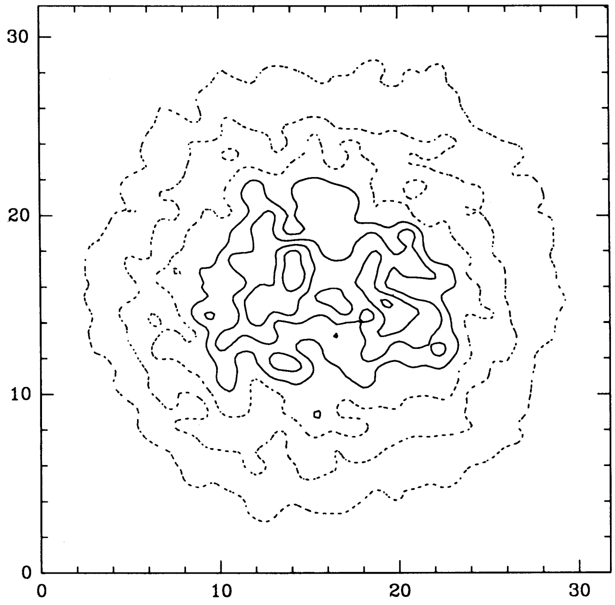

Figure 2. A simulation of the Coma radio halo at 1 arcmin = 40-kpc resolution, for a magnetic field scalelength of 25 kpc.

traction errors should not be a problem as affected areas of the map were not used, although one or two sources with fluxes of 1mJy or less might be present. The observed low level of fluctuations rules out a significant contribution from individual sources to the halo. According to Hanisch (1980), no more than 10 mJy of the total halo flux is due to cluster sources, which would lead to fluctuations smaller than the thermal noise level. Weak background sources could also lead to small fluctuations.

As some of the observed contrast in the image might be due to other causes than granularity in the halo, I take the observed 5 per cent contrast level as an upper limit and ask what field configurations are consistent with this limit. Generally, the number of cells must be large enough to reduce the noise below this level, so that an upper limit may be set on the field scale size.

The simplest possible model is that in which the field is uniform over a cell of size r_0 with random strength and direction. The observed fluctuations will be the cell to cell fluctuations reduced by averaging over the number of cells in the beam, both along the line of sight and in the plane of the sky. The intrinsic fluctuation level for a spectral index of unity is 1.1, so that for the depth of 640 kpc, the field scale size must be less than 16 kpc. This simple-minded estimate agrees with that obtained by taking the modelled emission in a box smoothed to the required resolution, and is shown in Fig. 4.

A second method is to calculate the fluctuations in my models with the same profile as the Coma radio halo, in the same way as for the observational data. I show, in Fig. 4, the fractional fluctuations of the simulations as a function of the scale size. Again, the scale length of the field must be less than about 15 kpc. This conclusion is also obtained by comparing the simulations presented in this paper with the map of Kim *et al.* (1990). The simulations with the larger scalelengths are much less smooth than the observations, and those simulations with $r_0 > 30$ kpc tend to be multiply peaked. The emission in the two simulations with magnetic field scalelengths of 40 and 25 kpc is clearly far more clumped than the Coma radio halo.

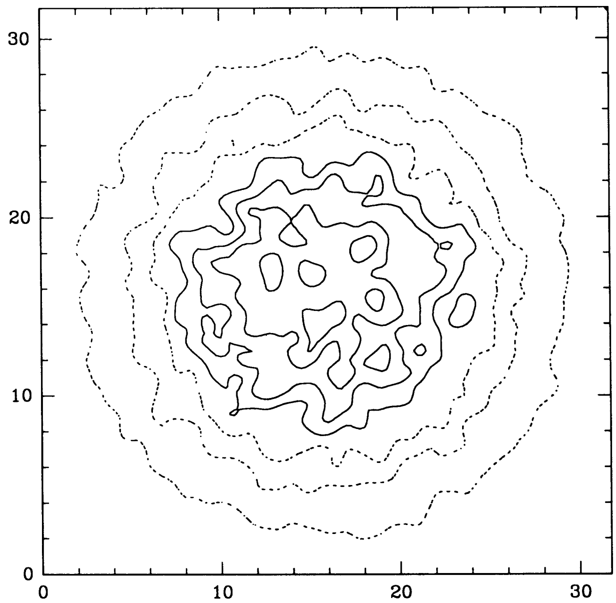

Figure 3. A simulation of the Coma radio halo at 1 arcmin = 40-kpc resolution, for a magnetic field scalelength of 16 kpc.

5 GALACTIC WAKES

It has been suggested that turbulent wakes are excited by galaxies as they move thrugh the intracluster medium and that these wakes tangle the field (Jaffe 1980), accelerate electrons (Roland 1981) or drive a dynamo (Ruzmaikin, Sokoloff & Shukurov 1989).

If the wake does no more than generate turbulence in the gas which winds a pre-existing field then the field will straighten and return to its original configuration on timescales of only a few 10^8 yr (Jaffe 1980). If the induced turbulence drives a dynamo then the effect is completely different as an irreversible change in the field's topology occurs. In either case, the filling factor of the wakes is rather small so that a filamentary network of enhanced field is set up throughout the cluster. One should see, at high enough resolution, a patchy halo with holes (Roland 1981). Furthermore, because the density of galaxies falls with increasing radius, the halo should become more ragged at the edges.

The characteristics of a halo from the wakes proposed by Roland (1981) will now be examined. The halo contains approximately 200 cells at 1-arcmin resolution, and each of 150 galaxies has a tail about 200 kpc or 5 resolution elements long. Each beam contains a contribution from 4 ± 2 wakes, leading to a predicted halo contrast of 50 per cent. This is very much larger than the granularity observed, so that the Coma radio halo cannot be made up solely of wakes behind galaxies. Using the upper limit of 5 per cent for the contrast derived in the previous section and adding a

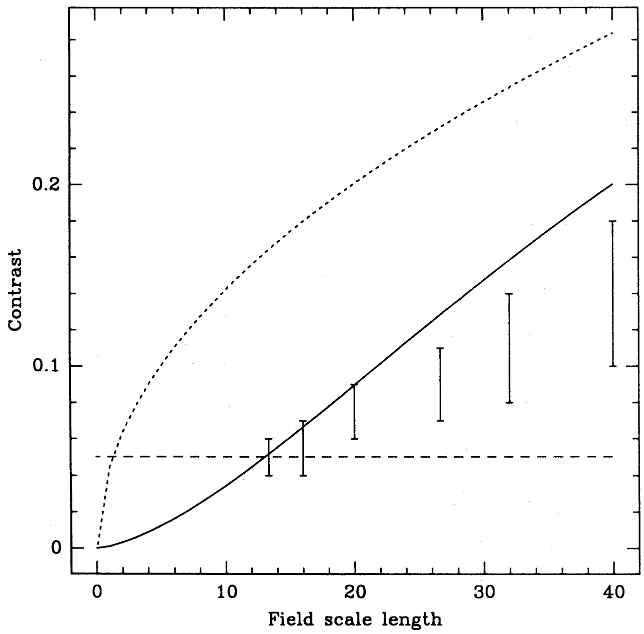

Figure 4. The predicted halo contrast as a function of field scalelength r_0 smoothed to 1-arcmin resolution from the models discussed in the text (solid line). The short-dashed line shows the intrinsic contrast (unaffected by smoothing). Error bars are the fluctuation levels calculated from the simulated haloes. The long-dashed line shows the upper limit on the contrast in the Coma radio halo, leading to an upper limit on the field scale size.

uniform background to reduce the contrast, I find that the wakes can be responsible for no more than 10 per cent of the total halo emission.

This last conclusion does not mean that wakes are ruled out or even unimportant. The emission contrast between the wakes and the general cluster could still be large if the filling factor of the wakes is correspondingly small. If d is the average diameter of the wake then the filling factor is

$$f \approx (d/50\ \mathrm{kpc})^2. \tag{13}$$

For example, if $d=5$ kpc then the wakes only fill 1 per cent of the cluster and could have volume emissivities 10 times greater than the more diffuse halo material while still being responsible for only 10 per cent of the total emission, so that it is difficult to set limits on the field or emission enhancement in galactic wakes. It is clear, however, that galactic wakes as envisaged by Roland (1981) are responsible for only a small fraction of the observed emission. In a similar fashion, the limits on the contribution of discrete cluster sources are even stronger, in agreement with Hanisch (1980) who found that less than 1 or 2 per cent of the halo emission could be due to cluster galaxies.

This last conclusion, that at least 90 per cent of the halo is due to truly diffuse emission, has important consequences. It shows that the relativistic electrons and magnetic fields are truly a cluster-wide phenomenon, not just local to specific regions. The inverse Compton lower limit on the field strength applies throughout the cluster volume. The field is not just localized to specific regions such as wakes or galactic debris. In addition, it constrains models of the origin of the relativistic electrons to allow only those which would fill the cluster. This agrees with the results of Valtaoja (1984) who found that diffusion models best fitted the extent of the halo, albeit with a large diffusion coefficient. As noted by Hanisch (1982), the electrons are guided by the field and must also diffuse through the tangled field structure in a similar way to thermal electrons (Tribble 1989). In the context of the present results, a large diffusion coefficient would help smooth out a set of initially localized sources of relativistic electrons.

The dynamo model of Ruzmaikin *et al.* (1989) leads to a set of turbulence scales. The constraint derived in the previous section cannot be directly applied to a field tangled on several different scales because, although the field might be correlated on scales of 20 kpc, the dominant field inhomogeneities are those on much smaller scales. In a similar manner, the observations cannot rule out a uniform component of the field, as long as small-scale structure is present. The dynamo model of Ruzmaikin *et al.* goes further and requires the field to be concentrated in intense ropes with a small filling factor. The emission contrast between the ropes and the background is much stronger than the contrast between strong and weak field regions for a Gaussian field, although the filling factor is small so that the bulk of the emission might still come from the low-field regions. The important factor here is that the small-scale structure is not random but is strongly correlated on larger scales, so that the constraint derived on the coherent structure applies in this case also and limits the outer scale of the turbulence. Indeed, the constraint is even stronger because the intrinsic contrast given by this kind of structure is greater than for a Gaussian field.

The electrons in the intense ropes, with field strengths of perhaps 10 μG, will lose their energy much faster than the electrons outside the ropes which have their energy losses dominated by inverse Compton losses. Thus the ropes will fade and all the emission will come from the low-field-strength regions. The constraint on the cell size still applies, however, because the field in the cells will be correlated on the same scale as the ropes.

6 RELATION TO ROTATION MEASURE AND POLARIZATION DATA

Kim *et al.* (1990) and Kim *et al.* (1991) derived magnetic field properties from RM observations. The excess RM due to a magnetic field in the intracluster gas gives a measurement of the quantity $B^2 r_0$. To derive the magnetic field strength B or the scalelength of the field r_0 requires another measurement of a different combination of these two quantities. Radio source depolarization is one possibility (Tribble 1991). The observation of granularity in a radio halo is another that I consider here. The observations require that $r_0 < 15$ kpc. This scalelength differs from that used in the RM analysis which is $r_0' = \int f\, dz$; the two are related by a factor of order unity. If f is Gaussian then $r_0' = \sqrt{2\pi} r_0$ so the limit translates to $r_0' < 40$ kpc. Even for this relatively weak limit on the scale length of the field, the magnetic field in the Coma cluster is still required to have an rms strength exceeding 1 μG.

No polarized emission has been detected from the halo. Kim *et al.* (1990) report that the polarization at 20 cm and 1-arcmin resolution varies from less than 1 per cent at the centre to less than 30 per cent in the outer halo. As discussed in Section 2.1, the simplest models predict that the halo polarization should be about the same as the intensity contrast, so that polarization about the 5 per cent level might be expected. This is not observed and one might be tempted to conclude that the intensity contrast is really only about 1 per cent. Unfortunately this conclusion is not very robust, as both galactic wakes and dynamo fields give low levels of polarization. In the case of the dynamo field, the field is wound into ropes from which the polarization cancels by symmetry – the same would be true of a single turbulent vortex.

7 CONCLUSIONS

I have considered the small-scale structure of radio haloes from analytic estimates and simulations. Because the magnetic field is not uniform, the strength of emission varies from one coherent cell of the field to another. This leads to granularity in the observed emission, which is reduced if the cell size of the field is smaller. The smoothness of the central parts of the Coma radio halo, having fluctuations of approximately 5 per cent or less about the mean level, indicates that the scalelength of the field is less than about 15 kpc. Further high-resolution observations would be valuable, as substantial emission structure and possibly polarization will be present in the halo even for a scalelength of 15 kpc that is presently smeared by poor resolution, and the limits derived here are close to the currently accepted values for the scale of the field, namely 10–20 kpc. In combination with RM data, a lower limit of approximately 1 μG can be set on the rms magnetic field strength in the Coma cluster.

Models in which the halo is due to overlapping wakes were considered and the wakes shown to emit less than 10 per cent of the total flux, showing that the halo emission and the magnetic field are spread rather uniformly throughout the entire cluster rather than being localized. Dynamo models in which the field is concentrated in intense ropes are also constrained to have an outer turbulent scale less than 15 kpc.

The shortcomings and limitations of the models must be taken into account. The structure of the field is simple, being characterized by a single scale. A range of scales, as might be expected in turbulence, could look different. Even here, though, the field has coherence up to the outer scale on which energy is put into the gas, so that structure is expected on these scales.

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