

Heroin Model

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April 17, 2018

Background

*Talk about fentanyl ("America's") The misuse of opioids, a drug class including prescription pain relievers and the illegal drug heroin, is rampant in today's society. The opioid crisis was declared a public health emergency in October 2017 by the United States Department of Health and Human Sciences [15]. In 2016, there were 11.8 million opioid misusers 12 years of age or older with 948,000 of these being heroin users [10]. In addition, there was an estimated 13,219 heroin deaths in 2016, a more than six-fold increase from the year 2002 [7].

Prescription pain relievers are misused for a variety of reasons with the most prominent being to relieve physical pain, to feel good or get high and to relax or relieve tension [10]. Individuals that misuse prescription opioids mostly obtain them from friends/relatives or from a healthcare provider, with misuse defined as taking the prescription at a higher dose or more frequently than prescribed, or taking someone else's medication [10].

To address this apparent problem in today's society, we have formulated a population level model to investigate the dynamics among individuals taking prescription opioids, addicted to opioids, using heroin and recovering from addiction. We aim to find ways to reduce opioid addiction and heroin use and plan to investigate how to optimally treat pain while reducing the risk of addiction lower than it's current state. This model was motivated by a previous model focusing solely on opioid addicts through prescriptions or via the black market; the purpose of formulating a separate model is to be able to understand the more complicated dynamics that arise among opioid abuse and heroin use [1].

Model Formulation

Our model consists of five subgroups of the national population:

1. Susceptibles (S): This portion of the population consists of individuals who are not taking prescription opioids of any kind, nor are using heroin.
2. Prescription opioid users (P): This class of individuals consists of individuals who are prescribed opioids by a health care provider and take the opioids as recommended by their doctor, so they are not considered addicted.
3. Opioid addicts (A): This group of individuals are addicted to opioids. Although the opioid class of drug includes heroin, here we will take opioids to mean non-heroin. In addition, opioid misuse, abuse and addiction will be used interchangeably throughout.
4. Heroin users (H): This class is composed of individuals who use heroin, which is

implicitly understood to be additive.

5. Individuals in treatment/rehabilitation (R): This class consists of individuals undergoing treatment for their addiction to opioids and heroin.

We denote the initial conditions as $S(0) = S_0$, $P(0) = P_0$, $A(0) = A_0$, $H(0) = H_0$, and $R(0) = R_0$, and we assume all of these values are positive. From data, the initial values for each of the classes are the following proportions of the entire population: $S_0 = 0.6221$, $P_0 = 0.37$, $A_0 = 0.0062$, $H_0 = 0.0014$, and $R_0 = 0.0003$ [4], **Will show that starting with positive initial conditions, will stay positive for all time.

$$\frac{dS}{dt} = -\alpha S - \beta(1-\xi)SA - \beta\xi SP - \theta_1 SH + \epsilon P + \delta R + \mu(P+R) + (\mu + \mu_A)A + (\mu + \mu_H)H \quad (1)$$

$$\frac{dP}{dt} = \alpha S - \epsilon P - \gamma P - \theta_2 PH - \mu P \quad (2)$$

$$\frac{dA}{dt} = \gamma P + \sigma_A R + \beta(1-\xi)SA + \beta\xi SP - \zeta A - \theta_3 AH - (\mu + \mu_A)A \quad (3)$$

$$\frac{dH}{dt} = \theta_1 SH + \theta_2 PH + \theta_3 AH + \sigma_H R - \nu H - (\mu + \mu_H)H \quad (4)$$

$$\frac{dR}{dt} = \zeta A + \nu H - \delta R - \sigma_A R - \sigma_H R - \mu R \quad (5)$$

The parameters involved in this model represent transition rates from one class to another; specifically:

α : the rate at which people are prescribed opioids

β : total probability of becoming addicted to opioids other than by prescription

$\beta(1-\xi)$: proportion of which the non-prescribed, susceptible population becomes addicted to opioids by black market drugs and other addicts

$\beta\xi$: proportion of which the non-prescribed, susceptible population obtains extra prescription opioids and becomes addicted

θ_1 : rate at which the non-prescribed, susceptible population becomes addicted to heroin by black market drugs and other addicts

ϵ : rate at which people come back to the susceptible class after being prescribed opioids and did not develop an addiction

δ : rate at which people come back to the susceptible class after successfully finishing treatment

μ : natural death rate

μ_A : enhanced death rate for opioid addicts (overdose rate which results in death)

μ_H : enhanced death rate for heroin addicts (overdose rate which results in death)

γ : rate at which prescribed opioid users become addicted

θ_2 : rate at which opioid prescribed user population becomes addicted to heroin

σ_A : rate at which people relapse from treatment into the opioid addicted class

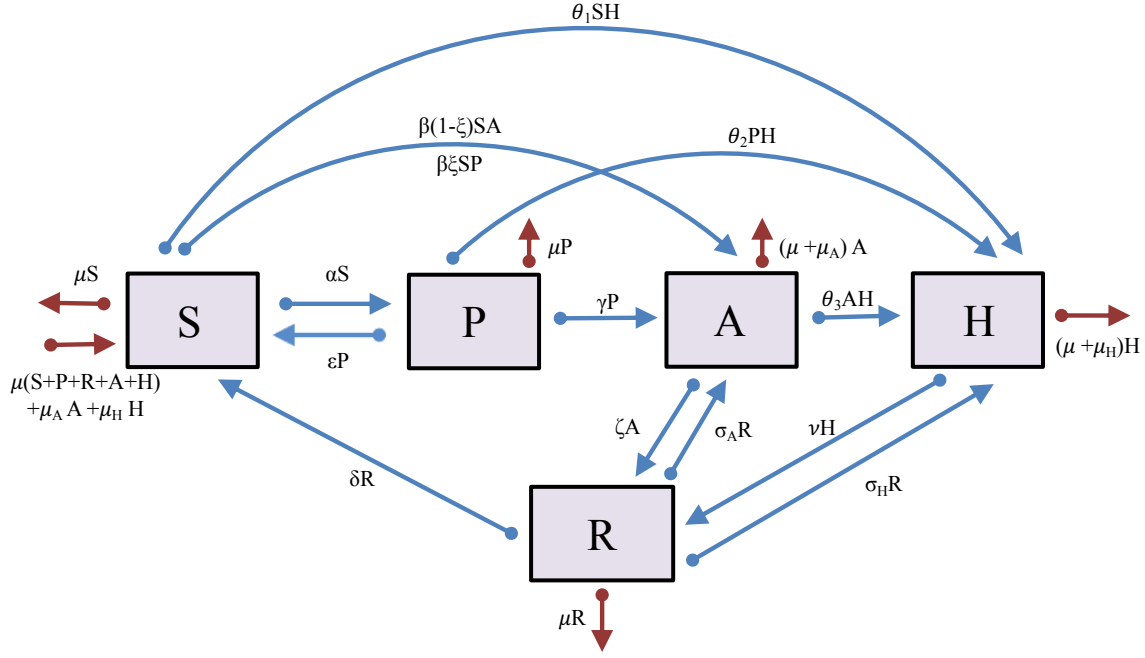
ζ : rate at which addicted opioid users enter treatment/rehabilitation

θ_3 : rate at which the opioid addicted population becomes addicted to heroin

σ_H : rate at which people relapse from treatment into the heroin addicted class

ν : rate at which heroin users enter treatment/rehabilitation

This model can be represented by the following flow diagram:



Equilibrium Analysis

**Find other equilibrium?? To find the addiction-free equilibrium, we set equations (1)-(5) equal to zero and require that $A = H = R = 0$. We are left with the system:

$$\begin{aligned}
 0 &= -\alpha S^* - \beta \xi S^* P^* + \epsilon P^* + \mu P^* \\
 0 &= \alpha S^* - \epsilon P^* - \gamma P^* - \mu P^* \\
 0 &= \gamma P^* + \beta \xi S^* P^*
 \end{aligned}$$

If $P = 0$, then the only solution is $S^* = P^* = H^* = R^* = 0$. Thus, will assume $P \neq 0$. This forces $\gamma + \beta \xi S^* = 0$ and since all of our parameters and variables are non-negative, then it must be $\gamma = 0$ and either $\beta = 0$ or $\xi = 0$. Under the assumption that $\gamma = 0 = \xi$ to ensure the existence of our AFE and that $1 = S + P + A + H + R$, we calculate the AFE to be

$$\begin{aligned}
 S^* &= \frac{\epsilon + \mu}{\alpha + \epsilon + \mu} \\
 P^* &= \frac{\alpha}{\alpha + \epsilon + \mu} \\
 A^* &= 0 \\
 H^* &= 0 \\
 R^* &= 0
 \end{aligned}$$

Basic Reproduction Number, \mathcal{R}_0

For the purposes of calculating \mathcal{R}_0 , we will assume $\gamma = 0$ and $\xi = 0$ (thus, $\beta \neq 0$) in order to ensure the existence of the addiction-free equilibrium. This results in the infected compartment (3)-(5) reducing to:

$$\begin{aligned}\frac{dA}{dt} &= \sigma_A R + \beta S A - \zeta A - \theta_3 A H - (\mu + \mu_A) A \\ \frac{dH}{dt} &= \theta_1 S H + \theta_2 P H + \theta_3 A H + \sigma_H R - \nu H - (\mu + \mu_H) H \\ \frac{dR}{dt} &= \zeta A + \nu H - \delta R - \sigma_A R - \sigma_H R - \mu R\end{aligned}$$

Thus, under the assumption of A, H and R as the infected compartments and parameter restrictions stated above, the assumptions of the Next Generation Method are satisfied for matrices \mathcal{F} and \mathcal{V} shown below. Note that \mathcal{F}_i represents the rate that secondary infections enter infected compartment i and \mathcal{V}_i represents the difference between the rate of transfer out of compartment i and the rate of transfer into compartment i by means different than a secondary infection. Using this method results in the following matrices:

$$\begin{aligned}\mathcal{F} &= \begin{pmatrix} 0 \\ 0 \\ \beta S A \\ \theta_1 S H + \theta_2 P H \\ 0 \end{pmatrix} \\ \mathcal{V} &= \begin{pmatrix} \alpha S + \beta S A + \theta_1 S H - \epsilon P - \delta R - \mu(P + R + A + H) - \mu_A A - \mu_H H \\ -\alpha S + \epsilon P + \theta_2 P H + \mu P \\ -\sigma_A R + \zeta A + \theta_3 A H + (\mu + \mu_A) A \\ -\theta_3 A H - \sigma_H R + \nu H + (\mu + \mu_H) H \\ -\zeta A - \nu H + \delta R + \sigma_A R + \sigma_H R + \mu R \end{pmatrix} \\ F &= \begin{pmatrix} \beta S^* & 0 & 0 \\ 0 & \theta_1 S^* + \theta_2 P^* & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ V &= \begin{pmatrix} \zeta + \mu + \mu_A & 0 & -\sigma_A \\ 0 & \nu + \mu + \mu_H & -\sigma_H \\ -\zeta & -\nu & \delta + \sigma_A + \sigma_H + \mu \end{pmatrix}\end{aligned}$$

The eigenvalues of FV^{-1} are calculated to be:

$$\sigma(FV^{-1}) = \left\{ 0, \frac{(r+s) - \sqrt{(r-s)^2 + 4\beta S^* z \sigma_A \zeta \sigma_H \nu}}{2\det(V)}, \frac{(r+s) + \sqrt{(r-s)^2 + 4\beta S^* z \sigma_A \zeta \sigma_H \nu}}{2\det(V)} \right\}$$

\mathcal{R}_0 may then be determined as the spectral radius of FV^{-1} :

$$\mathcal{R}_0 = \frac{(r+s) + \sqrt{(r-s)^2 + 4\beta S^* z \sigma_A \zeta \sigma_H \nu}}{2 \det(V)}$$

where $a = \zeta + \mu + \mu_A$, $b = \nu + \mu + \mu_H$, $c = \delta + \sigma_A + \sigma_H + \mu$, $z = \theta_1 S^* + \theta_2 P^*$, $r = \beta S^*(bc - \sigma_H \nu)$, $s = z(ac - \sigma_A \zeta)$, and $\det(V) = a(bc - \sigma_H \nu) - \sigma_A \zeta b$.

We note that the radicand $(r-s)^2 + 4\beta S^* z \sigma_A \zeta \sigma_H \nu$ is positive, since all parameters are positive. In addition, r is positive since bc contains the term that cancels with $-\sigma_H \nu$, s is positive since ac contains the term that cancels with $-\sigma_A \zeta$ and finally, $\det(V)$ is positive since abc contains terms that cancel with $-\sigma_A \zeta(\nu + \mu + \mu_H) - \sigma_H \nu$.

Sensitivity Analysis

Saltelli/sobol sensitivity analysis Finds nice points in the parameter space to test and records S, P, A, H and R at the final time (t=10 years) for the parameter choices.

References

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