Arithmetic expression geometry with an application on learnable non-linearity

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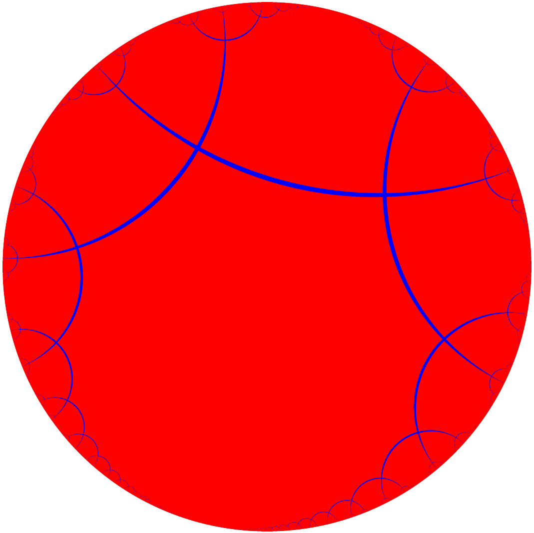
Arithmetic expression geometry: the first glimpse



The famous example of word2vec

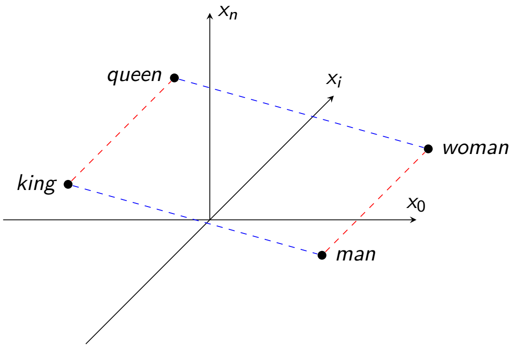


Figure: regulairty of word2vec

$$(\alpha + 1) \times 2 \neq \alpha \times 2 + 1$$

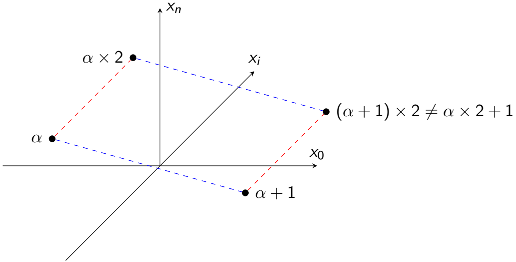
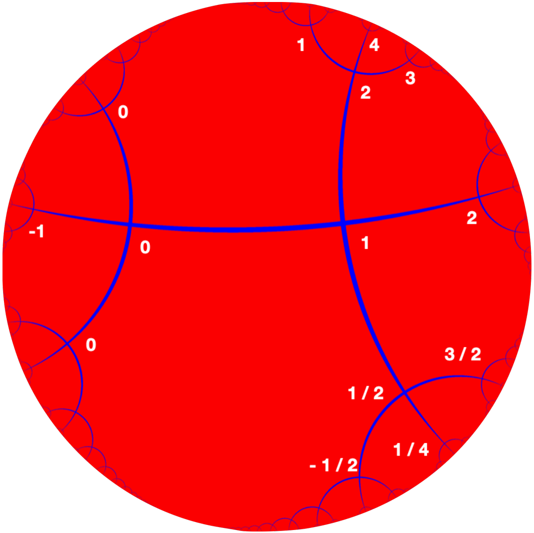


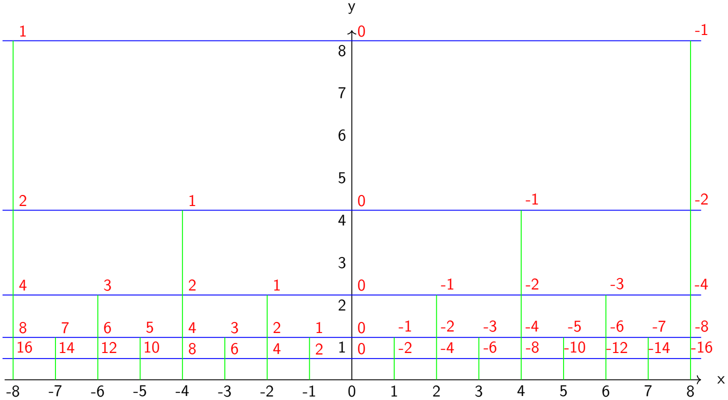
Figure: contradiction of numbers in Euclidean space

One arrangement in hyperbolic space



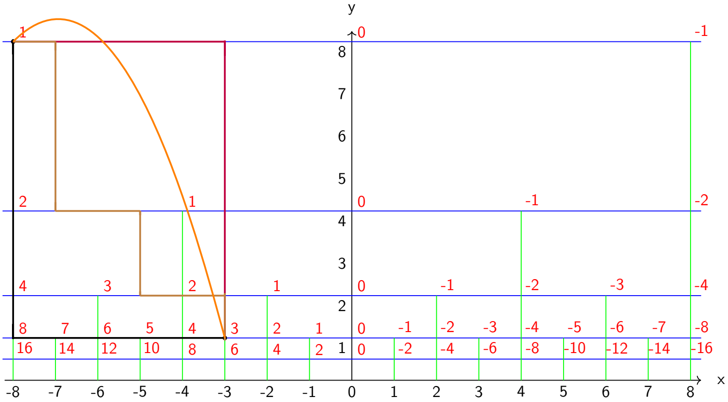
Another arrangement in hyperbolic space

$$a = -\frac{x}{y}$$



Encoding threadlike expressions as paths

- black line $1 \times 8 - 5 = 3$



$$a_\delta = (a_0 + \mu\epsilon\cos\theta)e^{\lambda\epsilon\sin\theta}$$

$$a_\delta = a_0 e^{\lambda\epsilon\sin\theta} + \mu\epsilon\cos\theta$$

Both formula can be simplified to the same result:

$$a_\delta = a_0 + \epsilon(a_0 \lambda \sin\theta + \mu \cos\theta)$$

Then, we have the following equation:

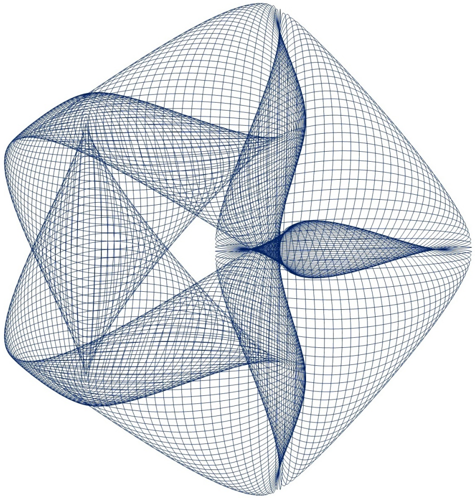
$$\frac{1}{\delta}(a_\delta - a_0) = \frac{\epsilon}{\delta}(\mu \cos\theta + x_0 \lambda \sin\theta)$$

When both δ and ϵ are towards zero, we get da/dt, and hence

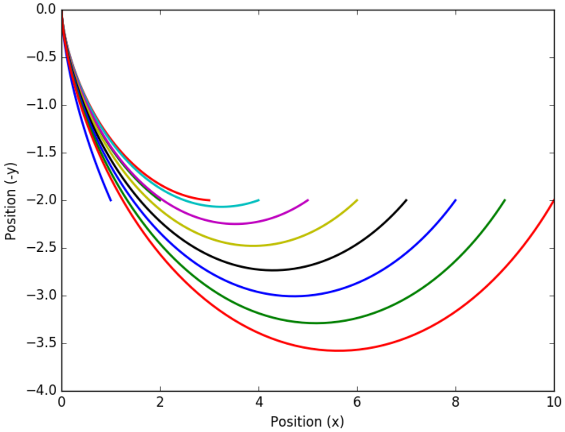
$$\frac{da}{dt} = u(\mu \cos\theta + a\lambda \sin\theta)$$

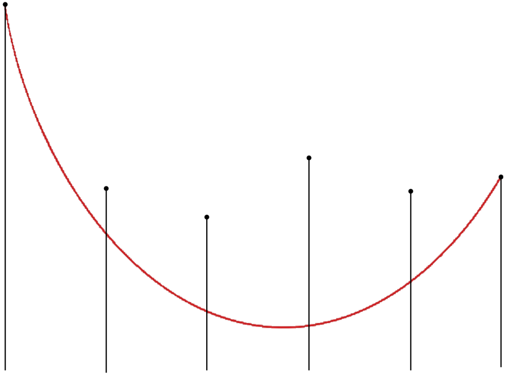
Or, we can change it to another form

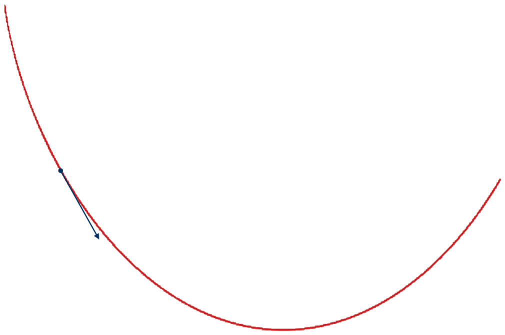
$$\frac{da}{ds} = \mu \cos\theta + a\lambda \sin\theta \tag{1}$$

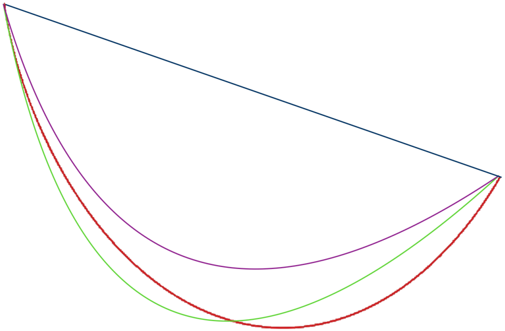


Brachistochrone problem

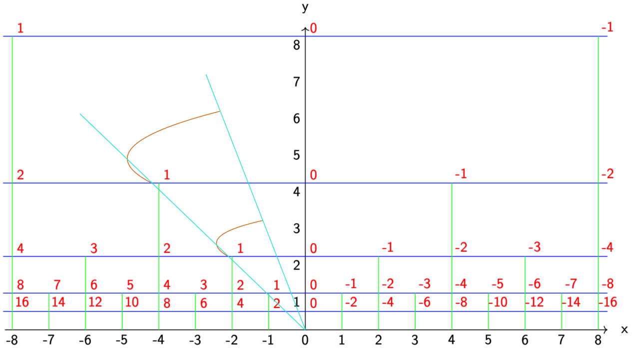




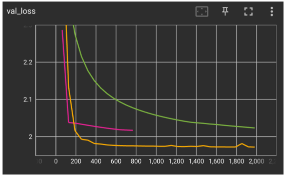


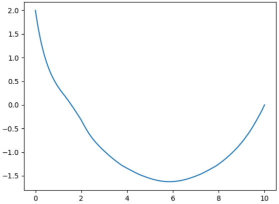


One way reachable



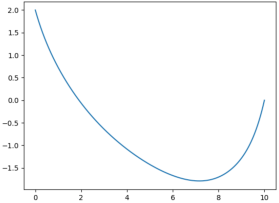
Experiment results

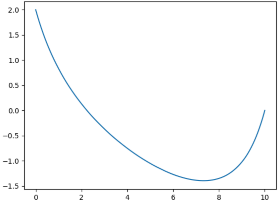




Learnable non-linearity
+ tensor linearity

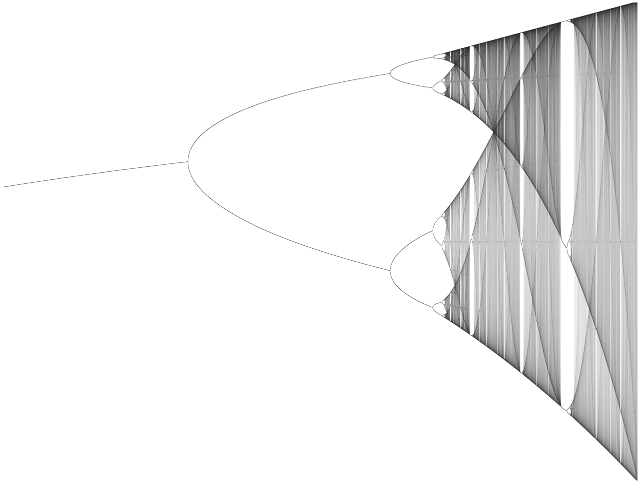
LSTM





Learnable non-linearity
+ matrix linearity

A learnable non-linearity



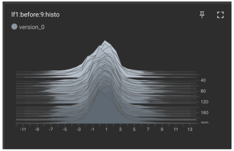
	logistic map p = 3.5	logistic map p = 3.8	logistic map p = 3.9	spline	learnable non-linearity
task success	✗	✓	✓	✗	✓
parameter learnable	✗	✗	✗	✗	✓

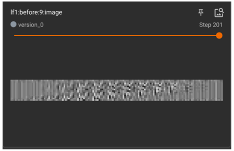
Hebbian?

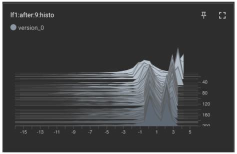
Conv

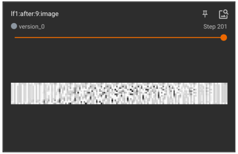
Learnable non-linearity

Conv

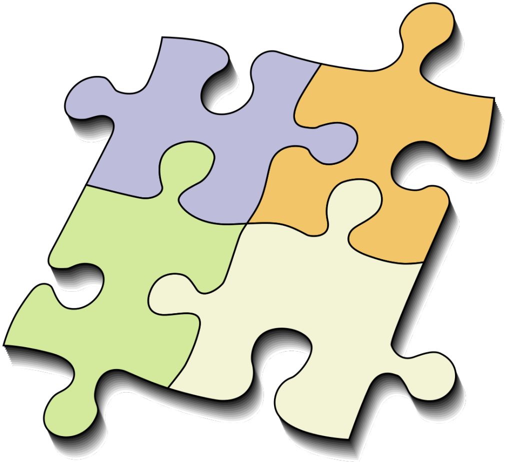








Arithmetic expression geometry: more topics



On the hyperbolic plane

$$ds^2 = \frac{dx^2 + dy^2}{y^2}$$

the Laplacian is

$$\Delta = -y^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$$

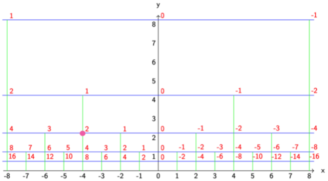
Given

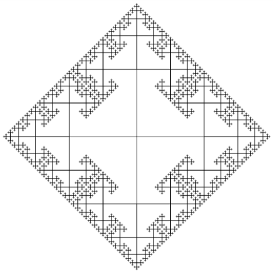
$$A = -\frac{x}{y} \tag{2}$$

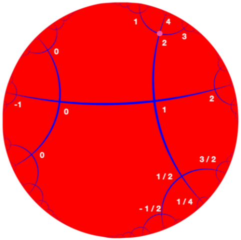
We have

$$\Delta A = -y^2\left(\frac{\partial^2}{\partial x^2}A + \frac{\partial^2}{\partial y^2}A\right) = y^2\left(\frac{1}{\partial y}\left(\frac{1}{\partial y}\frac{x}{y}\right)\right) = 2A$$

A jigsaw by Riemann mapping







Tube structure