

Parameter assumptions from calculations done from literature information:

$$\beta_A = 0.000273$$

$$\beta_P = 0.000777$$

$$\mu = 0.00868$$

$$\mu_A = 0.00870$$

$$\mu_H = 0.0507$$

$$\theta_2 = 3\theta_1$$

$$\theta_3 = 16\theta_1$$

$$\nu = 0.0155$$

$$\zeta = 0.0241$$

$$\omega = 10^{-10}$$

Used MultiStart to obtain 10 parameter values; these are the results we are going with for now as “good enough” (although still have parameters hitting bounds, this was the lowest objective function value we could seem to get):

$$\text{LowerBounds} = [-0.1 \ 0.0001 \ 0.2 \ 0.0001 \ 0.00000001 \ 0.000001 \ 0.0001 \ 0.0001 \ 0.0001 \ 0.0001]$$

$$\text{UpperBounds} = [0.1 \ 1 \ 6 \ 0.2 \ 0.1 \ 1 \ 0.5 \ 0.5 \ 0.5 \ 0.5]$$

Parameters estimating:  $[m \ \theta_1 \ \epsilon \ \gamma \ \sigma \ b \ P_0 \ A_0 \ H_0 \ R_0]$

$$\text{Parameter estimates: } [-0.0286 \ 0.313 \ \mathbf{5.953} \ \mathbf{0.000105} \ \mathbf{0.000000667} \ 0.602 \ 0.0369 \ 0.0078 \ \mathbf{0.446}]$$

where  $\alpha = mt + b$ ,  $\theta_2 = 3\theta_1$ ,  $\theta_3 = 16\theta_1$ , and  $S_0 = 1 - P_0 - A_0 - H_0 - R_0$  (and thus, a total of 12 parameter values relied on MultiStart least squares, and the **bolded** values are those that hit the upper/lower bounds when do several different runs).

Objective function value: 0.1309.

Thus, we have the following parameter estimates overall (**red** values most concerning):

$$\alpha = [0.602 \ 0.574 \ 0.545 \ 0.516 \ 0.488 \ 0.459]$$

$$\beta_A = 0.000273$$

$$\beta_P = 0.000777$$

$$\theta_1 = 0.313$$

$$\epsilon = 5.953$$

$$\gamma = 0.000105$$

$$\sigma = 0.000000667$$

$$\mu = 0.00868$$

$$\mu_A = 0.00870$$

$$\mu_H = 0.0507$$

$$\theta_2 = 0.939$$

$$\zeta = 0.0241$$

$$\theta_3 = 5.008$$

$$\nu = 0.0155$$

$$\omega = 10^{-10}$$

$$P_0 = 0.0369$$

$$A_0 = 0.0078$$

$$H_0 = 0.00856$$

$$R_0 = 0.446$$

$$S_0 = 0.508$$

Parameter	Description	Units
$\mu$	natural mortality rate	$\frac{1}{\text{year}}$
$\mu_A$	opioid addict overdose death rate	$\frac{1}{\text{year}}$
$\mu_H$	heroin addict overdose death rate	$\frac{1}{\text{year}}$
$\alpha$	prescription rate	$\frac{1}{\text{year}}$
$\beta_A$	illicit addiction rate from the black market	$\frac{1}{\text{year}}$
$\beta_P$	illicit addiction rate from availability of excess pills	$\frac{1}{\text{year}}$
$\theta_1$	heroin addiction rate for susceptible individuals	$\frac{1}{\text{year}}$
$\epsilon$	rate of finishing prescription addiction-free	$\frac{1}{\text{year}}$
$\gamma$	opioid addiction rate from prescription	$\frac{1}{\text{year}}$
$\theta_2$	heroin addiction rate for prescription opioids users	$\frac{1}{\text{year}}$
$\sigma$	relapse to addiction	$\frac{1}{\text{year}}$
$\zeta$	rate of stable recovery for opioid addict	$\frac{1}{\text{year}}$
$\theta_3$	heroin addiction rate for opioid addicts	$\frac{1}{\text{year}}$
$\nu$	rate of stable recovery for heroin addict	$\frac{1}{\text{year}}$
$\omega$	perturbation term	dimensionless
S	proportion of susceptible individuals	dimensionless
P	proportion of susceptible individuals	dimensionless
A	proportion of susceptible individuals	dimensionless
H	proportion of susceptible individuals	dimensionless
R	proportion of susceptible individuals	dimensionless

Table 1: Number of individuals in each category, 2013-2018

	2013	2014	2015	2016	2017	2018
Total population	6,490,795	6,540,007	6,590,726	6,649,404	6,715,984	6,770,010
Population 12 and older	5,517,176	5,559,006	5,602,117	5,651,993	5,708,586	-
Heroin users	-	14,000	14,000	19,000	-	-
Heroin addicts	-	<b>7,560</b>	<b>7,560</b>	<b>10,260</b>	-	-
Prescription opioid addicts (includes heroin addicts)	-	-	48,000	42,000	-	-
Prescription opioid addicts (excludes heroin)	<i>43,418</i>	<i>42,928</i>	<b>42,816</b>	<b>37,464</b>	<i>34,805</i>	-
Prescribed opioid users (includes those addicted)	1,845,144	1,824,342	1,819,581	1,761,363	1,636,374	-
Prescribed opioid users (excludes those addicted)	<i>1,825,910</i>	<i>1,805,325</i>	<b>1,800,613</b>	<b>1,744,766</b>	<i>1,620,955</i>	-
Prescription opioid overdose deaths	-	-	679	-	-	-
Heroin/fentanyl overdose deaths	-	-	374	-	-	-
Prescription opioid treatment admissions	4,485	4,530	4,326	-	-	-
Heroin treatment admissions	555	743	1,083	-	-	-

*italicized* values are numbers that we estimated by extrapolating from a different year

**bolded** values are numbers that we estimated by information within the same year

the rest of the numbers are actual data

the numbers in **blue** are data used for parameter estimation