

# Heroin Model Summary

## Original

$$\begin{aligned}
 \frac{dS}{dt} &= \beta E - \alpha S - \underbrace{S(\sigma + \xi(I + H))}_{\text{to resistant}} - \underbrace{S(\delta_1 E + \delta_2 I)}_{\text{to opioid addiction}} - \underbrace{S(\mu_1 + \mu_2 H)}_{\text{to heroin addiction}} \\
 &\quad + d_I I + d_H H + d_r R + d_e E \\
 \frac{dE}{dt} &= -\beta E + \alpha S - \underbrace{E(\sigma + \xi(I + H))}_{\text{to resistant}} - \underbrace{E(\mu_1 + \mu_2 H)}_{\text{to heroin addiction}} - \underbrace{\epsilon E}_{\text{to I}} - d_e E \\
 \frac{dI}{dt} &= \underbrace{\epsilon E}_{\text{from E}} + \underbrace{S(\delta_1 E + \delta_2 I)}_{\text{from S}} - \underbrace{\gamma(H)I}_{\text{to H}} - \underbrace{iI}_{\text{to R}} - d_I I \\
 \frac{dH}{dt} &= \underbrace{\gamma(H)I}_{\text{from I}} + \underbrace{(S + E)(\mu_1 + \mu_2 H)}_{\text{from S and E}} - \underbrace{hH}_{\text{to R}} - d_H H \\
 \frac{dR}{dt} &= \underbrace{(S + E)(\sigma + \xi(I + H))}_{\text{from S and E}} + \underbrace{iI}_{\text{from I}} + \underbrace{hH}_{\text{from H}} - d_r R \\
 \gamma(H) &= \gamma_0(\mu_1 + \mu_2 H)
 \end{aligned}$$

We assume that  $0 \leq \delta_1 \leq \delta_2 \leq 1$ ,  $\mu_1 < \mu_2$ , and  $d_r$  includes both natural death and backsliding from drug resistant to susceptible.

## I and H feeds S

$$\begin{aligned}
\frac{dS}{dt} &= \beta E - \alpha S - \underbrace{S(\sigma + \xi(I + H))}_{\text{to resistant}} - \underbrace{S(\delta_1 E + \delta_2 I)}_{\text{to opioid addiction}} - \underbrace{S(\mu_1 + \mu_2 H)}_{\text{to heroin addiction}} \\
&\quad + \underbrace{iI}_{\text{from I}} + \underbrace{hH}_{\text{from H}} + d_I I + d_H H + d_r R + d_e E \\
\frac{dE}{dt} &= -\beta E + \alpha S - \underbrace{E(\sigma + \xi(I + H))}_{\text{to resistant}} - \underbrace{E(\mu_1 + \mu_2 H)}_{\text{to heroin addiction}} - \underbrace{\epsilon E}_{\text{to I}} - d_e E \\
\frac{dI}{dt} &= \underbrace{\epsilon E}_{\text{from E}} + \underbrace{S(\delta_1 E + \delta_2 I)}_{\text{from S}} - \underbrace{\gamma(H)I}_{\text{to H}} - \underbrace{(i + d_I)I}_{\text{to S, rec. \& death}} \\
\frac{dH}{dt} &= \underbrace{\gamma(H)I}_{\text{from I}} + \underbrace{(S + E)(\mu_1 + \mu_2 H)}_{\text{from S and E}} - \underbrace{(h + d_H)H}_{\text{to S, rec. \& death}} \\
\frac{dR}{dt} &= \underbrace{(S + E)(\sigma + \xi(I + H))}_{\text{from S and E}} - d_r R \\
\gamma(H) &= \gamma_0(\mu_1 + \mu_2 H)
\end{aligned}$$

## No R

$$\begin{aligned}
\frac{dS}{dt} &= \beta E - \alpha S - \underbrace{S(\delta_1 E + \delta_2 I)}_{\text{to opioid addiction}} - \underbrace{S(\mu_1 + \mu_2 H)}_{\text{to heroin addiction}} + \underbrace{iI}_{\text{from I}} + \underbrace{hH}_{\text{from H}} + d_I I + d_H H + d_e E \\
\frac{dE}{dt} &= -\beta E + \alpha S - \underbrace{E(\mu_1 + \mu_2 H)}_{\text{to heroin addiction}} - \underbrace{\epsilon E}_{\text{to I}} - d_e E \\
\frac{dI}{dt} &= \underbrace{\epsilon E}_{\text{from E}} + \underbrace{S(\delta_1 E + \delta_2 I)}_{\text{from S}} - \underbrace{\gamma(H)I}_{\text{to H}} - \underbrace{(i + d_I)I}_{\text{to S, rec. \& death}} \\
\frac{dH}{dt} &= \underbrace{\gamma(H)I}_{\text{from I}} + \underbrace{(S + E)(\mu_1 + \mu_2 H)}_{\text{from S and E}} - \underbrace{(h + d_H)H}_{\text{to S, rec. \& death}} \\
\gamma(H) &= \gamma_0(\mu_1 + \mu_2 H)
\end{aligned}$$