Store Visits, Locations, and Customer Perceptions: Market Structure Analysis with Customer Trajectories in Shopping Malls

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Abstract

This paper conducts market structure analysis in retail agglomerations using unique trajectory data from 10 million customers across two shopping malls. We use a scalable deep embedding method to construct "store co-visit similarity" based on co-visitation patterns. This metric captures both consumer brand preferences and the impact of store locations, summarizing realistic inter-brand relationships for brick-and-mortar stores. We augment the trajectory data with consumer surveys and mall layouts, allowing us to decompose the store co-visit similarity into stated brand preferences (28%), store locations (22%), and other unobservables (50%). This decomposition suggests the substantial role of factors beyond stated preferences and location in shaping inter-brand relationships. We demonstrate our measure's effectiveness in capturing substitution patterns through two applications. First, we show how our measure captures consumer substitution in response to temporary store closures. Second, we conduct a counterfactual analysis of changing store locations, showing that co-visit similarity better predicts transition probabilities between stores compared to a model that simply combines perception and location data.

Keywords: Market structure analysis; Deep learning; Shopping trajectories; Retail agglomeration

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1 Introduction

Market structure analysis, which captures the substitution and complementarity between products or brands (Elrod et al., 2002), is fundamental to informing marketing strategies, such as identifying key competitors and optimizing pricing. A widely adopted approach in the literature is to derive market structure insights from consumer stated or revealed preference data. Stated preference data, such as surveys (DeSarbo and Grewal, 2007; John et al., 2006), product reviews (Lee and Bradlow, 2011), online discussion forums (Netzer et al., 2012), and social media engagement (Culotta and Cutler, 2016), directly reflect consumer perceptions of brand relationships. Revealed preference data, such as brand switching (Chintagunta, 1998), shopping basket (Gabel et al., 2019), and online search data (Ringel and Skiera, 2016), capture consumers' actual behavior and have the potential to uncover genuine brand relationships. These papers employ perceptual maps or market structure maps to assess closeness among products or brands, which implicitly informs their substitution or complementarity.

However, the relevance of these findings to physical retail settings may be limited. Brick-and-mortar stores often locate close to each other, creating spatial clusters known as retail agglomerations. Store locations can influence consumer behavior and inter-brand relationships in ways that may not be captured by stated preference data alone. For instance, two seemingly unrelated stores in the perception space, such as a cafe and a cinema, if located in close proximity (either deliberately or by chance), may lead customers to co-visit these stores. This proximity might foster meaningful inter-brand relationships.[1] While some revealed preference data used in the literature implicitly capture location elements (e.g., online product ranking or product assortment in shelf space), they do not indicate to what extent the revealed preferences are affected by the

[1] We posit that market structure or brand relationships are dynamic and vary with the specific context, as supported by Elrod et al. (2002). For example, Matthe et al. (2023) map the evolution of market structure over time, demonstrating its fluidity. Netzer et al. (2012) find that the market structure derived from online discussion forums sometimes differs from that based on brand-switching data, because the former react faster to a marketing campaign.

physical arrangements of brands or products, as location information is usually not observed. Without a clear understanding of how much observed brand relationships are driven by consumer preferences versus physical distance, many managerial decisions, such as store location or product assortment, may be suboptimal. Ideally, a combination of consumer revealed preferences, stated preferences, and location data is needed to fully understand the market structure for brick-and-mortar stores.

This paper leverages unique, population-level customer trajectory data from shopping malls to obtain market structure insights within retail agglomerations. Shopping malls or shopping centers have long been a cornerstone of the retail landscape.[2] Our trajectory data, which represents revealed preference, tracks granular store visits for approximately 10 million customers across two malls over an eight-month period. This rich dataset allows us to uncover realistic inter-brand relationships by capturing both consumer brand preferences and store locations. We employ a scalable deep embedding method that integrates the sequence and duration of store visits to construct a comprehensive measure of store similarity (we use "store" and "brand" interchangeably), termed *store co-visit similarity* hereafter. This metric portrays how similar stores are to each other in terms of both consumer brand preferences and proximity of store locations. This approach effectively overcomes the challenges posed by the granularity and scale of the data, and accurately captures inter-brand relationships within retail clusters in a way that no prior studies have investigated. We augment the trajectory data with surveys of consumer perception and layout plans of the malls, and quantify the relative importance of each factor in shaping the co-visit similarity.

We draw inspiration from natural language processing (NLP) to develop a store embedding model based on store co-visitation patterns. The embedded vector implicitly

[2]In 2021, the global shopping mall market was valued at $5.331 trillion (Straits Research, 2022). As of 2022, the United States alone hosted over 116,000 shopping centers (Statista, 2023). Sources:

Straits Research (2022). Global Shopping Centres Market Size, Share, Growth Analysis - Forecast 2022-2030. Retrieved from https://straitsresearch.com/report/shopping-centres-market Statista (2023). Number of shopping centers in the United States from 1970 to 2022. Retrieved from https://www.statista.com/statistics/208059/total-shopping-centers-in-the-us/

summarizes latent characteristics of the store, and the proximity between vectors captures the underlying inter-brand relationships. We conceptualize each customer's store visit as a "word," and their entire trip as a "sentence." We adopt the Word2Vec model to train store embeddings, wherein the sequential pattern of store visits is inherently captured by the model's context window. We further integrate visit duration into the model to create a more comprehensive representation of shopping behavior. Next, we create a market structure map by compressing the learned store representations into a two-dimensional space. Brands that are close on this map are likely to be substitutes or complements.[3] This visualization offers stores insights into their competitive landscape and market positioning.

To understand what the store co-visit similarity captures, we conceptualize it as a composite of consumer intrinsic brand preferences, physical store locations, and other unexplained heterogeneity. We first estimate the effect of store locations on co-visit similarity. This question is conceptually similar to examining the impact of product rankings on consumer search in online settings. However, research in the domain of brick-and-mortar retail is limited due to data scarcity (Honka et al., 2024). Fortunately, our data from two shopping malls allows us to observe changes in the relative spatial positions of the same brands. These variations in distance of a store pair across malls are plausibly exogenous because stores often enter the mall at different times, and their exact locations cannot be precisely predicted in advance due to imperfect coordination. This is supported by the negligible correlation of physical distances for the same store pairs between malls. We find converging evidence that reducing the distance between two stores increases their co-visit similarity.

Next, we assess the extent to which co-visit similarity reflects consumer brand preferences through two analyses. First, we focus on common store pairs across two malls and calibrate the co-visit similarity in one mall to account for differences in physical

[3]Distinguishing whether they are substitutes or complements, however, would require further purchase data, as discussed in Chen et al. (2020), and is beyond the scope of this paper. We conduct some preliminary analysis and discuss the results in Section 5.4.

distances using our estimate of the impact of distance. The correlation between calibrated similarity in one mall and observed similarity in the other is high at 0.64, indicating that store co-visit patterns are relatively stable across different spatial arrangements. Second, we compare the co-visit similarity derived from our revealed preference data with stated brand similarity obtained from survey data. Although the correlation is substantial at 0.53, our analysis reveals a key insight: survey data tend to underrate across-category brand relatedness. This finding highlights the limitations of traditional stated preference methods in capturing complex inter-brand relationships within retail agglomerations.

In a unified framework, we decompose store co-visit similarity into three components: stated brand similarity, physical distance, and other unobserved factors. Stated brand similarity, derived from survey data, reflects demand-side consumer preferences, while physical distance captures consumer travel costs. We show that stated brand similarity accounts for 28% of the variation in store co-visit similarity. Distance explains 22% of the variation. The remaining 50% of the variation is attributed to factors beyond these two. This decomposition exercise indicates that relying solely on consumer stated preference data is insufficient to characterize inter-brand relationships in retail agglomerations. While augmenting it with location information is helpful, substantial variation in store co-visit similarity remains unexplained. This highlights the complexity of consumer behavior in retail environments and the value of trajectory data and our approach in capturing nuanced aspects of shopping patterns.

To demonstrate the value of store co-visit similarity in capturing substitution patterns, we present two applications. First, we leverage quasi-experimental variation from store temporary closures and find that following the closure of the most similar store by co-visits, traffic to the focal store increases significantly after controlling for store fixed effects and time fixed effects. In contrast, relying solely on physical proximity does not predict brand substitution, as traffic to physically nearest stores does not change significantly. These results also suggest a significant mismatch between co-location and co-visitation patterns.

Second, we conduct a counterfactual analysis to show that store co-visit similarity better predicts transition probabilities under changing store locations compared to a model that simply augments stated preference data with location information. We model the share of consumers transitioning from a competitor to a focal store as a function of store co-visit similarity. We then calibrate the co-visit similarity in one mall to match the observed distance in the other mall using the estimated impact of distance. With this counterfactual co-visit similarity, we obtain predicted transition probabilities and compare them with the observed ones in the other mall. In a parallel scenario, we model the transition probability as a function of stated brand preferences and physical distance. Our results show that the model based on co-visit similarity reduces out-of-sample RMSE by a significant margin (9.8%) relative to the model that combines perception and location data. This exercise also suggests that our co-visit similarity measure, while seemingly context-specific, has potential for out-of-sample predictions. This potential is realized when we have a reasonable understanding of what the measure captures and can credibly calibrate it using the plausible causal impact of physical distance.

This paper makes three contributions. First, we adapt language models to non-textual data, compressing large-scale consumer trajectory information into store embeddings. This approach constructs a rich representation of store co-visitation patterns, implicitly revealing realistic inter-brand relationships in physical retail environments. Second, we go beyond merely introducing novel revealed preference data to uncover market structure. We augment our trajectory data with survey and location data, decomposing store co-visit similarity into three components: consumer stated preferences (perceptions), physical locations, and other factors. We demonstrate that our measure captures not only perceptions and locations but also unobserved heterogeneity shaping inter-brand relationships. More importantly, our measure more accurately captures substitution patterns that are not evident from stated preferences or location data alone. Third, we provide insights into generic brand substitutability beyond specific retail settings. We show that our calibrated co-visit similarity is robust across different malls and significantly improves out-of-sample predictions of consumer behavior, suggesting its potential for policy learning.

The remainder of the paper is organized as follows: The next section provides an overview of related literature. Section 3 introduces the data. Second 4 details our approach of using customer trajectory to uncover market structure. Section 5 delves into the interpretation of co-visit similarity, discussing its components and relevance for understanding inter-brand relationships. Section 6 explores two applications of our model, showing its value in predicting substitution patterns. Section 7 concludes.

2 Related Literature

Our paper contributes to the literature on market structure analysis, a research area with a rich history in marketing. Traditionally, this body of work relies on surveys (e.g., DeSarbo et al., 2006; John et al., 2006) or purchase data (e.g., Kannan and Sanchez, 1994; Erdem, 1996; Chintagunta, 1998) to define market structure. The advent of the Internet has steered researchers towards using user-generated content, often in the form of consumer stated preference data, to uncover and visualize market structures. For instance, Netzer et al. (2012) extract automobile co-occurrences from online discussion forums as a measure of brand similarity, while Lee and Bradlow (2011) use customer reviews to reveal brand substitution. Other data sources include social media engagement (Culotta and Cutler, 2016; Yang et al., 2022), consumer-posted images (Liu et al., 2020), large language models Li et al., 2024, etc. Given the absence of price and purchase data, these studies do not estimate price elasticity but rather adopt perceptual maps or market structure maps to implicitly infer substitution or complementarity patterns.[4]

[4]For example, Kim et al. (2011) explore the market structure in the camcorder category, positing that product proximity on perceptual maps sheds light on consideration sets that are searched together, thereby implying a higher level of competition. Amano et al. (2022) present a parallel argument regarding the link between market structure maps and substitution patterns. In our study, which spans multiple store

The three papers most closely related to ours are Kim et al. (2011), Ringel and Skiera (2016) and Amano et al. (2022), which leverage online search data to understand product substitution. While our paper also utilizes search data and employs co-occurrence to inform brand relationships, it differs in several aspects. First, we focus on brick-and-mortar retail environments, where consumer behavior can largely differ from online contexts, potentially due to higher search costs. Second, our method captures rich consumer heterogeneity in search sequences and durations to derive the market structure. In contrast, Kim et al. (2011) rely on aggregate product search data from Amazon, while Ringel and Skiera (2016) and Amano et al. (2022) use clickstream data that lacks information on the search sequences and durations. Third, we observe store locations, akin to product rankings in online data, allowing us to assess the impact of physical distances on store co-visitation. Lastly, by integrating multiple data sources (trajectory, survey, and location data), we decompose our co-visit similarity measure into consumer stated brand preferences, locations, and other factors, and quantify the contribution of each factor.

Our paper also relates to a small stream of literature using offline path tracking data. These papers use three types of data to answer different questions than ours. The first type is based on radio-frequency identification (RFID) tags (e.g., Hui et al., 2009; Seiler and Pinna, 2017; Seiler and Yao, 2017). The second type is from video tracking (e.g., Hui et al., 2013; Zhang et al., 2014; Jain et al., 2020; Musalem et al., 2021). Both RFID and video tracking primarily focus on in-store consumer movements and are limited to a single-store environment. While our data is from video tracking, it allows us to observe how consumers visit multiple stores. The third category employs cellphone tracking data, such as that from telecom services (Andrews et al., 2016) or SafeGraph (Yavorsky et al., 2021; Knight, 2022), which tracks consumers across different locations including retail stores. Unlike the SafeGraph data, which provides only aggregate location visit information, our data captures detailed consumer trajectories. This granularity allows us

categories, co-search patterns suggest potential relationships between brands, whether they are substitutes or complements.

to explore the inter-brand relationships within retail agglomerations. Overall, our paper uses new data to gain new insights into traditional marketing questions.

3 Data and Descriptive Patterns

3.1 Data

We collaborate with a large mall group in China and directly obtain the proprietary data from its data warehouse. The main dataset consists of population-level customer trajectories from two shopping malls in different cities, labeled as Mall A and Mall B. Besides, the company shares with us layout plans of these two malls, allowing us to reconstruct the 3-dimensional mall spaces and compute the distances between stores. The two malls host 201 and 139 stores, respectively, and each store belongs to one of the four main categories defined by the company: Retail, Food, Kids, and Entertainment. Stores from the same category are typically situated on the same floor, though mixed-category clusters are also common.

Customer trajectory data. This data captures the complete trajectory of *every* customer from their initial entry to the exit from the mall. In other words, the trajectory data captures the entire population of the visitors' movements within each mall during the study period. The data includes approximately 6.6 million customers from Mall A and 3.1 million from Mall B during our sample period from January 1, 2023 to August 31, 2023. All customer identities have been anonymized to ensure privacy compliance. The data is structured at the customer–store–action level, and each row represents either a store entry or store exit action. Each action is timestamped, allowing us to reconstruct the complete paths that customers navigate through the mall.

Our trajectory data offers three advantages for understanding inter-brand relationships within retail agglomerations. First, it captures granular offline customer search behavior, including the sequence of store visits and the time spent at each store. Such rich heterogeneity can be informative about brand relationships. Second, the data is representative and not susceptible to sample selection bias because it includes every visitor to the two malls during the sample period. Third, the dataset is large. For example, our customer base is approximately 100 times larger than that in Culotta and Cutler (2016), which uses online search data to explore market structures. A large sample size is important because we need a sufficient number of store co-visits to reasonably infer brand relationships.

Layout plans. We access the *scanned* layout plans of two malls (see Figure 1 for illustration). These layouts are scaled representations; thus, the distances between two stores on the layout are proportional to their real-world distances. From these scanned plans, we build *digitized* layout plans (see Figure 3) and then calculate the physical distances between stores.

Figure 1: **Scanned Layout Plan of Mall A, Floor 1**

The details of distance calculation are as follows. We use AutoCAD to process the layout data. For each mall, we mark the 2D coordinates of every store and escalator on each floor. Next, we compute the physical distances between stores. For stores on the same floor, we use Euclidean geometry for distance calculation. This approach is suitable due to the malls' rectangular design and minimal obstructions that would otherwise hinder a direct path between stores. For stores on different floors, the distance between store A on floor i and store B on floor j is the sum of the distance from store A to the nearest escalator on floor i, the distance from the same escalator on floor j to store B, and an additional component accounting for the psychological cost of taking the escalator.

Store characteristics. Due to inadequate observed brand attributes, we use a large language model (LLM) to generate brand characteristics (see similar practice in Chakraborty et al., 2024, Lee, 2024). We ask Baidu's LLM to describe each brand's type and product characteristics.[5] Then we employ pre-trained sentence-BERT (Reimers and Gurevych, 2019) to generate embeddings of each brand's description. Next, we calculate the brand description similarity for any pair of stores using cosine similarity. Typically, competing brands exhibit similar brand descriptions. For complementary brands, their shared audience might also lead them to adopt similar styles of language and branding elements in their product descriptions.

In addition, we have the physical size of each store in square meters. These store characteristics will be used as controls in Section 5.1.

3.2 Descriptive Patterns

Table 1 presents trajectory-level summary statistics. On average, customers visit three stores per trip (3.57 in Mall A and 2.68 in Mall B). Approximately 40% (37% in Mall A and 45% in Mall B) of customers visit only one store per trip. Among customers visiting more than one store, the average number of store visits is around 5 in Mall A and 4 in

[5]We prefer a Chinese LLM because many brands are domestic Chinese brands that Chinese LLMs may be more familiar with.

Mall B. These multi-store visitors contribute to the construction of store co-visit similarity detailed in Section 4. Figure A5 in the Online Appendix plots the distribution of store visit size, revealing substantial consumer heterogeneity. Additionally, in Online Appendix Figure A6, we examine the store visit durations and find that consumers spend less time in stores as they visit more stores. This suggests that customers may experience decision fatigue or have time constraints, leading to shorter stays as they visit more stores.

Table 1: **Customer Trajectory Summary Statistics**

	Mall A		Mall B	
	Mean	SD	Mean	SD
Num of store visits	3.57	4.05	2.68	2.56
Num of unique store visits	2.21	2.08	2.19	1.94
Num of store visits when visits >1	5.12	4.46	4.07	2.78
Duration of store visits (mins)	24.14	29.09	21.65	27.23

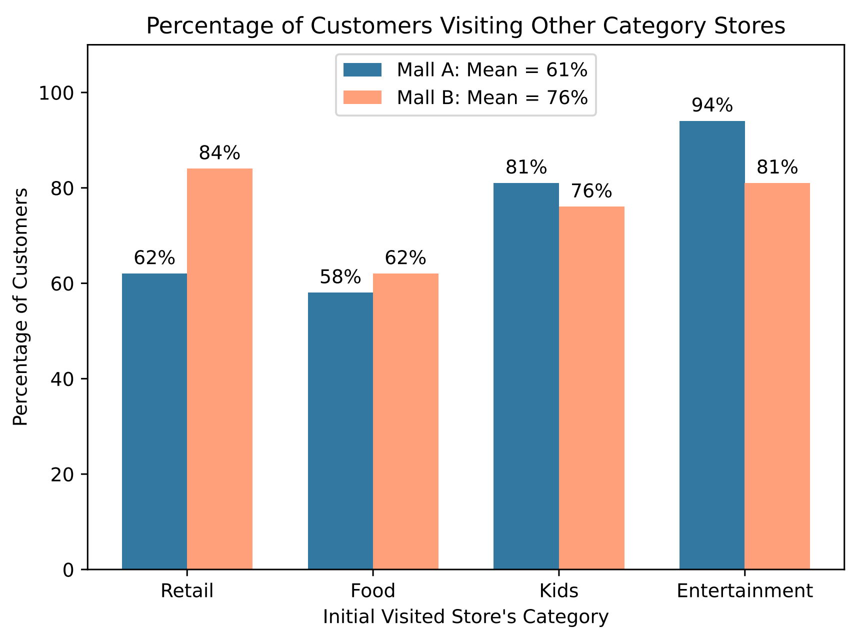


Figure 2: **Cross-category Visitation is Prevalent**
Notes: This figure shows the proportion of multi-store shoppers who visit stores across different categories. For example, among customers who initially visit a food store, 58% in Mall A and 62% in Mall B later visit stores from non-food categories.

One important observation is that cross-category visitation is prevalent. Figure 2 shows the proportion of multi-store visitors who visit stores across different categories. For customers visiting more than one store, on average, 61% in Mall A and 72% in Mall B explore stores from various categories. The mall-defined category is broad and coarse (i.e., both clothing and electronics are classified as retail), so cross-category visitation rates could be higher with a more granular category definition. The frequent cross-category visitation suggests potential meaningful relationships between stores from different categories. If we use traditional stated preference data (i.e., surveys), it will be difficult to capture inter-brand relationships within retail agglomerations because people often do not perceive stores from different categories as similar (Zhang et al., 2019).

Lastly, we visualize consumer movements within Mall A, Floor 1, on the digitized layout plan in Figure 3. It illustrates customer navigation through the mall and highlights the connections between stores based on customer co-visits. Co-visits play an important role to understand in-brand relationships. In Figure 3, square sizes correspond to store sizes, the color intensity of each square reflects the store's average daily traffic, and the thickness of the line connecting two stores is proportional to the volume of customer trips that include co-visits to those stores. To enhance visualization clarity, we only show connections with more than 3,0000 co-visits.

In Figure 3, brands offering similar products and/or targeting overlapping consumer profiles tend to be densely connected. For example, Zara, a leading fast-fashion brand, stands out as the most popular store, forming dense connections with its direct competitor, Charles & Keith, a women's fashion brand. Zara also maintains a strong link to Hey Tea, a popular milk tea brand, likely reflecting shared consumer demographics. Xiaomi and Huawei, two smartphone brands, though positioned far apart, still attract significant numbers of customers visiting both stores. In Section 4, we will employ more rigorous methods to investigate the inter-brand relationships.

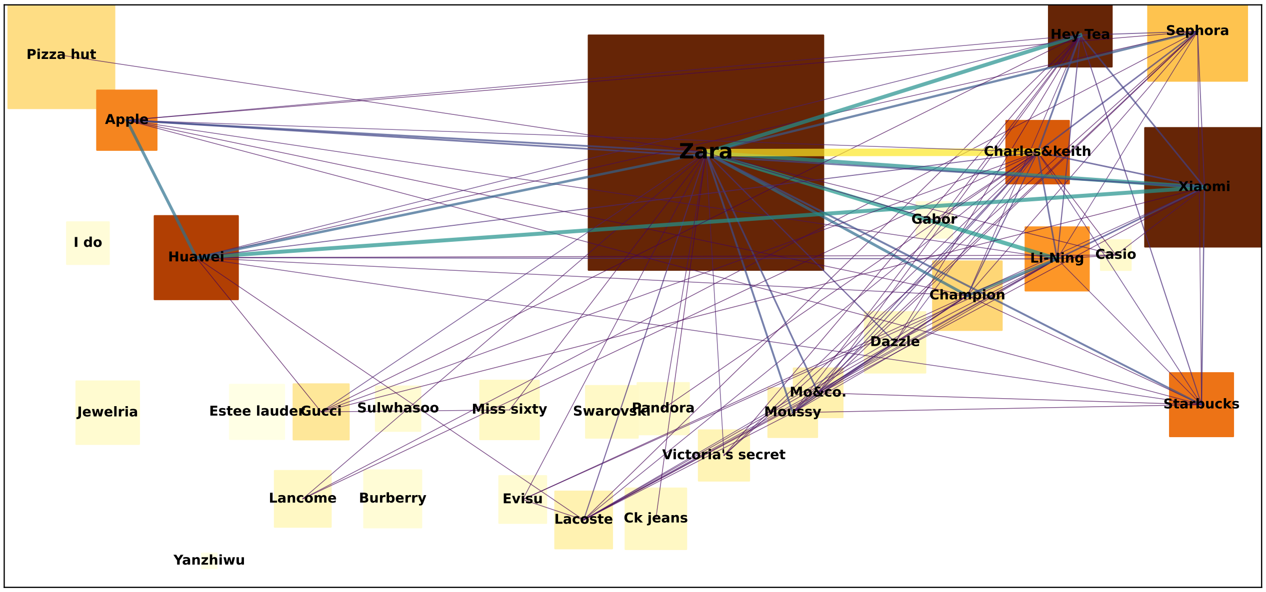


Figure 3: **Consumer Movement Patterns and Store Co-Visits in Mall A, Floor 1**
Notes: This figure visualizes consumer trajectories on the digitized layout plan, with the spatial arrangement corresponding to the scanned layout plan (See Figure 1). Each store is represented by a square, where the size reflects the store's physical size. The intensity of the square's color indicates the store's average daily traffic, with darker shades denoting higher traffic. Lines connecting the stores are proportional to the total number of co-visits. Thicker lines represent a greater number of co-visits. To improve the clarity, we limit the connections to co-visits exceeding 3,000.

4 Mining Market Structure with Customer Trajectory Data

In this section, we introduce our methods to uncover inter-brand relationships. We begin with a baseline measure–normalized co-visit counts (also referred to as *lift*). Then we use embedding techniques to condense customer trajectories into a low-dimensional representation of stores and construct store co-visit similarity from these embeddings, which serves as the foundation for the market structure analysis in our paper.

4.1 Baseline Measure of Store Co-visitation

We first use association rules from market basket analysis to identify relationships between store pairs.[6] The idea is that frequent co-visits to two stores indicate a similarity between them. Such similarity can be driven by consumer preferences, physical proximity, or both. However, a limitation of co-visit count is that stores with high foot traffic automatically have higher co-visits with any other store. To mitigate this bias, we use the *lift* metric to normalize co-visits. This measure is widely used in the market structure analysis literature (e.g, Netzer et al., 2012). *Lift* calculates the ratio of the observed co-visit frequency of two stores to the expected frequency under the assumption of independence, defined as:

$$\text{lift}(A, B) = \frac{P(A, B)}{P(A) \times P(B)}, \tag{1}$$

where $P(A, B)$ is the probability of co-visits to both stores A and B together, while $P(A)$ and $P(B)$ are the probabilities that stores A and B are individually visited, respectively. A *lift* ratio greater than 1 indicates that the two stores are co-visited more often than expected if they were independent.

Before computing *lift*, we process the data as follows. We exclude customers who visited only one unique store in a single trip, as visits to just one single store do not contain information about co-visitation. For each customer's trip, repeat visits to the same store are excluded. When calculating *lift*, neither the sequence nor the duration of store visits is considered.

4.2 A Machine Learning Approach: Store2Vec

Our preferred model incorporates sequence and duration of store visits to refine the construction of store co-visit similarity. Consider a customer's trip: Nike(20mins) $\rightarrow$

[6] For a detailed understanding of the application of market basket analysis, readers are referred to Hemalatha (2012).

Adidas(20mins) → Dior(5mins). Under the *lift* measure, the store pairs (Nike, Adidas) and (Nike, Dior) would be considered equally similar, as the association rule focuses exclusively on the occurrence of store visits. Our objective is to develop a model to recognize two stores as more similar if they are sequentially proximate in a customer's journey, for example, Sim(Nike, Adidas) > Sim(Nike, Dior). An even more ideal model would predict higher similarity between two stores when the time spent in these two stores are similar, for example, Sim(Nike, Adidas) > Sim(Adidas, Dior). We tailor a scalable Store2Vec model that integrates the store identities, as well as the order and duration of store visits, to achieve this.

Inspired by the Word2Vec model (Mikolov et al., 2013), we treat each store visit in a customer's trip as a "word", and the entire trip as a "sentence." This analogy lays the foundation for our Store2Vec model.[7] Store2Vec compresses customer trajectories into a lower-dimensional space, and the resulting embeddings represent all store characteristics implicitly. Stores with common features are closer in the embedding space. Similar to how Word2Vec trains word embeddings using a large corpus of text, Store2Vec trains store embeddings using large amount of shopping trajectories. The model's window size enables the incorporation of short-term sequential patterns in customer trajectories. For instance, in a trajectory like Nike → Adidas → Dior, when training a skip-gram model, a variant of Word2Vec, if Nike is the target store and the window size is 1, Adidas is set as the context store, leading the model to generate Sim(Nike, Adidas) > Sim(Nike, Dior).

Tokenizion. To further incorporate store visit durations, we segment visit durations into quartiles by store category and tokenize by store and duration quartile. Specifically, each store visit, along with its duration quartile, forms a unique token. The same customer journey, using the previous example, is represented as: [(Nike, Q4), (Adidas, Q4), (Dior, Q1)]. As a robustness test, we also tokenize based on the store, repeating the token

[7] Our work joins the burgeoning marketing papers applying embeddings to non-textual data. For instance, Gabel et al. (2019), Chen et al. (2020), and Chintala et al. (2024) utilize Word2Vec to summarize shopping basket data.

according to the visit duration quartile. The same customer journey therefore becomes: [Nike, Nike, Nike, Nike, Adidas, Adidas, Adidas, Adidas, Dior]. We observe quantitatively similar results under different operationalizations. For details, see Online Appendix A.1.

Data pre-processing. We exclude stores that appear in fewer than 1,000 trips during the sample period. This is because during training, embeddings for such rarely visited stores often stagnate at their random initialization values, potentially introducing noise into the results. In addition, we drop single-store visitors as we did using the *lift* metric. We retain repeat visits to the same store, as such repetition mirrors word recurrence in natural language and provides an realistic representation of consumer shopping experience.

Model training with hyperparameter selection. We train the Store2Vec model using the skip-gram architecture (Mikolov et al., 2013), which predicts context words (surrounding stores) given a target word (store). This model provides robust representations for less frequented stores by sampling each observation multiple times. We train skip-gram model separately for each mall. We also train Continuous Bag of Words (CBOW) model, an alternative variant of Word2Vec which predicts the target word (store) based on the context words (surrounding stores), to generate store embeddings. The substantive results does not change (see online Appendix A.2).

During model training, the two most relevant hyperparameters are the embedding dimension and the window size. We find that resulting store co-visit similarity shows limited sensitivity to the embedding dimension, so we fix the dimension at 100. This size is sufficient to capture the implicit store features without introducing unnecessary complexity.

The choice of contextual window size is more important, as it defines the range of stores around a target store that are considered as its context. Selecting a small window size ensures that we automatically attribute more weight to stores that are closer to the focal target store in the customer trajectory. In other words, a small window size guarantees that our embedding vectors capture the sequence of store visits. For this reason, we use a relatively small window size of 2. In an intrinsic model validation, we demonstrate that this small window size outperforms a larger window size in store analogy prediction task (for details, see Online Appendix A.3).

Store embeddings and similarity computation. After training the model, we extract the embedding vectors, which are at the store-duration quartile level. To derive the store embeddings, we average the embeddings across all duration quartiles for each store. We then use cosine similarity to measure the similarity between any store pairs. For each mall, we construct a $J \times J$ similarity matrix, where J is the number of stores, summarizing pairwise similarities between all store combinations. We refer to this measure as "store co-visit similarity." It captures not just which stores are visited together but also the sequence of these visits and the time spent at each store.

Discussion. Our Store2Vec model offers a scalable approach to incorporating rich consumer heterogeneity into the analysis of inter-brand relationships. We compare the preferred store co-visit similarity against the baseline *lift* measure. Although the insights from our preferred model do not significantly diverge from those obtained through the *lift* measure (see Online Appendix A.4), we find that the co-visit similarity more closely aligns with results based on consumer stated preference data (see Section 5.2). Furthermore, we show that the store co-visit similarities remain stable between weekday and weekend training samples (refer to Figure A7 in Appendix B), indicating that our model preserves the inter-brand relationships across different temporal contexts.

While Store2Vec is not designed to capture long-term dependencies as effectively as state-of-the-art transformer models, the typically short nature of consumer shopping paths in retail contexts renders the complexity of these transformer models unnecessary. Store2Vec provides an interpretable, computationally light, and directly relevant approach to analyzing customer trajectories.

4.3 Visualization of the Market Structure

We provide a visualization of store co-visit similarity. This "Market Structure Map" is informative about the inter-brand relationships in the shopping mall.

To visualize the inter-brand relationships, we use t-SNE, a widely used method for visualizing market structure maps or perceptual maps (e.g., Li et al., 2024; Yang et al., 2022). t-SNE is a nonlinear dimensionality reduction technique and can largely maintain local structures, thereby uncovering inter-brand relationships that might be hidden in higher-dimensional spaces. The proximity of points within the t-SNE plot corresponds to the degree of co-visit similarity between stores. Given the stochastic nature of t-SNE, which may yield varying results with each run, we repeat the procedure 100 times and select the iteration with the lowest Kullback–Leibler divergence, which has the greatest fidelity in terms of variation. We find that the choice of perplexity has minimal impact on the visualization outcome, so we settle on a perplexity level of 30. The input for t-SNE is the co-visit similarity matrix, and we focus on Mall A for illustration.

Market structure map by store category. We first present the market structure map by store categories in Figure 4. In this figure, the bubble size is proportional to the total traffic of each store, and the text labels represent the largest stores in each category. Figure 4 reveals that stores within the same category tend to cluster together. Interestingly, some stores from different categories are also positioned closely. For instance, Walmart, the *Retail* giant, is situated near a mix of stores in the *Food* and *Kids* categories. This pattern is consistent with the observation that customers often visit various types of stores in a single trip (see Figure 2).

To examine inter-brand relationships more closely, Figure 5 zooms in two stores and their top 10 nearest neighbors in the market structure map. Figure 5 (a) shows that Dior, a luxury brand specializing in makeup and skincare in Mall A, is surrounded by its widely recognized direct competitors such as Lancôme, Estée Lauder, Burberry, Clarins,

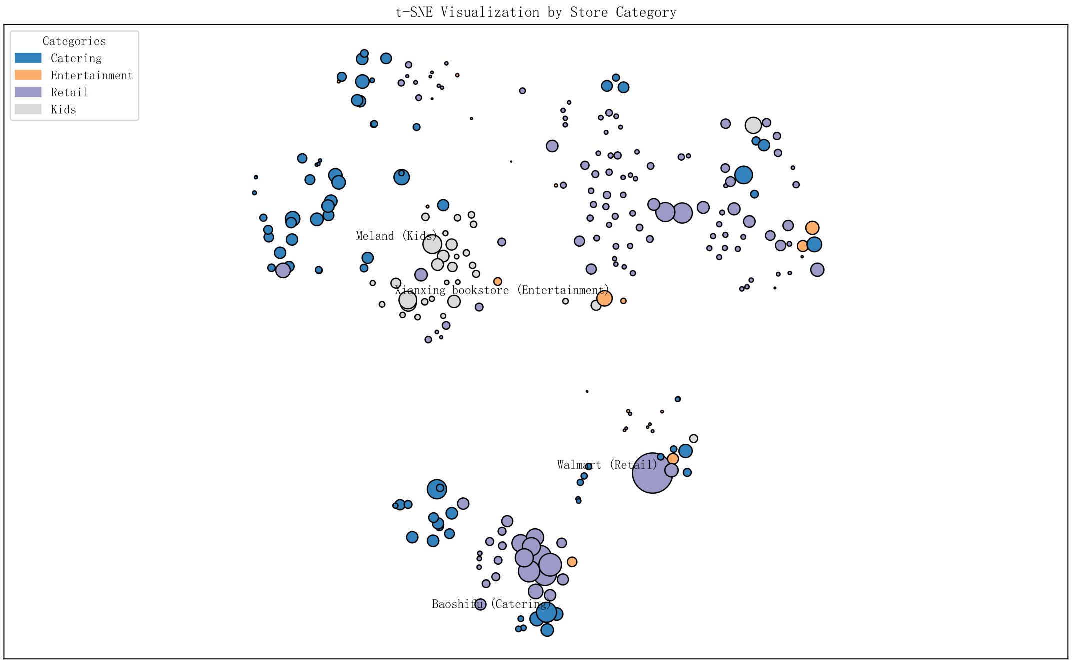
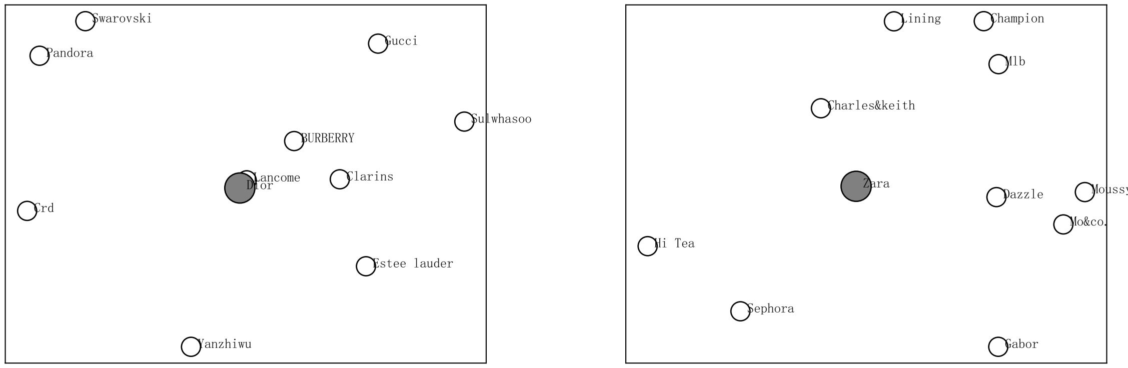


Figure 4: **Market Structure Map by Store Category**

Sulwhasoo, and Gucci. Dior is also near jewelry brands like Pandora, Swarovski, and CRD, which are traditionally not considered its direct competitors. Figure 5 (b) shifts focus to ZARA. As a fast fashion brand popular among younger consumers, ZARA is close to similar fashion brands, including Charles&Keith, MLB, Champion, and Li-Ning. But it is also near Sephora, a beauty and personal care retailer, and Hey Tea, a milk tea brand.

These associations between cross-category stores may reflect similar customer preferences, and/or physical proximity. In Section 5, we provide evidence that after accounting for the influence of physical proximity, the overlap in consumer bases and similarities in product characteristics are predictive of co-visit similarity among cross-category stores. This suggests that high cross-category similarity might be indicative of complementarity.

Market structure map with detected communities. We also refine the market structure visualization by identifying submarkets through community detection methods, as de-



(a) Dior

(b) ZARA

Figure 5: **Focal Brand and Its Closest Stores on Market Structure Map**
Note: Panel (a) and (b) zoom into two focal stores and their neighbors in the market structure map.

scribed by Netzer et al. (2012) and Ringel and Skiera (2016). Community detection aims to uncover submarkets with stronger internal relationships compared to their external connections. Formally, a community is visualized as a subgraph in a network wherein its vertices exhibit a tighter internal interconnectedness than with external vertices. Unlike traditional clustering techniques like K-means, community detection can autonomously discover the number of communities. We use Louvain community clustering method to identify submarkets.

Figure 6 displays the market structure map with detected communities. The number of detected submarkets aligns with the pre-defined store categories. We next examine the degree of overlap between these detected submarkets and the pre-defined store categories. In Figure A8 of Online Appendix, we zoom into a specific submarket as highlighted in yellow in Figure 6. We observe a dominant presence of retail stores. This finding suggests a significant overlap between the sub-markets detected by the community clustering method and pre-defined store categories. This submarket also includes some brands from *Food*, *Entertainment*, and *Kids*, reinforcing our earlier observation that brand relationships extend beyond store category boundaries.

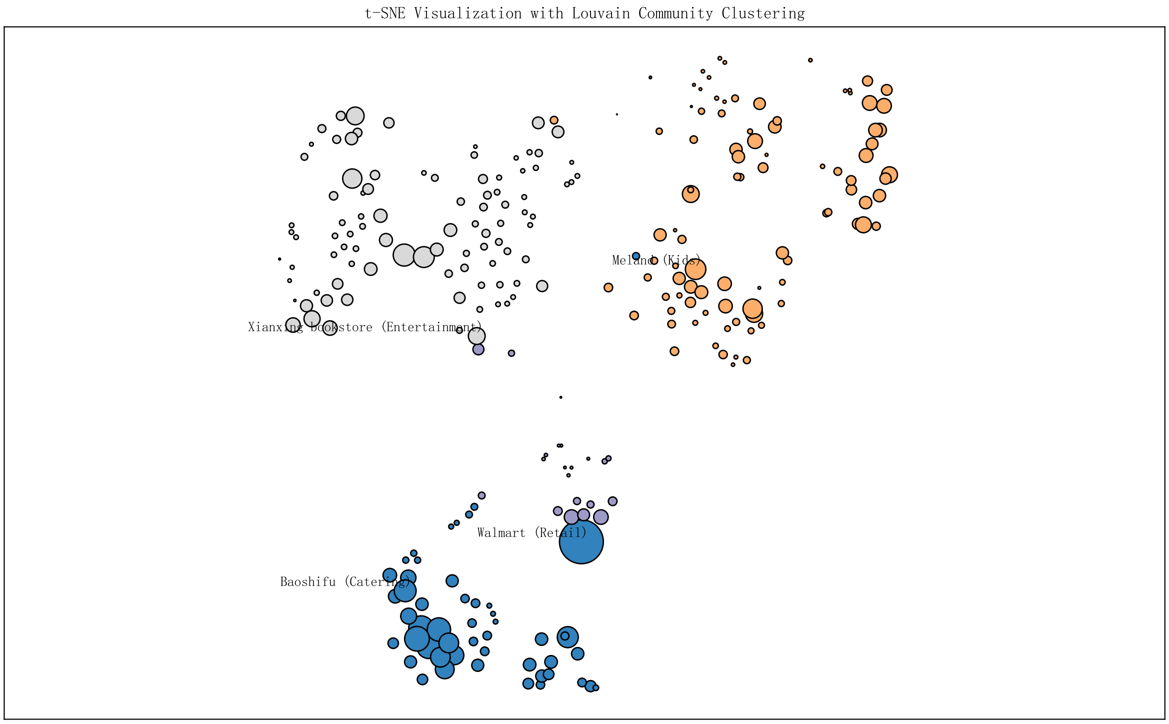


Figure 6: **Market Structure Map with Detected Communities**

5 What Does the Store Co-visit Similarity Captures?

In this section, we examine what the store co-visit similarity captures. Understanding this helps determine the extent to which it reflects inter-brand relationships and its potential applications. Customer trajectories are influenced by both consumer preferences and store locations, and the corresponding store co-visit similarity between store i and j in mall m can be expressed as follows:

$$\text{Co-visit Similarity}_{ij}^{m} = F(\text{Consumer Preference}_{ij}, \text{Store Distance}_{ij}^{m}, \xi_{ij}^{m}), \tag{2}$$

where Consumer Preference$_{ij}$ represents an intrinsic demand-side primitive that remains constant across different malls. Store Distance$_{ij}^{m}$, arising from endogenous, mall-specific locations of stores, captures consumer travel costs and can shift the co-visit similarity. ξ_{ij}^{m} represents other unobserved heterogeneity, such as co-marketing campaigns, complementary services (e.g., shared fitting rooms), etc.

We first estimate the causal impact of store distance on co-visit similarity. Following this, we provide external validation for co-visit similarity to gauge how well it reflects consumer brand preferences and its ability to generalize across different locations. We then use stated brand similarity from surveys as a proxy for consumer preferences and decompose co-visit similarity into three components: (1) stated brand similarity, (2) store distance, and (3) other factors.

5.1 The Role of Store Locations

We first examine how store locations affect store co-visit similarity. The physical distance between two stores, reflecting their placement within the mall, does not directly change consumer preferences, at least in the short run. Obtaining the causal impact of distance helps calibrate the co-visit similarity we will show in Section 5.2 and conduct counterfactual analysis related to store location choice in Section 6.2.

We start by simply regressing co-visit similarity on physical distance separately for Mall A and Mall B (columns 1 and 3 of Table 2). The R^2 are 0.295 and 0.338, respectively. A substantial portion of the variation in co-visit similarity remains unexplained by physical distance. In columns 2 and 4, we further control for store closeness in characteristic space. Specifically, we include a same-category dummy that is 1 if the two stores in a given pair are from the same category. We also include brand description similarity as detailed in Section 3.1. We adjust for differences in store physical size by calculating the absolute differences between paired stores. Incorporating these controls improves the R^2 by 7% - 9%. While the coefficient of physical distance diminishes slightly, it remains statistically significant and negative. The effect size is consistent across the two malls, suggesting internal validity and external generalizability to some extent. In Online Appendix A.5, we also use average walking time as a proxy for physical distance and observe similar findings.

To control for additional unobserved heterogeneity at the store-pair level, we use a store-pair fixed effect model. To this end, we limit the sample to store pairs that appear in both malls. We leverage the plausibly exogenous variation in spatial layouts of identical store pairs across the two malls. After consulting with the collaborating company, we confirm that while the general area of a store's location can be predicted, the exact placement within that area remains largely unpredictable due to a lack of coordination among stores, particularly as different stores enter the mall at various times. Therefore, the variations in distance between two identical stores across these two malls are largely driven by random factors. We also find that the correlation of physical distance for the same store pairs between two malls is negligible (see Section 5.2), further supporting our identification assumption. We run the following regression model:

$$\text{Co-visit similarity}_{ij}^{m} = \phi \text{Physical Distance}_{ij}^{m} + \alpha_{ij} + \gamma_m + \epsilon_{ij}^{m}, \tag{3}$$

where α_{ij} is the store-pair fixed effect, and including it ensures that the source of identification comes from the observed changes in the physical distance between two identical stores across two malls. γ_m is the mall fixed effect, controlling for mall-level aggregate heterogeneity.

Column 5 presents the estimation results. The effect size shows a substantial decrease to 0.094. To put this in context, increasing the distance between two stores by about half the mall's length (e.g., the distance from Huawei to Zara in Figure 3) would lower co-visit similarity by its mean value. Collectively, our analyses show that while co-visit similarity reflects the impact of store locations to some extent, a significant portion of the variation is still unexplained by store locations.

Table 2: **Regression of Co-visit Similarity on Physical Distance**

	Mall A		Mall B		Mall A&B
	(1)	(2)	(3)	(4)	(5)
Physical distance	-0.148***	-0.140***	-0.170***	-0.155***	-0.094***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.014)
Brand description similarity		0.029***		0.048***	
		(0.002)		(0.002)	
Same category		0.051***		0.079***	
		(0.004)		(0.005)	
Store size difference		-0.001**		-0.001	
		(0.000)		(0.005)	
Mall FE					Yes
Store Pair FEs					Yes
N	7,186	7,186	6,706	6,706	473
R^2 or within R^2	0.295	0.369	0.338	0.427	0.184

Notes: In this regression, each observation is a store pair (e.g, "Nike, Adidas"). Standard errors in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

5.2 External Validation: Assessing the Extent to Which It Captures Consumer Preferences

Cross-mall comparison. We focus on stores common in both Malls A and B. For these common store pairs, the Pearson correlation of co-visit similarity between the two malls is 0.53 with p-value < 0.01. However, this is not an apples-to-apples comparison since the physical distances between corresponding stores across the two malls are different. To address this, we calibrate the co-visit similarity by accounting for differences in distance between the two malls using the following equation (we omit store pair notation for simplicity):

$$\text{Co-visit Similarity}'_A = \text{Co-visit Similarity}_A + \hat{\phi} \cdot (\text{Distance}_B - \text{Distance}_A), \quad (4)$$

where $\hat{\phi}$ is the estimated impact of distance on co-visit similarity (see Section 5.1). After this adjustment, the correlation of calibrated co-visit similarity in Mall A with the observed similarity in Mall B increases to 0.64. This suggests that after accounting for differences in physical distance, customers' store co-visitation patterns are relatively stable across different malls. By contrast, the correlation of physical distance for the same store pairs between the two malls is negligible ($r = 0.08$, p-value = 0.23), which means that a store pair's physical distance in one mall does not necessarily predict their distance in the other. The consistency in co-visit similarity strengthens our confidence in its ability to reflect collective consumer brand preferences. However, one needs to be cautious in interpreting these results due to the modest sample size of 210 common store pairs, which could limit the generalizability of our findings.

Comparison with survey. Validating market structure through survey designs is a common practice in the literature (e.g, Culotta and Cutler 2016; Yang et al. 2022). Our survey design is similar to Yang et al. (2022), who ask participants to rate the similarity between a focal automobile brand and 27 other brands. In our context, it is impractical to assess all possible store pairs. We therefore selected 70 stores across all categories from Mall A. We exclude local, small brands as they are less familiar to survey respondents. Ultimately, we obtain 2,415 unique store pairs for evaluation. The survey design has two parts:

1. *Public survey:* We conducted a public survey on *Wenjuan.com*, a popular Chinese survey platform. We recruited 500 participants who lived in major Chinese cities and had visited a mall at least once in the past six months. They were asked to rate the similarity of 50 random store pairs on a scale from 1 to 5. An option to indicate unfamiliarity with either of the stores in a given pair was available.

2. *Research assistant (RA) assessment:* We also recruited 20 RAs who were regular mall visitors from several Chinese universities. Each RA rated 500 random pairs, with 100 pairs per day to mitigate fatigue due to information overload.

While increasing the number of public survey participants could improve data quality, the large size of our selected store pairs makes this option costly. We thus complement the public survey with RA assessment to adds reliability due to easier monitoring of RAs. The final similarity score combines these two sources: Survey-based similarity = $0.5 \times$ Average public survey rating + $0.5 \times$ Average RA rating.[8]

The survey-based similarity represents consumer stated preferences. The correlation between the co-visit similarity and survey-based similarity is 0.53, which is on par with those reported in the literature. For context, in Netzer et al. (2012), their text-mining-based similarity has a correlation of 0.55 with survey-based similarity, while in Yang et al. (2022), the correlation is 0.39. Additionally, store co-visit similarity shows a stronger correlation (0.53) with survey-based similarity than the *lift* measure (0.35). This suggests that the co-visit similarity, which incorporates store visit sequences and durations, may more accurately reflect consumer brand preferences than methods based only on co-visit counts.

Figure 7 narrows the focus to two specific stores, Starbucks and Li-Ning, and shows the interplay between survey-based similarity and co-visit similarity. The circle size reflects the physical proximity to the focal store (Starbucks or Li-Ning), with larger circles denoting closer proximity.

Beyond the positive correlation between these two similarity measures, Figure 7 suggests that the survey method tends to underrepresent across-category brand relatedness. For example, Xiaomi (a smartphone brand) and Sephora (a personal care retailer) are perceived as dissimilar to Starbucks by survey respondents, but they exhibit high co-visit similarity to Starbucks. This disparity might stem from survey respondents ignoring consumer overlap, or it may result from physical proximity that increases the co-visitation of two seemingly irrelevant brands. We will discuss this in more detail in Section 5.4. Another observation is that the spatial arrangement of stores within malls exhibits some level of randomness. For example, Anta and Vans, which are close competitors to Li-Ning, are located far from it. This finding aligns with the observed low correlation in physical distances between identical store pairs across different malls.

[8] We assign more weight to individual RA ratings than public survey responses, as RAs' assessments can be more reliable. The results are qualitatively consistent if the two inputs are used separately.

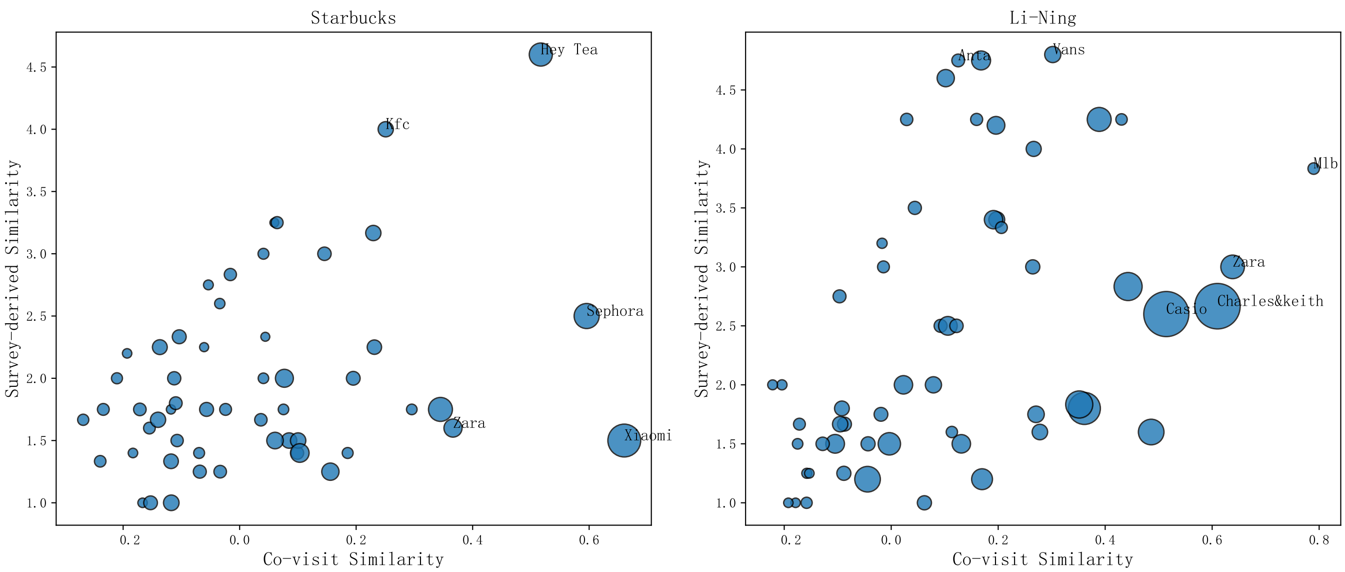


Figure 7: **Co-visit Similarity and Survey-based Similarity**
Notes: The size of each circle is proportionate to the physical proximity to the focal store (Starbucks or Li-Ning).

5.3 Co-visit Similarity Decomposition

To better interpret the store co-visit similarity, we decompose it into three components: (1) stated brand similarity, based on consumer stated preferences data from the survey (refer to Section 5.2), (2) physical distance, and (3) other factors. We perform a Shapley decomposition of R^2, which quantifies the contribution of each factor to the overall model fit.

Table 3: Co-visit Similarity Decomposition

	R^2 Shapley value
Stated brand similarity	0.283
Physical distance	0.217
Other factors	0.499
N	2,187

Table 3 presents the results. Stated brand similarity accounts for 28% of the variation in co-visit similarity, while physical distance explains 22%. The remaining 50% of the variation is attributed to factors beyond these two. Although consumer stated preferences and physical distance play significant roles, they do not completely explain co-visit similarity. This raises a critical question: How important is the remaining variation in capturing inter-brand relationships in brick-and-mortar settings? To what extent might this unexplained variation be due to noise and measurement error? If this remaining variation is noise or non-essential, combining stated preference data with location data might suffice to understand market structure. Conversely, if it captures meaningful aspects of consumer behavior, it could provide valuable insights beyond traditional measures. We will provide further discussion in Section 6.

5.4 Connections to Substitutes or Complements

While firmly establishing whether two brands are substitutes or complements based on their co-visit similarity is beyond the scope of this paper, understanding if this similarity might indicate such relationships is managerially important. Consider Figure 5, where Zara and Sephora, from different categories, exhibit high co-visit similarity. This similarity could be entirely driven by physical proximity and there is no deeper linkage between the two brands, or it could reflect a shared target demographic that seeks a holistic shopping experience including both apparel and cosmetics.

If two brands have meaningful relationships independent of spatial arrangement, after excluding the influence of physical distance, consumer overlap and similarity in brand description are expected to be strong predictors of the residual of co-visit similarity, for both intra-category and cross-category store pairs. In this case, we could reasonably interpret two brands from the same category with high co-visit similarity as an indication of substitutes, and two from different categories with high similarity as an indication of complements.

We combine LLM with human evaluations to build a proxy for consumer overlap between two brands. We ask Baidu's LLM to describe each brand's consumer demographics in China, based on gender, age, income, and occupation. Then we recruit 10 RAs to assess the consumer overlap between brand pairs based on these descriptions, rating them from 1 to 3. For this analysis, we focus on 31 stores located on the first floor of Mall A (refer to Figure 1). This floor houses well-known brands, which ensures that the LLM can provide reasonable descriptions of their consumer demographics. We further categorize these brands into four refined groups: electronics, jewelry, beauty and makeup, and clothing. The analysis proceeds in two steps: first obtaining the residuals from regressing the co-visit similarity on physical distance, then regressing the residuals on brand description similarity and consumer overlap.

Table 4: **Regression of Residuals in Co-visit similarity on Consumer Overlap and Brand Description**

	Same-category pairs (1)	Across-category pairs (2)
Brand description similarity	0.047*	0.039***
	(0.026)	(0.011)
	[31.5%]	[33.1%]
Consumer overlap	0.066***	0.049***
	(0.021)	(0.010)
	[68.5%]	[66.9%]
N	87	264
R^2	0.190	0.157

Notes: In this regression, each observation is a store pair (e.g, "Nike, Adidas"). Standard errors are in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Shapley value decomposition percentages are in brackets.

Table 4 shows results for two sub-samples: one comprising pairs from the same category and another including pairs from different categories. In both samples, brand description similarity and consumer overlap are statistically significant and positively correlated with the residuals of co-visit similarity. We also perform a Shapley decomposition of R^2 and find that consumer overlap has greater explanatory power than brand description similarity across both samples. While these decomposition results should be interpreted with caution due to potential measurement errors, they provide suggestive evidence that the high co-visit similarity between cross-category stores not merely captures physical proximity, but also reflects deeper connections stemming from shared consumer bases, consistent with the idea of complementarity. Similarly, high co-visit similarity between same-category stores likely indicates substitutes.

6 Applications

We provide two applications of co-visit similarity to show its effectiveness in capturing inter-brand relationships within retail settings. First, co-visit similarity better predicts where customers substitute following store closures compared to physical distance. Second, it provides more accurate out-of-sample predictions of store transition probabilities when changing the location of a focal store, compared to merely augmenting stated preference data with location information.

6.1 Using Store Closures to Understand Substitution Patterns

We exploit natural experiments provided by temporary store closures to show that co-visit similarity captures consumer substitution and provides a robust summary of inter-brand relationships within retail agglomerations. We find that consumers are more likely to substitute to stores with high co-visit similarity to the closed stores, rather than to those that are merely physically close.

Specifically, we examine the impact of temporary store closures on customer traffic to the nearest focal stores based on either co-visit similarity or physical distance. Store temporary closures are often driven by supply-side factors such as renovations, and thus exogenous to demand-side consumer responses. Essentially, we compare the average traffic to a focal store between periods with and without nearest store closures after adjusting for seasonal trends and other aggregate shocks. Our regression model is as follows:

$$log(\text{Traffic}_{it}) = \beta \text{Closest store temporarily closed}_{it} + \alpha_i + \gamma_{w(t)} + \epsilon_{it}, \quad (5)$$

where $log(\text{traffic}_{it})$ is the log traffic to store i on day t, and Closest store temporarily closed$_{it}$

is whether the closest stores to store i is temporarily closed on day t. A store closure is defined as a period where there are no customer visits for more than 14 days. α_i is the store fixed effect controlling for store time-invariant heterogeneity, $\gamma_{w(t)}$ is the week fixed effect and controls for aggregate demand shocks. For this analysis, we exclude stores that have no within-store variation in Closest store temporarily closure$_{it}$, as they do not provide any identification variation.

Table 5: **The Impacts of the Nearest Store's Closure on Focal Store Traffic**

	Dep Var: log(Traffic)	
Closeness based on:	Co-visit similarity	Physical distance
	(1)	(2)
Closest store temporarily closed	0.047***	0.008
	(0.016)	(0.019)
Store FE	Yes	Yes
Week FE	Yes	Yes
N	10,233	10,233
R^2	0.745	0.724

Notes: The variable "Closest store temporarily closure" equals 1 for each store on a given day if its nearest stores, determined by either co-visit similarity or physical proximity, are temporarily closed. A store is defined as temporarily closed if it experiences more than 14 consecutive days with zero customer visits. 54 stores out of 201 stores experienced temporarily closure during the sample period. We exclude stores that did not experience closest store closure. Standard errors are clustered at store level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5 presents our analysis of store closure impacts. Traffic to a focal store increases by a statistically significant 4.7% following the closure of the most similar store in terms of store co-visit similarity (column 1). In contrast, the closure of physically nearby stores has an insignificant and economically negligible impact on focal store traffic (column 2). In Table A2 in the Online Appendix, we redefine Closest store temporarily closed$_{it}$ to consider whether any of the three closest stores, in terms of co-visit similarity or physical proximity, are closed, and find similar patterns. We also examine the impacts of closures of stores with high co-visit similarity but that are not physically close, and those that are physically close but do not have high co-visit similarity (see Table A3 in Online Appendix). The findings are consistent.

Our analysis also suggests significant mismatches between the physical layout of stores and their co-visit patterns (see the scatter plot of store pairs regarding their co-visit similarity and physical distance in Online Appendix Figure A9). For a more concrete illustration, we randomly draw and list eight store pairs situated at the extreme ends of the co-visit similarity and physical distance spectrum (see Online Appendix Figure A10). These examples demonstrate that stores that are physically distant can have high co-visit similarity, and conversely, proximate stores can have low co-visit similarity.

Table 6: **Effects of Nearest Store Closures: Same vs. Cross-Category Comparison**

	Dep Var: log(Traffic)	
	Same category	Different categories
	(1)	(2)
Closest store temporarily closed	0.046***	0.047
	(0.018)	(0.039)
Store FE	Yes	Yes
Week FE	Yes	Yes
N	9,428	2,762
R^2	0.731	0.792

Notes: Column 1 estimates the effect of closures for stores with the highest co-visit similarity to the focal store within the same category. Column 2 estimates the effect for stores in different categories. We exclude stores that did not experience closest store closure. Standard errors are clustered at store level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6 focuses on co-visit similarity and reports heterogeneity in effects between same-category and cross-category stores. The closure of most similar stores in terms of co-visit similarity results in a statistically significant increase in traffic for the focal store in the same category by 4.6%. Interestingly, the effect size is similar for stores in different categories, though the coefficient is not statistically significant, likely due to the relatively small sample size. In other words, the closure of a store like Apple may lead customers to visit more Starbucks. One explanation is that customers generally remain in the mall even if a specific store is closed; upon discovering that Apple is closed, they may choose to visit Starbucks instead of exiting the mall. Table A4 in the Online Appendix shows that overall mall traffic did not decrease during store closure periods, consistent with this explanation.

6.2 Counterfactual Store Location Choice

Brands often face challenges in selecting store locations in new environments or adjusting existing layouts. Consider a store in Mall A contemplating a change in location. In such scenarios, a key metric is the counterfactual probability of consumers transitioning from other stores to the focal store. This probability can significantly impact overall store performance. To assess the predictive power of co-visit similarity, we conduct external validation tests. We show that store co-visit similarity more accurately predicts the counterfactual transition probabilities compared to a model that simply augments stated brand similarity with location information. This demonstrates the value of leveraging trajectory data in capturing nuanced brand substitution patterns.

In the first exercise, we model the transition probability from store i to j in mall m as a linear function of co-visit similarity between i and j in mall m, which depends on consumer preference$_{ij}^{m}$, physical distance$_{ij}^{m}$ and other factors ξ_{ij}^{m}:

$$P^{m}(\text{Store}_j|\text{Store}_i) = \alpha_0 + \alpha_1 \text{Co-visit Similarity}_{ij}^{m}(\text{Preference}_{ij},\ \text{Distance}_{ij}^{m}, \xi_{ij}^{m}) + \epsilon_{ij}^{m}, \quad (6)$$

We estimate Equation 6 using all data from Mall A. Next, we focus on common store pairs in both malls and calculate the counterfactual co-visit similarity in Mall A using Equation (4), which adjusts for the differences in physical distance between the two malls. This adjustment calibrates the co-visit similarity in Mall A to match the observed distance in Mall B. We then compute the predicted transition probability in Mall A based on this adjusted co-visit similarity and compare it with the observed transition probability in Mall B.

For illustration, we present a case study of two anchor stores, Huawei and Xiaomi (see Online appendix A.6). These stores are far apart in Mall A (see Figure 1) but close in Mall B. We show that after adjusting the distance between Huawei and Xiaomi in Mall A to match the level observed in Mall B, the predicted transition probability in Mall A closely aligns with the observed probability in Mall B. We also discuss the broader implications of location changes on non-focal stores in Online Appendix A.6. We do not find evidence that placing Huawei and Xiaomi far apart is associated with increased store visits or longer dwell times in the mall.

In the second analysis, we model the transition probability as a linear function of consumer stated similarity and physical distance:

$$P^m(\text{Store}_j|\text{Store}_i) = \beta_0 + \beta_1 \text{Stated similarity}_{ij} + \beta_2 \text{Distance}^m_{ij} + \epsilon^m_{ij}, \tag{7}$$

After estimating Equation (7) using data from Mall A, we adjust the distance to match the level observed in Mall B and calculate the predicted transition probability. This provides a baseline model to assess the relative effectiveness of our co-visit similarity metric.

Table 7: Out-of-sample Validation

	RMSE	RMSE improvement
Stated Similarity + Physical Distance	0.0069	-
Co-visit Similarity	0.0064	9.8%

Table 7 provides out-of-sample RMSE as a performance measure for how well the predicted transition probabilities in Mall A match the observed data in Mall B. Modeling the transition probabilities using co-visit similarity reduces the RMSE by a significant margin (9.8%) relative to the baseline model that relies on stated brand similarity and physical distance.

There are two takeaways: First, the use of customer trajectory data and our store co-visit similarity measure provides a more accurate representation of inter-brand relationships in retail agglomerations than the model that merely augment stated preference data with location data. This highlights the importance of leveraging real behavioral data when investigating consumer substitution patterns. Second, while the co-visit similarity may initially seem specific to a particular mall, this measure, with credible estimates of physical distance and a clear understanding of its composition, can be adjusted and applied to a wider array of contexts, making it an valuable tool for strategic decision-making in brick-and-mortar settings.

7 Conclusion

Despite physical stores accounting for over 80% of total retail sales,[9] recent literature on market structure analysis has primarily focused on using online user-generated content to derive brand relationships. Such approaches may not be fully applicable to brick-and-mortar stores, where spatial locations, a defining characteristic of retail agglomerations, can reshape the inter-brand relationships. To our best knowledge, this paper is the first using large-scale offline customer trajectory data to gain market structure insights for brick-and-mortar stores. Considering the significant challenges physical stores face from e-commerce, providing realistic and applicable market structure insights is important for their strategic decision-making.

Utilizing a scalable Store2Vec model, we incorporate rich consumer heterogeneity in store visit behavior to construct a rich representation of store co-visitation patterns that reveals realistic inter-brand relationships in physical retail environments. This approach offers a nuanced view of market structure that captures both consumer preferences and the spatial locations of retail stores. Our research goes beyond simply introducing novel revealed preference data to market structure analysis literature. By augmenting our trajectory data with survey and location data, we decompose store co-visit similarity into three key components: consumer stated preferences, physical locations, and other factors. This exercise suggests the complexity of consumer behavior in retail environments and highlights the limitations of relying solely on stated preference data or location information

[9] https://www.statista.com/statistics/534123/e-commerce-share-of-retail-sales-worldwide/

to understand inter-brand relationships.

We provide evidence that our constructed co-visit similarity more accurately captures substitution patterns compared to models based on stated preferences or location data alone. Through quasi-experimental analysis of store closures and counterfactual predictions of transition probabilities, we demonstrate the superior predictive power of the new measure we introduce in this study.

Our paper has important implications for retailers and mall operators. The ability to accurately uncover market structure and predict consumer behavior in physical retail environments can inform a wide range of strategic decisions, from store placement and layout optimization to targeted marketing campaigns and competitor analysis.

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Online Appendix

A Additional Tests

A.1 Alternative Tokenization Method

We adopt a different tokenization approach that focuses solely on the store, repeating the token in accordance with the visit duration quartile. To accommodate this method, we train skip-gram models with a range of window sizes from 4 to 8. This adjustment is necessary because the token repetition method inherently requires a larger window size to accurately capture the brand relationships, compared to our preferred model's window size of 2. Upon training the store embeddings, we compile a pairwise store similarity matrix. The correlation between the matrices—derived from our initial tokenization by store and duration quartile method and the alternative tokenization by store with duration repetition method—remains stable, ranging from 0.59 to 0.61 across different window sizes. This consistency suggests that both tokenization approaches yield comparable results in terms of elucidating store co-visit similarities.

A.2 Alternative Embedding Architectures

We also employ the Continuous Bag of Words (CBOW) model, an alternative variant of Word2Vec, to generate store embeddings. The CBOW model predicts a target word (store) based on its surrounding words (stores). For apple-to-apple comparison, we use the same tokenization method and hyperparameters as for the skip-gram model (refer to Section 4.2). For each store, we identify the top 10 most similar stores using cosine similarity of embeddings derived from the Skip-Gram model and subsequently determine their corresponding ranks in the CBOW model. Figure A1 shows the distribution of rank differences for these top 10 similar stores between the two models. The observed average rank difference is 5.96, implying a substantial agreement between the models in terms of identifying similar neighboring stores. Overall, the co-visit similarity between stores is robust to different word embedding architectures.

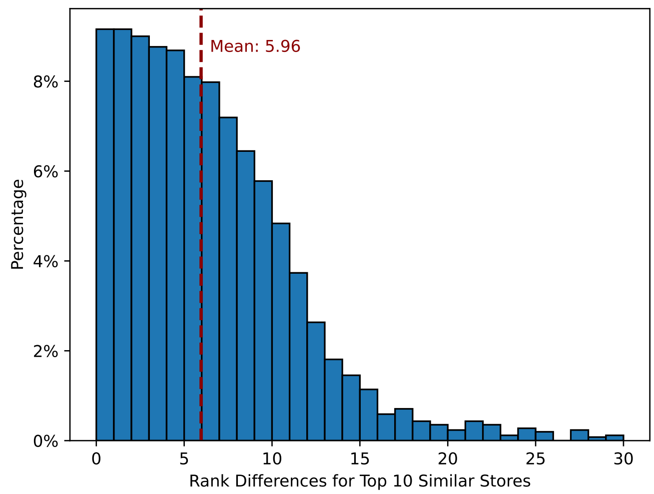


Figure A1: **Distribution of Rank Differences for Store Co-visit Similarities: Skip-Gram vs. CBOW**
Notes: This figure represents the rank differences of the top 10 most similar stores as identified by the Skip-Gram model, compared against their corresponding ranks in the CBOW model. The rank differences were calculated based on cosine similarity measures of the store embeddings generated by each model.

A.3 Store Analogy Prediction

Word analogy tasks are commonly used for intrinsic evaluation of word embeddings in NLP. This task predicts a missing element in a proportional relationship between words, or in our case, stores. A classic word analogy is: "King is to Queen as Man is to Woman." Similarly in our context, we construct store analogies, such as: "Nike (Store a) is to Adidas (Store b) as Hugo Boss (Store c) is to ? (Store d)." The underlying vector arithmetic for this is:

$$\text{vec(Store } d) \approx \text{vec(Store } c) + \text{vec(Store } b) - \text{vec(Store } a),$$

We assess the accuracy of these predictions by comparing the cosine similarity between the predicted and actual embeddings of Store d. Store analogy tasks allows us to evaluate the embeddings' capability to mirror human-like reasoning about store relationships, thereby offering insights into market structure.

We explore three types of store analogy tasks: customer demographic analogies, category-based analogies, and price range analogies. Each type reflects different market dimensions, such as demographic preferences, product categories, or pricing strategies. We manually curate ten sets of stores for each analogy type. For instance, one set of category-based analogies is: Huawei and Apple as stores a and b, and H&M and UR as stores c and d.

Figure A2 compares the average cosine similarity scores between the predicted and actual embeddings for Store d. Across all analogy tasks, a window size of 2 consistently shows better performance.

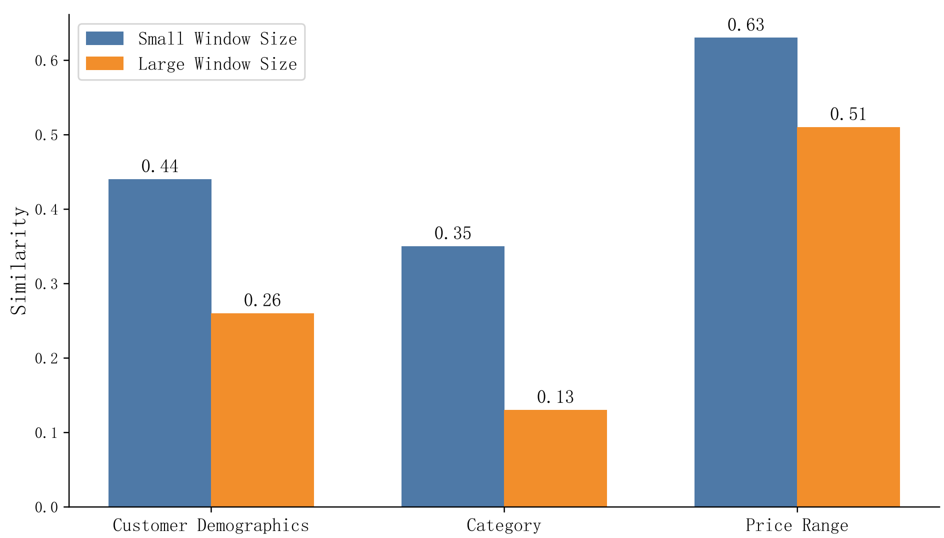


Figure A2: **Average Cosine Similarity of the Predicted vs Actual Embeddings in Store Analogy Tasks**
Notes: Small window size = 2, large window size = 4. We compute the predicted embeddings using Equation A.3 and compare the cosine similarity between actual and predicted embeddings. We use data from Mall A for this exercise.

A.4 Comparing Store2Vec with the Baseline Lift Method

We compare the store co-visit similarity with the baseleine *lift* measure. The correlation is 0.50 (p-value < 0.01) in Mall A. In addition, for each store in Mall A, we identify the top 10 most similar stores based on co-visit similarity and then find their corresponding ranks based on the *lift*. Figure A3 shows the distribution of rank differences for these top 10 similar stores between the two models. The average rank difference is 9.70. It seems that while our co-visit similarity can capture more nuanced aspects of store relationships, it also largely aligns with the insights provided by store co-visits as reflected in the *lift*.

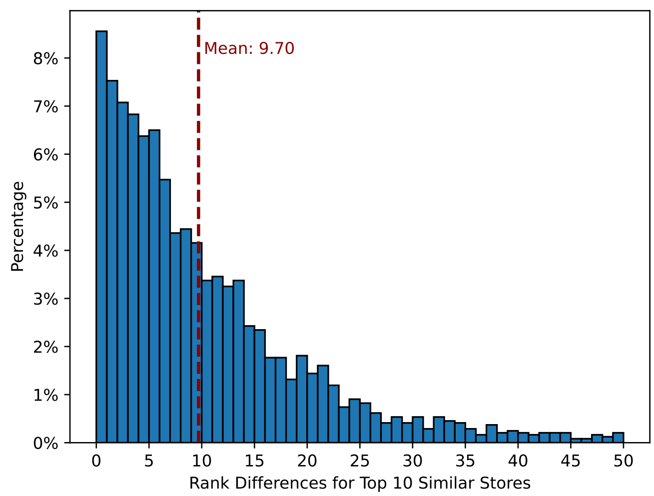


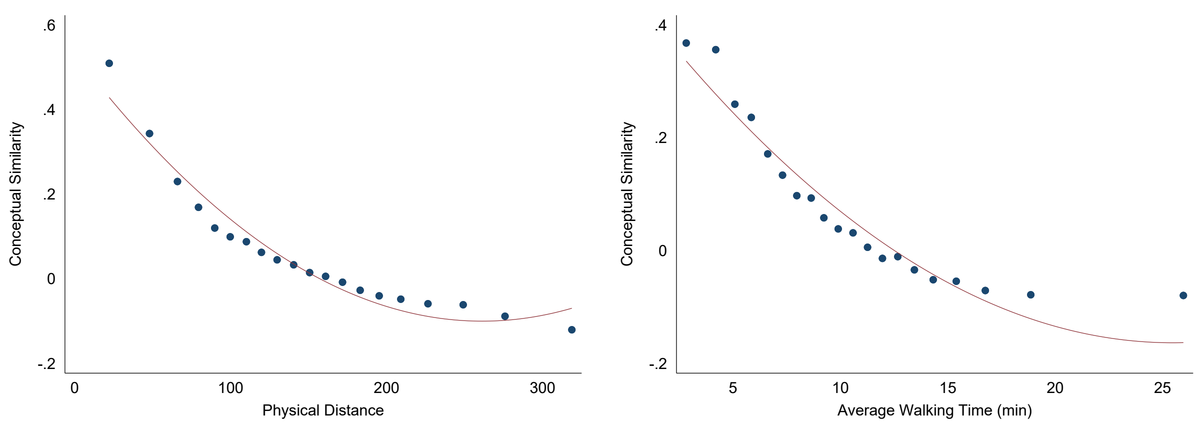
Figure A3: **Distribution of Rank Differences: co-visit Similarity vs. Lift**
Notes: This figure represents the rank differences of the top 10 most similar stores based on co-visit similarity, compared against their corresponding ranks based on *lift*.

A.5 Alternative Measure of Physical Distance

We alternatively employ average walking time between stores as a proxy for physical distance. In our trajectory data, we observe the timing of a customer's entry and exit from stores. By aggregating data from a substantial number of customers who travel directly from one store to another within specific a store pair, we calculate the average time taken to walk between these two stores. For instance, if Customer 1 visits Nike, then

Adidas, followed by Dior, and Customer 2 goes from Adidas to Nike, we can obtain the time Customer 1 takes from Nike to Adidas, and from Adidas to Dior, as well as the time Customer 2 takes from Adidas to Nike. Then, the average walking time from Nike to Adidas (and vice versa) is the average walking time across Customers 1 and 2.

The walking time between two stores is a strong indicator of their physical distance. In Figure A4, we visualize the correlation between physical distance and co-visit similarity using two different measures: the distance as per the digitized mall layout and the average walking time between stores. Both measures exhibit very similar correlation patterns. This consistency affirms that our conclusions regarding the relationships between physical distance and co-visit similarity are robust to the measure of physical distance used.



(a) Physical distance in layout

(b) Walking time

Figure A4: **Binscatter: The Correlation between Physical Distance and co-visit similarity**

Notes: In Panel a, we use distance in the digitized mall layout to measure physical distance; In Pannel b, we use average walking time to measure physical distance.

A.6 Counterfactual Store Location Choice: Using Huawei and Xiaomi as a Case Study

The observed transition probability from Huawei to Xiaomi is 0.129, and the predicted probability is 0.115. In Section 6.2, the estimated impact of physical distance on co-visit similarity ranges between -0.094 and -0.149. And reducing the distance to the level observed between Huawei and Xiaomi in Mall B, the predicted transition probability in Mall A ranges between 0.161 and 0.186. This estimate closely aligns with the observed transition probability of 0.180 in Mall B, suggesting that our co-visit similarity captures meaningful brand relationships.

We also explore the broader implications of placing Huawei and Xiaomi far apart on non-focal stores. Separating these anchor stores might encourage customers to discover other stores when they travel between them. To this end, we focus on customers who begin their trajectory at Huawei and later visit Xiaomi, and compare customers' average total number of store visits and average total visit duration between two malls in Table A1. For customers patronizing both stores, the average number of total store visits between two malls is similar. For a more precise comparison, we normalize these numbers against the average store visit size within each mall and find similar results. This consistency extends to the average total duration of store visits. Thus, we do not find evidence that distancing similar stores is associated with increased store visits or longer time spent in malls. Intuitively, the arrangement of similar stores impacts the order in which stores are visited. In Mall B, where Huawei and Xiaomi are in proximity, a majority of customers (70%) move directly from Huawei to Xiaomi. Conversely, in Mall A, where the stores are apart, 45% of customers choose to visit Xiaomi after exiting Huawei, and 44% head to Xiaomi as their last stop.

One concern with this exercise is sample selection bias. For Mall A, we may mechanically include consumers who have fewer time constraints and are willing to travel longer distances to visit Xiaomi from Huawei. These customers are more likely to explore other stores and stay longer in the mall. However, even with this potentially selected sample, their exploration behavior is still not more prevalent than that of Mall B visitors. Another issue is that this analysis just focuses on two stores, which limits its generalizability. We believe Huawei and Xiaomi provide a clear example for illustrative purposes, because they are anchor stores that typically receive more consideration in layout design, and they happen to be situated very differently across the two malls. We leave a more rigorous treatment of the exploration effect for future studies.

Table A1: **Differences in Customer Visit Behaviors between Malls**

	Initial Visit Huawei and Later visit Xiaomi	
	Mall A (Far Apart)	Mall B (Close)
Average store visit size	4.5	5
Normalized store visit size	1.6	1.6
Mean total duration of store visits	47.9	50.2
Normalized total store visit time	0.96	0.99
Mean total duration (exclude Huawei and Xiaomi)	36.6	38.4
$Prob$(Huawei → Xiaomi → Others)	0.45	0.70
$Prob$(Huawei → Others → Xiaomi)	0.44	0.19

Notes: We restrict to customers who first visit Huawei (or Xiaomi) and later also visit Xiaomi (or Huawei). We normalize store visit size by taking the ratio of it to the average store visit size within each mall. Similar for the total duration of store visits.

Table A2: **The Impacts of the Nearest Store's Closure: Robustness Check 1**

	Dep Var: log(Traffic)	
Closeness based on:	Co-visit similarity	Physical distance
	(1)	(2)
Closest store temporarily closure	0.086***	0.017
	(0.011)	(0.020)
Store FE	Yes	Yes
Week FE	Yes	Yes
N	58,861	58,861
R^2	0.730	0.730

Notes: The variable "Closest store temporarily closure" equals 1 for each store on a given day if any of its three nearest stores, determined by either co-visit similarity or physical proximity, are temporarily closed. A store is defined as temporarily closed if it experiences more than 14 consecutive days with zero customer visits. 54 stores out of 201 stores experienced temporarily closure during the sample period. Standard errors are clustered at store level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A3: **The Impacts of the Nearest Store's Closure: : Robustness Check 2**

	Dep Var: log(daily traffic)			
Closeness based on:	Co-visit Not Physical	Physical Not co-visit	Co-visit Not Physical	Physical Not co-visit
	(1)	(2)	(3)	(4)
Closest store temporarily closure	0.101***	0.043	0.080**	0.050
	(0.047)	(0.081)	(0.047)	(0.064)
Time linear trend	✓	✓	✓	✓
Week FE			✓	✓
Store FE	✓	✓	✓	✓
N	58861	58861	58861	58861
R^2	0.720	0.720	0.730	0.728

Notes: This regression examines the impacts of closure of co-visitly similar but not physically close stores, and closure of physically close but not co-visitly similar stores. A store is defined as temporarily closed if it experiences more than 14 consecutive days with zero customer visits. Heteroscedasticity-robust Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A4: **No Overall Traffic Reduction Effect**

	Log(Traffic)		
	(1)	(2)	(3)
1(Days with store closure > median)	0.076	−0.023	−0.024
	(0.074)	(0.080)	(0.081)
Constant	11.395***	11.310***	11.311***
	(0.067)	(0.072)	(0.073)
Month FE	No	Yes	Yes
Linear Time trend	No	No	Yes
N	243	243	243
R^2	0.004	0.038	0.038

Notes: Heteroscedasticity-robust Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

B Additional Figures

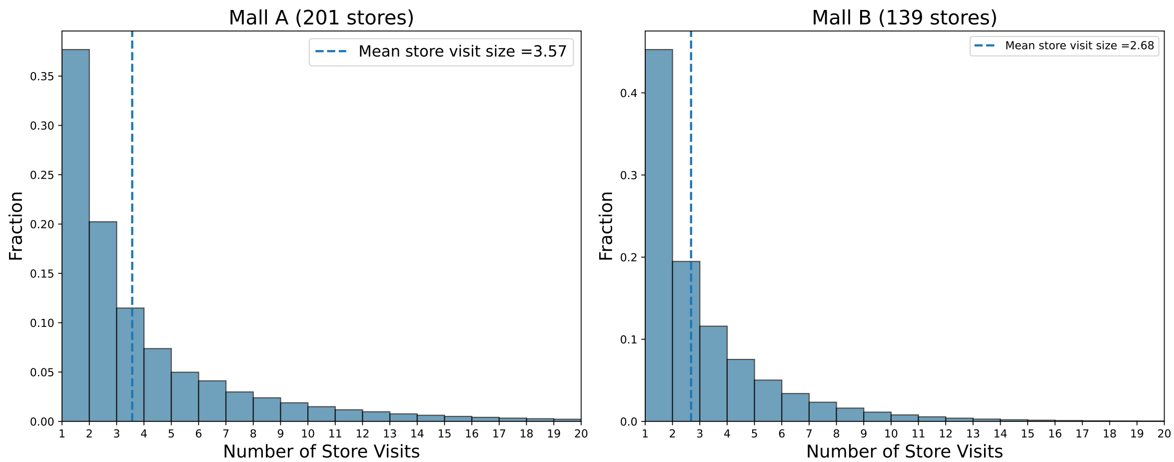


Figure A5: **Distribution of Customer Store Visit Size**

Note: The data is capped at 20. Any customer's store visit size beyond this threshold are excluded.

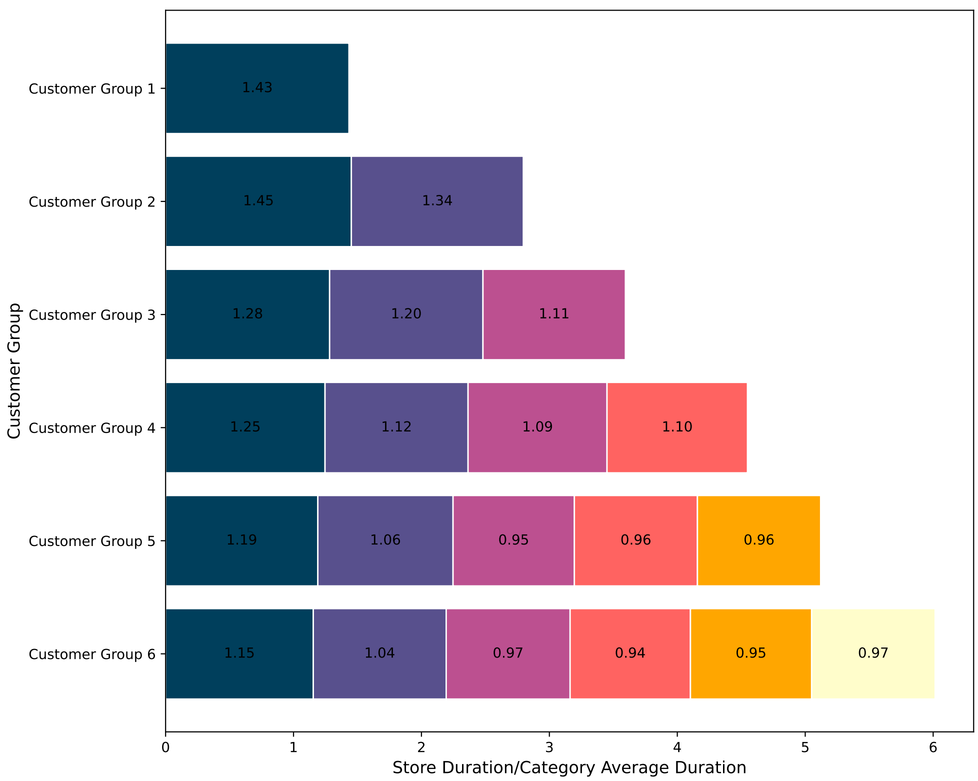


Figure A6: **Store Visit Sequence and Duration in Mall A**

Notes: The customer groups are defined by the number of store visits, and for illustration purpose are focused on consumers who visit 6 or fewer stores. The X-axis represents the sequence of store visits. For each group, we calculate the average time spent per visit sequence. To ensure comparability, we take the ratio of each store visit duration to the average duration of the store's category.

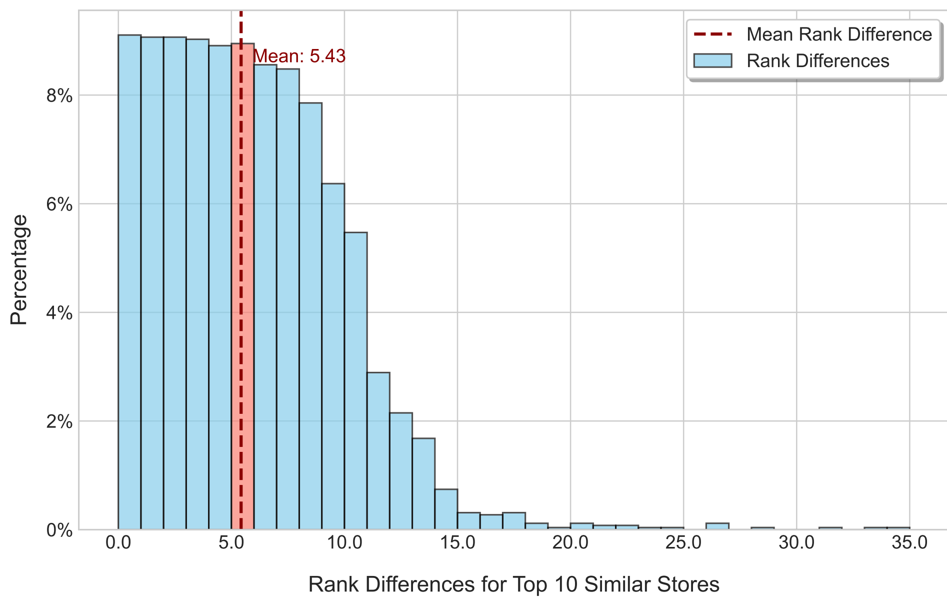


Figure A7: **Distribution of Rank Differences for Store Co-visit Similarities: Weekday Sample vs Weekend Sample**
Notes: This figure represents the rank differences of the top 10 most similar stores based on co-visit similarity calculated based on weekday sample, compared against their corresponding ranks based on weekday sample.

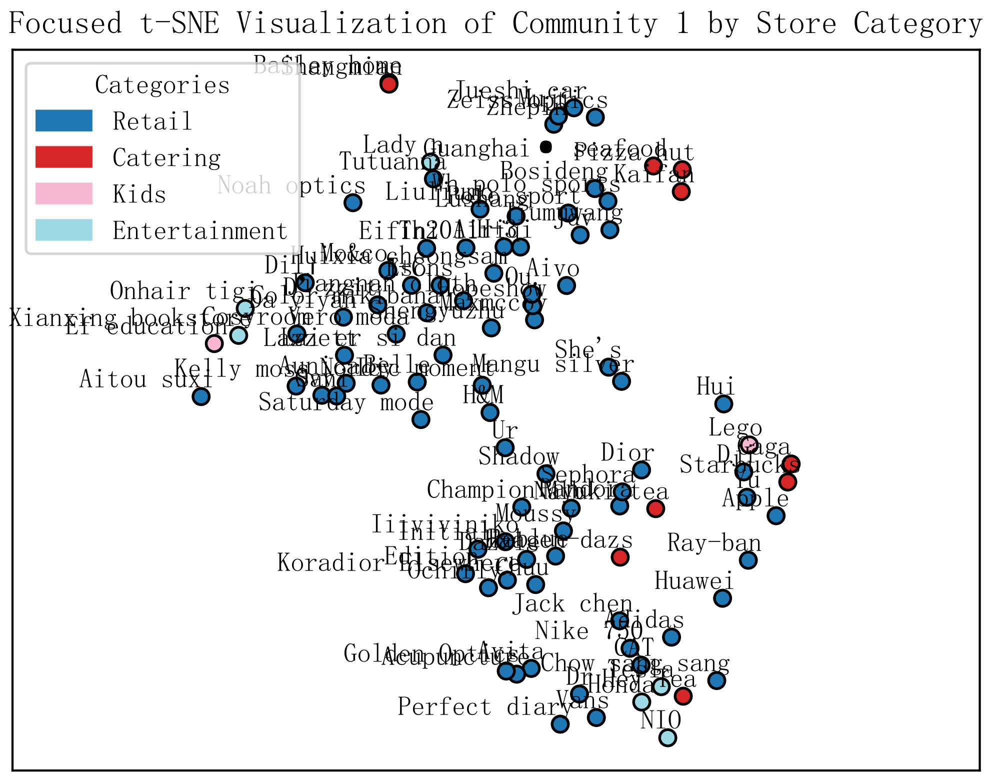


Figure A8: **Market Structure Map with Detected Communities: Community 1**

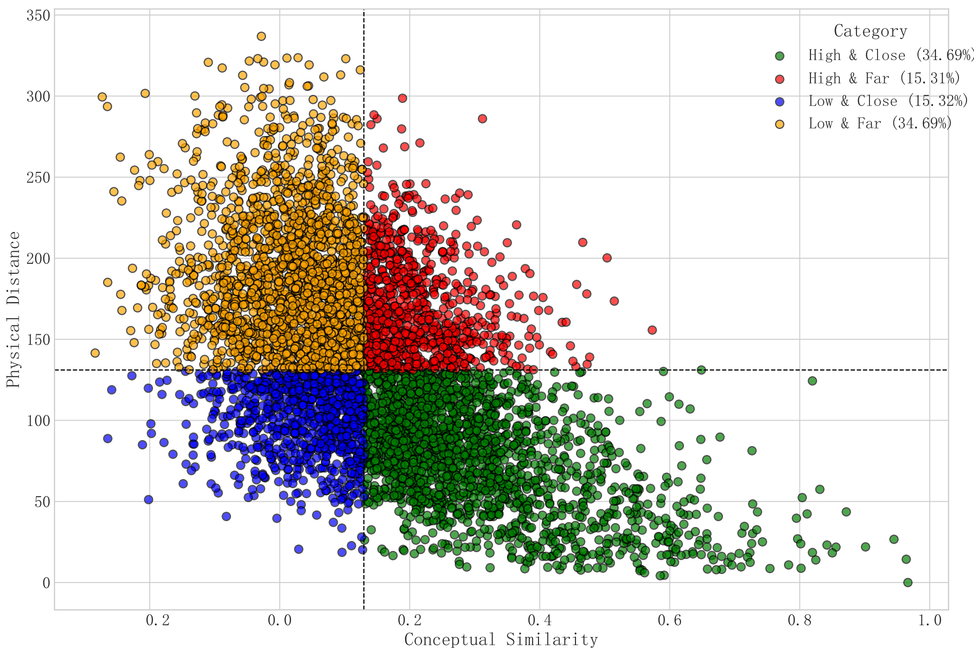


Figure A9: **Distribution of Store Pairs with respect to co-visit similarity and Physical Distance**

Notes: For enhanced visualization, we randomly select 50% of the store pairs within each category. The dashed lines represent the medians of physical distance and co-visit similarity, respectively. Based on these medians, we categorize the store pairs into four distinct groups. This figure showcases the distribution of store pairs in Mall A.

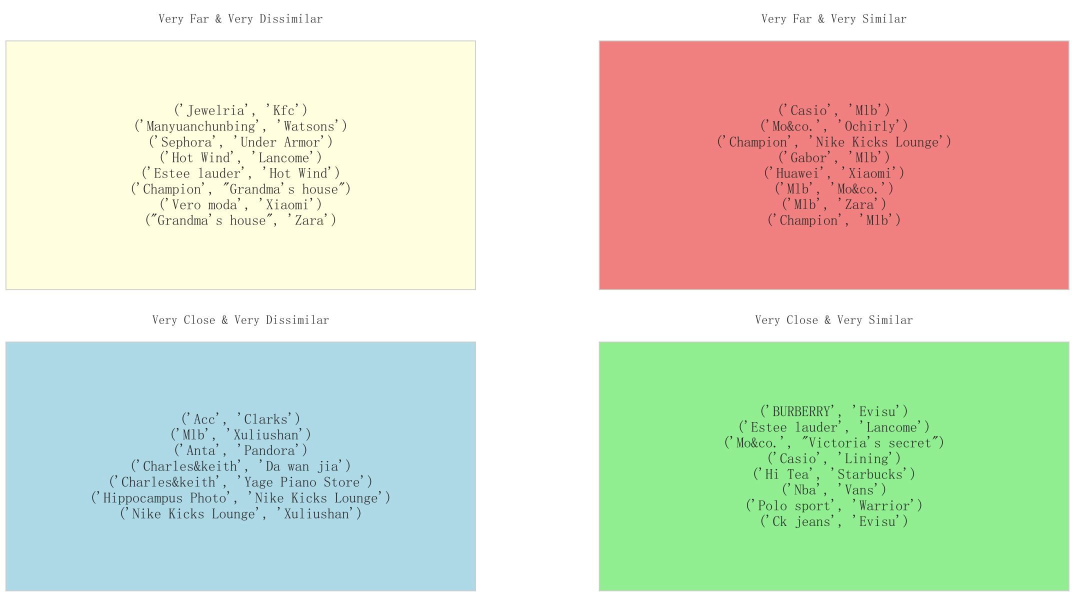


Figure A10: **Representative Stores**

This figure shows representative stores in Mall A. Very Far & Very Dissimilar store pairs are those ranked in the 10th decile for physical distance and the 1st for co-visit similarity. Very Far & Very Similar store pairs are those ranked in the 10th decile for physical distance and the 10th for co-visit similarity. Very Close & Very Dissimilar store pairs are those ranked in the 1st decile for physical distance and the 1st for co-visit similarity. Very Close & Very Similar store pairs are those ranked in the 1st decile for physical distance and the 10th for co-visit similarity.