1 Knot 4_1

- Generators: a, b
- Relator: $abbbaBAAB$
- Alexander polynomial: $t^2 - 3t + 1$

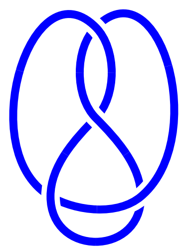

Figure 1: Knot 4_1

In this section, we consider the knot 4_1 with generators a, b and the relator $abbbaBAAB$. The associated Alexander polynomial is $t^2 - 3t + 1$, and Figure 1 illustrates the geometric representation of this knot. We interpret each symbol in the relator $abbbaBAAB$ as an operator: a as a multiplicator $\otimes_t$, b as an additor $\oplus_1$, A as a divisor $\oslash_t$, and B as a subtractor $\ominus_1$, where t is an indeterminate variable. In the arithmetic expression space H, this relator corresponds to a closed loop path, viewed as a threadlike expression.

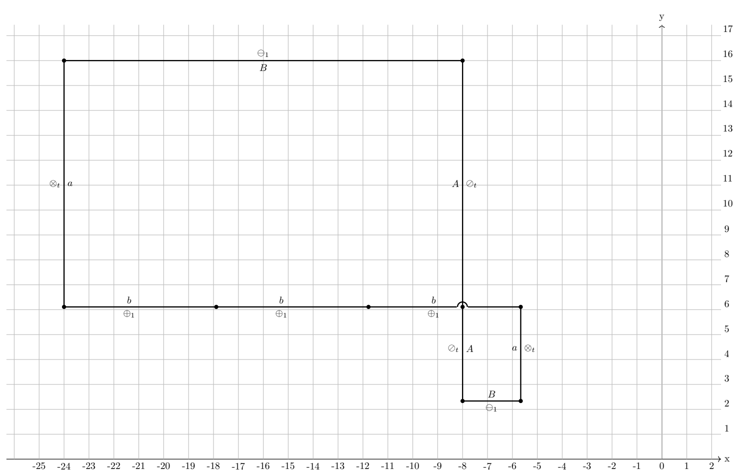

Figure 2: Relator in the arithmetic expression space H

Applying the group action on a real number x from right to left gives:

$$(((x-1)t^{-1}t^{-1}-1)t+1+1+1)t=x,$$

which forces the Alexander polynomial t^2-3t+1 to vanish. Solving $t^2-3t+1=0$ yields $t=\frac{3\pm\sqrt{5}}{2}$. Equivalently, letting ϕ denote the golden ratio, we have $t=\phi^2$ or $t=\phi^{-2}$.

Although we have shown how applying the relator $abbbaBAAB$ as an arithmetic operation leads to the equation $t^2-3t+1=0$, which is a vanishing Alexander polynomial for the knot 4_1, the deeper reason behind this connection remains unclear. We conjecture that a more geometric interpretation of the Alexander polynomial—and how it encodes the knot's topological features—will illuminate why this algebraic path in H corresponds precisely to the polynomial that characterizes the knot's properties.