Two Models: Heroin Epidemic and Harvesting Forest Products

Tricia Phillips

Advised by: Dr. Suzanne Lenhart and
Dr. Christopher Strickland

May 25, 2018

Overview

Overview

Motivation

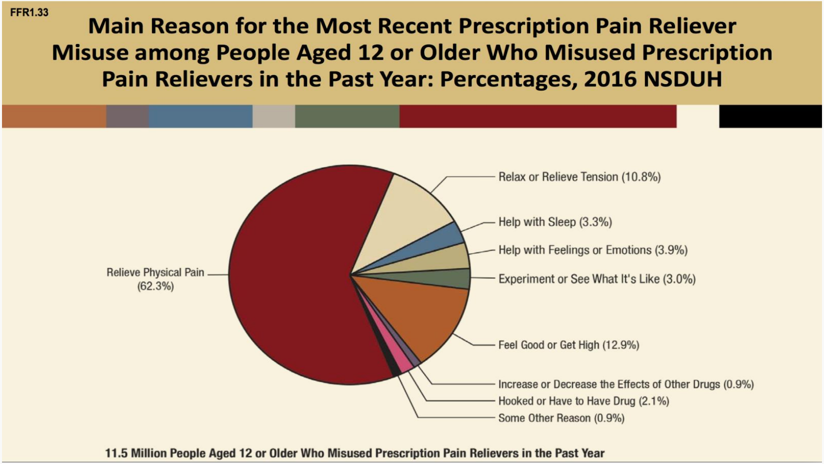
Opioids

- American Pain Society aggressively pushed idea of pain as the fifth vital sign in mid 1990's as they believed pain was being undertreated in doctor offices and hospitals.
- In 2000, Joint Commission required physicians to accept and respect the self-reporting of pain by patients.
- Early 2000s, drug manufacturers funded publications and physicians to support opioid use for pain control.
- Number of opioid prescriptions that pharmacies distributed in 2011 was almost triple that of 1991.

Motivation

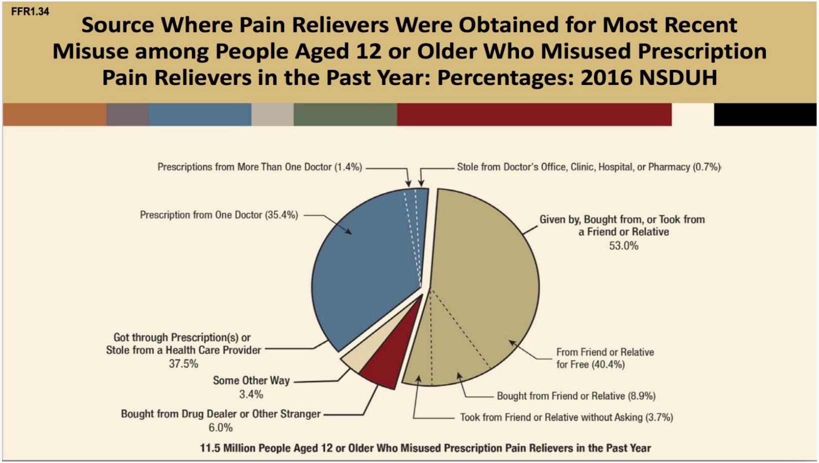
- The misuse of opioids, a drug class including prescription pain relievers and heroin, is rampant in today's society.
- The opioid crisis was declared a public health emergency in October 2017 by the United States Department of Health and Human Sciences.
- Treatments available for opioid and heroin use; involves medications (methadone, naltrexone, etc.), counseling, behavioral therapies.

Motivation



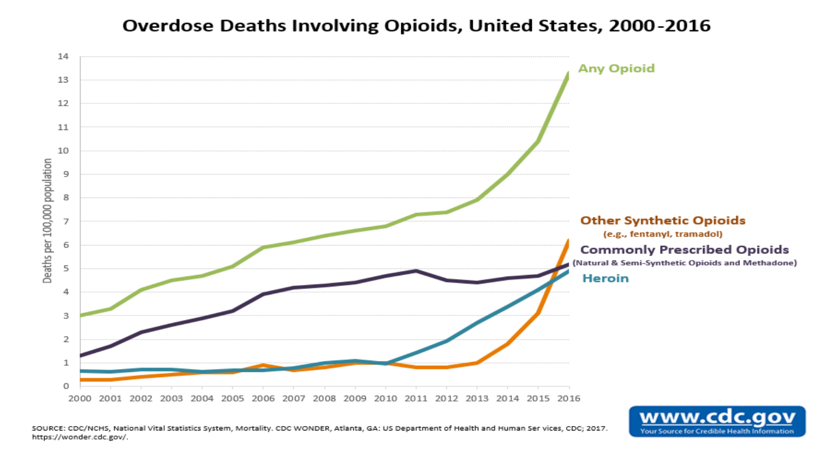
Source: 2016 National Survey on Drug Use and Health

Motivation



Source: 2016 National Survey on Drug Use and Health

Motivation



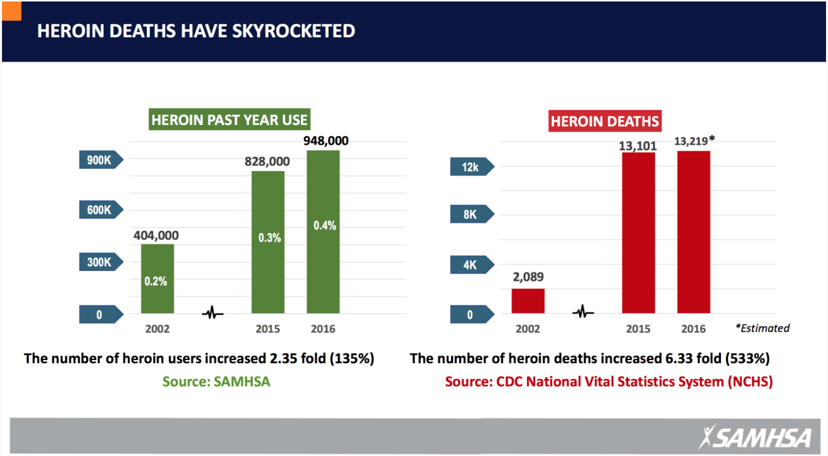
Source: Centers for Disease Control and Prevention

Motivation

Heroin

- Dramatic increase in accessibility to heroin and the lower cost of the drug has influenced prescription opioid users to turn to heroin.
- Based on 2002-2012 NSDUH data, study found heroin initiation 19 times more likely for non-medical opioid users than non-users.
- Estimated 80% of heroin users at the national level used prescription opioids previously.
- Heroin overdose deaths have increased significantly in recent years.
- In 1960s, heroin users composed mainly of young, non-white men in urban areas with initial opioid being heroin; present day, shifted to older, white, rural/suburban, men and women.

Motivation

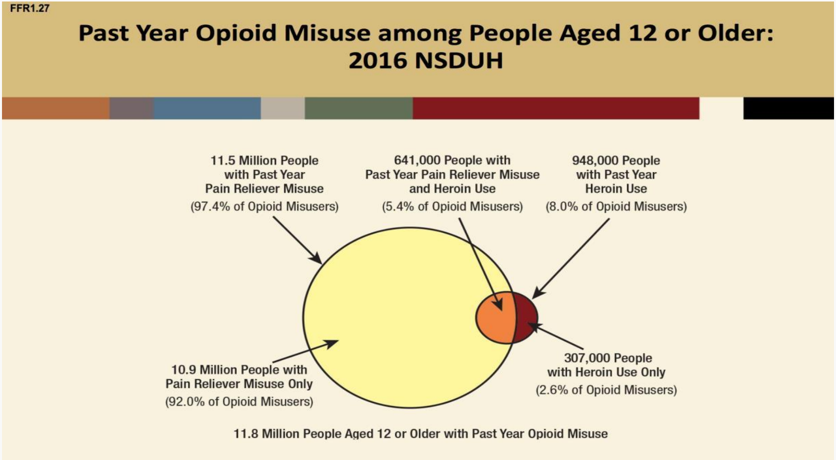


Source: 2016 National Survey on Drug Use and Health Report from SAMSHA.gov

Challenges

- Fentanyl is a surgical-grade synthetic opioid up to 50 times more potent than heroin.
- Fentanyl is mixed with heroin to increase effect; unknown purity increases overdose risk.
- Difficulty in modeling due to variability of the purity of heroin.
- 1 in every 5 overdose deaths have multiple drugs present, difficult to determine actual cause of death.
- Opioid misuse, abuse, dependency, addiction and use disorder often not clearly defined in literature/difficult to know exactly what is intended.

Motivation



Source: 2016 National Survey on Drug Use and Health Report

Previous Work

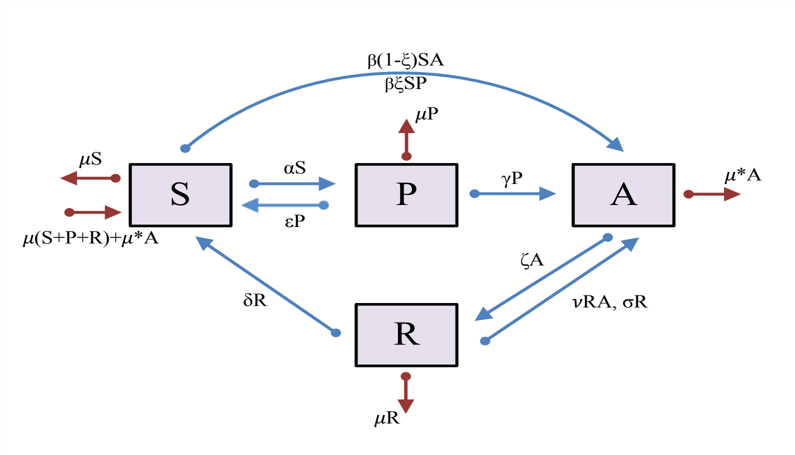
Opioid Model

- Dr. Christopher Strickland and collaborators, Nicholas Battista and Leigh Pearcy, developed a population-level model for the opioid epidemic (excluding heroin) using a system of ODE's.

Population Classes

- Susceptibles (S): not taking prescription opioids, nor recovering from opioid addiction.
- Prescription opioid users (P): opioid-prescribed individuals not considered addicted.
- Opioid addicts (A): addicted to opioids.
- Individuals in treatment/rehabilitation (R): undergoing treatment for their addiction to opioids.

Previous Work



Schematic diagram for opioid-only model

Previous Work

$$\frac{dS}{dt} = -\alpha S - \beta(1-\xi)SA - \beta\xi SP + \epsilon P + \delta R + \mu(P+R) + \mu^* A$$

$$\frac{dP}{dt} = \alpha S - \epsilon P - \gamma P - \mu P$$

$$\frac{dA}{dt} = \gamma P + \sigma R + \beta(1-\xi)SA + \beta\xi SP + \nu RA - \zeta A - \mu^* A$$

$$\frac{dR}{dt} = \zeta A - \nu RA - \delta R - \sigma R - \mu R$$

Previous Work

Main Results

- In order to have an addiction-free equilibrium, both addictions that come from prescriptions and addictions from accessibility to excess drugs must be eliminated ($\gamma = 0 = \xi$).
- Near addiction-free state: prevention of prescription opioid users becoming addicted is more important than reducing prescriptions getting into the hands of non-prescribed users to combat the epidemic.
- More realistically, outside of addiction-free state: reducing prescription-user addictions, decreasing prescriptions dispensed and increasing entry into treatment are most important to reduce number of addicted.

Overview

Model Formulation

- **Goals**:
  - Investigate the dynamics behind the opioid and heroin epidemic and identify important conditions relating to the reduction of opioid and heroin addicted individuals.
  - Develop a system of ODEs model consisting of classes of individuals taking prescription opioids, addicted to opioids, using heroin and recovering from opioid addiction, including heroin, and analyze it.
  - Investigate management strategies for how to best treat pain with prescriptions while reducing opioid addiction and heroin use.

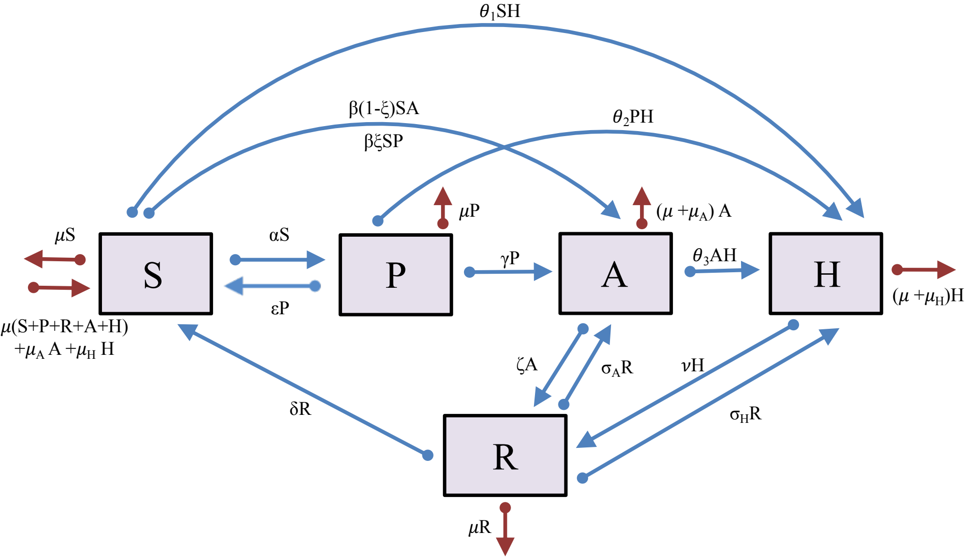
Model Formulation

- We formulated a five class compartmental population model.
- Population Classes
  - Susceptibles (S): not taking prescription opioids, nor using heroin.
  - Prescription opioid users (P): opioid-prescribed individuals not considered addicted.
  - Opioid addicts (A): addicted to opioids.
  - Heroin users (H): addicted to heroin.
  - Individuals in treatment/rehabilitation (R): undergoing treatment for their addiction to opioids or heroin.

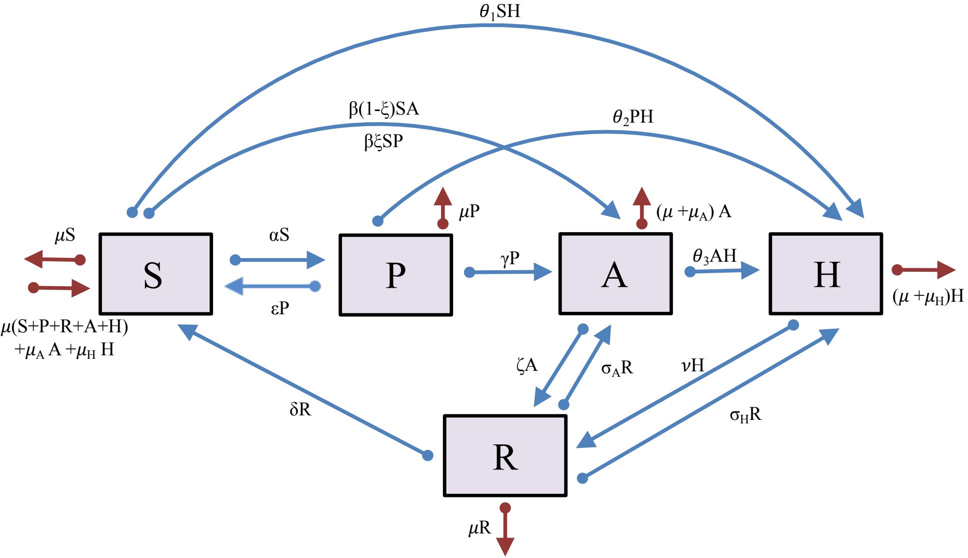
Model Formulation

Assumptions

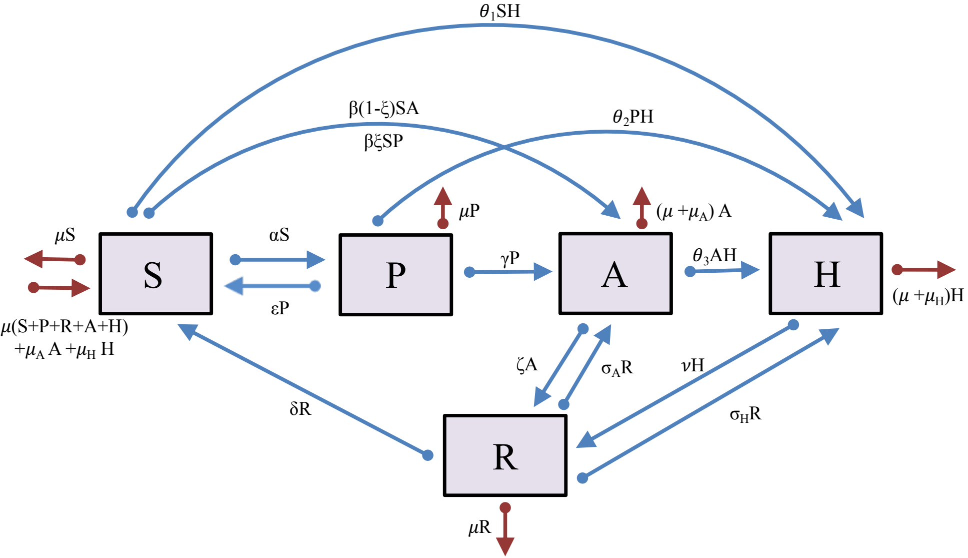
- Constant population so total death rate is equal to the incoming rates for the susceptible class.
- Only considering individuals who are addicted to opioids (not just any type of misuse); addiction defined as individuals with a pattern of continued non-medical use with the potential for harm.
- Assume there is no permanent recovery class or immunity to addiction, so recovered individuals go back to the susceptible class.



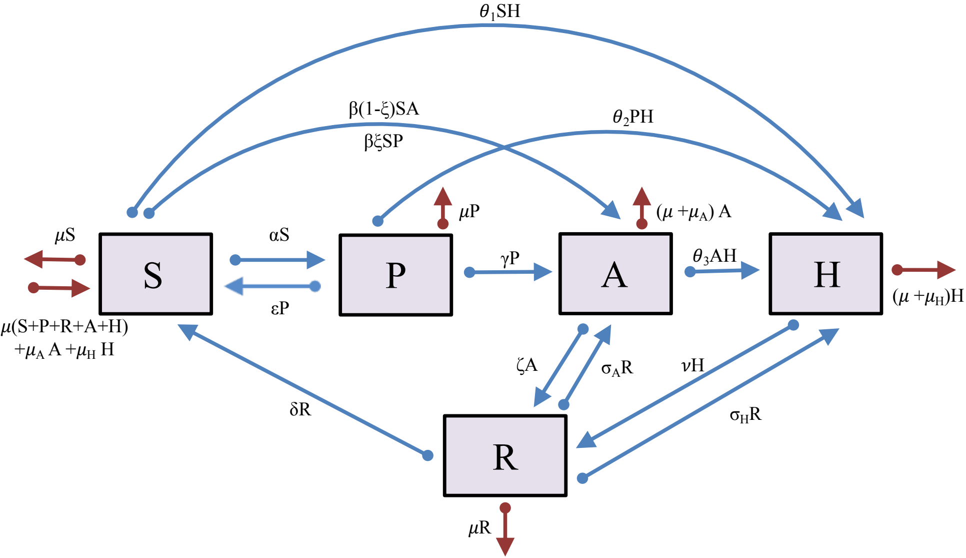
αS: prescription rate



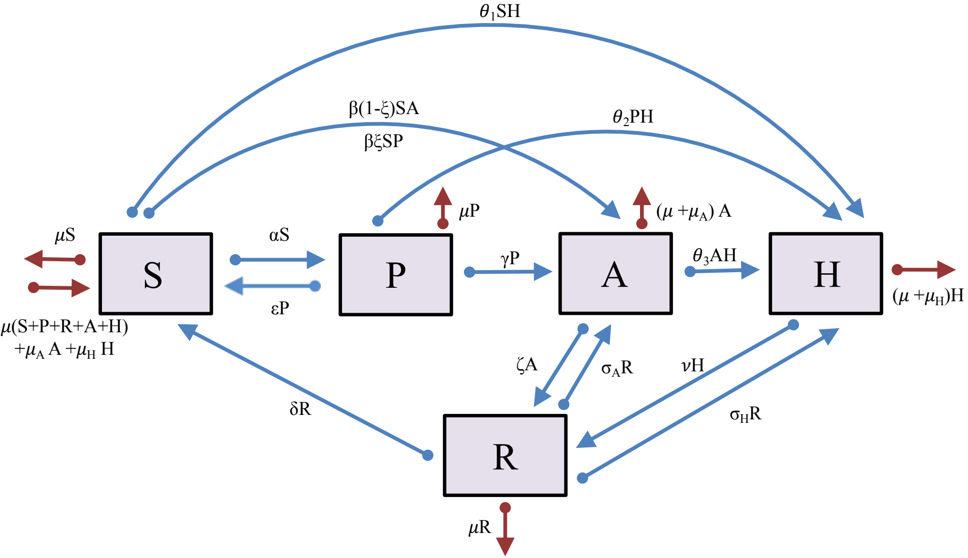
$\beta(1-\xi)SA$: opioid addiction rate by black market drugs/interaction with other addicts



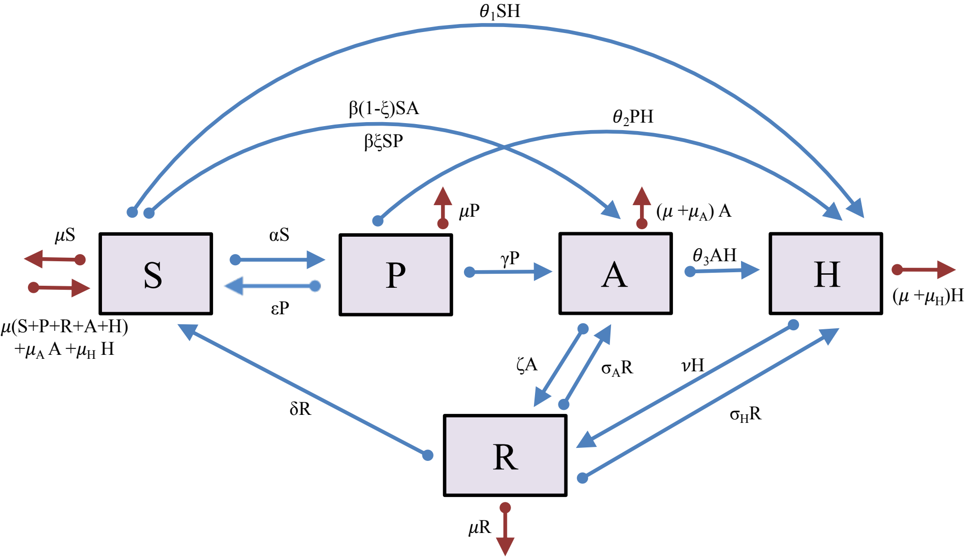
$\beta\xi SP$: opioid addiction rate by obtaining extra prescription opioids



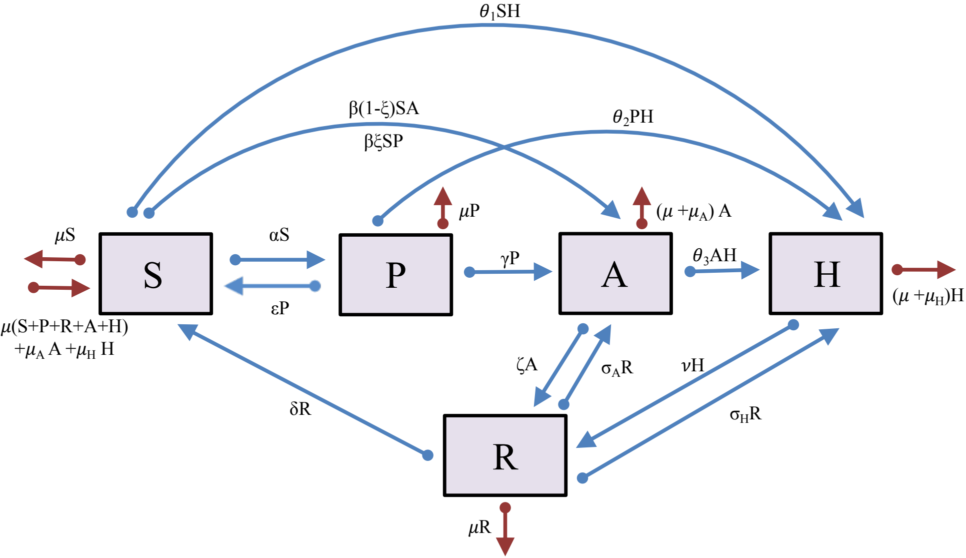
$\theta_1 SH$: rate of addiction to heroin by black market availability/ interaction with other users



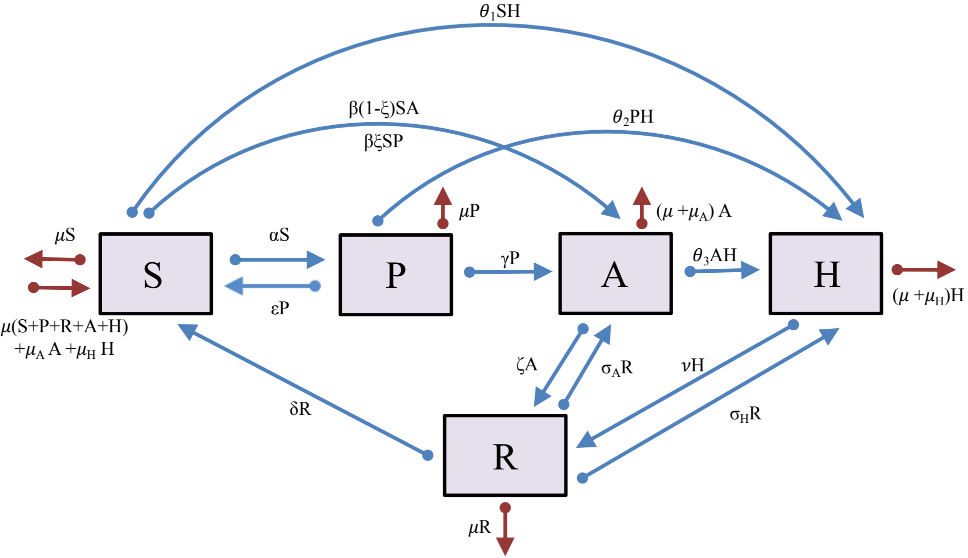
ϵP: rate of non-addicted opioid prescribed users back to susceptible



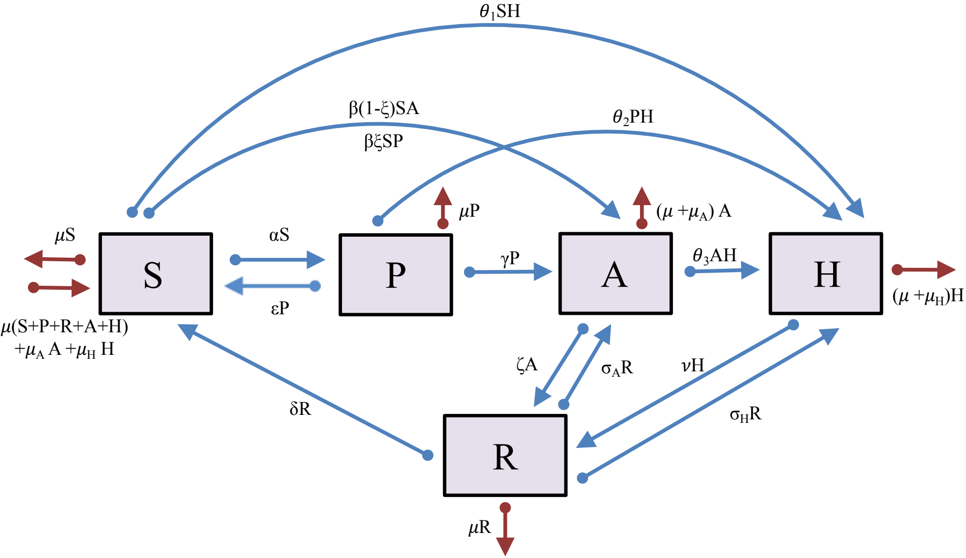
δR: rate of opioid and heroin addicts successfully finishing treatment



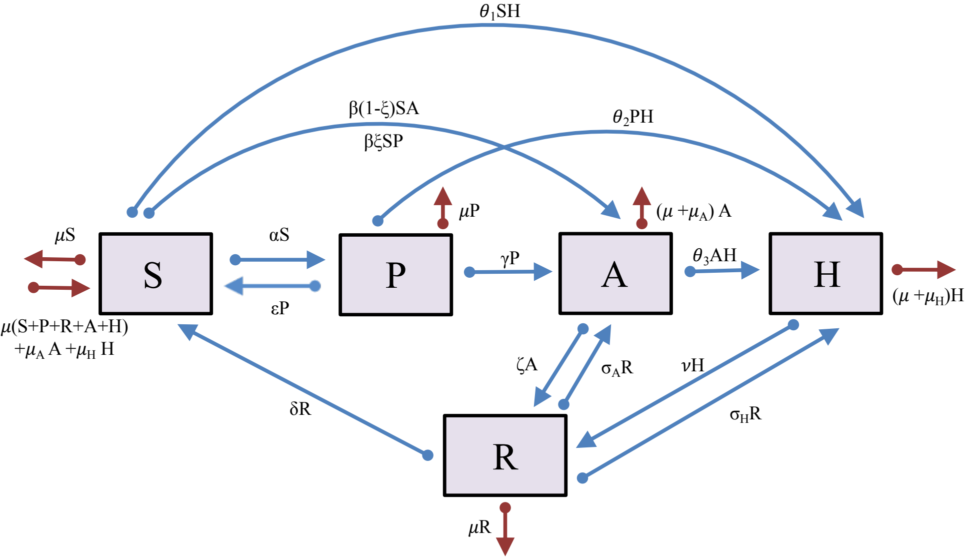
μS, μP, μA, μH, μR: natural death rates



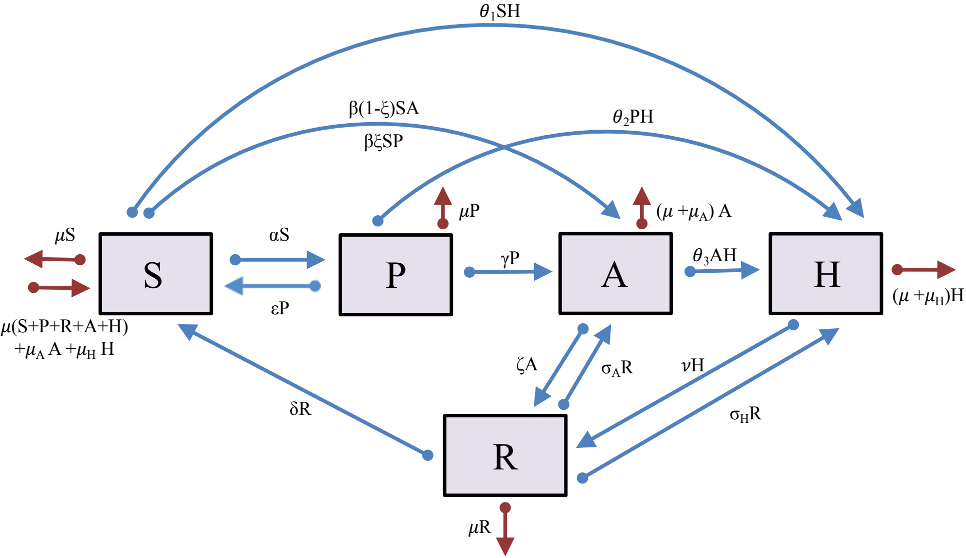
$\mu_A A$: opioid addict overdose death rate



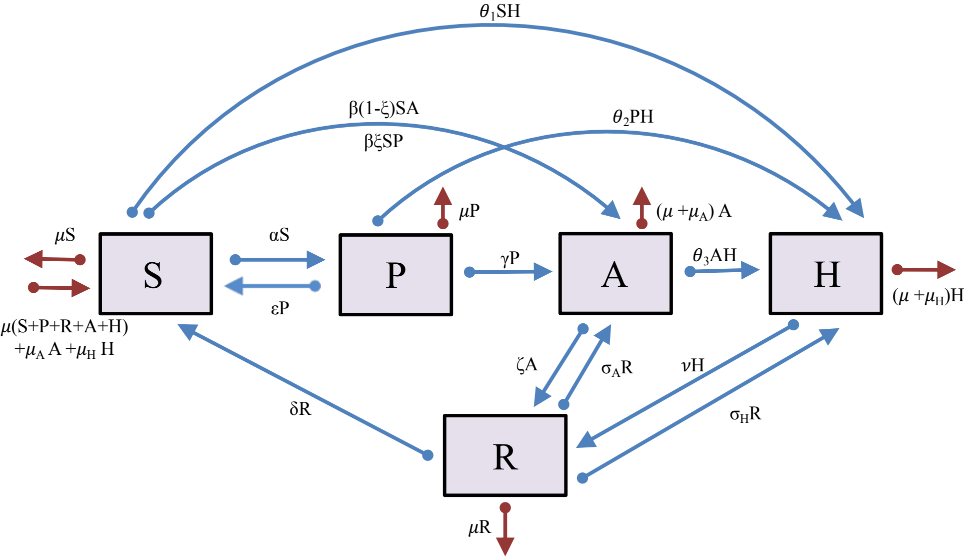
$\mu_H H$: heroin user overdose death rate



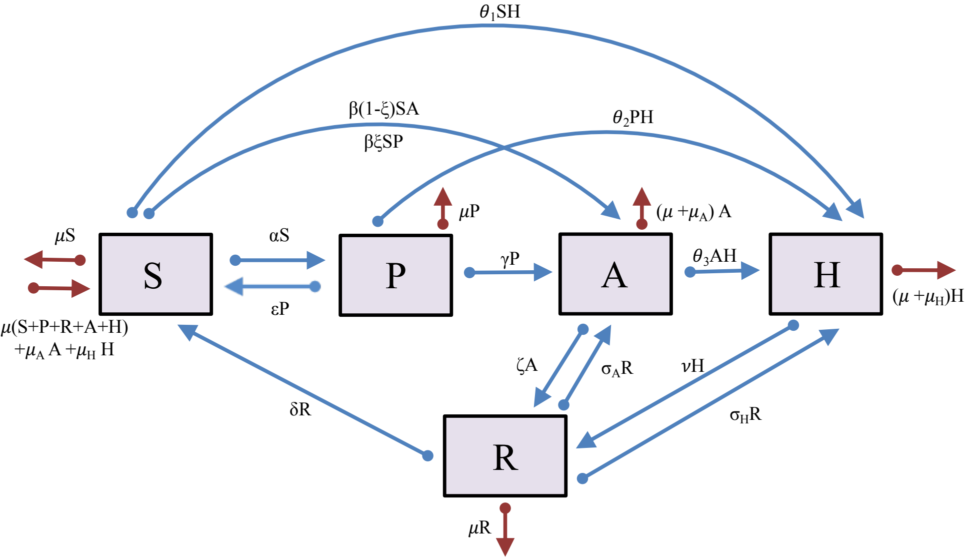
γP: rate of opioid addiction for prescribed users



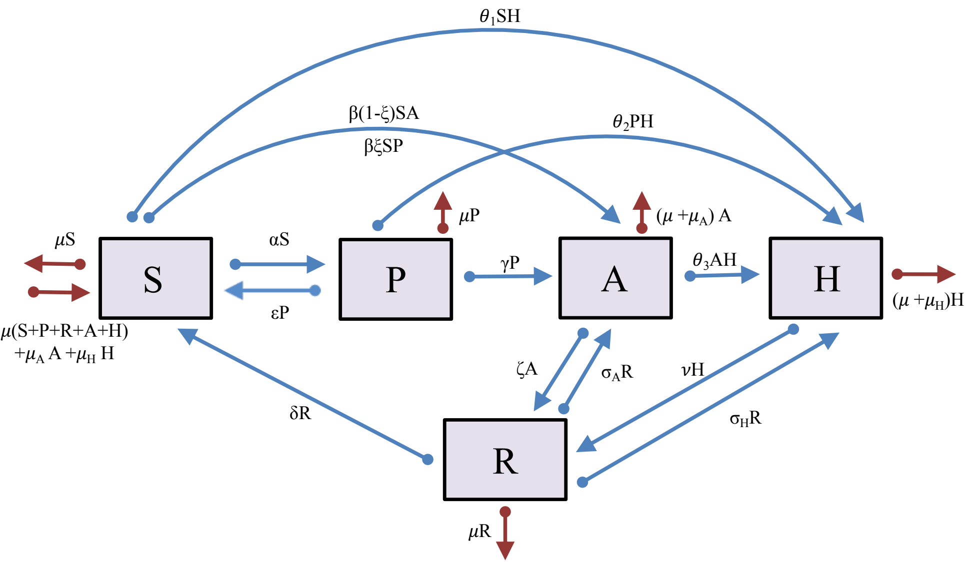
$\theta_2 PH$: rate of heroin addiction for prescribed users



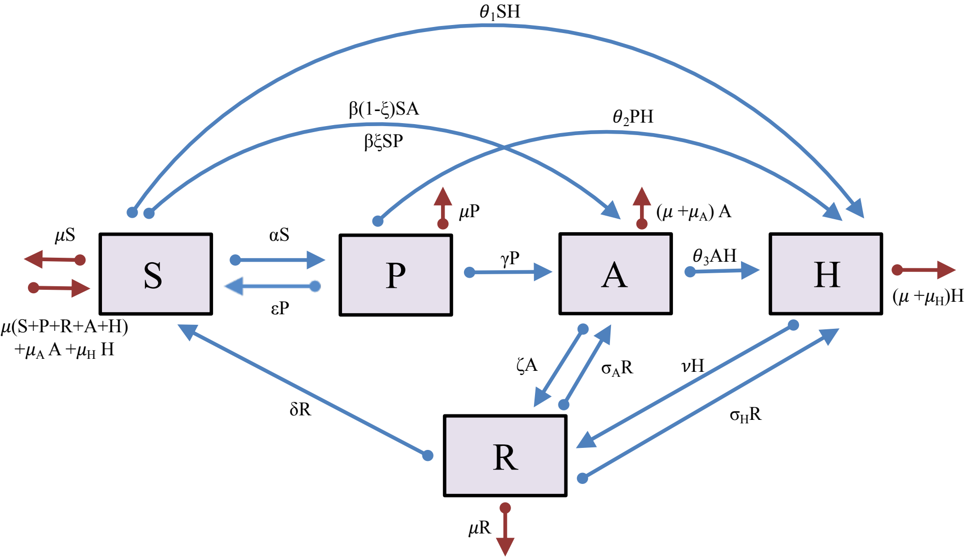
$\sigma_A R$: transition rate from treatment into the opioid addicted class



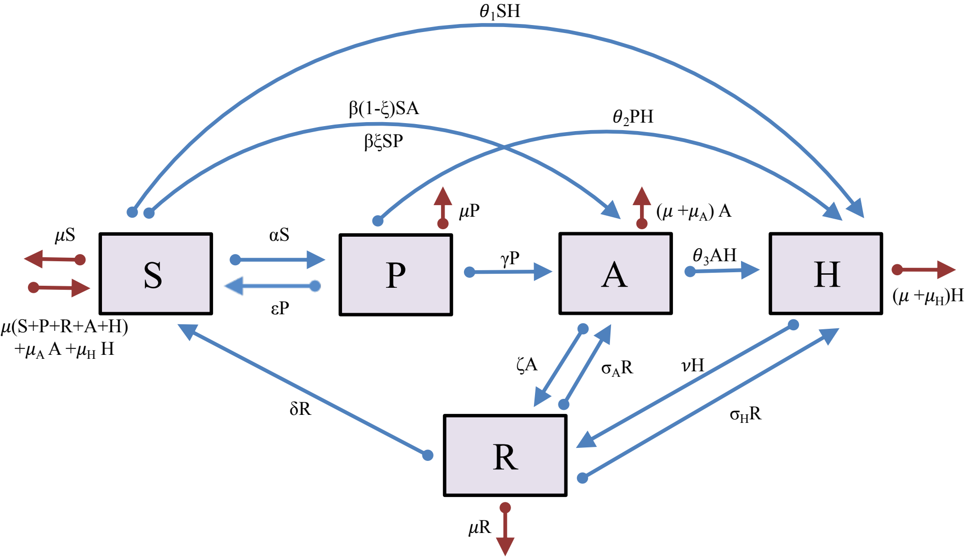
$\sigma_H R$: transition rate from treatment into the heroin addicted class



ζA: rate addicted opioid users enter treatment



νH: rate heroin users enter treatment



$\theta_3 AH$: heroin addiction rate from opioid addicted

System of ODEs Model

$$\frac{dS}{dt} = -\alpha S - \beta(1-\xi)SA - \beta\xi SP - \theta_1 SH + \epsilon P + \delta R + \mu(P+R) + (\mu+\mu_A)A + (\mu+\mu_H)H$$

$$\frac{dP}{dt} = \alpha S - \epsilon P - \gamma P - \theta_2 PH - \mu P$$

$$\frac{dA}{dt} = \gamma P + \sigma_A R + \beta(1-\xi)SA + \beta\xi SP - \zeta A - \theta_3 AH - (\mu + \mu_A)A$$

$$\frac{dH}{dt} = \theta_1 SH + \theta_2 PH + \theta_3 AH + \sigma_H R - \nu H - (\mu + \mu_H)H$$

$$\frac{dR}{dt} = \zeta A + \nu H - \delta R - \sigma_A R - \sigma_H R - \mu R$$

Data

- We have made contact with individuals in diverse fields in order to obtain data on the opioid and heroin epidemic specifically in Knox County and East Tennessee:
  - Dr. Paul Erwin, Head of the Department of Public Health
  - Dr. Agricola Odoi, Associate Professor of Epidemiology
  - Dr. Kelly Cooper, Director of Clinical Services and Assistant Public Health Officer at the Knox County Health Department

Overview

Addiction-Free Equilibrium

To find the addiction-free equilibrium, require $A = H = R = 0 \implies$

$$\frac{dS}{dt} = 0 = -\alpha S^* - \beta\xi S^* P^* + \epsilon P^* + \mu P^*$$

$$\frac{dP}{dt} = 0 = \alpha S^* - \epsilon P^* - \gamma P^* - \mu P^*$$

$$\frac{dA}{dt} = 0 = \gamma P^* + \beta\xi S^* P^*.$$

- Note:
  - If $P^* = 0 \implies$ the only solution is $S^* = P^* = A^* = H^* = R^* = 0$, but $S^* + P^* + A^* + H^* + R^* = 1$, not possible.

Addiction-Free Equilibrium

$$0 = -\alpha S^* - \beta\xi S^* P^* + \epsilon P^* + \mu P^*$$

$$0 = \alpha S^* - \epsilon P^* - \gamma P^* - \mu P^*$$

$$\boxed{0 = \gamma P^* + \beta\xi S^* P^*}$$

- Solving:
  - Will assume $P^* \neq 0 \implies \gamma + \beta\xi S^* = 0$ and since all of our parameters and variables are non-negative $\implies \gamma = 0$ and either $\beta = 0$ or $\xi = 0$.
  - $\gamma = 0$ means that individuals who are prescribed opioids cannot become addicted to opioids.
  - $\xi = 0$ means that only black market opioids are available and there are no excess prescription drugs available.
  - $\beta = 0$ means that susceptibles are unable to become addicted to opioids at all (less realistic).

Addiction-Free Equilibrium

- Solving continued:
  - Assuming $\gamma = 0 = \xi$ to ensure the existence of our addiction-free equilibrium and that $1 = S + P + A + H + R$, we calculate the addiction-free equilibrium to be:

$$S^* = \frac{\epsilon + \mu}{\alpha + \epsilon + \mu}$$
$$P^* = \frac{\alpha}{\alpha + \epsilon + \mu}$$
$$A^* = 0$$
$$H^* = 0$$
$$R^* = 0$$

  - Note that enforcing $P^* \neq 0$ implies that $\alpha \neq 0$, as well.

Basic Reproduction Number, $\mathcal{R}_0$

- In general, $\mathcal{R}_0$ gives the expected number of secondary infected cases that result from the introduction of a disease to a susceptible population.
- Since $\gamma = 0$ and $\xi = 0$ for addiction-free equilibrium, individuals can become addicted only with interactions with addicted individuals or heroin users so takes the form of an infectious disease $\implies$ can calculate $\mathcal{R}_0$.
- Our model has three addiction compartments, A, H and R since these all consist of opioid and/or heroin addicted individuals.
- Will utilize the Next Generation Matrix Method in order to calculate $\mathcal{R}_0$.

Calculating $\mathcal{R}_0$

- $\gamma = 0$ and $\xi = 0$ (thus, $\beta \neq 0$) results in:

$$\frac{dS}{dt} = -\alpha S - \beta SA - \theta_1 SH + \epsilon P + \delta R + \mu(P+R) + (\mu + \mu_A)A + (\mu + \mu_H)H$$

$$\frac{dP}{dt} = \alpha S - \epsilon P - \theta_2 PH - \mu P$$

$$\frac{dA}{dt} = \sigma_A R + \beta SA - \zeta A - \theta_3 AH - (\mu + \mu_A)A$$

$$\frac{dH}{dt} = \theta_1 SH + \theta_2 PH + \theta_3 AH + \sigma_H R - \nu H - (\mu + \mu_H)H$$

$$\frac{dR}{dt} = \zeta A + \nu H - \delta R - \sigma_A R - \sigma_H R - \mu R.$$

Calculating $\mathcal{R}_0$

- We may write the differential equations of the addicted compartments, A, H and R as:

$$\frac{dA}{dt} = \mathcal{F}_1(x,y) - \mathcal{V}_1(x,y)$$
$$\frac{dH}{dt} = \mathcal{F}_2(x,y) - \mathcal{V}_2(x,y)$$
$$\frac{dR}{dt} = \mathcal{F}_3(x,y) - \mathcal{V}_3(x,y),$$

where $x = [A \quad H \quad R]^T$, $y = [S \quad P]^T$, $\mathcal{F}_i$ represents rate that new addicted cases contribute to addicted compartment i and $\mathcal{V}_i$ represents rate of transitions, i.e. rate the addicted compartment i is decreased by means of death, recovery and progression of the addiction (for $i = 1, 2, 3$).

Calculating $\mathcal{R}_0$

- Assuming A, H and R are the addicted compartments, and with $\gamma = 0$ and $\xi = 0$, we have the following matrices that meet the assumptions of the Next Generation Matrix Method:

$$\mathcal{F} = \begin{pmatrix} 0 \\ 0 \\ \beta SA \\ \theta_{\mathbf{1}} SH + \theta_{\mathbf{2}} PH \\ 0 \end{pmatrix}$$

$$\mathcal{V} = \begin{pmatrix} \alpha S + \beta SA + \theta_{\mathbf{1}} SH - \epsilon P - \delta R - \mu(P + R + A + H) - \mu_A A - \mu_H H \\ -\alpha S + \epsilon P + \theta_{\mathbf{2}} PH + \mu P \\ -\sigma_A R + \zeta A + \theta_{\mathbf{3}} AH + (\mu + \mu_A) A \\ -\theta_{\mathbf{3}} AH - \sigma_H R + \nu H + (\mu + \mu_H) H \\ -\zeta A - \nu H + \delta R + \sigma_A R + \sigma_H R + \mu R \end{pmatrix}.$$

Calculating $\mathcal{R}_0$

- Taking $F = \frac{\partial \mathcal{F}_i}{\partial x_j}(0, y_0)$ and $V = \frac{\partial \mathcal{V}_i}{\partial x_j}(0, y_0)$, i, j= $1, 2, 3$, where $(0,\ y_0) = (\frac{\epsilon+\mu}{\alpha+\epsilon+\mu}, \frac{\alpha}{\alpha+\epsilon+\mu}, 0, 0, 0)$ is the addiction-free equilibrium:

$$F = \begin{pmatrix} \beta S^* & 0 & 0 \\ 0 & \theta_1 S^* + \theta_2 P^* & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} \zeta + \mu + \mu_A & 0 & -\sigma_A \\ 0 & \nu + \mu + \mu_H & -\sigma_H \\ -\zeta & -\nu & \delta + \sigma_A + \sigma_H + \mu \end{pmatrix}.$$

Calculating $\mathcal{R}_0$

- The eigenvalues of FV^{-1} are calculated to be:

$$\{0, \frac{(r+s)-\sqrt{(r-s)^2+4\beta S^* z\sigma_A\zeta\sigma_H\nu}}{2det(V)}, \frac{(r+s)+\sqrt{(r-s)^2+4\beta S^* z\sigma_A\zeta\sigma_H\nu}}{2det(V)}\}$$

  where $a = \zeta + \mu + \mu_A$, $b = \nu + \mu + \mu_H$, $c = \delta + \sigma_A + \sigma_H + \mu$, $z = \theta_1 S^* + \theta_2 P^*$, $r = \beta S^*(bc - \sigma_H\nu), s = z(ac - \sigma_A\zeta)$, and $det(V) = a(bc - \sigma_H\nu) - \sigma_A\zeta b$.

- $\mathcal{R}_0$ may then be determined as the spectral radius of FV^{-1}:

$$\mathcal{R}_0 = \frac{(r+s)+\sqrt{(r-s)^2+4\beta S^* z\sigma_A\zeta\sigma_H\nu}}{2det(V)}$$

- If $\mathcal{R}_0 < 1$, the AFE will be locally stable and addiction will die out; if $\mathcal{R}_0 > 1$, the AFE will be unstable and addiction will persist.

Future Tasks

- Obtain local data from Knox County/East Tennessee.
- Perform sensitivity analysis to determine the sensitivity of each of the classes to the parameters (i.e. the contribution of each of the parameters to the sizes of the classes).
- Fit parameters to data that are difficult to find in literature.
- Explore management strategies for how to best treat pain with prescriptions while reducing opioid addiction and heroin use.
- Break recovery class into two different classes for opioid addicts and heroin users.
- Look into gender, race, socioeconomic class or rural versus urban location to investigate differences.

Overview

Background

- In collaboration with Dr. Orou Gaoue.
- *Khaya senegalensis* (African Mahogany) is a large tree species, typically 30 meters high with a 3 meter diameter, found in parts of Western Africa.
- Focus on two areas of Benin where this tree species grows:
  - Sudanian northern dry region with a shorter growing season, lower rainfall, higher temperatures, lower diversity of habitats
  - Sudano-Guinean central moist region with a longer growing season, higher rainfall, lower temperatures, higher diversity of habitats
- Local cattle-herders, called *Fulani*, defoliate the trees in the dry season in order to feed their livestock.
- Due to the risk of climbing for harvesting, they maximize the amount of foliage they obtain which results in almost full defoliation, usually more than 80%.

- *K. senegalensis* is harvested lethally for its timber, in which the entire plant is removed, and non-lethally for both its leaves and bark.
- Non-lethal harvest of non-timber forest products (NTFPs) holds economic and cultural significance.
- Lethal harvest removes individuals but also affects the growth rate of the population.
- Non-lethal harvesting does not directly kill the tree but results in a reduction in reproduction and the population growth rate (indirect effects).

Previous Work

System of ODEs model:

- Dr. Orou Gaoue, Dr. Suzanne Lenhart and collaborators developed a model that incorporated the effect of both types of harvesting on plant population dynamics and population growth rate for general plant species experiencing timber and/or NTFP harvesting.
- Plant population density: $x(t)$, population intrinsic growth rate: $r(t)$

$$\frac{dx(t)}{dt} = r(t)x(t)(1 - \frac{x(t)}{K}) - h_L(t)x(t)$$

$$\tau\frac{dr(t)}{dt} = r_e - r(t) - (\alpha h_N(t) + \beta h_L(t))$$

- Optimal control applied to determine optimal time-dependent nonlethal and lethal harvest strategies for population, maximizing conservation and benefits to harvesters, while minimizing cost.

Previous Work

- Main results:
  - Optimal strategy is to perform nonlethal NTFP harvesting and then after a few years, begin lethal harvesting.
  - Lethal or non-lethal harvesting rates must be $< 40\%$ of the population density, lower than most sustainable harvest rates reported for NTFPs.
- Prior work with varying harvest, however, did not include size structure, which provides the motivation for the development of our model.

Previous Work

Stage-structured model:

- Dr. Orou Gaoue developed a discrete harvesting model that incorporated non-lethal harvesting of adults.
- Population Classes
  - Seedlings (SDL): 0 cm $<$ basal diameter $<$ 2 cm
  - Saplings (SAP): 2 cm $\leq$ basal diameter $<$ 5 cm
  - Juveniles (JUV): 5 cm $\leq$ diameter at breast height $<$ 20 cm
  - Small-reproductive adults (AD1): 20cm $\leq$ diameter at breast height $<$ 40 cm
  - Large-reproductive adults (AD2): diameter at breast height $\geq$ 40 cm
- Harvested mostly reproductive (AD1 and AD2) trees
  - High harvest: $>$ 50% of trees defoliated, $>$ 10% tree bark removed
  - Low harvest: $<$25% of trees defoliated, $<$ 10% tree bark removed

Previous Work

$$\begin{pmatrix} \sigma_1(1-\gamma_{12}) & \sigma_2\rho_{21} & 0 & \sigma_4\phi_4 & \sigma_5\phi_5 \\ \sigma_1\gamma_{12} & \sigma_2(1-\gamma_{23}-\rho_{21}) & \sigma_3\rho_{32} & 0 & 0 \\ 0 & \sigma_2\gamma_{23} & \sigma_3(1-\gamma_{34}-\rho_{32}) & \sigma_4\rho_{43} & 0 \\ 0 & 0 & \sigma_3\gamma_{34} & \sigma_4(1-\gamma_{45}-\rho_{43}) & \sigma_5\rho_{54} \\ 0 & 0 & 0 & \sigma_4\gamma_{45} & \sigma_5(1-\rho_{54}) \end{pmatrix}$$

- σ_i: survival probabilities
- γ_{ij}: probability of transitioning from class i to class j
- ϕ_i: fertility rate of adult class
- 6 populations in dry region: 3 high harvest, 3 low harvest;
  6 populations in moist region: 3 high harvest, 3 low harvest

Previous Work

- Main results:
  - Effect of NTFP harvest was greater in the short term on population growth rate than the long term, especially in moist region; using long-term growth rates only to inform management decisions for sustainable harvest in the short term can be misleading.
  - Survival of early, non-reproductive stages is more important for short-term dynamics than long-term dynamics (management decisions made short-term).
  - Information on both short and long-term population dynamics should be used for management plans.
- Generalized harvest of adults as high or low to explore short and long term population growth rates; not explicit with amount of harvest for each size class and harvest did not vary over time.
- Model did not incorporate the non-lethal harvest of juveniles or lethal harvest of adults.
- Provides motivation for the development of our model.

Overview

Harvesting Model Formulation

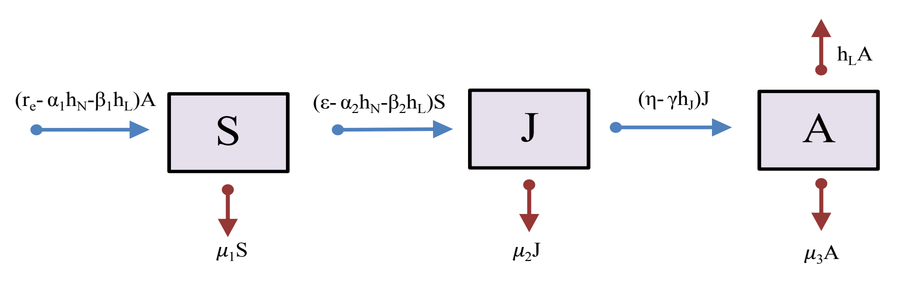
- **Goals**:
  - Develop a size-dependent, time-varying harvesting model for *K. senegalensis*.
  - Investigate optimal size-dependent harvesting strategies for *K. senegalensis* with the goal of maximizing benefits to the local population, while minimizing the cost of harvesting.

Harvesting Model Formulation

Preliminary idea: Continuous model

- Population Classes
  - Seedlings (S)
  - Juveniles (J)
  - Adults (A)
- Assumptions
  - Seedlings do not reproduce nor are harvested.
  - Juveniles do not reproduce and are non-lethally harvested.
  - Adults reproduce and are both non-lethally and lethally harvested.

Harvesting Model Formulation



Schematic diagram for harvesting model

System of ODEs Model

$$\frac{dS}{dt} = (r_e - \alpha_1 h_N - \beta_1 h_L)A - (\epsilon - \alpha_2 h_N - \beta_2 h_L)S - \mu_1 S$$

$$\frac{dJ}{dt} = (\epsilon - \alpha_2 h_N - \beta_2 h_L)S - (\eta - \gamma h_J)J - \mu_2 J$$

$$\frac{dA}{dt} = (\eta - \gamma h_J)J - h_L A - \mu_3 A,$$

h_L: function of time representing lethal harvest of adults
h_N: function of time representing nonlethal harvest of adults
h_J: function of time representing nonlethal harvest of juveniles
Assume $(r_e - \alpha_1 h_N - \beta_1 h_L)$, $(\epsilon - \alpha_2 h_N - \beta_2 h_L)$ and $(\eta - \gamma h_J)$ all > 0.

Harvesting Model Formulation

$$\frac{dS}{dt} = (r_e - \alpha_1 h_N - \beta_1 h_L)A - (\epsilon - \alpha_2 h_N - \beta_2 h_L)S - \mu_1 S$$

$$\frac{dJ}{dt} = (\epsilon - \alpha_2 h_N - \beta_2 h_L)S - (\eta - \gamma h_J)J - \mu_2 J$$

$$\frac{dA}{dt} = (\eta - \gamma h_J)J - h_L A - \mu_3 A$$

- Parameters:
  - $r_e A$: maximum rate A produce S with no harvest
  - $\alpha_1 h_N A$: rate A nonlethal harvest reduces S production
  - $\beta_1 h_L A$: rate A lethal harvest reduces S production
  - ϵS: rate S transition to J

Harvesting Model Formulation

$$\frac{dS}{dt} = (r_e - \alpha_1 h_N - \beta_1 h_L)A - (\epsilon - \alpha_2 h_N - \beta_2 h_L)S - \mu_1 S$$

$$\frac{dJ}{dt} = (\epsilon - \alpha_2 h_N - \beta_2 h_L)S - (\eta - \gamma h_J)J - \mu_2 J$$

$$\frac{dA}{dt} = (\eta - \gamma h_J)J - h_L A - \mu_3 A$$

- Parameters:
  - $\alpha_2 h_N S$: rate A nonlethal harvesting delays transition rate of S to J
  - $\beta_2 h_L S$: rate A lethal harvesting delays the transition rate of S to J
  - $\mu_1 S$: natural mortality rate of S
  - ηJ: rate J transition to A

Harvesting Model Formulation

$$\frac{dS}{dt} = (r_e - \alpha_1 h_N - \beta_1 h_L)A - (\epsilon - \alpha_2 h_N - \beta_2 h_L)S - \mu_1 S$$

$$\frac{dJ}{dt} = (\epsilon - \alpha_2 h_N - \beta_2 h_L)S - (\eta - \gamma h_J)J - \mu_2 J$$

$$\frac{dA}{dt} = (\eta - \gamma h_J)J - h_L A - \mu_3 A$$

- Parameters:
  - $\gamma h_J J$: rate J nonlethal harvesting reduces J growth and delays the transition rate of J to A
  - $\mu_2 J$: natural mortality rate of J
  - $\mu_3 A$: natural mortality rate of A

Future Tasks

- Further develop a harvesting model and analyze (currently deciding between continuous and discrete).
- Determine best size-dependent harvesting strategies for *K. senegalensis* to maximize benefits of harvest and conservation of the species, while minimizing harvesting costs.

Units

Heroin Model:

- S, P, A, H, R: unit-less because proportions
- t: year
- ξ: unit-less
- All other parameters: 1/year

Harvesting Model:

- S, J, A: number of trees
- t: year
- $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma$: unit-less
- h_N, h_L: 1/year
- All other parameters: 1/year

References

- Han B, Compton WM, Blanco C, Crane E, Lee J, Jones CM. (2017), Prescription Opioid Use, Misuse, and Use Disorders in U.S. Adults: 2015 National Survey on Drug Use and Health. Ann Intern Med,167:293-301. doi: 10.7326/M17-0865
- Potter, J. S., Marino, E. N., Hillhouse, M. P., Nielsen, S., Wiest, K., Canamar, C. P., ... Ling, W. (2013). Buprenorphine/naloxone and methadone maintenance treatment outcomes for opioid analgesic, heroin, and combined users: Findings from starting treatment with agonist replacement therapies (START). Journal of Studies on Alcohol and Drugs, 74(4), 605-613
- 2016 NSDUH Report: America's Behavioral Health Changes and Challenges. www.samhsa.gov/sites/default/files/topics/data outcomes quality/nsduh-ppt-09-2017.pdf
- Results from the 2016 National Survey on Drug Use and Health. www.samhsa.gov/sites/default/files/sites/default/files/2016ffr1slideshowv5.pdf
- Opioid Overdose. Centers for Disease Control and Prevention, Centers for Disease Control and Prevention, 16 Dec. 2016, www.cdc.gov/drugoverdose/data/fentanyl.html
- Key Substance Use and Mental Health Indicators in the United States: Results from the 2016 National Survey on Drug Use and Health. www.samhsa.gov/data/sites/default/files/NSDUH-FFR1-2016/NSDUH-FFR1-2016.pdf

References

- Driessche, P. van den, and James Watmough. "Further Notes on the Basic Reproduction Number." SpringerLink, Springer, Dordrecht, 1 Jan. 1970, link.springer.com/chapter/10.1007/978-3-540-78911-66
- Volkow, Nora D. "America's Addiction to Opioids: Heroin and Prescription Drug Abuse." NIDA, 14 May 2014, www.drugabuse.gov/about-nida/legislative-activities/testimony-to-congress/2016/americas-addiction-to-opioids-heroin-prescription-drug-abuseftn2
- National Institute on Drug Abuse (NIDA) Prescription Opioids and Heroin. d14rmgtrwzf5a.cloudfront.net/sites/default/files/19774-prescription-opioids-and-heroin.pdf
- Key Substance Use and Mental Health Indicators in the United States: Results from the 2015 National Survey on Drug Use and Health. www.samhsa.gov/data/sites/default/files/NSDUH-FFR1-2015/NSDUH-FFR1-2015/NSDUH-FFR1-2015.pdf
- U.S. and World Population Clock. www.census.gov/popclock/
- Medication-Assisted Treatment. www.samhsa.gov/medication-assisted-treatment
- Hughs, Arthur, et al. Prescription Drug Use and Misuse in the United States: Results from the 2015 National Survey on Drug Use and Health. Sept. 2016, www.samhsa.gov/data/sites/default/files/NSDUH-FFR2-2015/NSDUH-FFR2-2015.htm

References

- Mandell, Brian F. The Fifth Vital Sign: A Complex Story of Politics and Patient Care. June 2016, www.mdedge.com/sites/default/files/issues/articles/MandellJune16Blurb.pdf.
- Muhuri, Pradip K., et al. Associations of Nonmedical Pain Reliever Use and Initiation of Heroin Use in the United States. Aug. 2013, archive.samhsa.gov/data/2k13/DataReview/DR006/nonmedical-pain-reliever-use-2013.pdf.
- Gaoue, O. G., Ngonghala, C. N., Jiang, J. and Lelu, M. (2016), Towards a mechanistic understanding of the synergistic effects of harvesting timber and non-timber forest products. Methods Ecol Evol, 7: 398-406.
- Gaoue, O. G., Jiang, J., Ding, W., Agusto, F. B., Lenhart, S. (2016). Optimal harvesting strategies for timber and non-timber forest products in tropical ecosystems. Theoretical Ecology, 9(3), 287-297. doi:http://dx.doi.org/10.1007/s12080-015-0286-4
- Gaoue, O. G. (2016), Transient dynamics reveal the importance of early life survival to the response of a tropical tree to harvest. J Appl Ecol, 53: 112-119. doi:10.1111/1365-2664.12553
- Bufford, J. L. and Gaoue, O. G. (2015), Defoliation by pastoralists affects savanna tree seedling dynamics by limiting the facilitative role of canopy cover. Ecological Applications, 25: 1319-1329. doi:10.1890/14-0953.1
- Gaoue OG (2015) Data from: Transient dynamics reveal the importance of early life survival to the response of a tropical tree to harvest. Dryad Digital Repository. https://doi.org/10.5061/dryad.mb1d0