Modeling the Heroin Epidemic: A Preliminary Report

Presented by: Tricia Phillips, University of Tennessee, Knoxville

Collaborators: Dr. Suzanne Lenhart and Dr. Christopher Strickland

July 10, 2018

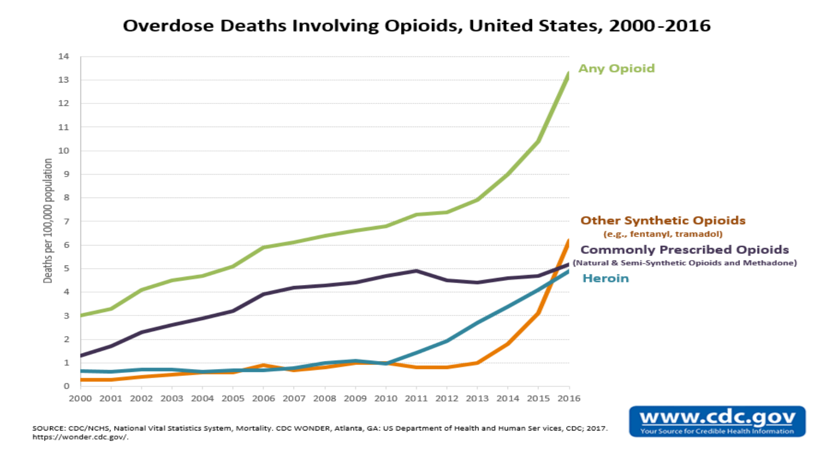
Overview

Motivation

Opioids

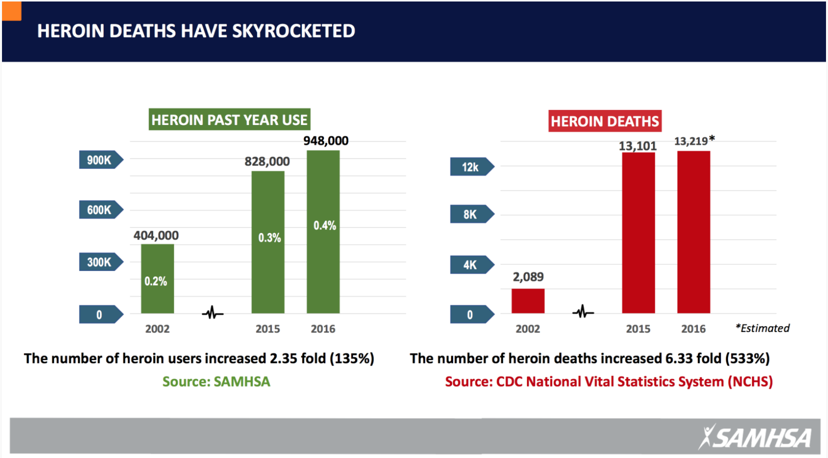
- The misuse of opioids, a drug class including prescription pain relievers and heroin, is rampant in today's society.
- Number of opioid prescriptions that pharmacies distributed tripled from 1991 to 2011.
- Dramatic increase in accessibility to heroin and the lower cost of the drug has influenced prescription opioid users to turn to heroin.
- Estimated 80% of heroin users at the national level used prescription opioids previously.
- The opioid crisis was declared a public health emergency in October 2017 by the United States Department of Health and Human Sciences.

Motivation



Source: Centers for Disease Control and Prevention

Motivation



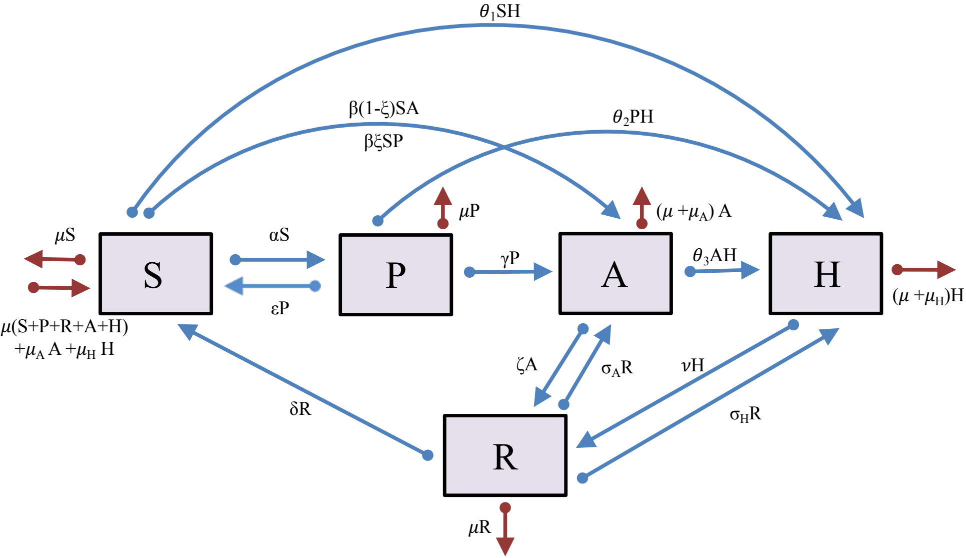
Source: 2016 National Survey on Drug Use and Health Report from SAMSHA.gov

Model Formulation

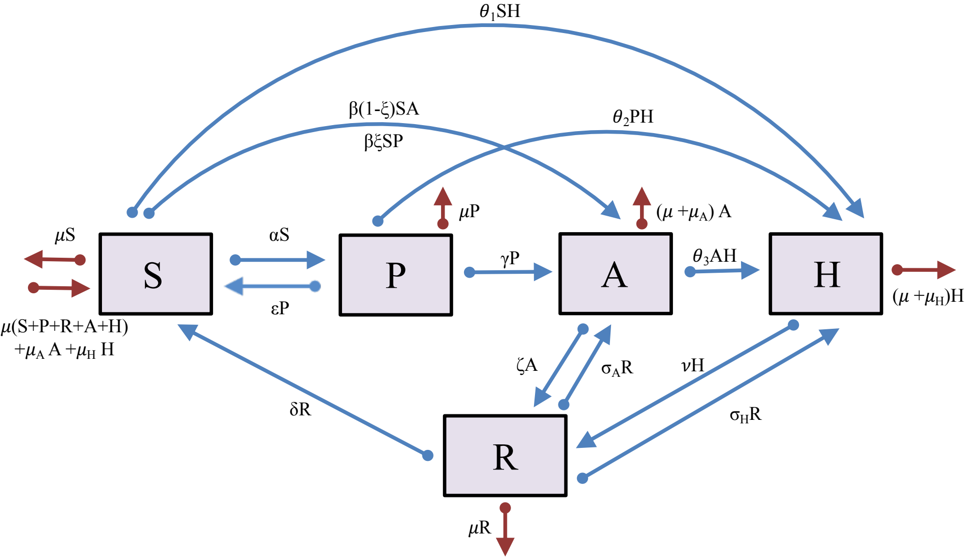
- **Goals**:
  - Investigate the dynamics behind the opioid (non-heroin) and heroin epidemic and identify important conditions relating to the reduction of opioid and heroin addicted individuals.
  - Develop a system of ODEs model consisting of classes of individuals taking prescription opioids, addicted to opioids, using heroin and recovering from opioid addiction, including heroin, and analyze it.
  - Investigate management strategies for how to best treat pain with prescriptions while reducing opioid addiction and heroin use.

Model Formulation

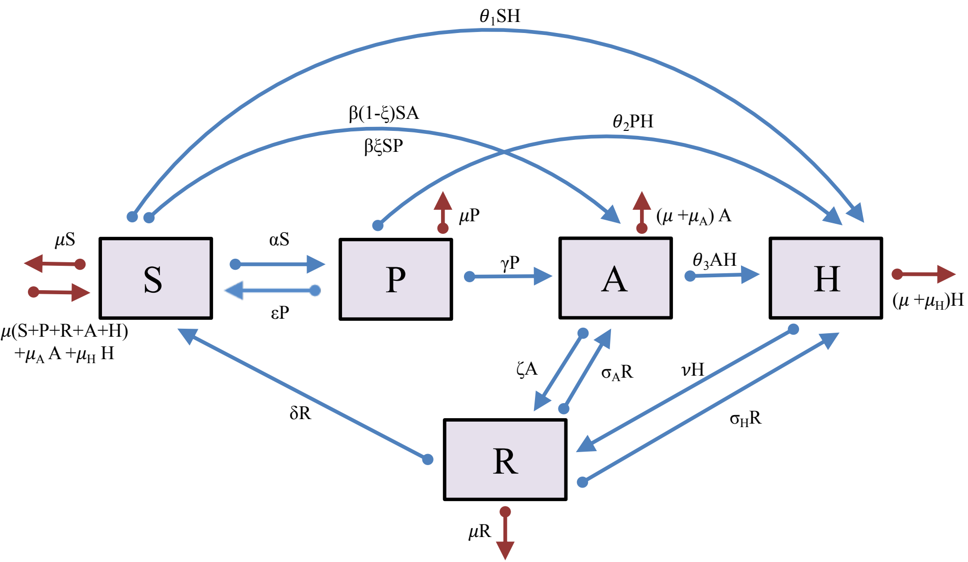
- Based on work by Battista, Pearcy, Strickland (preprint 2018).
- Formulated a five class compartmental population model (classes are fractions of the entire population).
- Population Classes
  - Susceptibles (S): not taking prescription opioids, nor using heroin.
  - Prescription opioid users (P): opioid-prescribed individuals not considered addicted.
  - Opioid addicts (A): addicted to opioids.
  - Heroin users (H): addicted to heroin.
  - Individuals in treatment/rehabilitation (R): undergoing treatment for their addiction to opioids or heroin.



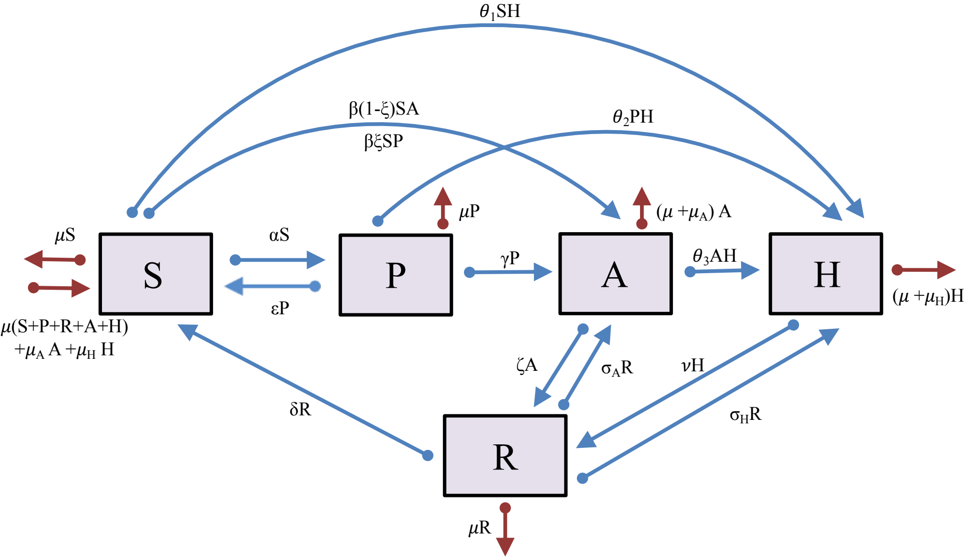
αS: prescription rate



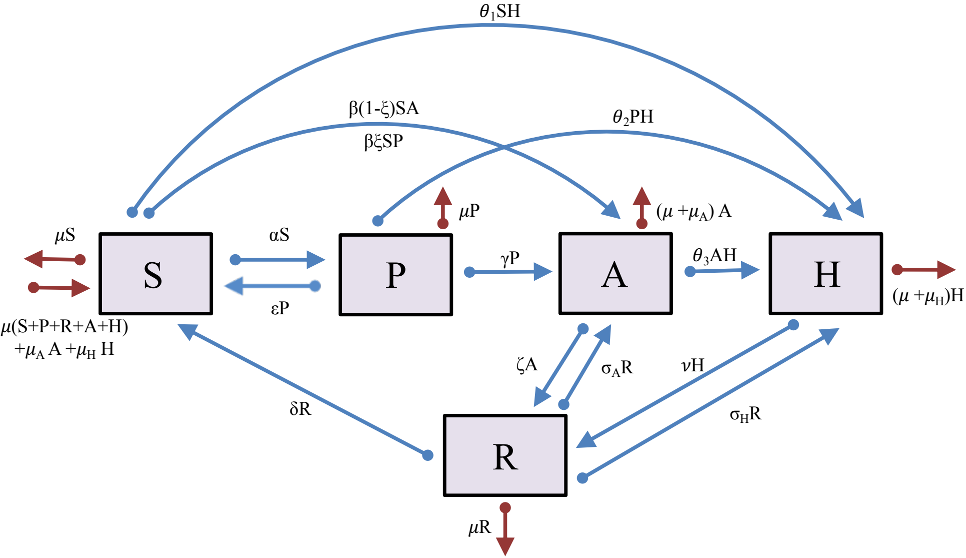
$\beta(1-\xi)SA$: opioid addiction rate by black market drugs/interaction with other addicts



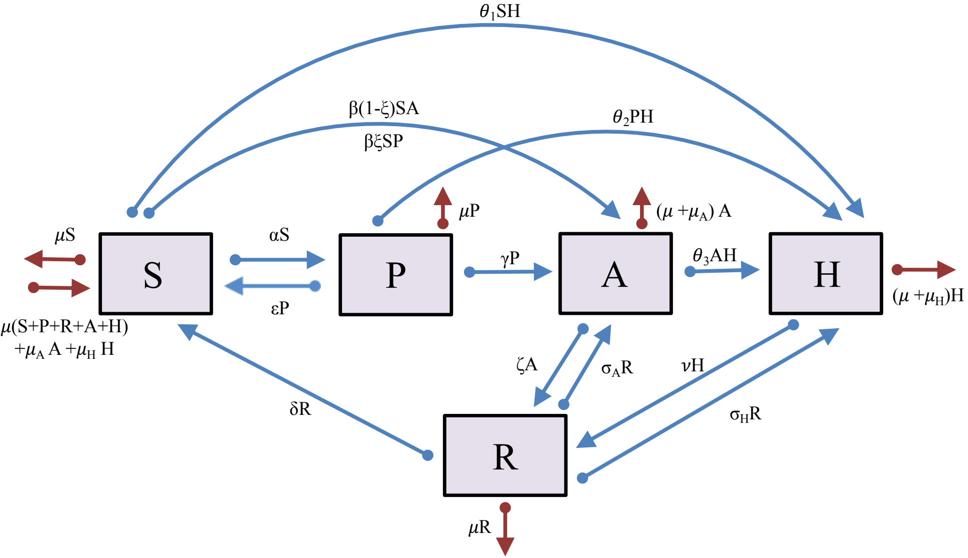
$\beta\xi SP$: opioid addiction rate by obtaining extra prescription opioids



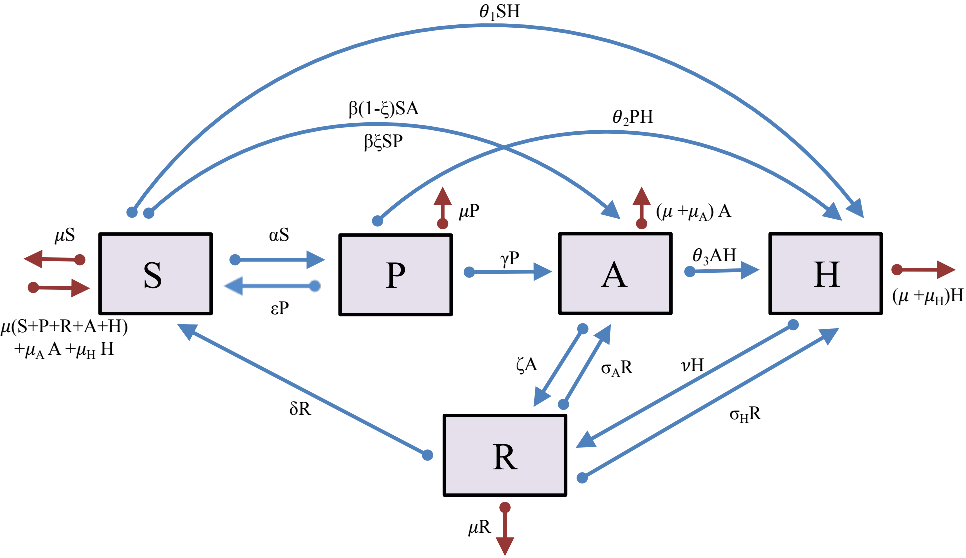
$\theta_1 SH$: rate of addiction to heroin by black market availability/ interaction with other users



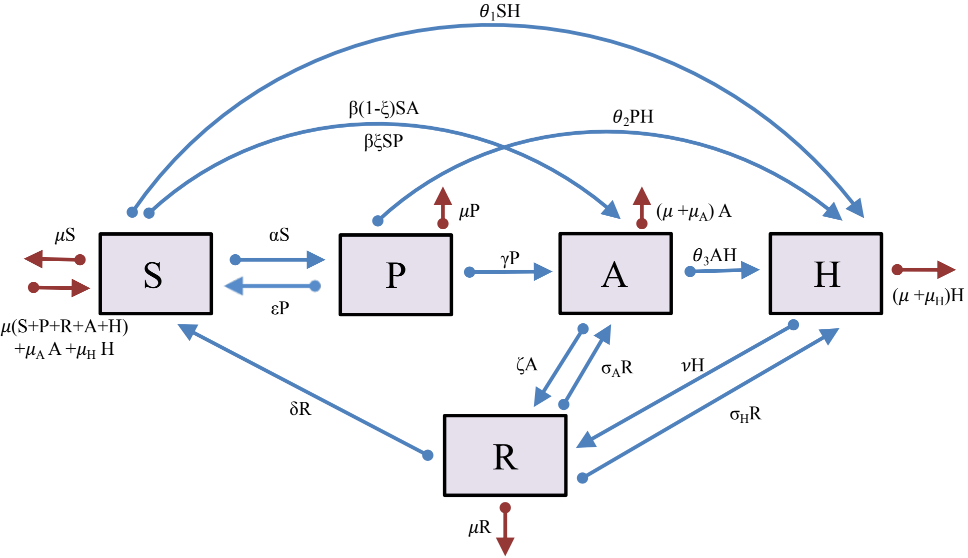
ϵP: rate of non-addicted opioid prescribed users back to susceptible



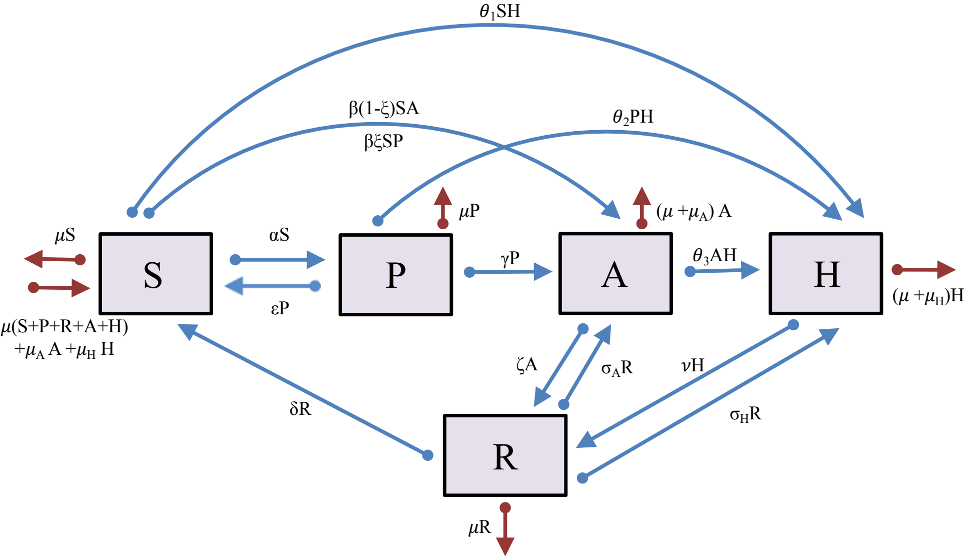
δR: rate of opioid and heroin addicts successfully finishing treatment



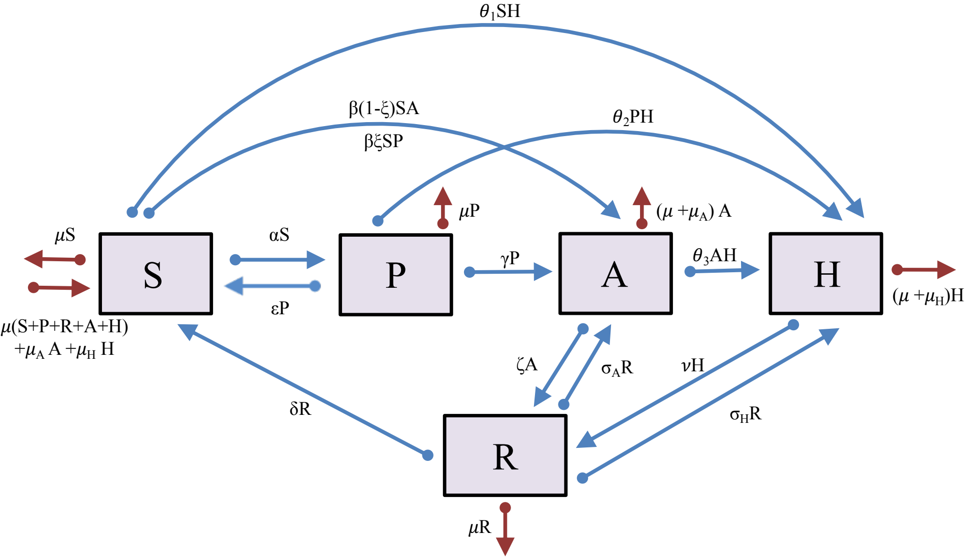
μS, μP, μA, μH, μR: natural death rates



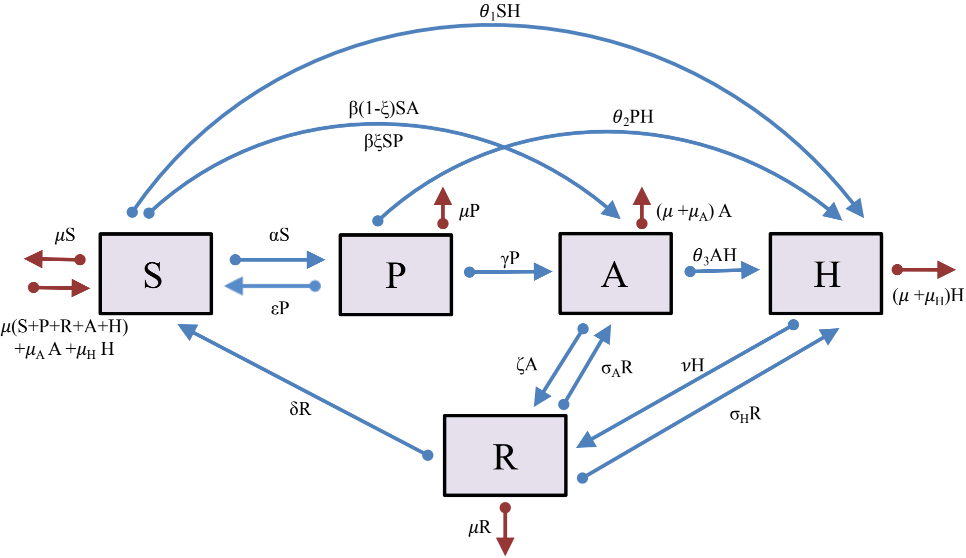
$\mu_A A$: opioid addict overdose death rate



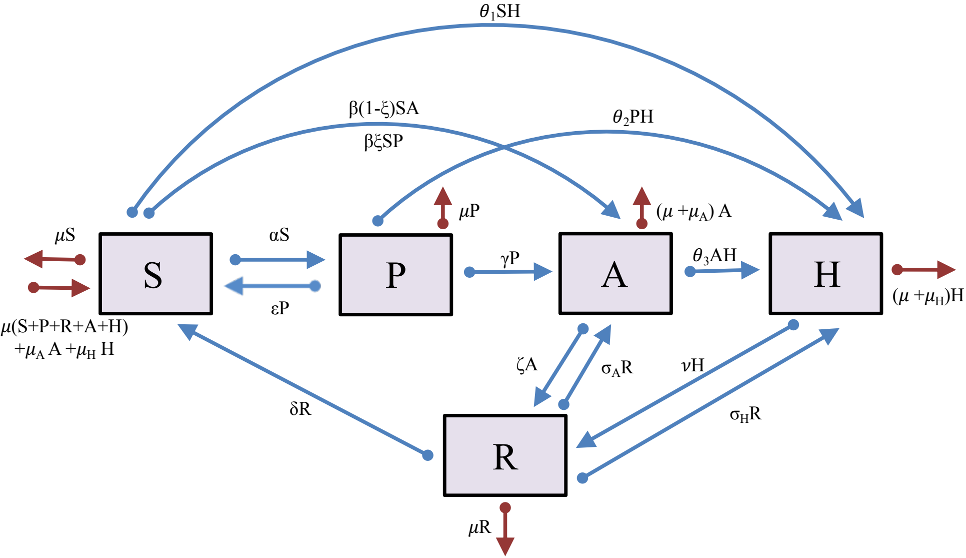
$\mu_H H$: heroin user overdose death rate



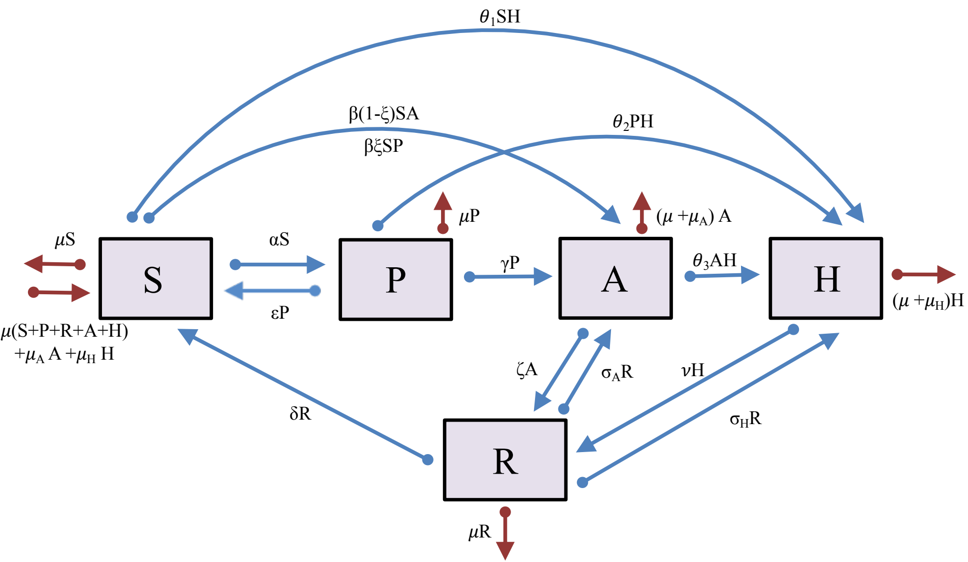
γP: rate of opioid addiction for prescribed users



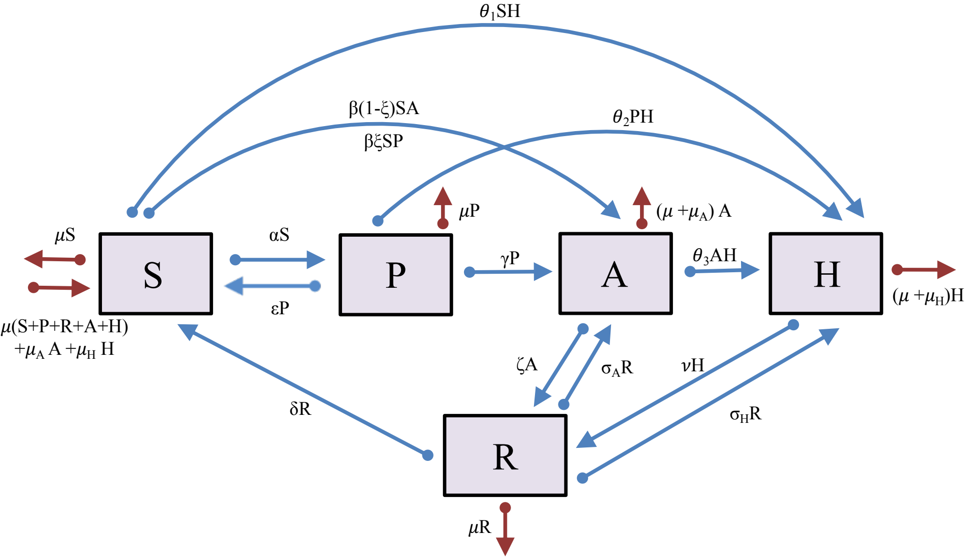
$\theta_2 PH$: rate of heroin addiction for prescribed users



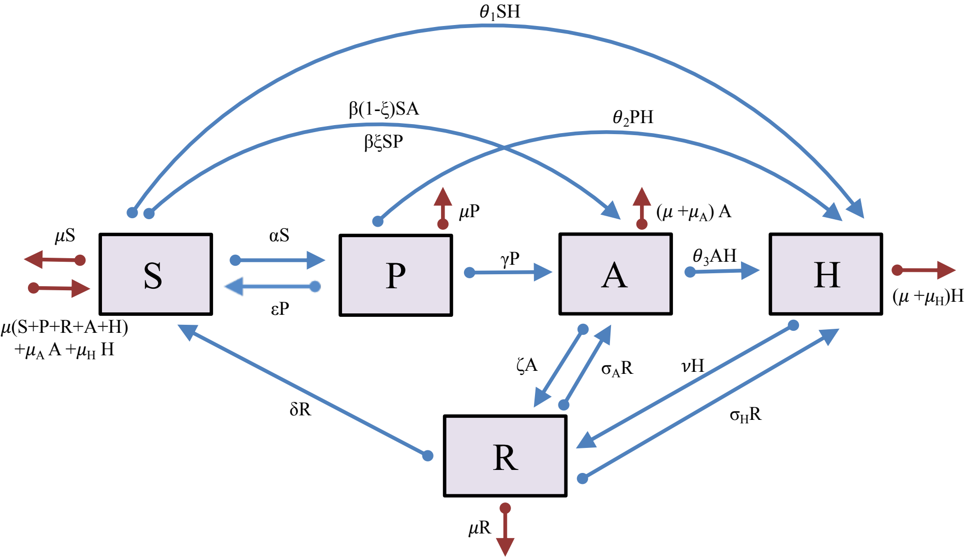
ζA: rate addicted opioid users enter treatment



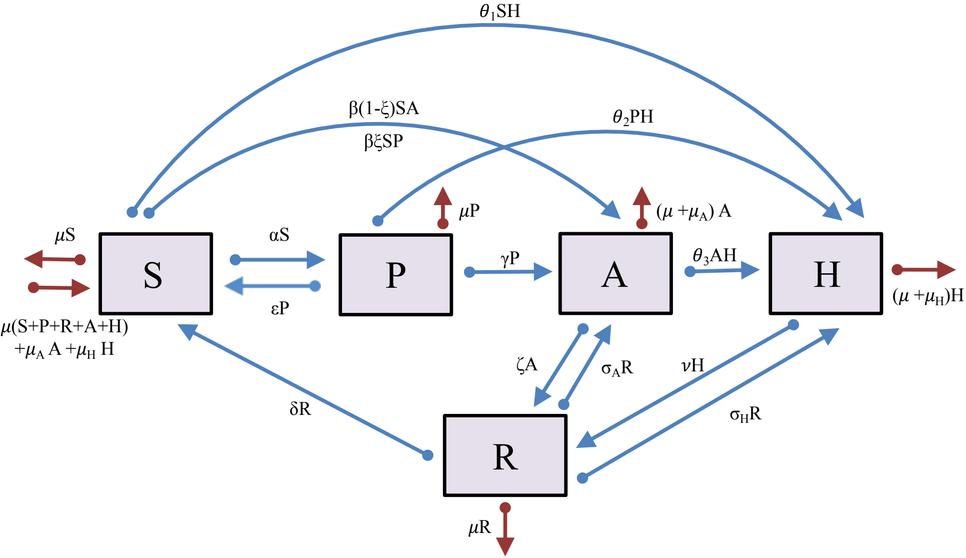
νH: rate heroin users enter treatment



$\sigma_A R$: transition rate from treatment into the opioid addicted class



$\sigma_H R$: transition rate from treatment into the heroin addicted class



$\theta_3 AH$: heroin addiction rate from opioid addicted

System of ODEs Model

$$\frac{dS}{dt} = -\alpha S - \beta(1-\xi)SA - \beta\xi SP - \theta_1 SH + \epsilon P + \delta R + \mu(P+R) + (\mu+\mu_A)A + (\mu+\mu_H)H$$

$$\frac{dP}{dt} = \alpha S - \epsilon P - \gamma P - \theta_2 PH - \mu P$$

$$\frac{dA}{dt} = \gamma P + \sigma_A R + \beta(1-\xi)SA + \beta\xi SP - \zeta A - \theta_3 AH - (\mu + \mu_A)A$$

$$\frac{dH}{dt} = \theta_1 SH + \theta_2 PH + \theta_3 AH + \sigma_H R - \nu H - (\mu + \mu_H)H$$

$$\frac{dR}{dt} = \zeta A + \nu H - \delta R - \sigma_A R - \sigma_H R - \mu R$$

Data

- We have made contact with individuals in diverse fields in order to obtain data on the opioid and heroin epidemic specifically in Knox County and East Tennessee:
  - Dr. Paul Erwin, Head of the Department of Public Health
  - Dr. Agricola Odoi, Associate Professor of Epidemiology
  - Dr. Kelly Cooper, Director of Clinical Services and Assistant Public Health Officer at the Knox County Health Department

Addiction-Free Equilibrium

- Require $A = H = R = 0 \implies$

$$\frac{dS}{dt} = 0 = -\alpha S^* - \beta\xi S^* P^* + \epsilon P^* + \mu P^*$$

$$\frac{dP}{dt} = 0 = \alpha S^* - \epsilon P^* - \gamma P^* - \mu P^*$$

$$\frac{dA}{dt} = 0 = \gamma P^* + \beta\xi S^* P^*.$$

- Forces $\gamma = 0$ (prescribed users cannot become addicted to opioids) and either $\beta = 0$ (susceptibles unable to become addicted to opioids at all) or $\xi = 0$ (only black market opioids available and no excess prescription drugs available).
- $S^* = \frac{\epsilon+\mu}{\alpha+\epsilon+\mu}$, $P^* = \frac{\alpha}{\alpha+\epsilon+\mu}$, $A^* = 0$, $H^* = 0$, $R^* = 0$.

Basic Reproduction Number, $\mathcal{R}_0$

- In general, $\mathcal{R}_0$ gives the expected number of secondary infected cases that result from the introduction of a disease to a susceptible population.
- For the addiction-free equilibrium ($\gamma = 0$ and $\xi = 0$), individuals can become addicted only with interactions with addicted individuals or heroin users $\implies$ can calculate $\mathcal{R}_0$ (using the Next Generation Matrix Method).
- Our model has three addiction compartments, A, H and R since these all consist of opioid and/or heroin addicted individuals.

Calculating $\mathcal{R}_0$

- The eigenvalues for the Next Generation Matrix Method are calculated to be:

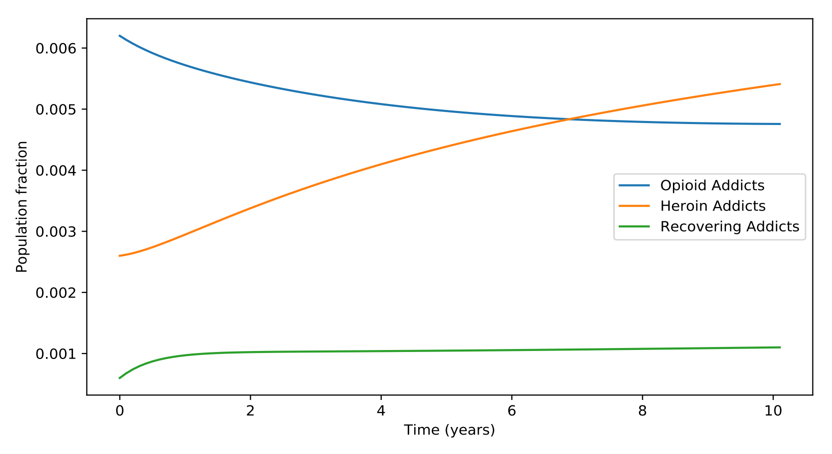
$$\{0, \frac{(r+s)-\sqrt{(r-s)^2+4\beta S^* z\sigma_A\zeta\sigma_H\nu}}{2det(V)}, \frac{(r+s)+\sqrt{(r-s)^2+4\beta S^* z\sigma_A\zeta\sigma_H\nu}}{2det(V)}\}$$

  where $a = \zeta + \mu + \mu_A$, $b = \nu + \mu + \mu_H$, $c = \delta + \sigma_A + \sigma_H + \mu$, $z = \theta_1 S^* + \theta_2 P^*$, $r = \beta S^*(bc - \sigma_H\nu), s = z(ac - \sigma_A\zeta)$, and $det(V) = a(bc - \sigma_H\nu) - \sigma_A\zeta b$.

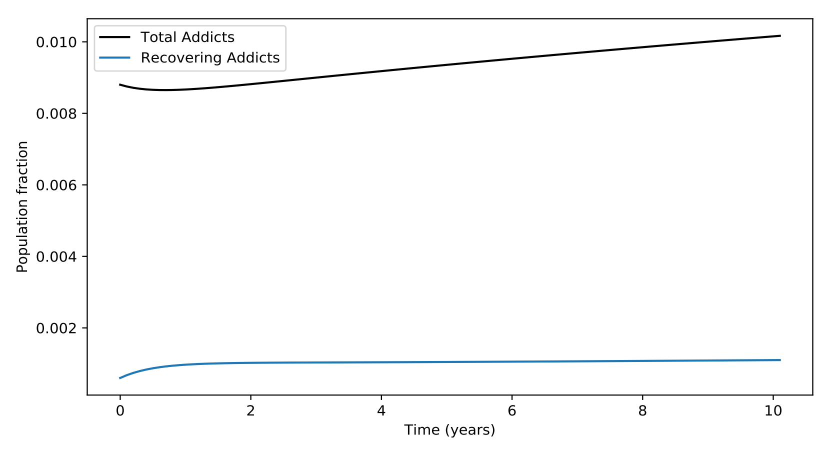
- $\mathcal{R}_0$ may then be determined as the spectral radius of FV^{-1}:

$$\mathcal{R}_0 = \frac{(r+s)+\sqrt{(r-s)^2+4\beta S^* z\sigma_A\zeta\sigma_H\nu}}{2det(V)}$$

Example Solution Curves



Example Solution Curves



Future Tasks/Possible Extensions

- Obtain local data from Knox County/East Tennessee and fit parameters to data.
- Perform sensitivity analysis to determine the sensitivity of each of the classes to the parameters (i.e. the contribution of each of the parameters to the sizes of the classes).
- Explore management strategies for how to best treat pain with prescriptions while reducing opioid addiction and heroin use.

Acknowledgements:

Discussions with Dr. Nicholas Battista and Leigh Pearcy

Calculating $\mathcal{R}_0$

- Assuming A, H and R are the addicted compartments, and with $\gamma = 0$ and $\xi = 0$, we have the following matrices that meet the assumptions of the Next Generation Matrix Method:

$$\mathcal{F} = \begin{pmatrix} \beta SA \\ \theta_1 SH + \theta_2 PH \\ 0 \end{pmatrix}$$

$$\mathcal{V} = \begin{pmatrix} -\sigma_A R + \zeta A + \theta_3 AH + (\mu + \mu_A)A \\ -\theta_3 AH - \sigma_H R + \nu H + (\mu + \mu_H)H \\ -\zeta A - \nu H + \delta R + \sigma_A R + \sigma_H R + \mu R \end{pmatrix}.$$

Calculating $\mathcal{R}_0$

- Taking $F = \frac{\partial \mathcal{F}_i}{\partial x_j}(0, y_0)$ and $V = \frac{\partial \mathcal{V}_i}{\partial x_j}(0, y_0)$, i, j$= 1, 2, 3$, where $(0,\ y_0) = (\frac{\epsilon+\mu}{\alpha+\epsilon+\mu}, \frac{\alpha}{\alpha+\epsilon+\mu}, 0, 0, 0)$ is the addiction-free equilibrium:

$$F = \begin{pmatrix} \beta S^* & 0 & 0 \\ 0 & \theta_1 S^* + \theta_2 P^* & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} \zeta + \mu + \mu_A & 0 & -\sigma_A \\ 0 & \nu + \mu + \mu_H & -\sigma_H \\ -\zeta & -\nu & \delta + \sigma_A + \sigma_H + \mu \end{pmatrix}.$$

References of other models

- Driessche, P. v. d. and Watmough, J. (2008). Mathematical Epidemiology, chapter 6: Further Notes on the Basic Reproduction Number, pages 159?178. Springer, Berlin, Heidelberg.
- (2017). HHS acting secretary declares public health emergency to address national opioid crisis.
- National Institute on Drug Abuse (NIDA) Prescription Opioids and Heroin. d14rmgtrwzf5a.cloudfront.net/sites/default/files/19774-prescription-opioids-and-heroin.pdf

Previous heroin models:

- White, E. and Comiskey, C. (2007). Heroin epidemics, treatment and ode modelling. Mathematical Biosciences, 208:312?324.
- Liu, J. and Zhang, T. (2011). Global behaviour of a heroin epidemic model with distributed delays. Applied Mathematics Letters, 24:1685?1692.