PHYS 375 Final Project
Nuclear Group

University of Waterloo
Department of Physics and Astronomy

Team Members

- J. F. Ducatel
- F. H. Khan
- G. Ko
- A. S. Padawer-Blatt
- G. Song
- X. Zhang

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Objectives

Study changes in nuclear processes allowed

Consider Variations in the specific energy generation rates of:

- PP-Chain
- CNO-Cycle

$$\varepsilon = \Lambda \rho X^2 T^{\lambda}$$

Energy per Reaction

How Many Atoms

Rate of Reaction

The Stars

$$\frac{d\rho}{dr} = -\left[\frac{GM\rho}{r^2} + \frac{\partial P}{\partial T}\frac{dT}{dr}\right] \Big/ \frac{\partial P}{\partial \rho}\,,$$

Hydrostatic Equilibrium

$$\frac{dT}{dr} = -\min\left[\frac{3\kappa\rho L}{16\pi a c T^3 r^2}, \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{GM\rho}{r^2}\right],$$

Energy Transport

$$\frac{dM}{dr} = 4\pi r^2 \rho\,,$$

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon\,,$$

Energy Generation

$$\frac{d\tau}{dr} = \kappa\rho\,.$$

Main Assumptions:

- Star composition is constant
- Use star composition of Sun, a common Main Sequence Star
- Assume Adiabatic Index of an Ideal Gas
- Stars is a Blackbody

Numerical Method

1. Code in **Python**
2. Used an **Adaptive Step-Sizing Runge Kutta 45**
   - Initial Conditions: $r_0 = 10\ \mu m \sim 0$ $\quad L_C = \frac{4\pi}{3} r_0{}^3 \rho_c \varepsilon(\rho_c, T_c)$

     $M_C = \frac{4\pi}{3} r_0{}^3 \rho_c$ $\quad L(R_*) = 4\pi\sigma R_*{}^2 T_*{}^4$ $\quad \tau(\infty) - \tau(R_*) = 2/3$
   - "Infinity" Limits: Stop when $\delta\tau = 0.01$ or $M = 1000M_{Sun}$
3. Find Radius using a **linear interpolation**
4. **Shooting Method** to find core density
   - **Bisection algorithm** until core density saturates
5. **REPEAT** for each Star with different core Temperatures!

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Coding Challenges

Optimization to reduce runtime

→ Down to ~10s per Main Sequence

Outlier Stars in Main Sequence

→ Potential Reason: Bisection did not converge to the right core density

→ Solution: Assume Continuous Main Sequence and Remove Outliers

Info on PP-Chain and CNO-Cycle

Overall Reaction

$$4H \rightarrow 1He \qquad \Delta E = 27\ \text{MeV!!}$$

PP Chain (PPI)

$$ {}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \nu_e $$
$$ {}^2_1H + {}^1_1H \rightarrow {}^3_2He + \gamma $$
$$ {}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2\,{}^1_1H $$

$$4\,{}^1_1H \rightarrow {}^4_2He + 2\gamma + 2e^+ + 2\nu_e$$

CNO Cycle

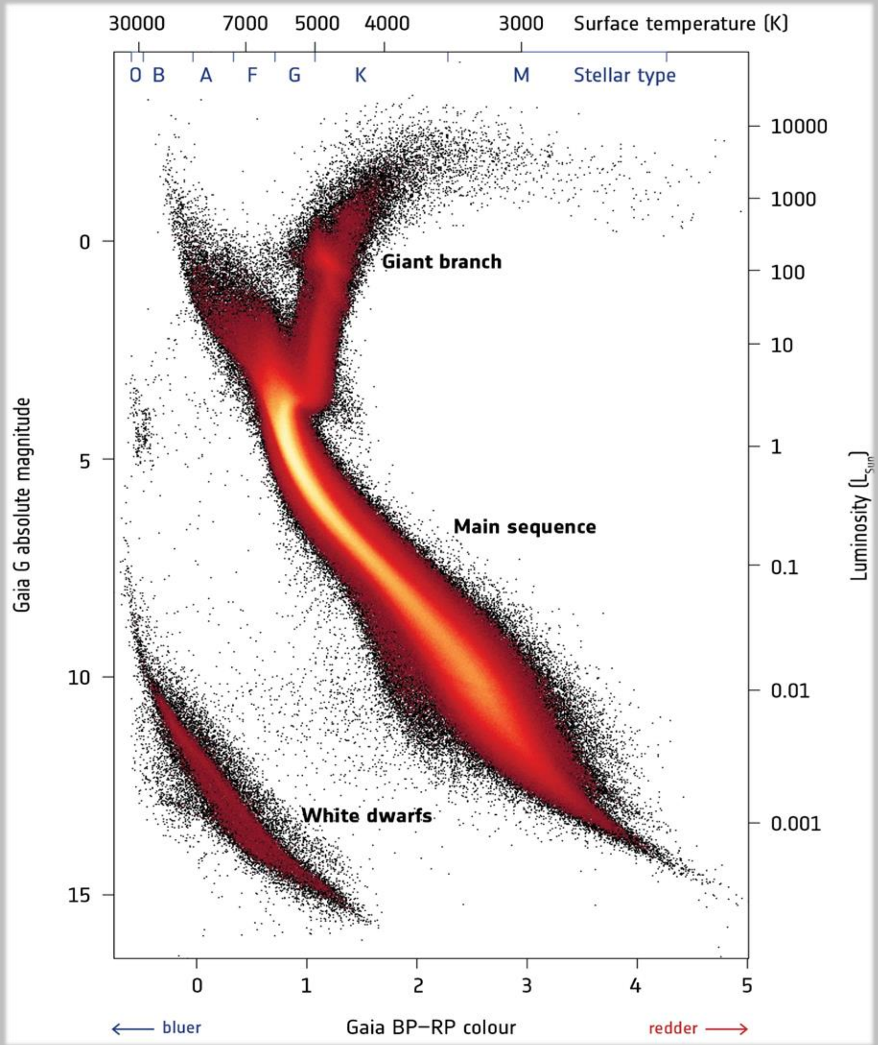
$$ {}^{12}_6C + {}^1_1H \rightarrow {}^{13}_7N + \gamma $$
$$ {}^{13}_7N \rightarrow {}^{13}_6C + e^+ + \nu_e $$
$$ {}^{13}_6C + {}^1_1H \rightarrow {}^{14}_7N + \gamma $$
$$ {}^{14}_7N + {}^1_1H \rightarrow {}^{15}_8O + \gamma $$
$$ {}^{15}_8O \rightarrow {}^{15}_7N + e^+ + \nu_e $$
$$ {}^{15}_7N + {}^1_1H \rightarrow {}^{12}_6C + {}^4_2He $$

$$4\,{}^1_1H \rightarrow {}^4_2He + 3\gamma + 2e^+ + 2\nu_e$$

$$\varepsilon_{PP} = (1.07 \times 10^{-7}) X^2 \rho_5 {T_6}^4\ W/kg \qquad \varepsilon_{CNO} = (8.24 \times 10^{-26}) X X_{CNO} \rho_5 {T_6}^{19.9}\ W/kg$$

$$\Lambda_{PP} = 1.07 \times 10^{-7} \qquad \lambda_{PP} = 4 \qquad \Lambda_{CNO} = 8.24 \times 10^{-26} \qquad \lambda_{CNO} = 19.9$$

Hertzsprung Russell Diagrams



Can measure:

Brightness vs Color

Equivalent to:

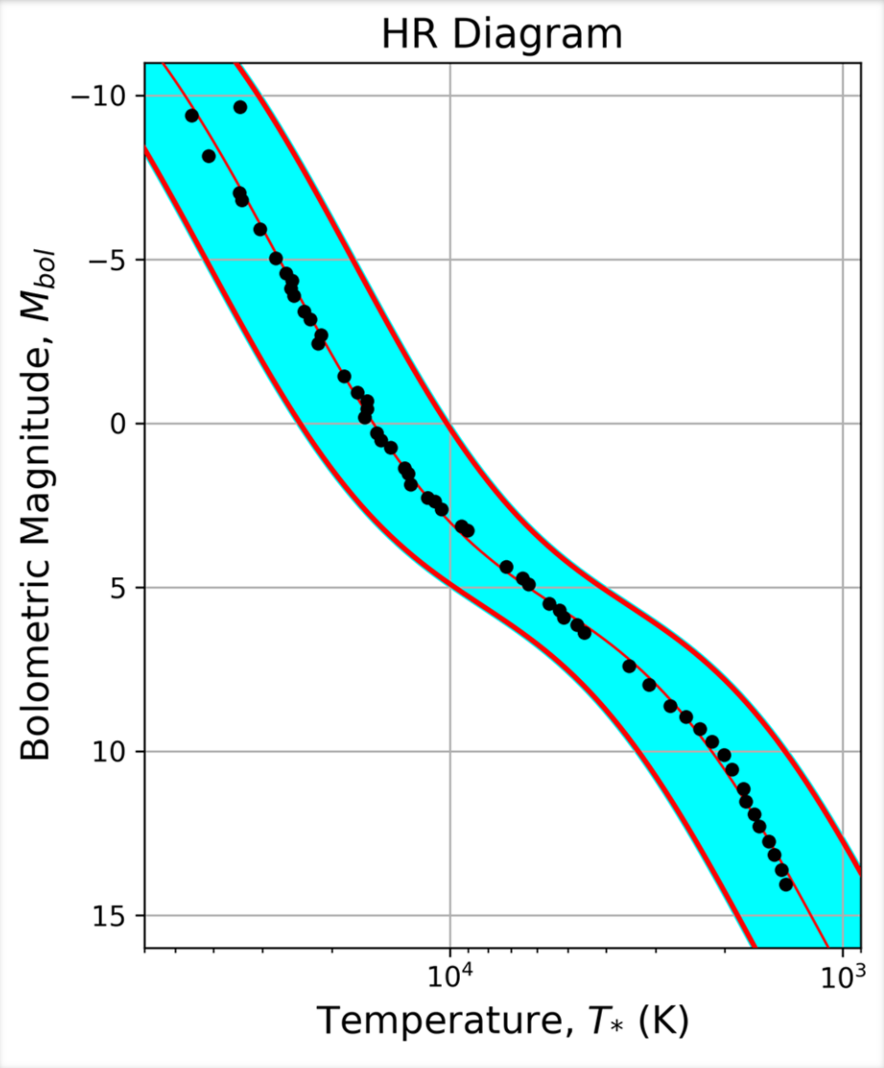
Bolometric Magnitude

Vs

Surface Temperature

Known to 10%

Main Sequence



Plot M_{bol} vs $T_{surface}$
(black dots)

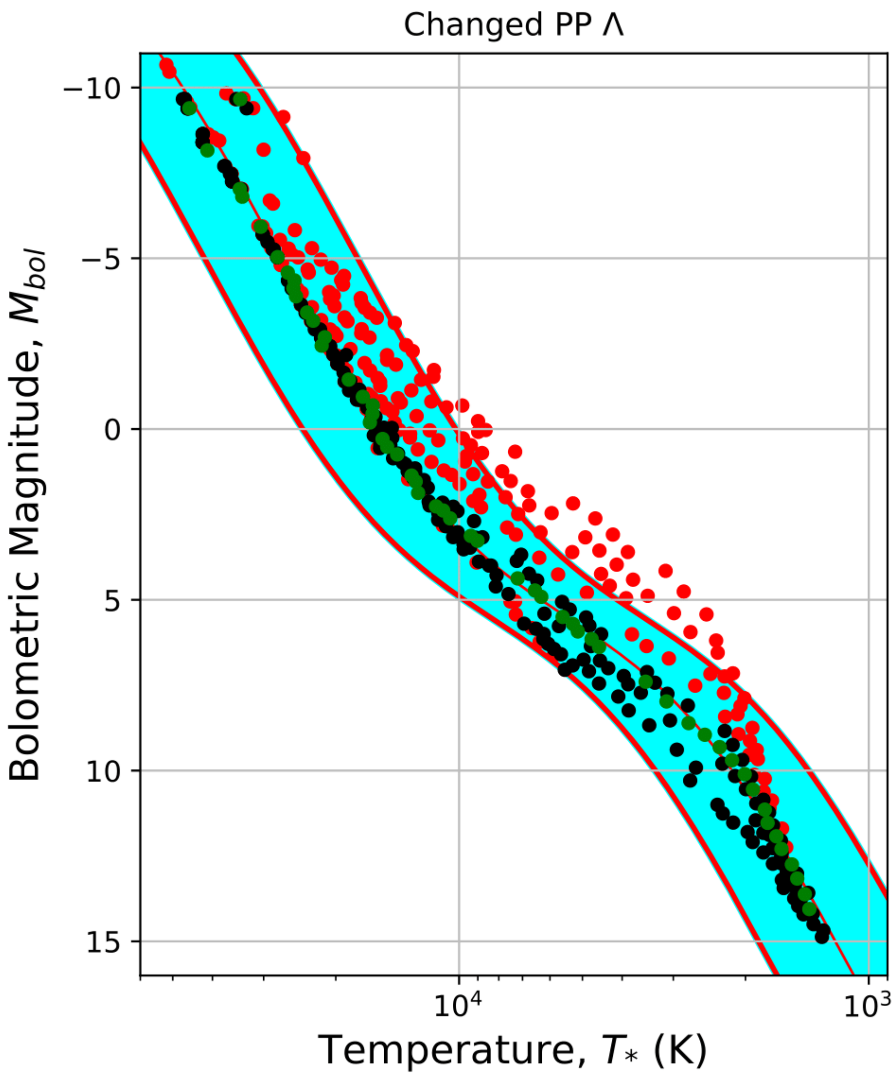
Fit to arbitrary function
(red line)

Main Sequence known within 10%
(blue region)

Permitted Range for Λ, λ

Nuclear Variations to PP-Chain

Legend	
Blue Region	10% boundary on the main sequence
Green Dots	Baseline main sequence
Black Dots	Main sequences inside 10% boundary
Red Dots	Main sequences outside 10% boundary



$$\varepsilon = \Lambda \rho X^2 T^\lambda$$

Changing Λ for PP-Chain

Allowed Range of Values:

6.4x10^{-10} – 2.4x10^{-6}

Can change by factors of about 1000!

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Nuclear Variations to PP-Chain

Legend	
Blue Region	10% boundary on the main sequence
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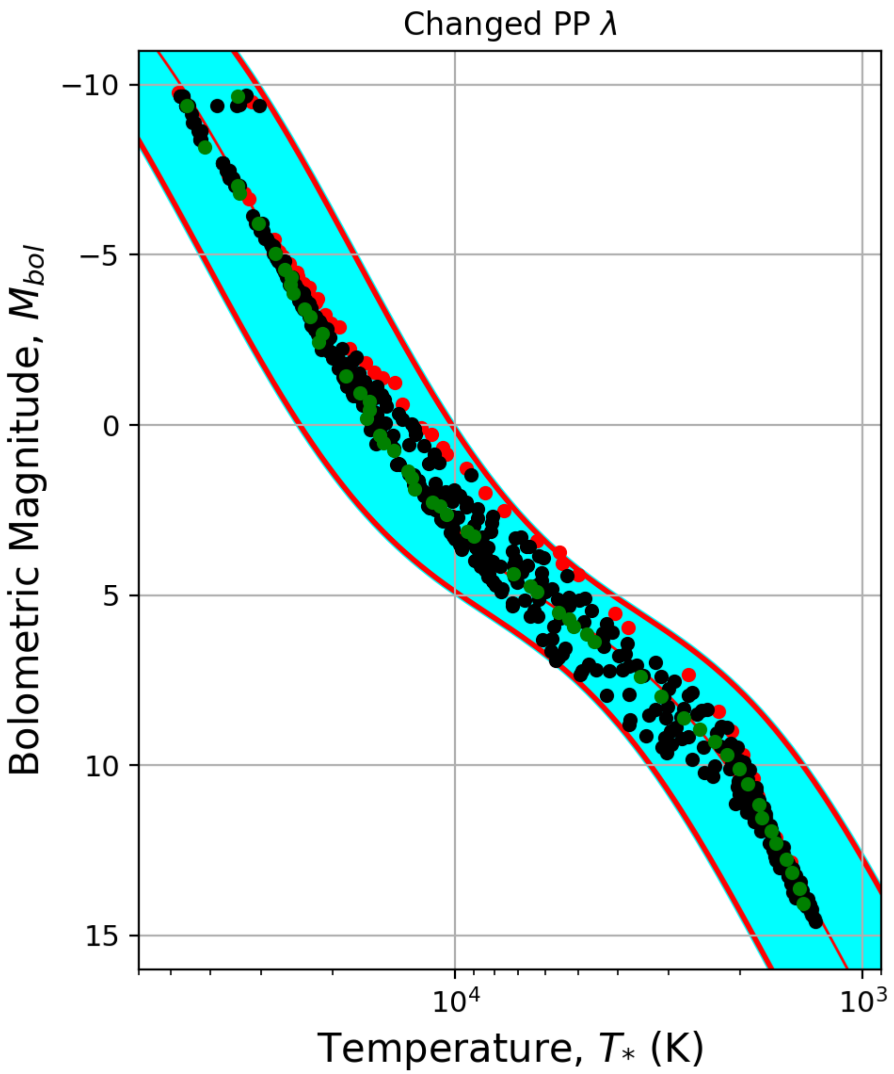
Black Dots	Main sequences inside 10% boundary
Red Dots	Main sequences outside 10% boundary



$$\varepsilon = \Lambda \rho X^2 T^{\lambda}$$

Changing λ for PP-Chain

Allowed Range of Values:
2 – 5.6 (no lower bound)

Note:
PP-Chain affects low-mid temperature Stars

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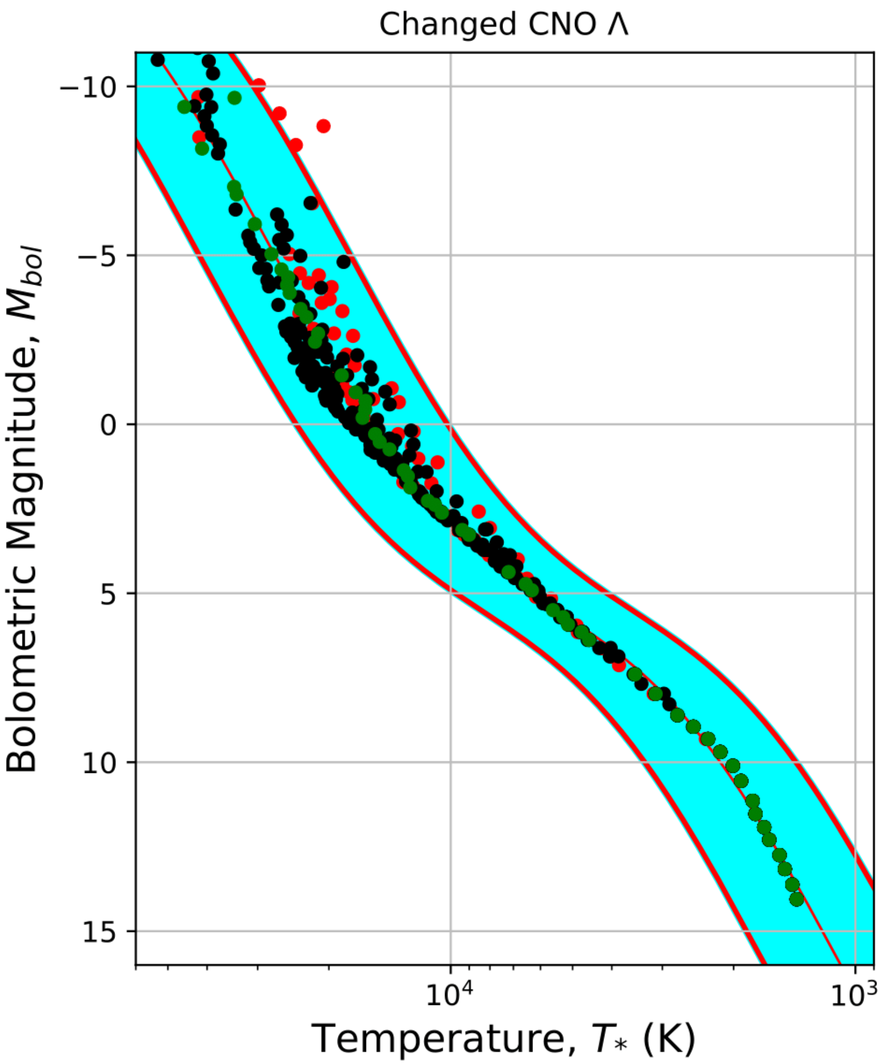
Nuclear Variations to CNO-Cycle

Legend	
Blue Region	10% boundary on the main sequence
Green Dots	Baseline main sequence
Black Dots	Main sequences inside 10% boundary
Red Dots	Main sequences outside 10% boundary



$$\varepsilon = \Lambda \rho X^2 T^{\lambda}$$

Changing Λ for CNO-Chain

Allowed Range of Values:

8.2×10^{-30} – 3.8×10^{-27}

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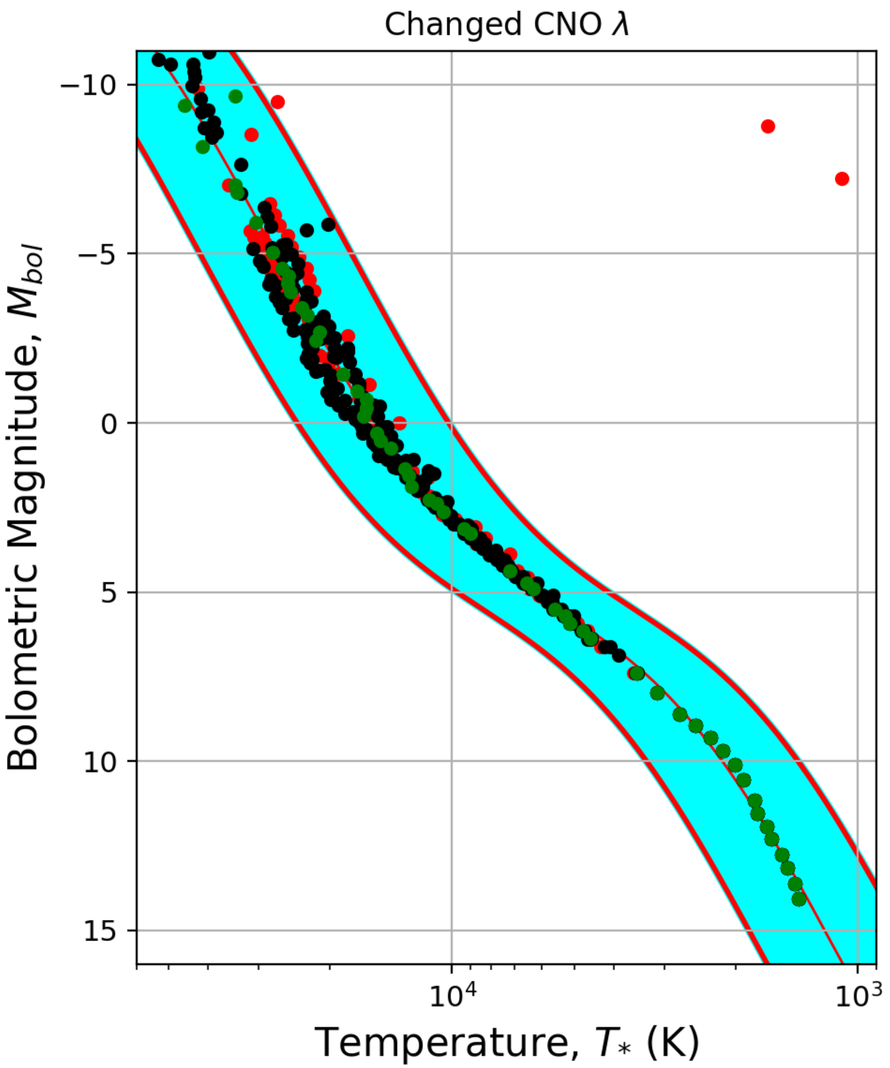
Nuclear Variations to CNO-Cycle

Legend	
Blue Region	10% boundary on the main sequence
Green Dots	Baseline main sequence
Black Dots	Main sequences inside 10% boundary
Red Dots	Main sequences outside 10% boundary



$$\varepsilon = \Lambda \rho X^2 T^\lambda$$

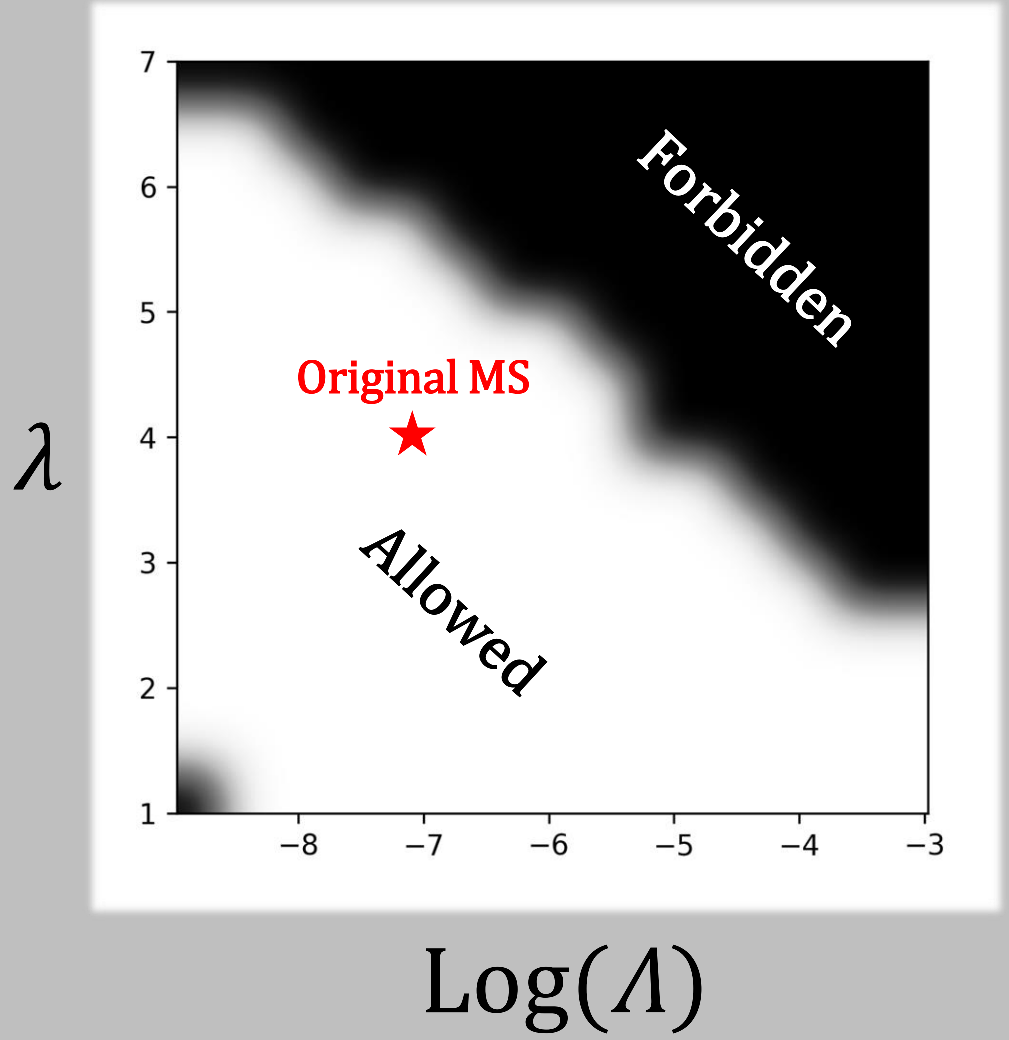
Changing λ for CNO-Chain

Allowed Range of Values:
19.8 - 21.3

Very Sensitive!

Note:
CNO-Cycle affects high temperature Stars

PP-Chain Λ, λ allowed combinations



Map of Allowed Value (2D Parameter Space)

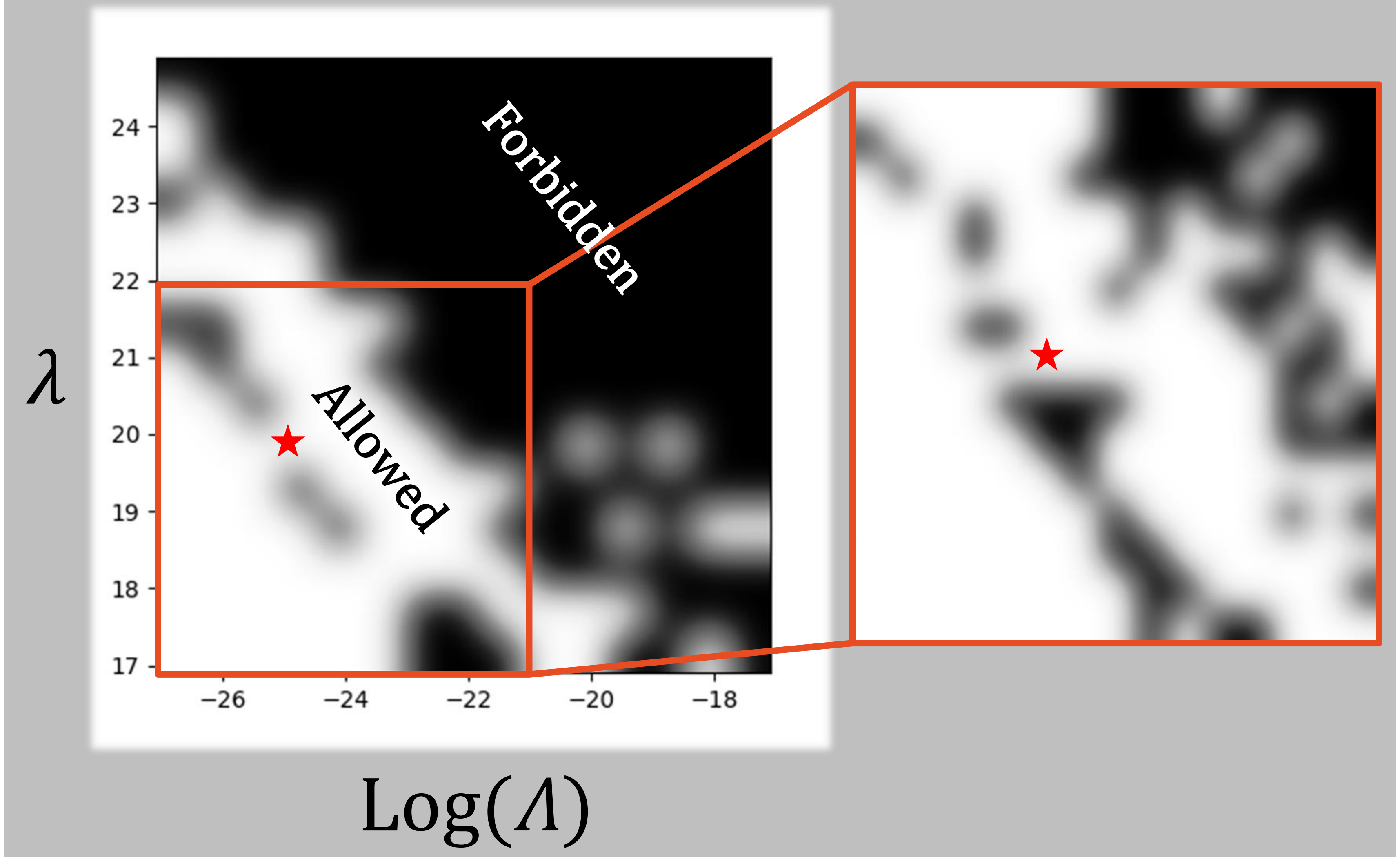
Each pixel is a Main Sequence

Large Allowed Region

↓

Not very sensitive to changes

CNO-Cycle Λ, λ allowed combinations



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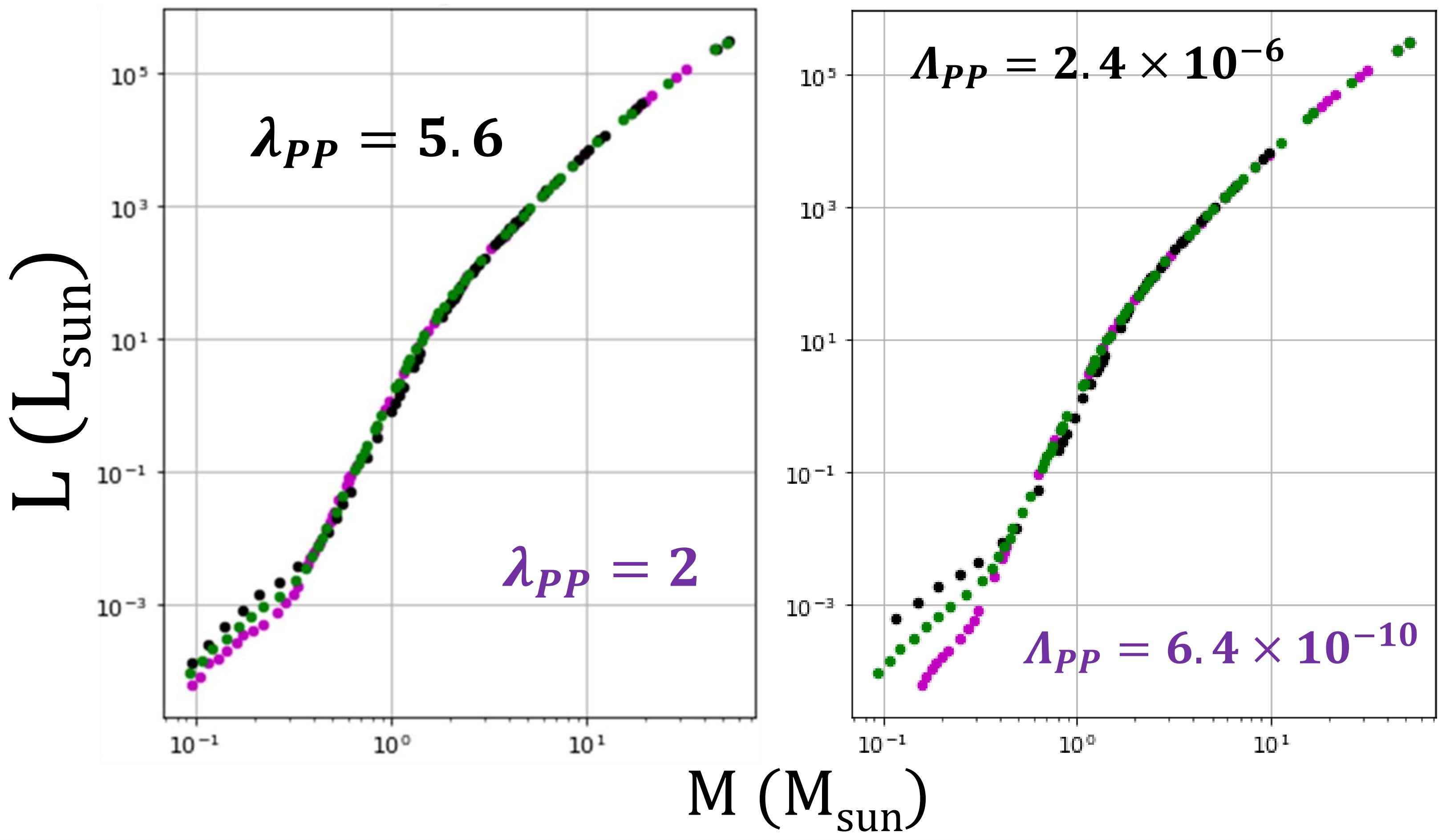
L-M Relation

PP Chain

Original MS

Minimum Bound

Maximum Bound



Note: PP chain parameters affect low mass luminosities

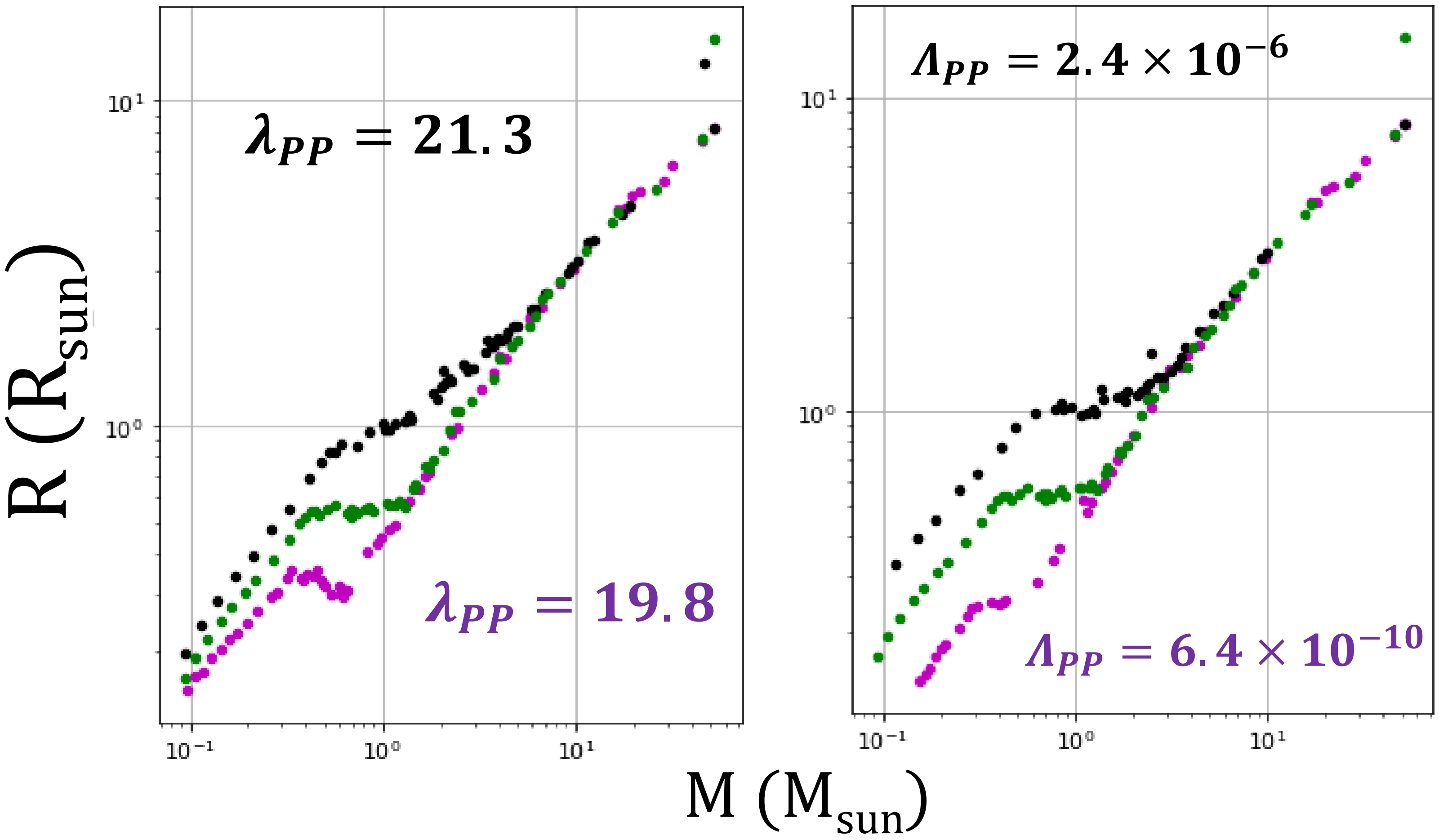
R-M Relation

PP Chain

Original MS

Minimum Bound

Maximum Bound



Note: PP chain parameters affect low mass radii

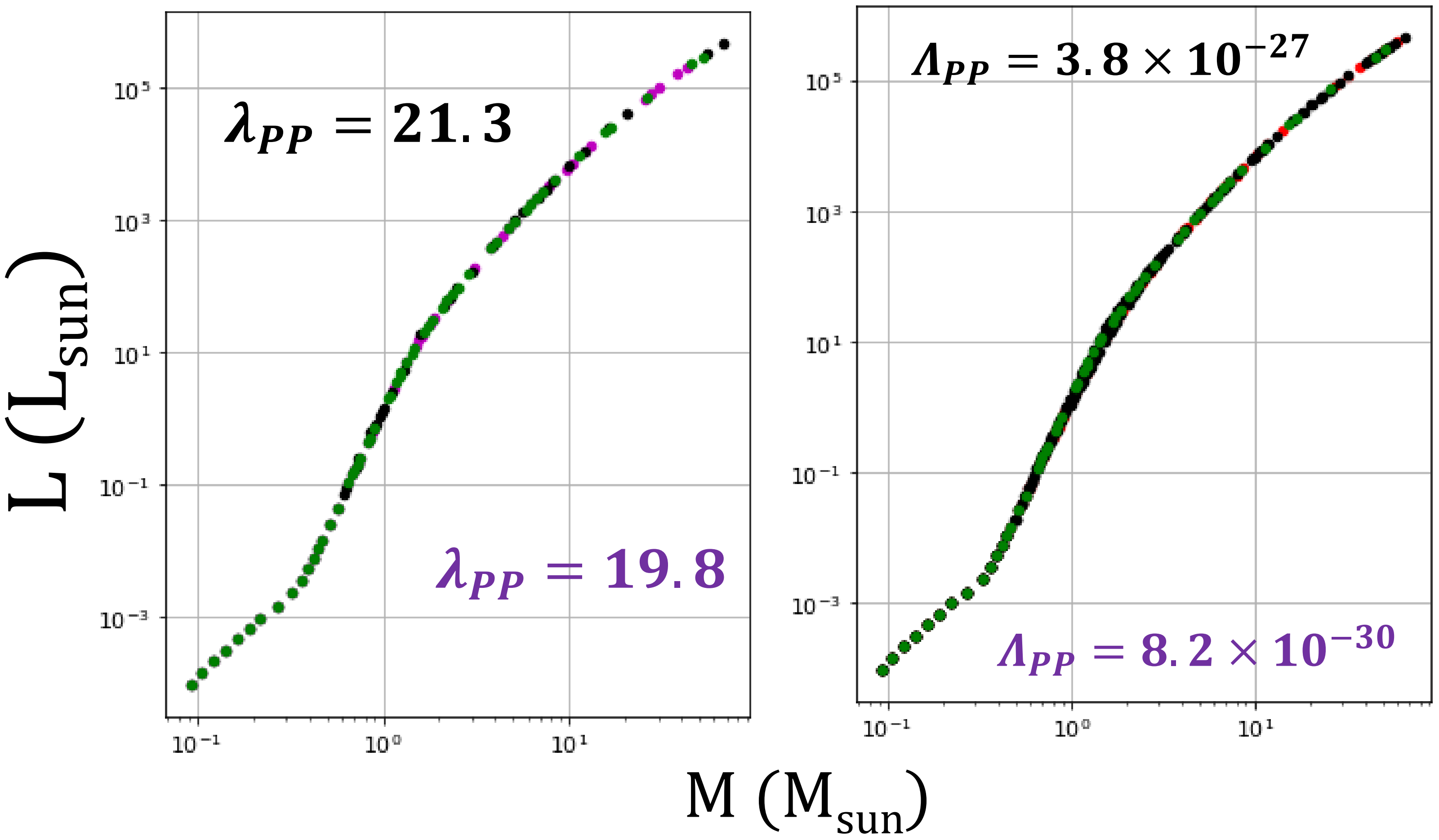
L-M Relation

CNO Cycle

Original MS

Minimum Bound

Maximum Bound



Note: CNO cycle parameters affect high mass luminosities

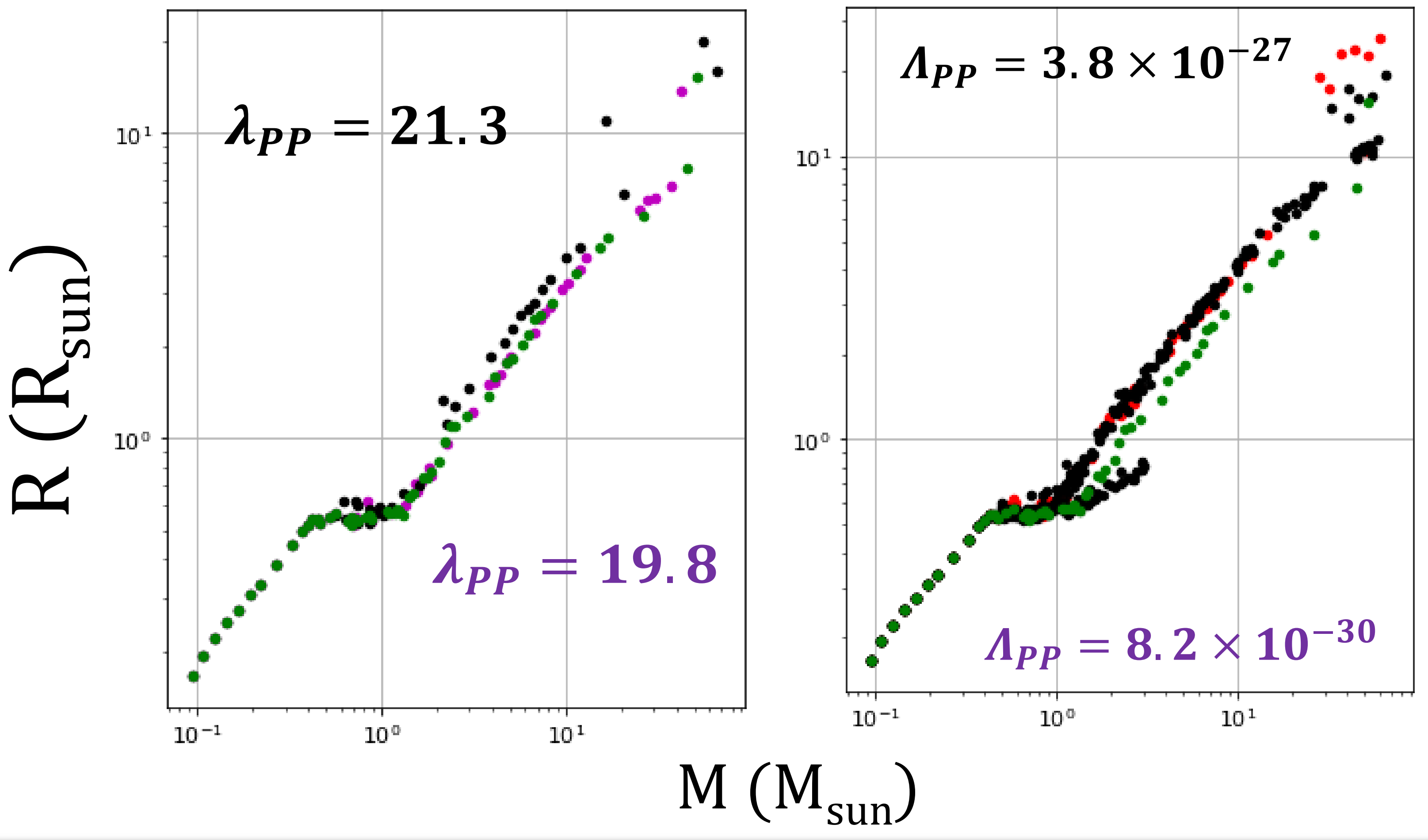
R-M Relation

CNO Cycle

Original MS

Minimum Bound

Maximum Bound



Note: CNO cycle parameters affect high mass radii

Conclusion

Question: We considered 'what if the efficiency and rate of known nuclear reactions (PP and CNO) were different, would we see it in the sky?

- Valid solutions were found within 10% of the main sequence
- PP chain affects low temperature stars and CNO affects high temperature stars
- L-M relationship showed low variation with changing parameters
- R-M relationship responded more to parameter variation, with CNO being more chaotic than PP and having few solutions at low mass
- More sensitive to CNO changes than PP

END

Thank You