Arithmetic expression geometry with an application on learnable non-linearity

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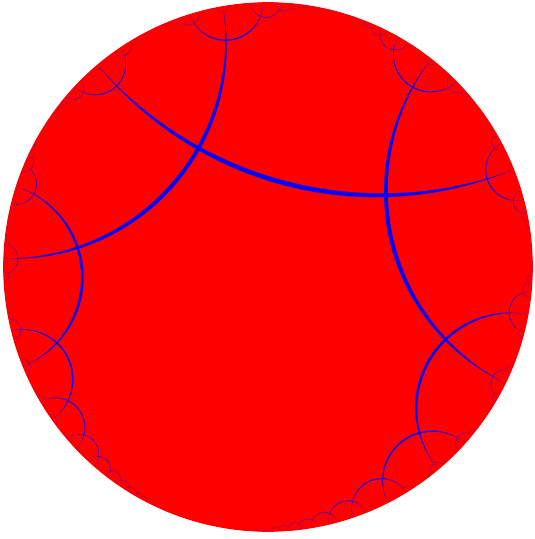


The famous example of word2vec

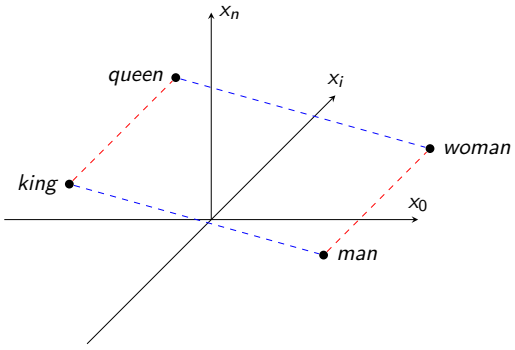


Figure: regulairty of word2vec

$$(\alpha + 1) \times 2 \neq \alpha \times 2 + 1$$

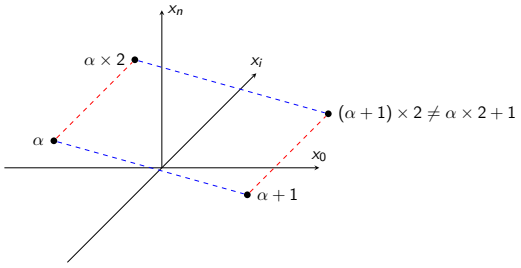
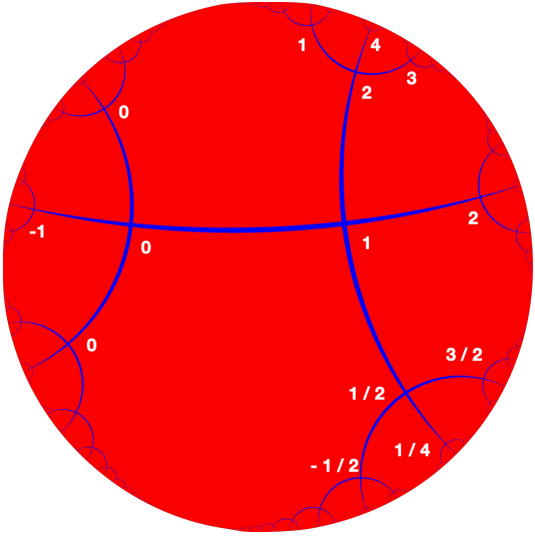
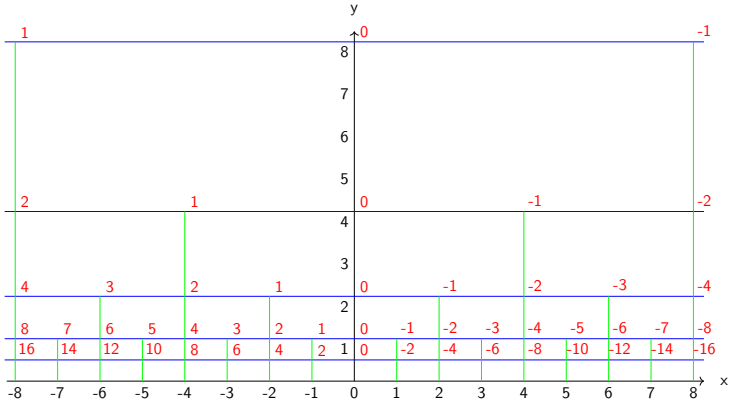


Figure: contradiction of numbers in Euclidean space

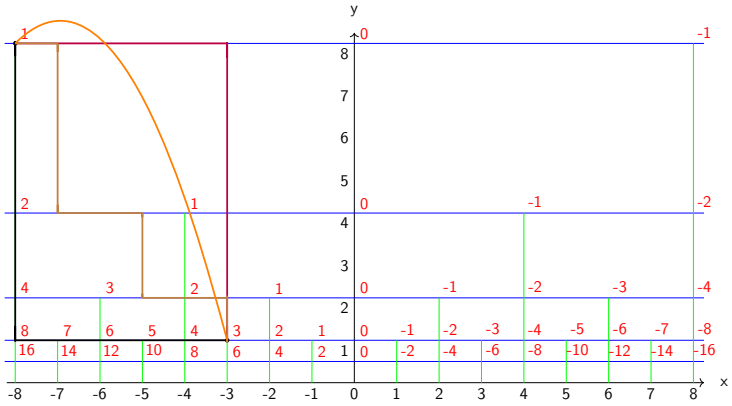
One arrangement in hyperbolic space



$$a = -\frac{y}{x}$$


y
x
1
0
-1
2
1
0
-1
-2
4
3
2
1
0
-1
-2
-3
-4
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
16
14
12
10
8
6
4
2
1
0
-2
-4
-6
-8
-10
-12
-14
-16
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8

- black line $1 \times 8 - 5 = 3$



$$a_\delta = (a_0 + \mu\epsilon\cos\theta)e^{\lambda\epsilon\sin\theta}$$

$$a_\delta = a_0 e^{\lambda\epsilon\sin\theta} + \mu\epsilon\cos\theta$$

Both formula can be simplified to the same result:

$$a_\delta = a_0 + \epsilon(a_0\lambda \sin\theta + \mu\cos\theta)$$

Then, we have the following equation:

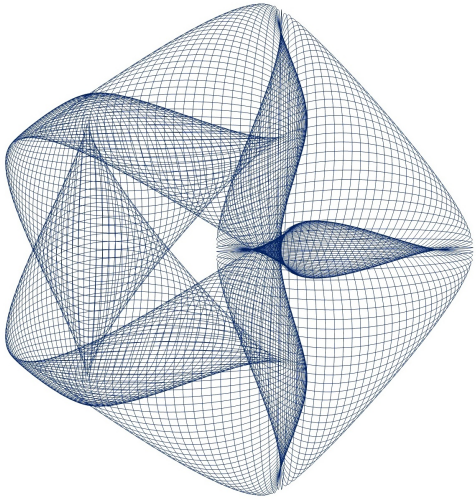
$$\frac{1}{\delta}(a_\delta - a_0) = \frac{\epsilon}{\delta}(\mu\cos\theta + x_0\lambda\sin\theta)$$

When both δ and ϵ are towards zero, we get da/dt, and hence

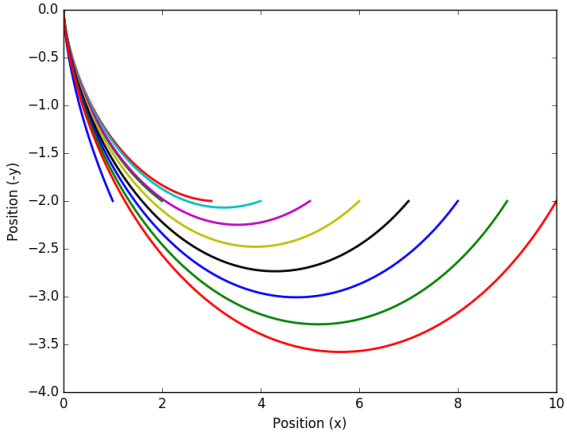
$$\frac{da}{dt} = u(\mu\cos\theta + a\lambda\sin\theta)$$

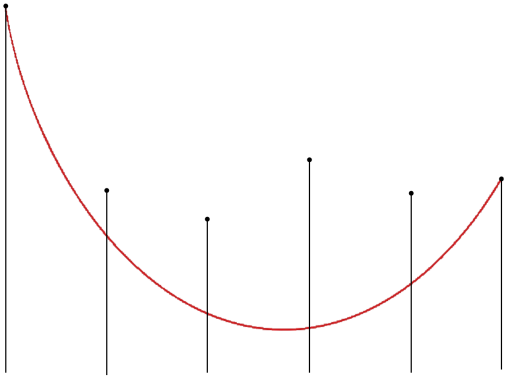
Or, we can change it to another form

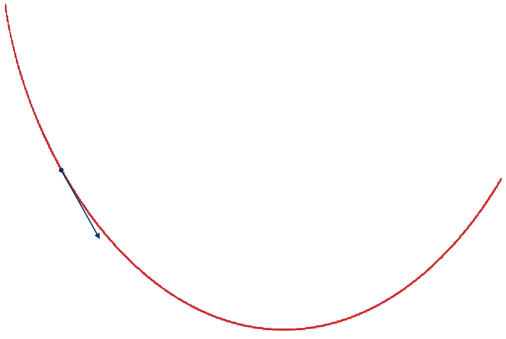
$$\frac{da}{ds} = \mu\cos\theta + a\lambda\sin\theta \tag{1}$$



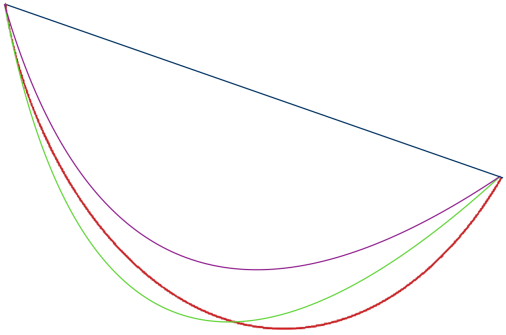
Brachistochrone problem



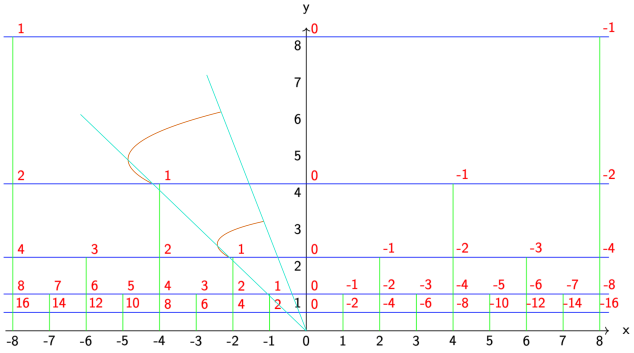




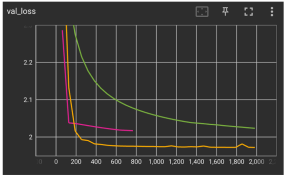
Trap

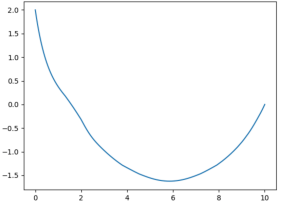


One way reachable



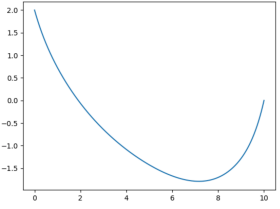
Experiment results

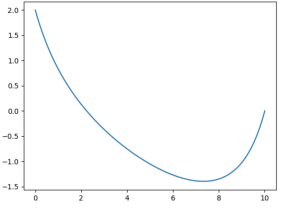




Learnable non-linearity
+ tensor linearity

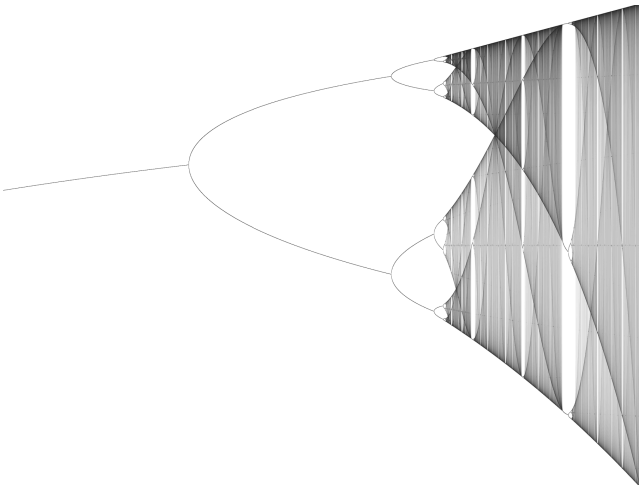
LSTM





Learnable non-linearity
+ matrix linearity

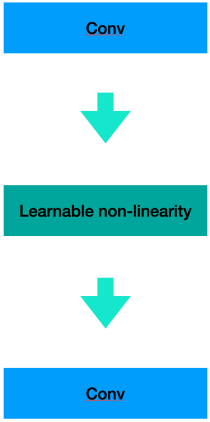
A learnable non-linearity

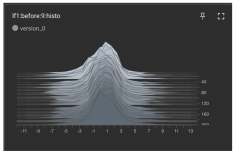


Experiment results

	logistic map p = 3.5	logistic map p = 3.8	logistic map p = 3.9	spline	learnable non-linearity
task success	✗	✓	✓	✗	✓
parameter learnable	✗	✗	✗	✗	✓

Hebbian?


Conv
Learnable non-linearity
Conv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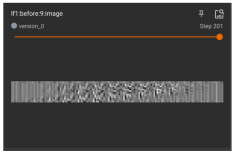
version_0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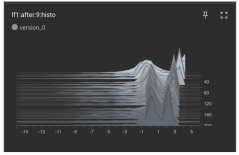
Step 201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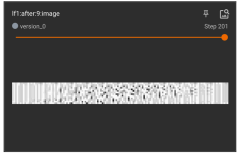
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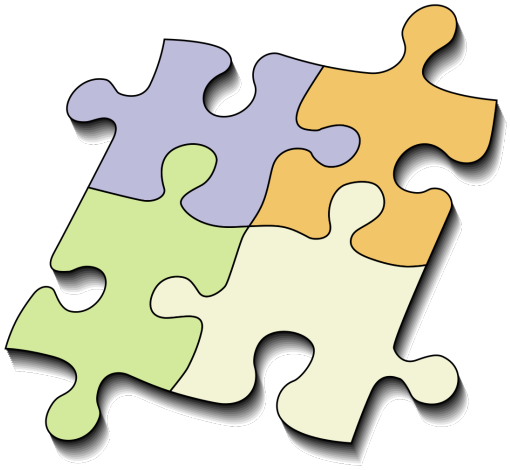
Step 201



Tube structure

- The efficiency of gradient learning