MM6761: Take-home Assignment 5

Dai Yao (dai@{yaod.ai, mle.bi})

June 17, 2024

1 Rust (ECMA 1987)

Compute the Harold Zurcher model.

- *Use the parameter estimates θ from the top of Table X in Rust's (1987) paper.*
- *Compute $EV(x, i; \theta)$ using the value iteration procedure, described in Rust paper.*
- *Graph $EV(x, i; \theta)$, separately for $i = 0, 1$.*

Answer:

Please refer to the following page for an excellent implementation (best I've ever seen) in Matlab: link. By twisting run_fxp.m a bit, you will be able to produce the following two plots.

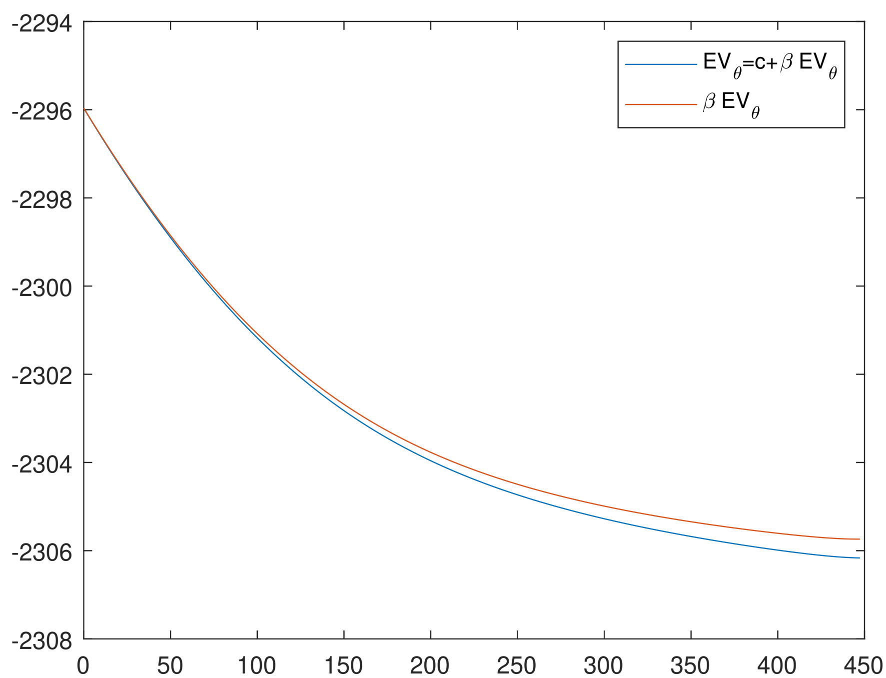

Figure 1: Replication of Figure 2 in Rust (1987): No normalization

If you don't know how to code in Matlab, or how to twist the code to produce the plots above, please check this link.

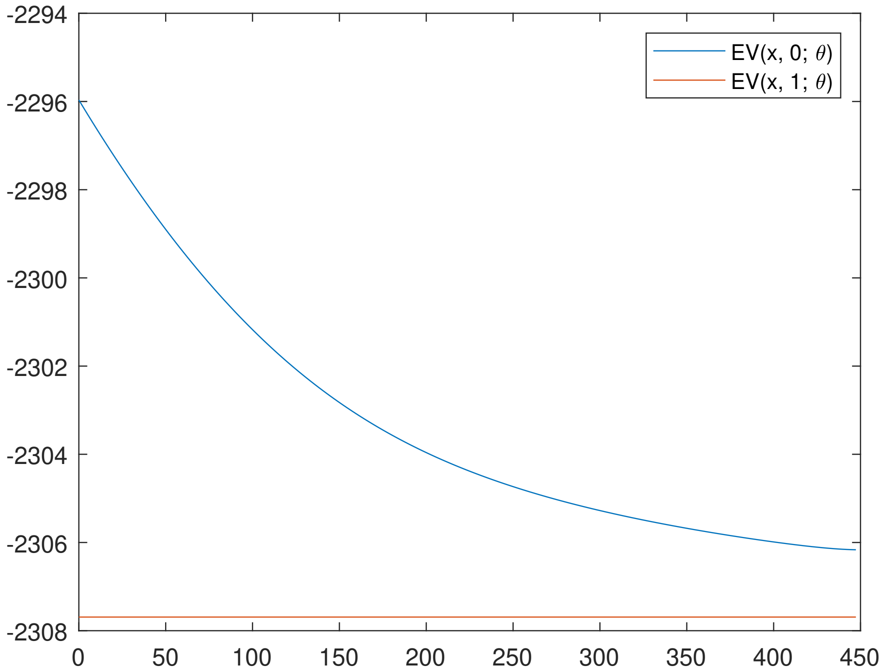

Figure 2: $EV(x,0;\theta)$ vs. $EV(x,1;\theta)$

2 Soysal and Krishnamurthi (MKSC 2012)

What's the main innovation in S & K (2012)? What are the challenges that the innovation imposes on estimation of the model, and how the authors handle the challenges?

Answer:

The main innovation of the paper is that they incorporate product availability into the dynamic choice model. With the possibility that a fashion product would become unavailable approaching the end of the selling season, and the assumption that consumers can foresee it and take it into their forward-looking decision making process, the Bellman equation changes to the following (right above Equation 6 on page 302):

$$V_{ij0t}(S_t) \quad = \quad \gamma \mathrm{E}[\lambda_{j,t+1} \max\{V_{ij,t+1}(S_{t+1}), V_{ij0,t+1}(S_{t+1})\} + (1-\lambda_{j,t+1}) \times 0 | S_t] + U_{ij0t}(S_t), \tag{1}$$

In the above equation, there are two components on the right-hand side inside the brackets of expectation, the first component $\lambda_{j,t+1}\max\{V_{ij,t+1}(S_{t+1}), V_{ij0,t+1}(S_{t+1})\}$ captures the expected utility from future purchase of the fashion project, and the second component $(1-\lambda_{j,t+1}) \times 0$, which is essentially 0, captures the expected utility from not being able to purchase the product due to unavailability. The last term on the right-hand side is the utility from the action (i.e., no purchase) in the current period, which is normalized as 0. With these, the Bellman equation becomes:

$$V_{ij0t}(S_t) \quad = \quad \gamma \mathrm{E}[\lambda_{j,t+1} \max\{V_{ij,t+1}(S_{t+1}), V_{ij0,t+1}(S_{t+1})\} | S_t], \tag{2}$$

This specification is pretty much the same as a standard dynamic choice model (e.g., Equation 3.5 on page 1007 in Rust [1987]), except that there is this $\lambda_{j,t+1}$ before the maximization inside the expectation.

There are not many challenges that the main innovation imposes on the estimation of the model, but there are some additional work to do. Specifically, the evolution of the availability state $\lambda_{j,t}$ needs to be estimated beforehand. Once its evolution is determined (together with the evolution of other state variables), we can follow the standard procedure to derive the purchase probability in any period for a representative customer (i.e., Equations 9 to 12 in the paper).

3 Yao, Tang, and Chu (MKSC 2023)

What's the main innovation in Yao et al. (2012)? What are the challenges that the innovation imposes on estimation of the model, and how the authors handle the challenges?

Answer:

The main innovation in the paper is that the authors developed a discrete time dynamic choice model which could accommodate the case wherein the decision maker does not necessarily need to make a decision in each period, while all the previous literature on this type of model assumes (implicitly) that the decision maker has to make a choice (even if the choice is the outside option) in each period.

The challenge that the innovation imposes on estimation is that the traditional Bellman equation, which does not accommodate "holes" in the time periods (i.e., when the decision maker does not have a decision to make), no longer works, hence, there is no way to derive the likelihood function (or to construct some auxiliary variables such as moment conditions to facilitate the estimation of the model). To overcome the challenges, the authors introduced the availability state of a car $B_{it} = (b_{it}, b_{i,t+1}, \cdots, b_{i,t+k})$, where each element $b_{i\tau}$ is a dummy which equals 1 if the car is available on day τ and 0 otherwise. They then introduced two operators: A forward operator $\digamma(\cdot)$, which rolls the availability state forward by one day, and an overlay operator $\oplus$, which updates the availability state by adding a specific rental request characterized by (c_{it}, d_{it}), where c_{it} represents the lead time, d_{it} is the duration, and both are defined in detail below. Formally,

$$\begin{aligned}
\digamma(B_{it}) \quad &= (\underbrace{b_{i,t+1}, \cdots, b_{i,t+4}}_{\text{elements 2 to 5 in } B_{it}}, 1), \\
B_{it} \oplus (c_{it}, d_{it}) \quad &= (b_{it}, \cdots, b_{i,t+(c_{it}-1)}, \underbrace{0, \cdots\cdots\cdots, 0}_{\text{elements } c_{it} \text{ to } c_{it}+(d_{it}-1)}, b_{i,t+c_{it}+d_{it}}, \cdots, b_{i,t+4}).
\end{aligned}$$

With the two operators, they can calibrate the evolution of B_{it} as follows:

$$B_{i,t+1} = \begin{cases} \digamma\left(B_{it} \oplus (c_{it}, d_{it})\right), & \text{if } r_{it} = 1 \text{ and } a_{it} = 1, \\ \digamma(B_{it}), & \text{otherwise.} \end{cases} \tag{3}$$

They also model the hazard rate of a request arriving at owner i in period t, given a host of covariates X_{it} and an inter-request time gap T_{it} as follows:

$$\lambda_{it}(T_{it}) = \bar{\lambda}(T_{it}) \exp(X_{it}^{r} \alpha), \tag{4}$$

Using the hazard model to connect the "holes" in the time periods when the decision maker (car owners in this context) does not need to make a decision, the authors were able to derive the likelihood of the observed arrivals of the rental requests at each owner and the acceptance/rejection decisions made by the owner (Equation 8 to 12 in the paper).