



机器能数数吗？

苑明理

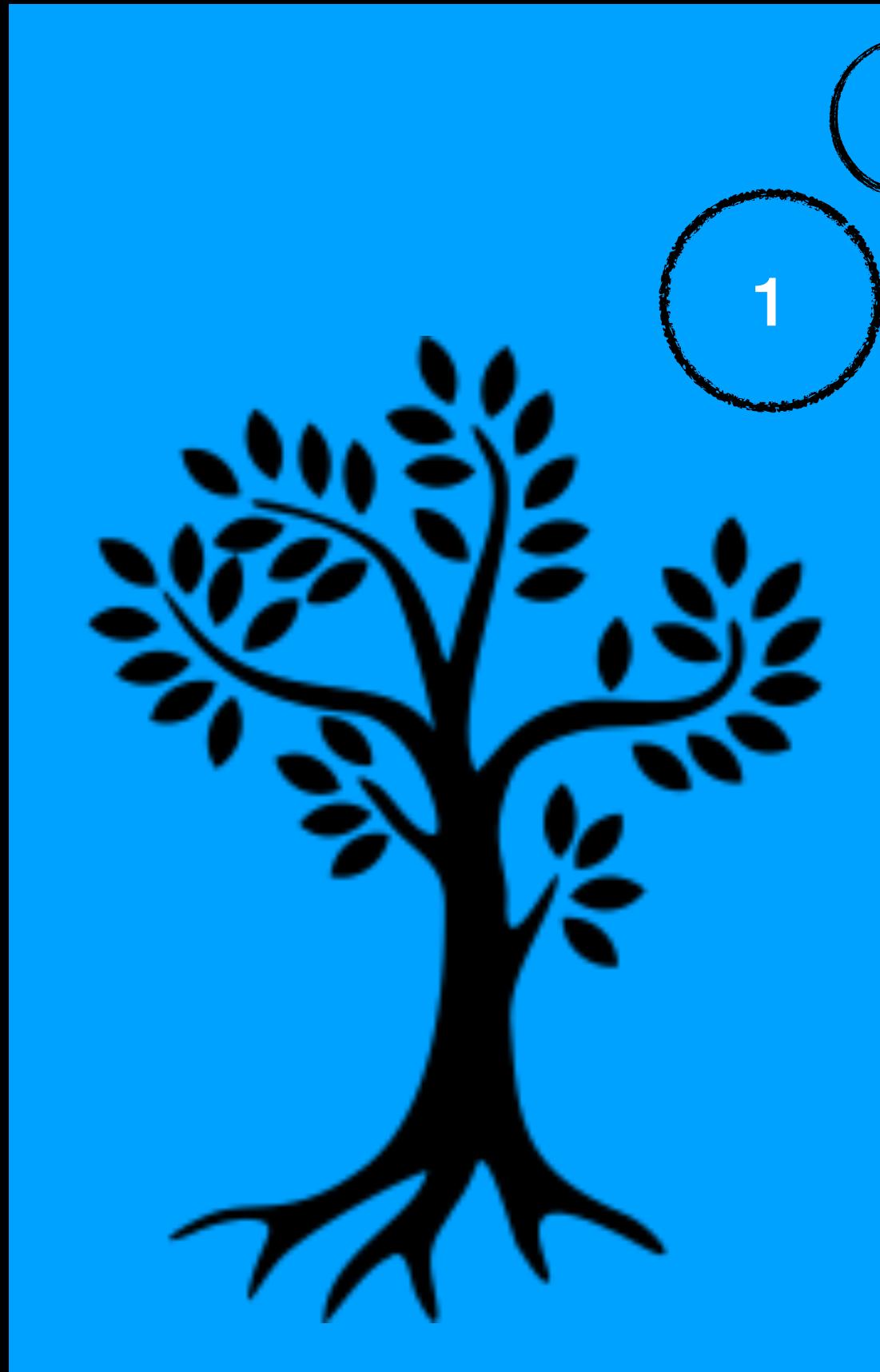
集智年会分享
2021 年 01 月

目录

- 数的演化史：生物、认知、考古
- 机器数数问题：什么是数数与几种形式定义
- 绕过困难的新思路：塞伦盖蒂游戏
- 致谢与参考文献

第一节

数的演化史



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- 生物学的例子
- 认知科学里的数感
- 考古学里的例子
- 复杂性的恰当估计

本节致谢

夏海宁（洛伦兹蝶）老师提供的翻译方案
王东老师的有趣讨论

捕蝇草

捕蝇草是一种食虫植物。它的捕捉机构由叶子的末端部分形成，通过其内表面的微小毛发触发。当昆虫爬行在页面时接触到一根触毛时，捕虫器就会准备关闭，只有在第一次接触后 20 秒内发生第二次接触时，捕鼠器才会关闭。在五次刺激之后捕蝇草才会开始消化过程。



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doi: 10.1016/j.cub.2015.11.057. PMID: 26854557

The Venus Flytrap *Dionaea muscipula* Counts Prey-Induced Action Potentials to Induce Sodium Uptake

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Summary
Carnivorous plants, such as the Venus flytrap (*Dionaea muscipula*), depend on an animal diet when grown in nutrient-poor soils. When an insect visits the trap and tilts prey repeatedly touch the trigger hairs over the subsequent hours, leading to a hermetically closed trap, via the gland-based endocrine system is flooded by a prey-decomposing acidic enzyme cocktail. Here, we asked the question as to how many times trigger hairs have to be stimulated (e.g., how many APs are required) for the flytrap to recognize an encaged object as potential food, thus making it worthwhile activating the glands. By applying a series of trigger-hair stimulations, we found that the touch hormone jasmonic acid (JA) signaling pathway is activated after the second stimulus, while more than three APs are required to trigger an expression of genes encoding prey-degrading hydrolases, and that this expression is proportional to the number of mechanical stimulations. A decomposing animal contains a sodium load, and we have found that these sodium ions enter the capture organ via glands. We identified a flytrap sodium channel (*DiSCK1*) as responsible for this sodium acquisition, with the number of transcripts expressed being dependent on the number of mechano-electric stimulations. Hence, the number of APs a victim triggers while trying to break out of the trap identifies the preying prey as a struggling “Na⁺-rich” animal and nutrition for the plant.

Video Abstract

Introduction
During evolution, plants developed sensors to recognize mechanical forces. Such forces can be imposed on plants from environmental clues such as wind and barriers in the soil to animal movement and herbivory. The touch causes an immediate rise in jasmonic acid (JA) biosynthesis [1–2, 13].

The model plant species *A. thaliana* touch without any specialist cell types or organs. Tendrils of climbing plants, however, have specialized buds that allow a conversion of weak mechanical forces into movement of the climbing organ toward a given support. A similar tendril response can be elicited when, in the absence of touch, jasmonates are administered [14]. Like climbing plants, some carnivorous plants are also equipped with the sophisticated mechano-sensitivity structures and touch signaling mechanisms [15–17].

When Charles Darwin observed the Venus flytrap, *Dionaea muscipula*, he was fascinated by this plant's ability to sense and catch animals to circumvent the limitations of its nutrient-poor habitat [1]. The tips of *Dionaea* leaves have developed into bladelike snap-type capture organs, equipped with a number of mechano-sensitive hairs and a densely packed array of glands. Insects are attracted to the capture organ by a fruity volatile blend emitted by starving flytraps [18]. Visitors searching for food eventually touch the trigger hairs leading to the electric excitation of the trap; the mechanical stimulus is converted into an all-or-nothing action potential (AP). Once two APs are elicited within 15–20 s of each other, the trap lobes close within a fraction of a second [19]. Ongoing mechanical stimulation of the trigger hairs by the insect as it tries to escape initiates the hermetical sealing of the capture organ into a green stomach. Secretion of acid and digestive enzymes into the stomach breaks the entire insect down. By establishing this outstanding apparatus, carnivorous plants are able to ingest the prey's components. In addition to carbon, nitrogen, phosphate, and prey-engorged APs as identity visitors as food sources, to capture and process them, and to innervate nutrient cations from the prey. We found that counting and integrating the mechano-electric signals elicited by the trapped prey, *Dionaea muscipula* triggers biosynthesis of the touch hormone jasmonate, secretion of lytic enzymes, and channel-mediated uptake of prey nutrient-associated sodium loads.

Results

Insect Capture Electrically Excites the Venus Flytrap

In order to investigate the Venus flytrap's signaling cascade involved in the prey capturing and digestion, we monitored the electrical activity of a prey-capturing flytrap via surface electrode measurements. For these experiments we used cricket (Acheta domesticus) 6–12 mm in length and with an average weight of 2.8 mg. It is well known that only two APs are sufficient for provoking fast trap closure, thereby capturing the insect [19]. Once trapped, the still-moving victim continues to activate the mechanism, prolonging the electrical stimulation for many hours (Figure 1A). In these experiments, we recorded 43 ± 13 APs during the first hour after prey capture (n = 5, mean ± SE).

Mechano-electric Signals Are Translated Into JA Biosynthesis and Hydrolase Production

In a previous study, we showed that when a second AP quickly follows the first, the rapid trap closure is provoked, but there is no increase in the cytosolic calcium level in the staled cells. The subsequent AP arrived at the dead, immobile, staled a calcium spike [19]. There is evidence from non-carnivorous plants that increases in

蚂蚁

约有 25 万个神经元

蚂蚁在归巢行为中需要寻找最短的回归路径，动物行为学的观察和实验，说明了某些沙漠蚁有计步和路径累积的能力。



Ant Odometer: Stepping on Logs and Stumps

¹ Philip Wexler,² Harold Wolf

Cataglyphis, cognitive in that it can learn about distance by path integration. They therefore discriminate distance, as demonstrated by their avoidance response and enhanced response to repeated non-reinforced distances. Here we find the hypothesis that ant navigation is achieved by using some form of path integration, or "dead reckoning". We found the lengths of the less and longer paths, the greater distances. In Figs 6(a) and 6(b), the total length of the paths (ℓ_{total}) was longer in the two trials, respectively, than the average travel distance. Travel distance (d_{travel}) is normalized by experimental time, and is path independent, so distance was not a factor.

cross state length, the logo was decreased by reversing the characters in the middle of the

The support by J. W. P. C. and the author supplied with a final letter and two

time paths integration [1,2] a more recent position relative to the next and

lipid migration might function as a means of avoiding phagocytosis, although this possibility has been ruled out by other studies.

This is similar to the *luteola* form described above. The note walking back from the C_4 note is also slightly different than the

The path integrals require two main assumptions: near constant movements over short periods of time, and periodic returns from the movements of other legs [15]. Although this may sound like a “pedestrian” assumption, we can make it more precisely by defining a step length as

These data were collected from the United States Geological Survey's National Water Information System (USGS NWS) and the USGS National Water Data Store (NWDS).

in which note measure used

ing locomotion has not yet been determined, there are several possibilities.

Benthic uniflorous reefs from their red seagrass meadows, over a distance of 10 m and

In contrast to most *in vitro* models of disease, this hypothesis is, if true, testable by a relatively simple experiment.

at the feeding site and transferred to a test channel, aligned parallel to the training channel

Fig. 1. Boxes. Two metric size metric boxes, the one permanent, the other household.

A "spike flow hypothesis" has been proposed, which integrates visual and auditory information.

Training

Time Step	Accuracy
50	~0.85

For further discussion of the issue from a Biblical perspective, see *What Is a Christian*?

After several hours of analysis, we can conclude that the main difference with the rotated case was around the interval $\pi/2$, still

Fig. 3. Behavior of an ion in a uniform magnetic field.

percentage increase of site-specific Institute of Science, Technology and
University.

Fig. 1. Schematic diagram of channel array, as well as training and testing procedures applied to each individual channel.

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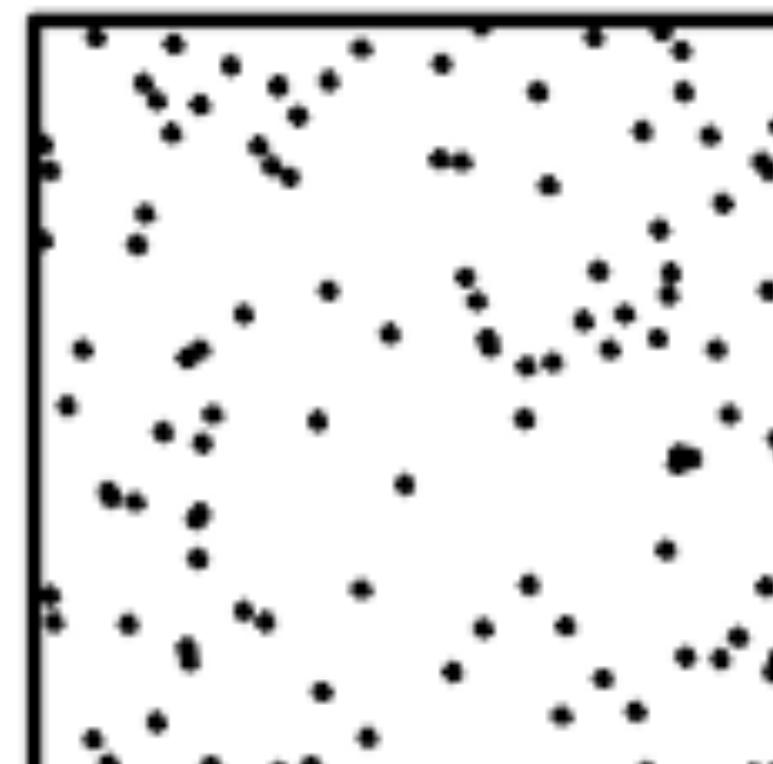
Digitized by srujanika@gmail.com

韦伯-费希纳定律

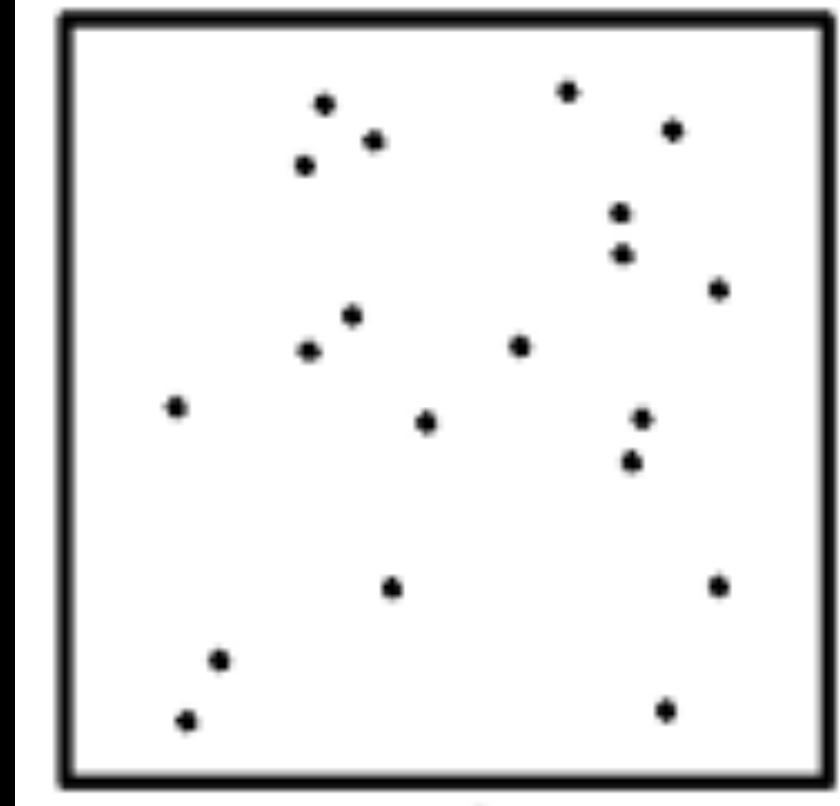
https://en.wikipedia.org/wiki/Weber%E2%80%93Fechner_law



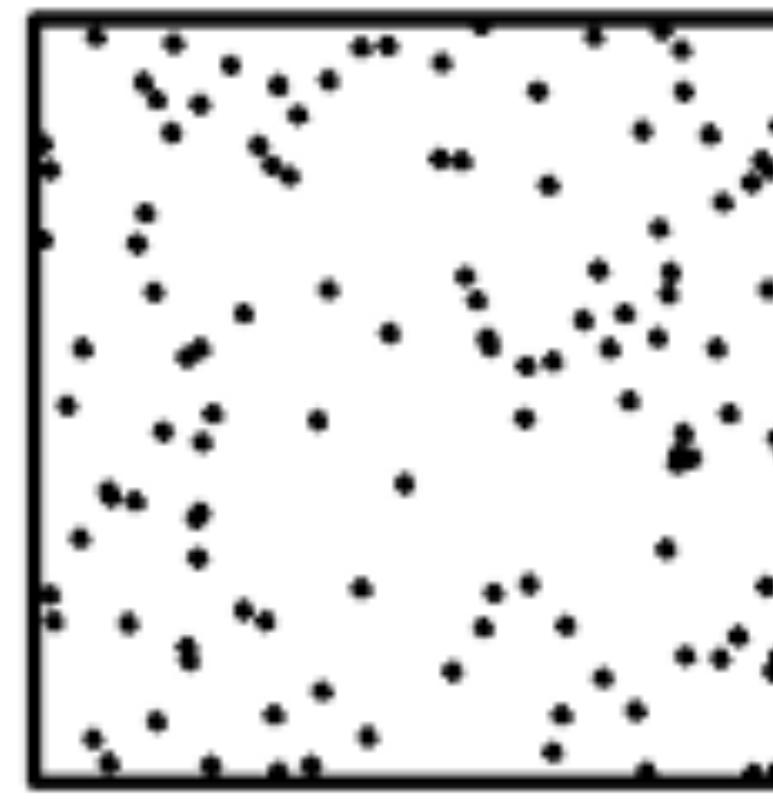
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110



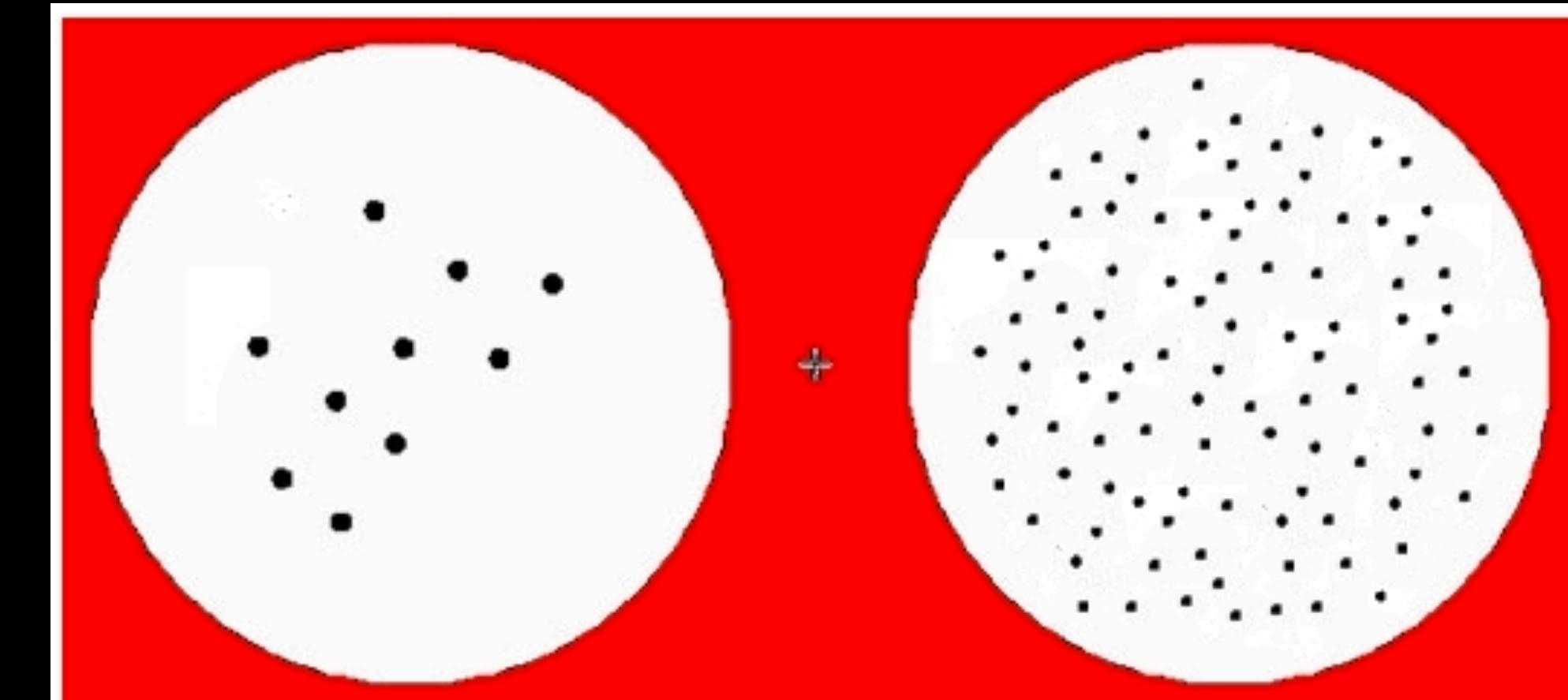
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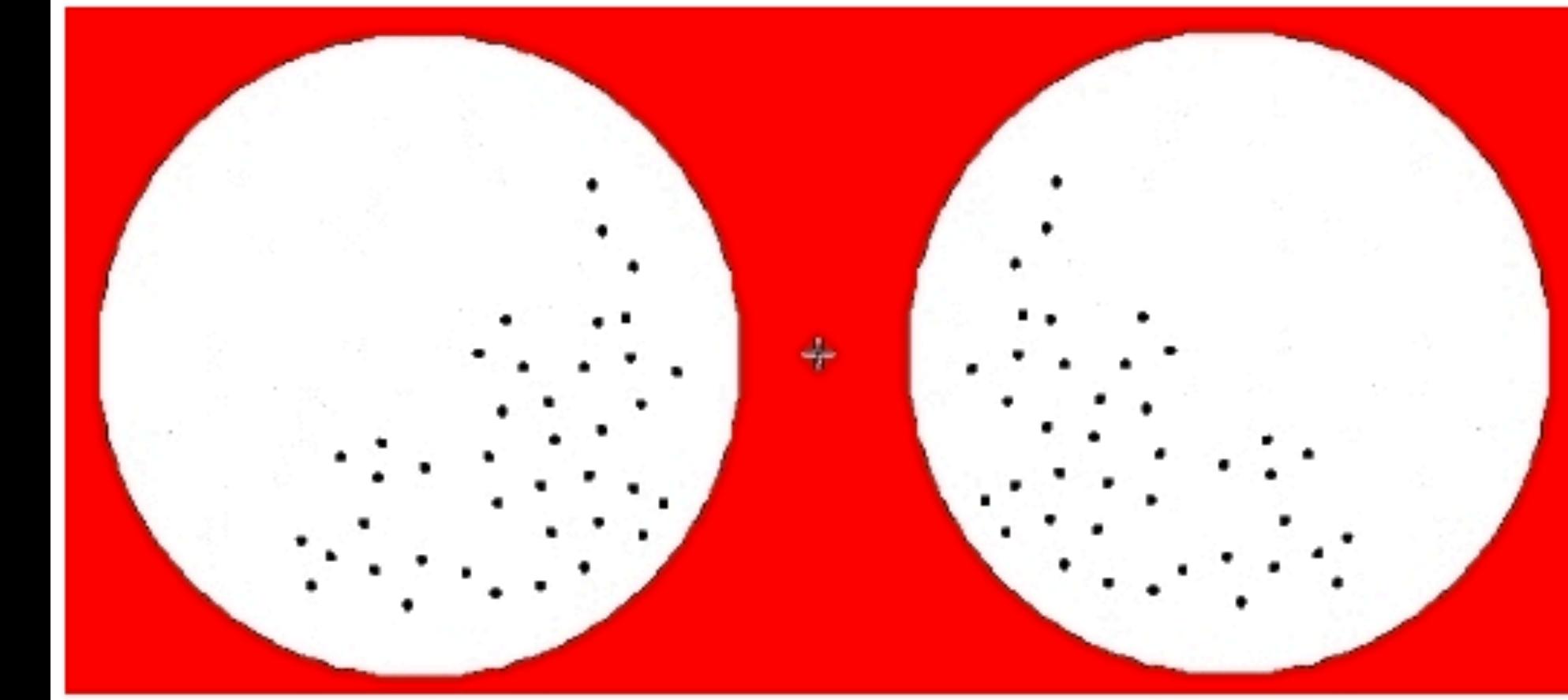
120

数量适应效应

https://en.wikipedia.org/wiki/Numerosity_adaptation_effect



Stare at the fixation "+" sign for 30 sec, then see the figure below.



数感

判断相等、多少的一种直感的能力

几个相关的概念

- 抽象性：能否跨越不同的标的物？
- 跨模态：能否跨越不同感知手段？

- 估数系统
- 循物系统
- 序数能力
- 快估

手、工具与语言

手、工具和火、大脑、社会行为、语言的协同演化历史

260万年前，能人(*Homo habilis*)时代，石器的使用兴起

The evolution of early symbolic behavior in *Homo sapiens*

by Kristian Tylén et al

关于智人早期的符号行为

Human language may have evolved to help our ancestors make tools

by Michael Batter

关于语言和工具制作

The co-evolution of number concepts and counting words

by Heike Wiese

关于数字概念和语言的协同演化

Empirical approaches to the study of language evolution

by W. Tecumseh Fitch

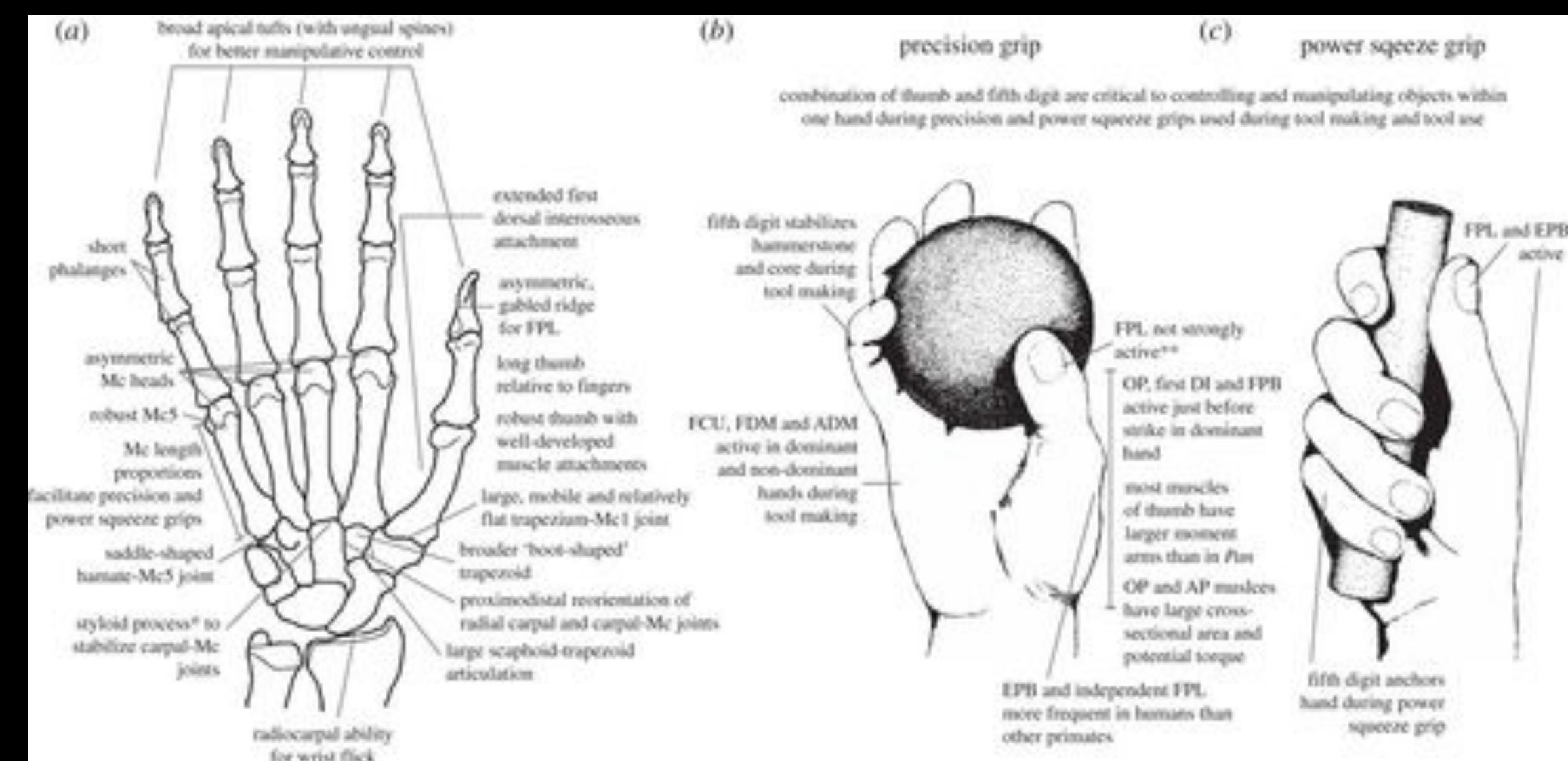
The mystery of language evolution

Review article by Marc Hauser et al

Origin of language 语言演化的各种假说

FOXP2 在基因层面的考察

语言的社会性



Evidence in hand

Recent discoveries and the early evolution of human manual manipulation

Review article by Tracy L. Kivell

Ishango 骨刻

旧石器时代晚期 约公元前 18,000 年 - 公元前 20,000 年



$$\begin{array}{ccccccc} \text{|||} & \text{|||} & \text{|||||} & | & \text{|||||} & || & \text{|||||} \\ 19 & + & 17 & + & 13 & + & 11 \\ = 60 \end{array}$$

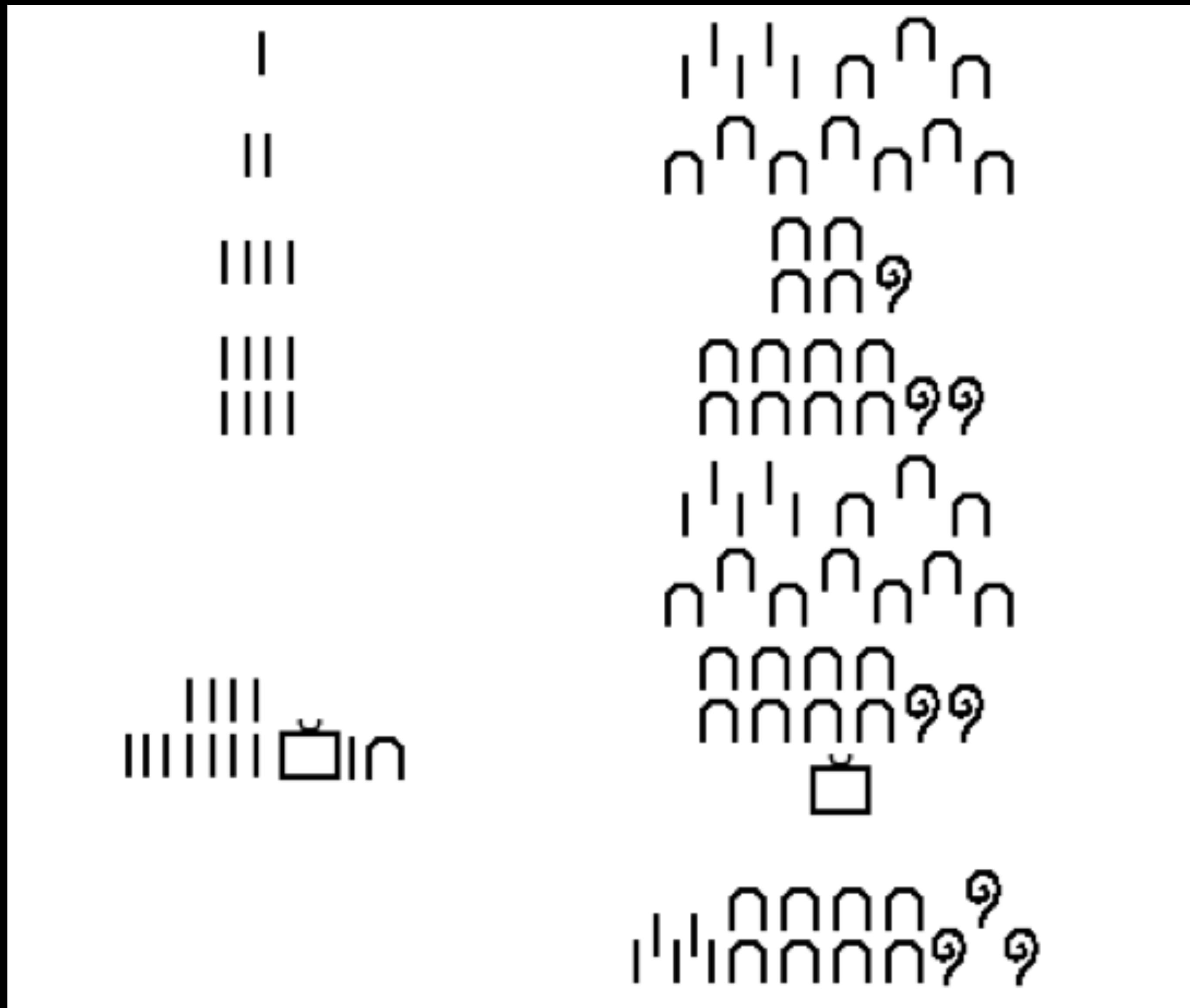
$$\begin{array}{ccccccc} \text{|||} & \text{|||} & \text{|||} & | & \text{|||||} & \text{|||} & \text{|||} \\ 7 & + & 5 & + & 4 & + & 1 \\ 5? & & 10 & & & & \\ + 1? & + & + 1 & + 9 & + & 8 & + 4 \\ = 48 \end{array}$$

$$\begin{array}{ccccccc} \text{|||} & \text{|||} & \text{|||} & | & \text{|||||} & \text{|||||} & \text{|||} \\ 9 & + & 19 & + & 21 & + & 11 \\ = 60 \end{array}$$

最原始的系统：一进数字系统

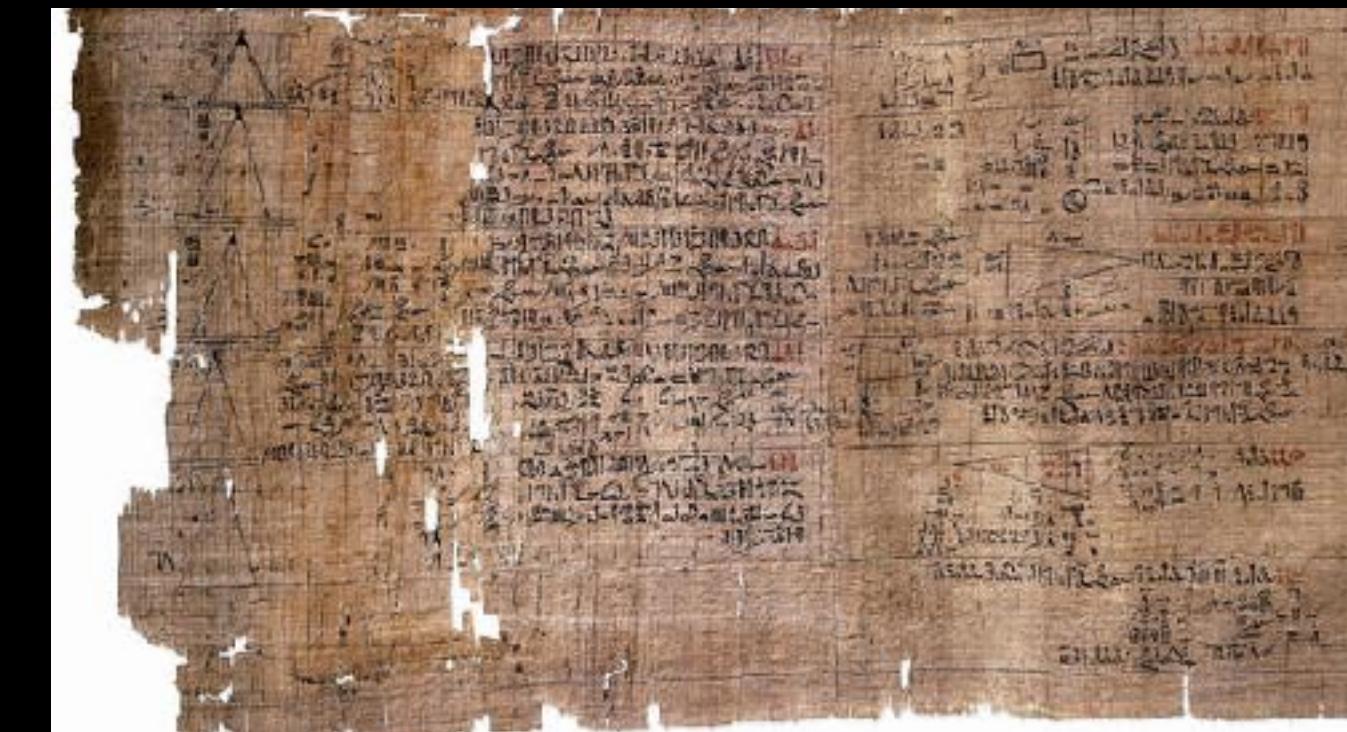
最自然的数字表示，但却难于表达大数字

古埃及的数



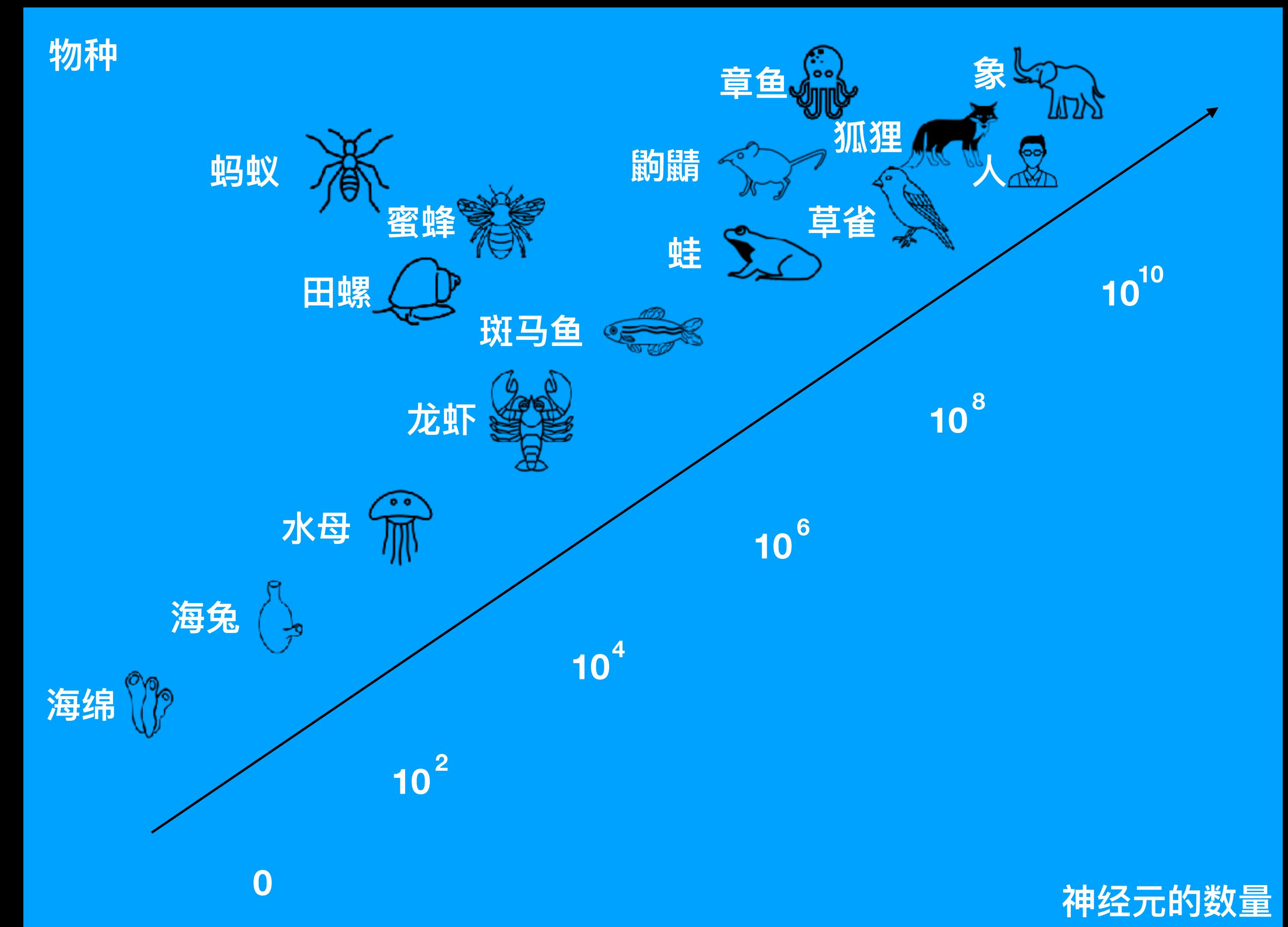
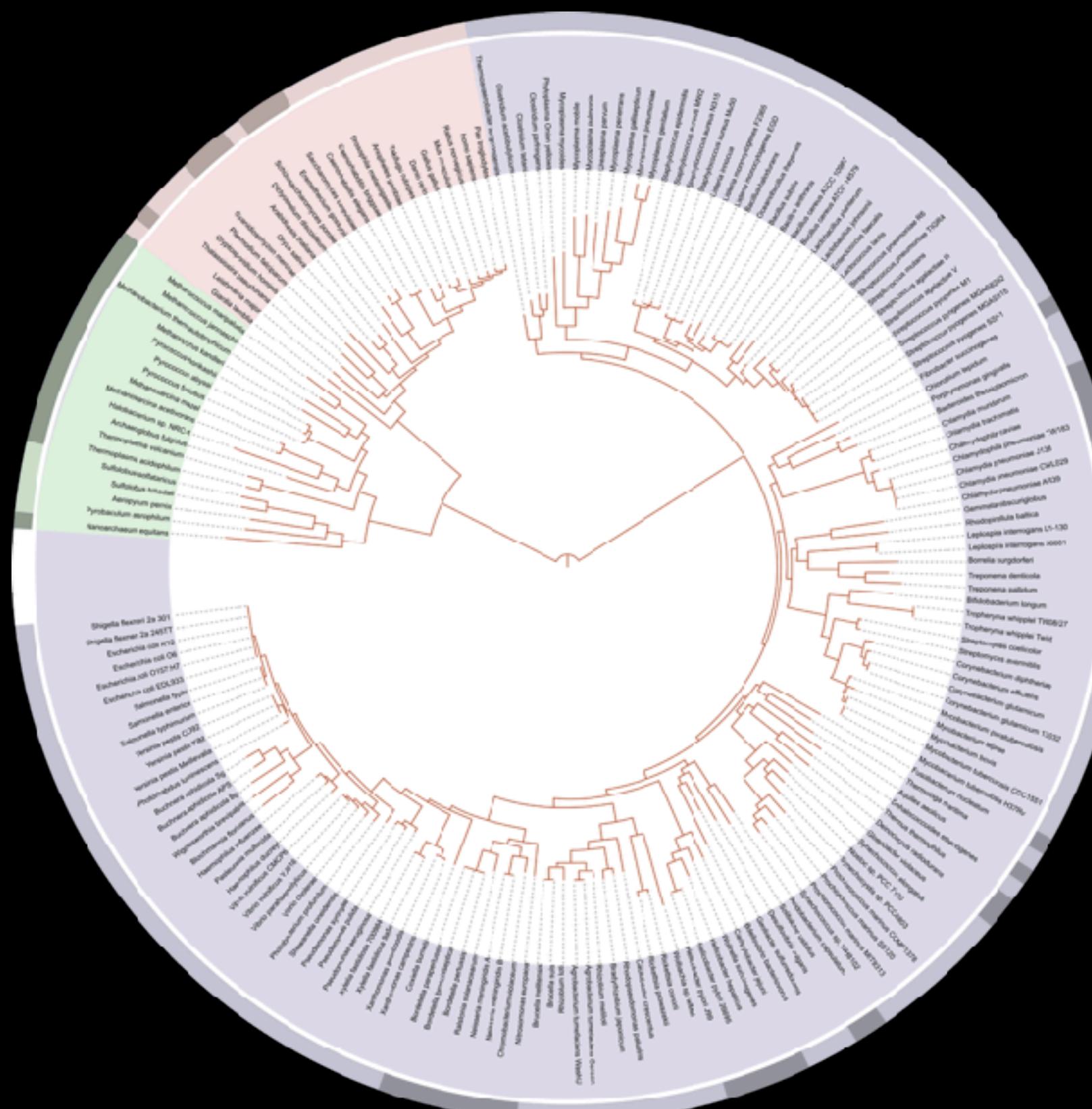
1*	35
2*	70
4	140
8*	280
$1+2+8=11$	$35+70+280=385$

在这个时期大数的
表示与运算都比较困难



Ahmes Papyrus

生物演化序作为复杂性度量？



一个设问方向

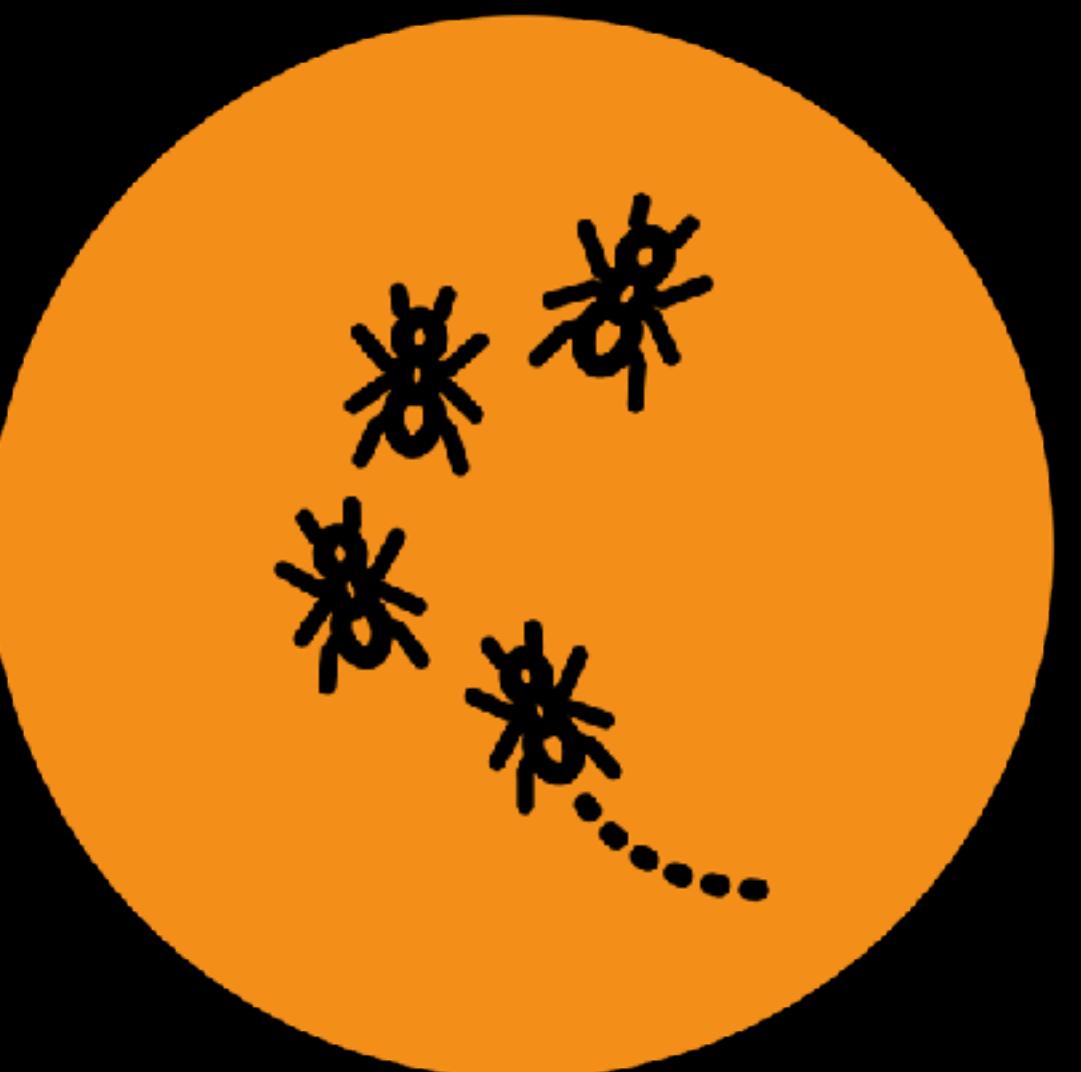
具身能力与外化工具扮演了什么角色？真的是数数内在的必须条件吗？

- 具身能力：一个复杂的脑、能写符号的手、能灵活发音的喉管
- 外化工具：图形符号、语音符号

从蚂蚁到人类的上亿年，有很多花在了具身条件的演化上

数数真正的复杂度究竟有多高？

我们能否给蚂蚁一些贴心又方便的辅助工具，使得它们可以操作数字符号？



一个相反的设问方向

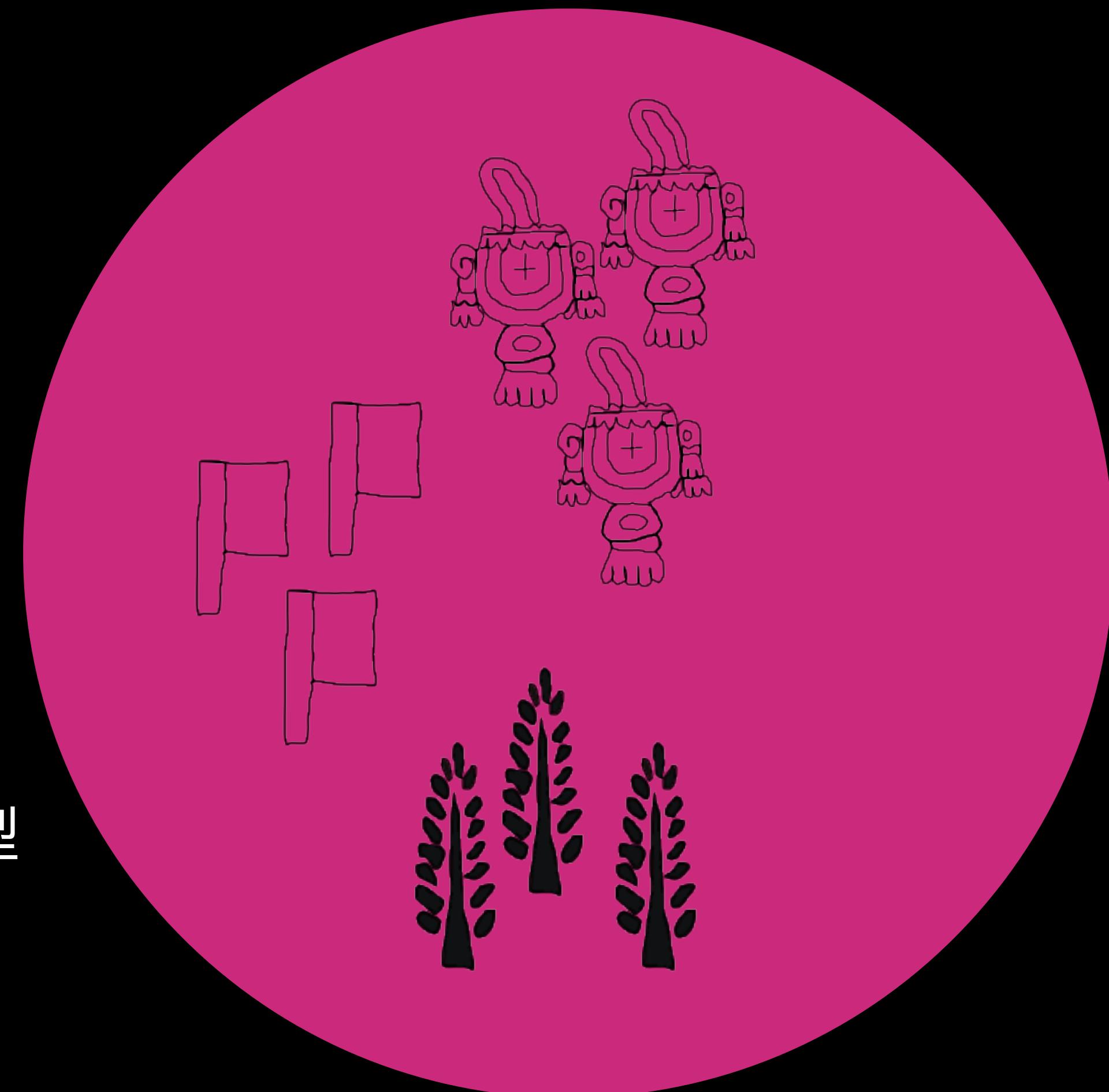
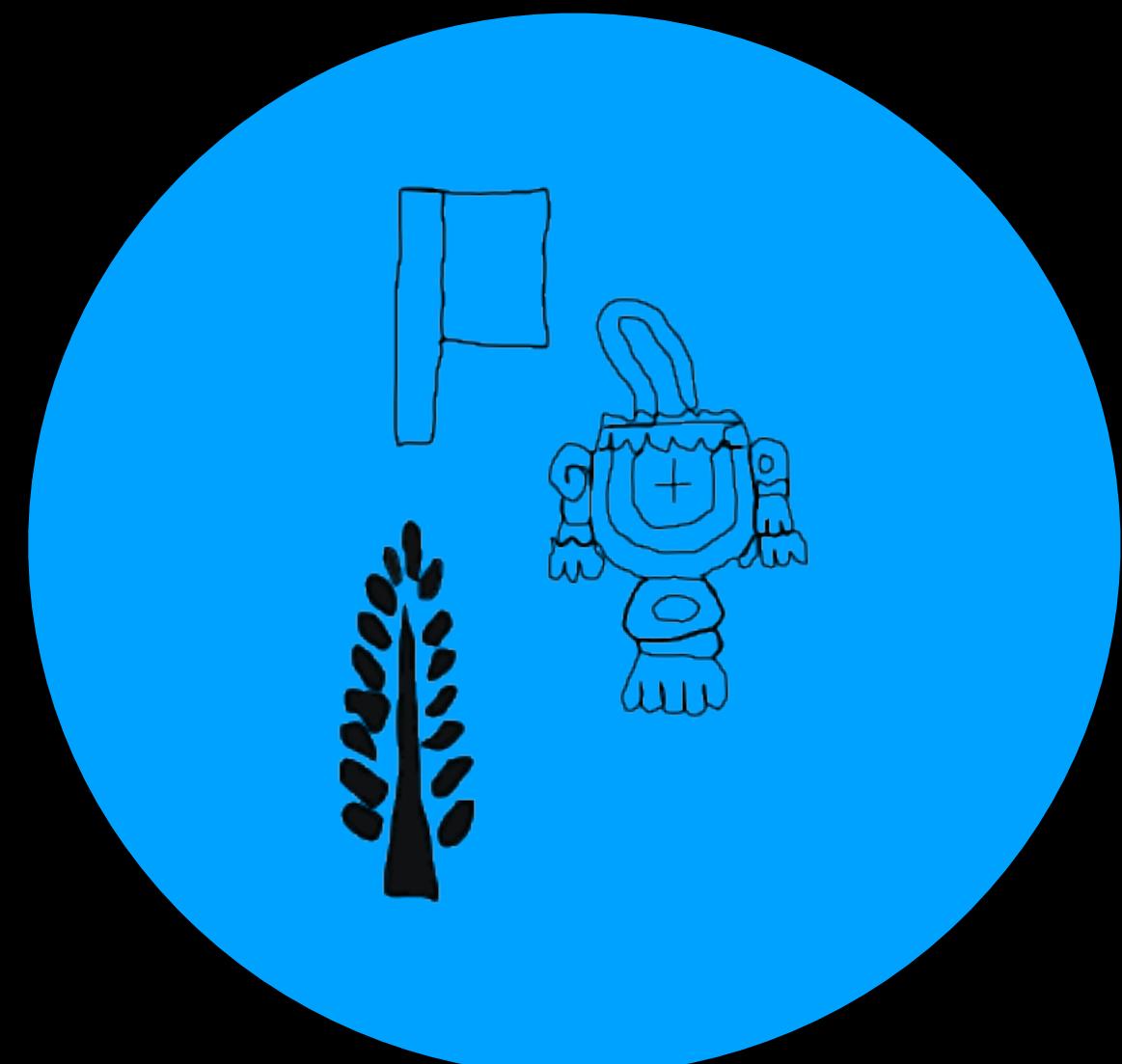
- 主流的方向：看到具身在理性里的作用，但假定理性的普遍
- 另类的方向：人类的理性是否只是人类具身形态的产物？
 - 是否存在其他形态的智能思维，我们还无法理解？

AI 是一种工具，可以让我们在两个方向上同时探索

第二节

什么是让机器数数？

- 让机器数数里的问题
- 为什么要考慮形式定义
 - 相等与比较
 - 基数与序数
 - 皮亚诺公理体系
 - 一进代数系统的初始模型
 - 有限性
 - 判定算法



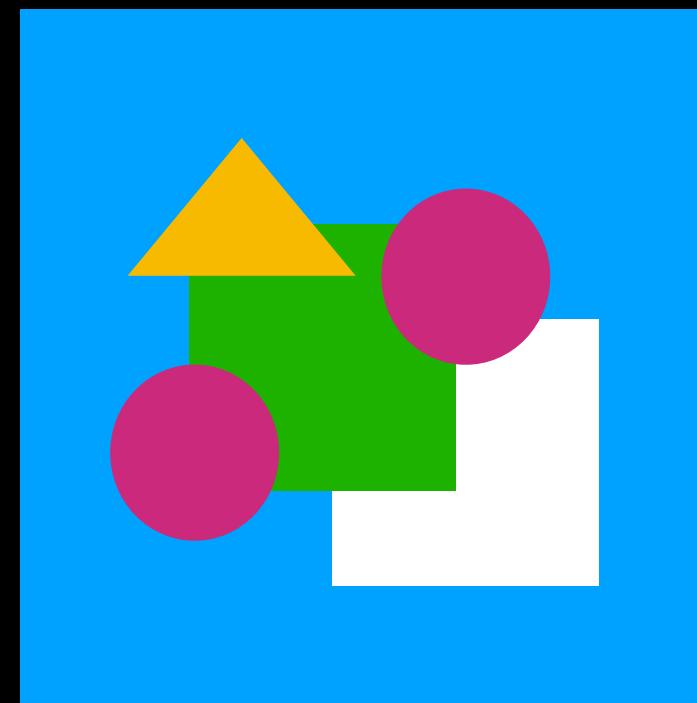
本节致谢李熙、晃晃老师的有益讨论和指导

可理解问题

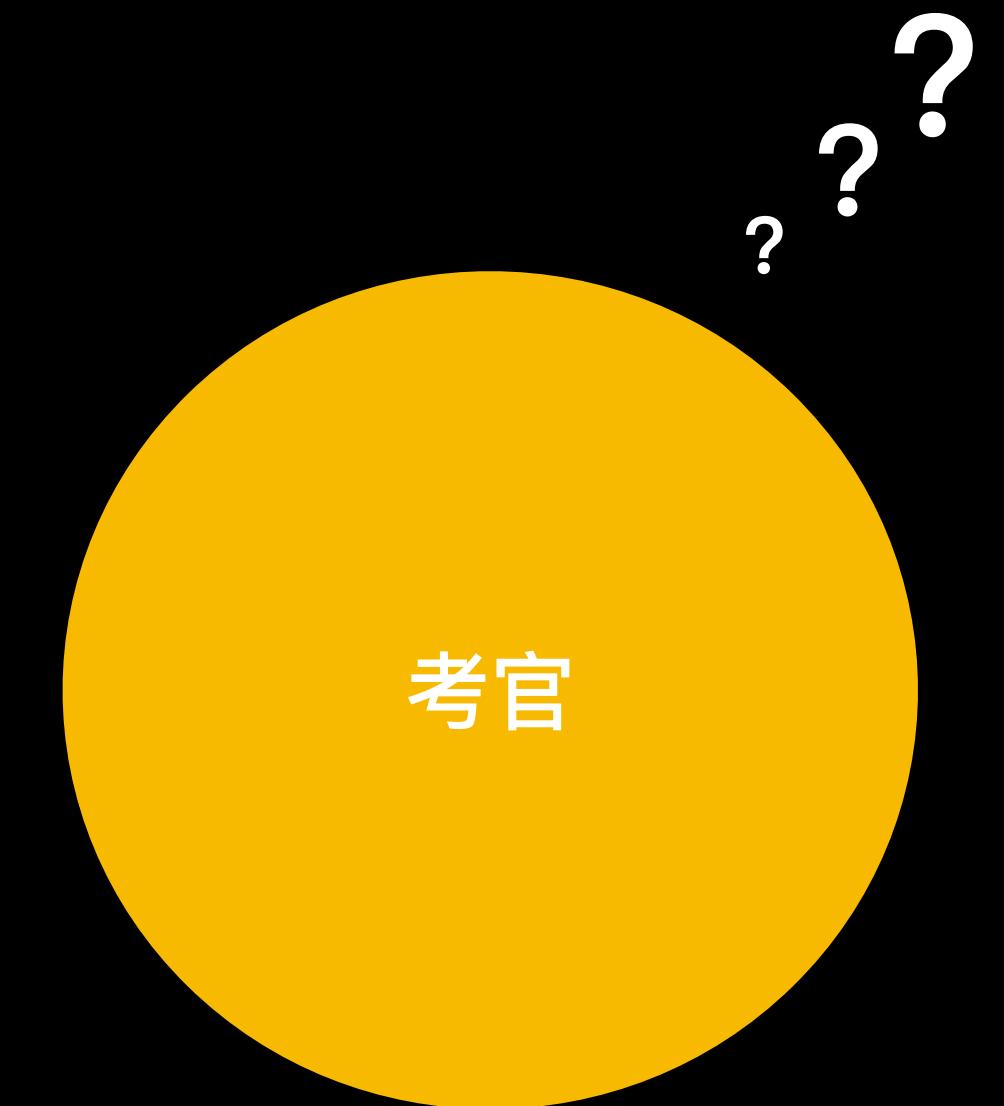
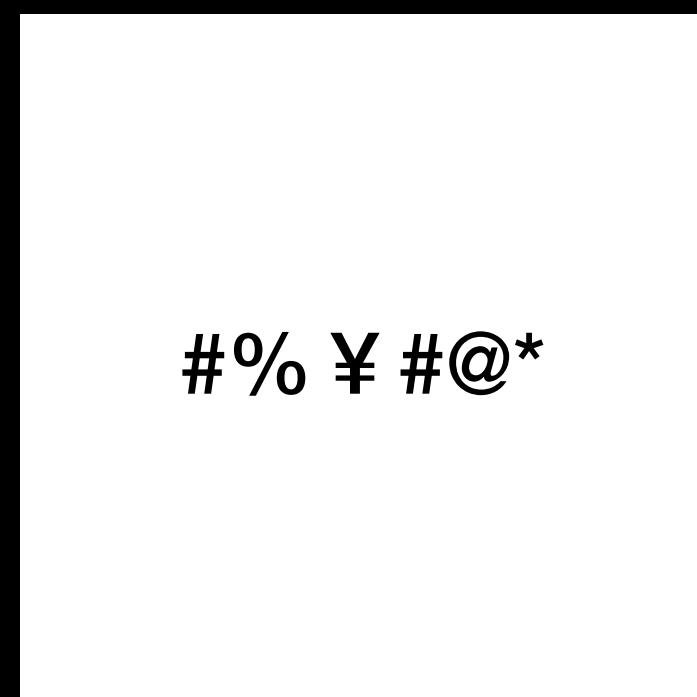
- 对图灵测试存在诸多的批评
- AI 数数能否检验？



考题



答卷



如果机器会自创符号了
考官会不会看不懂？

有限与无限的问题

困难在于

- 从 1 数到 100， 它会数数了吗？ 到千、万、亿.....就会了吗？
- “有限经验的学习可以泛化到无限？”这句话是什么意思？

请对比如下两个说法

- 有限经验的学习如何能跨越到无限？

- 有限经验的学习如何能跨越过任何给定边界？

请考虑下面的两个学习目标

- 一个个独立的数

- 一段递归程序且有递归起点

关于数数的学习

- 可以是有监督学习－学会人类已有的数字系统
- 最好是无监督学习－机器自己发明新的数字系统

学习的目标是什么

泛化
检验

经验 vs 先验

比图灵测试更难？

- 可理解问题，让数数问题的测试不同于图灵测试
- 创造能力也更接近于智能本质
- 数数问题的测试可能会比图灵测试更难吗？

为什么需要自然数定义？

最朴素的想法

- 有定义后，我们可以验证机器的输出是否满足定义

但是

- 合适的定义是什么？
- 合适的输出又是什么？
- 验证的算法是清晰的吗？

但如果 没有 定义，那就进入荒诞不经的领地了。

相等和比较

相等是具有如下性质的关系

- 自反性
- 对称性
- 传递性

某种归纳原理?

良序是具有如下性质的关系

- 三岐性
- 传递性
- 良基性

基数

- 数感是一种比较多少的能力，把相等的侧面上升为数学概念就是基数
- 全体自然数的基数是 \aleph_0 ，从 0 到 \aleph_0 只是全部基数的一小部分
- 能否通过增加定义中的条件，来从基数中限定出来我们想要的一部分呢？
比如，增加一个有限性的条件？

有限性 vs 有限

序数

- 数感是一种比较多少的能力，把比较的侧面上升为数学概念就是序数
- 第一个无穷序数是 ω
- 有趣的可数序数有很多

The screenshot shows a section of a Wikipedia article titled "Large countable ordinals". The title bar includes icons for Y·I·E and [hide]. The main content lists several large countable ordinals: First infinite ordinal ω , Epsilon numbers ε_0 , Feferman–Schütte ordinal Γ_0 , Ackermann ordinal $\theta(\Omega^3)$, small Veblen ordinal $\theta(\Omega^\omega)$, large Veblen ordinal $\theta(\Omega^\Omega)$, Bachmann–Howard ordinal $\psi(\varepsilon_{\Omega+1})$, $\Psi_0(\Omega_\omega)$, and Church–Kleene ordinal ω_1^{CK} .

可构造的从有限趋于无限的方式有很多

“数”可以被发明的空间也同样很大

皮亚诺公理

- P1: 0 是自然数

可引入后继函数 S

- P2: 自然数在 S 下封闭

成链

- P3: S 是入射

无叉

- P4: 不存在自然数 x 使得 $S(x) = 0$

无环

- P5: 归纳公理

自然数唯一性

结构归纳法

个体到全体的跨越

一进代数系统

如下的函数签名定义的代数系统

- $0 :: N$
- $S :: N \rightarrow N$

它的初始代数是自然数系统，初始代数模型可以由项代数给出

有限性

脱离了我们人类的自然数，我们能谈论有限吗？

有限的戴德金定义

- 一个集合有穷，当且仅当，不存在从它到它真子集的单射

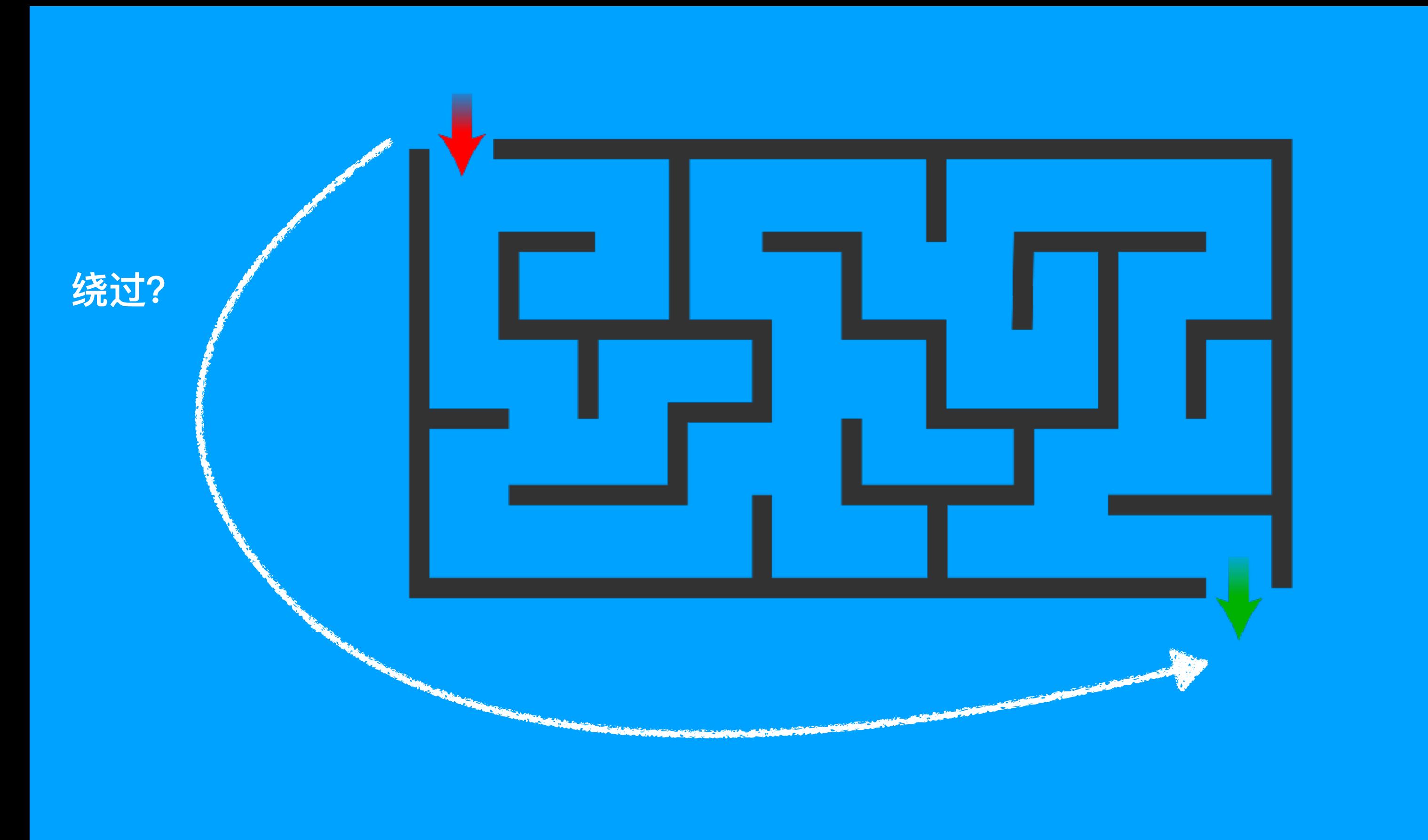
存在通用的判定算法吗？

- 如果
 - 接受有限性是数数问题的一个本质特征
 - 接受检测程序必须能停机才算通过检测
- 我们能看到一个非常危险的信号：
 - “判断任意一个生成结构是否是有限生成”可能会把我们导向停机问题

第三节

绕过困难的新思路吗？

- 塞伦盖蒂游戏的想法
- 游戏具体设计的尝试
- 基于几何的元理论？



游戏的想法

如果能找到一个游戏场景

- 1、智能体需要通过数数才能获得更好的生存
- 2、智能体可以自己画符号
- 3、为了达成 1，通过 2 发明的符号是必须的

初步的设计

塞伦盖蒂游戏

假想一个原始人部落在塞伦盖蒂草原逐水草而居。他们需要通过左前方和右前方采到的浆果数量，来判断部落移动的方向，获得最高分的部落是到达水草最丰美位置的部落。

采到的浆果的数量有一定的随机因素，原始人必须要累计多日的采集结果，才能获得稳定的信号。而多日的累计需要通过某种符号记录下来。

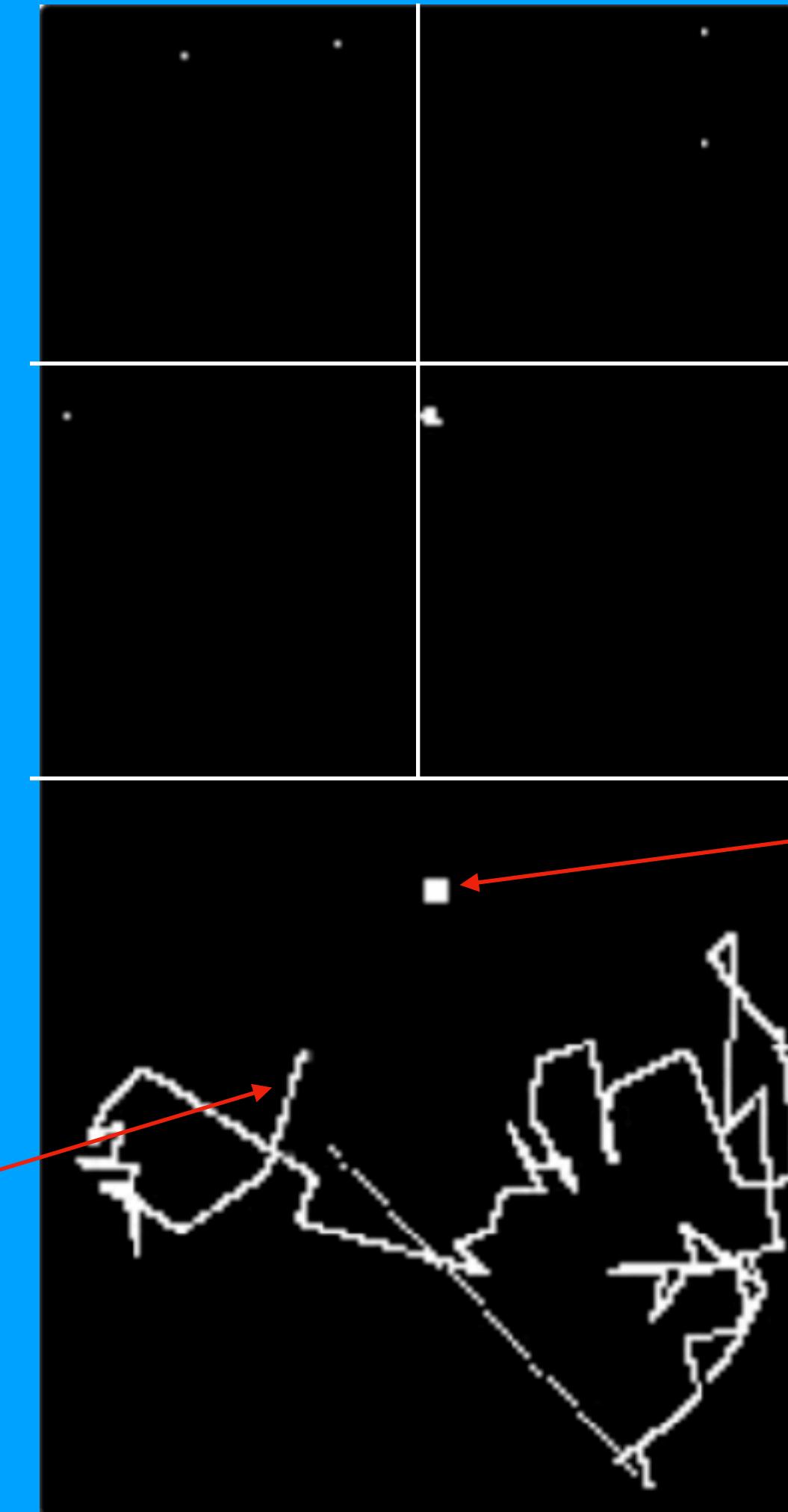
我们的 AI 原始人能自己发明出来一种计数的符号体系吗？这是我们想考察的问题。

今天左前方采到的果子

对左前方采果子的多日记录

今天右前方采到的果子

对右前方采果子的多日记录



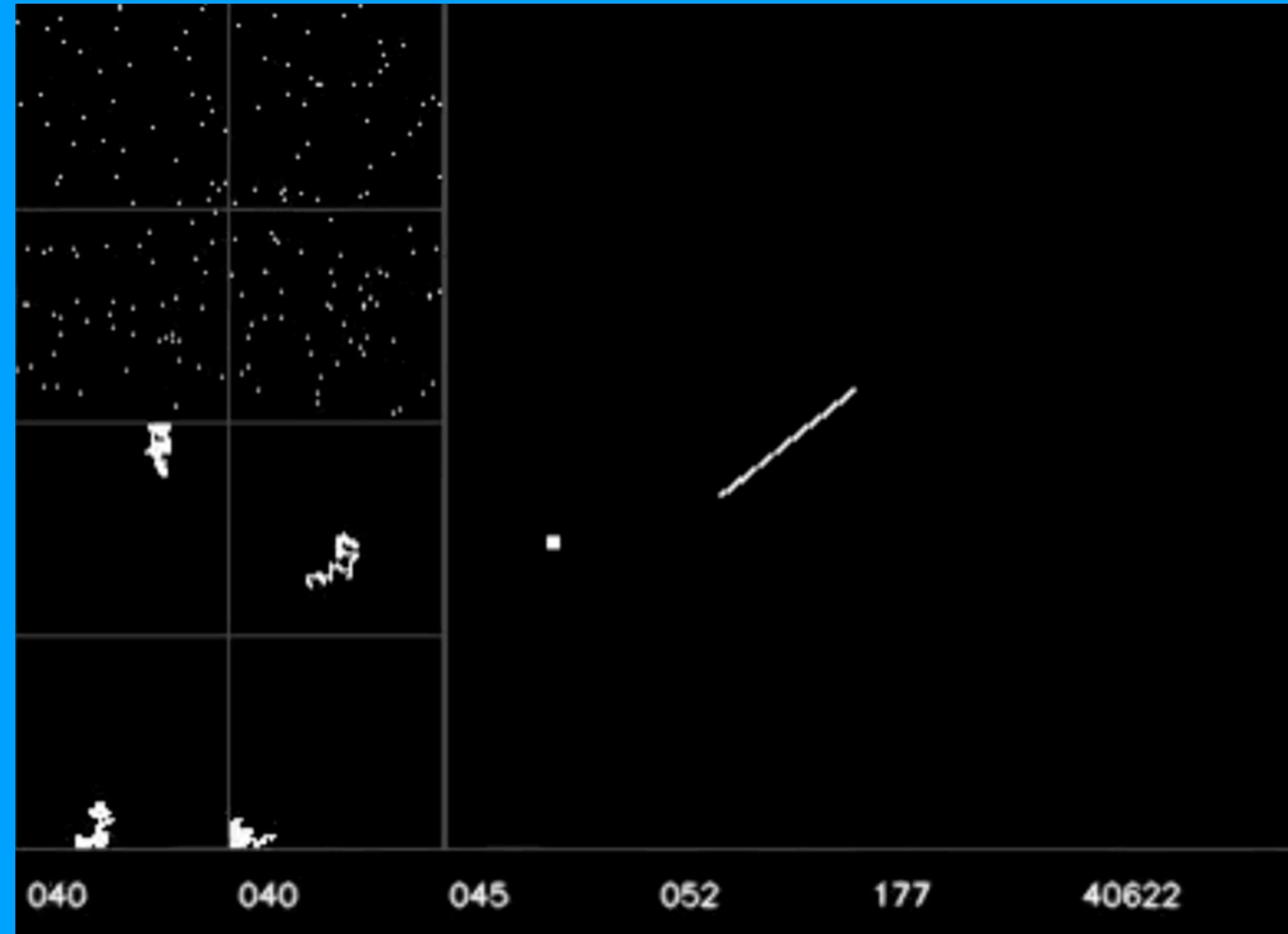
大草原地图

水草最丰美的位置

设计的改进

左前、右前、左后、右后
四个采集地

四块画板分别记录
四个采集地多日的收成



根据四个收成记录
可计算梯度
决定走向和步幅

设计里的体会

- 具身与世界：彼此融洽一致的多种具身性、主体与世界遭遇的方式
- 灵活控制：否则主体太笨拙，很难学习一个高技巧任务
- 最简化：随机游走里的复杂性，必须要简化设定到最必要的条件
- 基线对比：要有基线对比方案

规避了困难吗？

- 可理解性：可以被 RL 评分比较好的规避掉
- 无限性：
 - 可以通过可伸缩的大小的画布和可游走的视界窗口解决吗？
 - 会发现它和塞伦盖蒂游戏是类似的一个问题
 - RL 的无限叠加？还是别的可能？
 - 有没有可能构造同样一个 RL 的多层叠加，每次提升都缩小了数数的范围

攻克无限性（一）

- 方案一：无限大画布、可移动视窗和可移动的笔

攻克无限性（二）

攻克无限性（三）

基于几何的元理论

- 如果能用 RL 来很好的刻画数数问题
- RL 在优化过程中的每一步，都可以用几何的语言来理解
- 在寻求一个代价更低的势能面
- 这是否意味着，我们可以发展一种关于数的几何理论？

致谢与参考文献

- 致谢李熙、晃晃、王东和夏海宁老师的有益探讨和指导
- 动物的图标均来自 the Noun Project
- 其他的图片都来自维基共享资源项目

谢谢



NumberX 项目

塞伦盖蒂游戏是其中之一



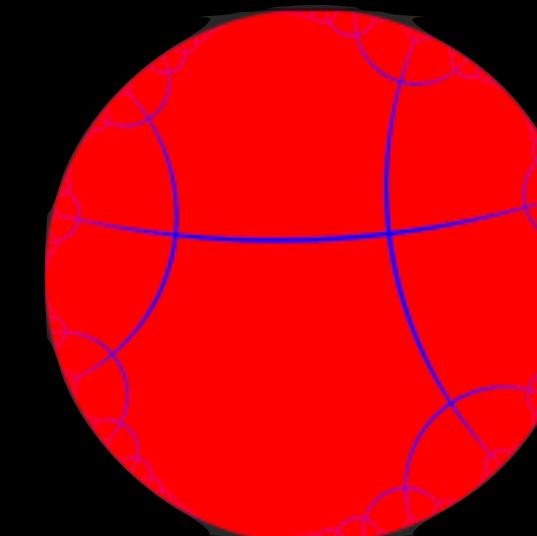
Skiverse 项目

另类数字自催化网络想法的探索



SelfX 项目

探讨自我概念的产生



更神秘的项目

请大家等待会在下面逐渐开启