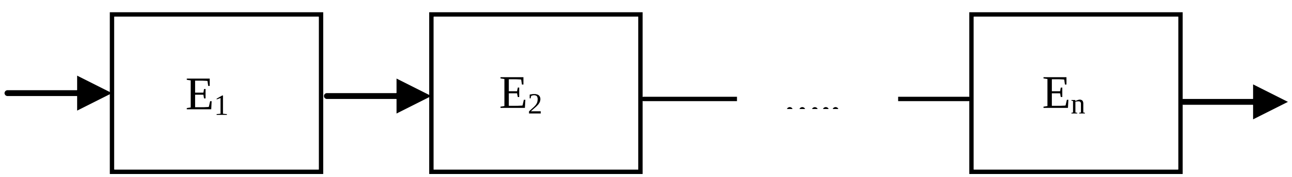
Reliability of Systems

In this section, we discuss the system reliability in which the components are connected in different fashions.

Series (nonredundant) System

Consider a system having a total of n components which are connected in series.



Let E_i denotes the event that the component i is functioning properly.

Let $R_i(t)$ be the reliability of the i th component in the series, that is,

$R_i(t) = P(E_i)$= probability that E_i fucntions

Then the reliability of the system is the probability that components $E_1, E_2, ..., E_n$ function properly is given by

$$R_s = P(E_1 \cap E_2 \cap ... \cap E_n)$$

$$= P(E_1)P(E_2)...P(E_n)$$

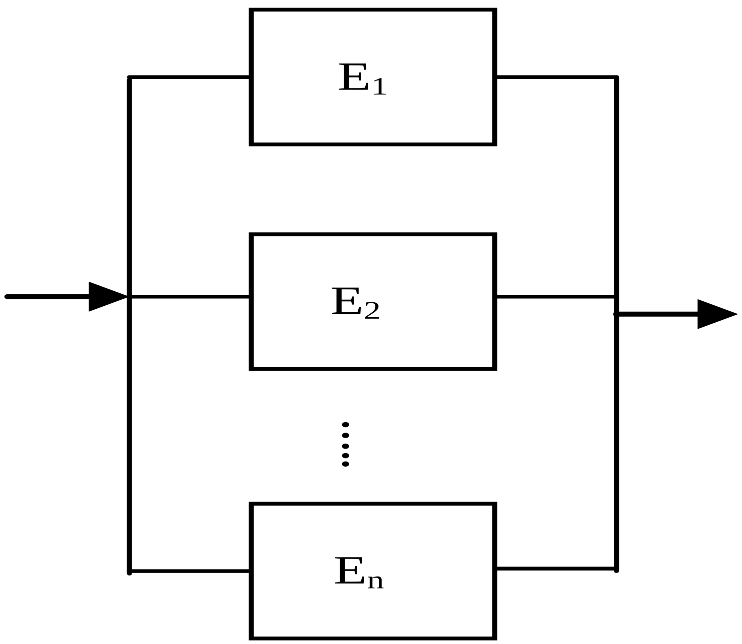
$$= R_1(t)R_2(t)...R_n(t)$$

(Assuming that the components function independently)

Parallel (Redundant) System

Parallel configuration is one in which the components of the system are connected in parallel..

In parallel system, if all components fail, then the system fails.



Consider two components

$$R_p = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= P(E_1) + P(E_2) - P(E_1).P(E_2)$$

(E1 and E2 are independent)

$$= R_1 + R_2 - R_1 R_2$$

$$= 1 - (1 - R_1)(1 - R_2)$$

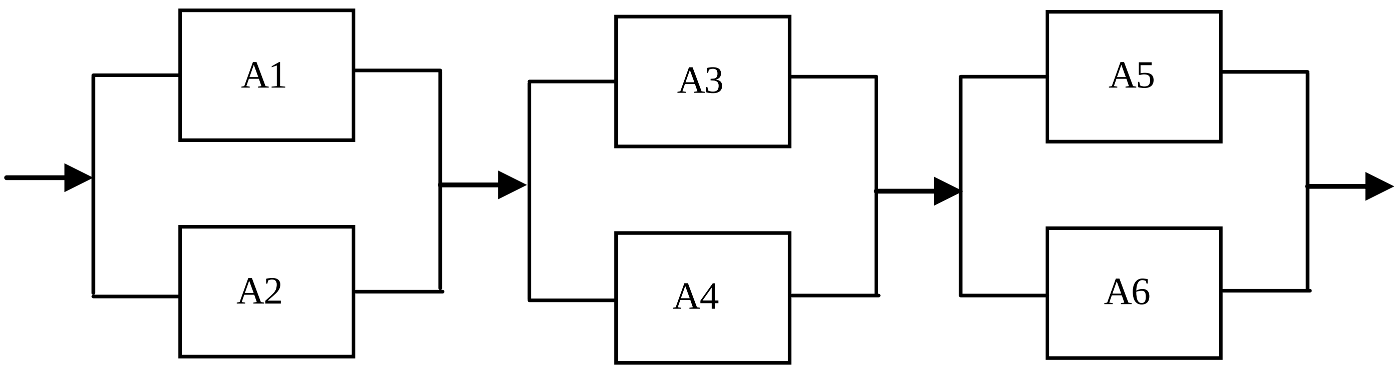
Now, extending to n components, we have

$$R_p = 1 - (1 - R_1)(1 - R_2)\ldots(1 - R_n)$$

Parallel -Series Configuration

A system in which m subsystems are connected in series where each subsystem has n components connected in parallel is said to be in parallel series configuration

Take m= 3 and n = 2



Now,

$$\text{Reliability of first subsystem} = 1-(1-R_1)(1-R_2)$$

$$\text{Reliability of second subsystem} = 1-(1-R_3)(1-R_4)$$

$$\text{Reliability of third subsystem} = 1-(1-R_5)(1-R_6)$$

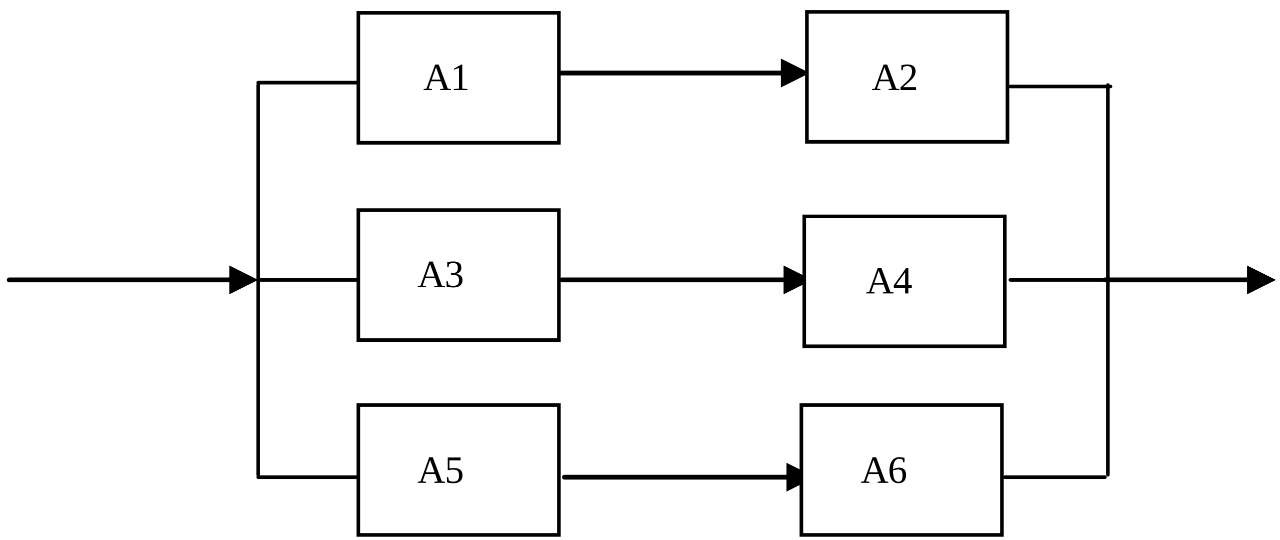
The system reliability

$$,\ R_S = (1-(1-R_1)(1-R_2))(1-(1-R_3)(1-R_4))(1-(1-R_5)(1-R_6))$$

Series-Parallel Configuration

A system, in which m subsystems are connected in parallel where each subsystem has n components connected in series is said to be in series–parallel configuration .

Take m=3 and n=2



Now,

the reliability of first subsystem = $R_1 R_2$.

the reliability of second subsystem = $R_3 R_4$.

the reliability of third subsystem = $R_5 R_6$.

The reliability of the system

$$R_S = 1 - (1 - R_1 R_2)(1 - R_3 R_4)(1 - R_5 R_6)$$

P1: A system has 100 units in series, each one has reliability 0.98. What is the reliability of the system?

Solution:

Since the each unit of the system has reliability 0,98,

$$R_i = 0.98, i = 1, ..., 100.$$

The reliability of the system

$$R_s(t) = \prod_{i=1}^{100} R_i = (0.98)^{100} = 0.1326$$

P2: Three lamps are connected in parallel to produce light in a hall. The reliabilities of the lamps are 0.92, 0.95 and 0.96. Find the reliability of the system. If the lamps are connected in series, calculate the reliability of the system.

Solution: Let $R_s(t)$ be the reliability of the system.

For parallel system:

$$R_s(t) = 1 - \prod_{i=1}^{3}(1 - R_i)$$

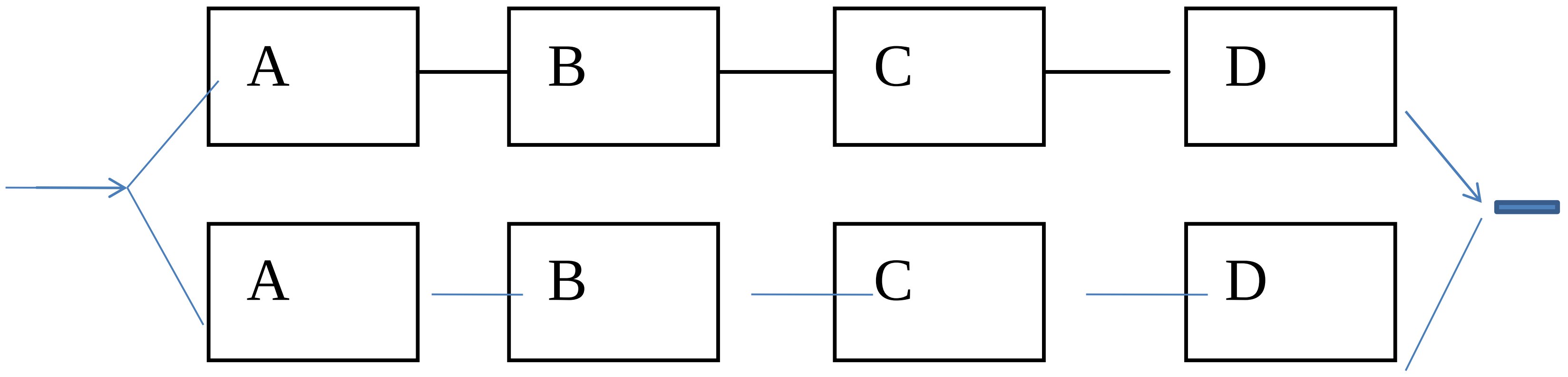
$$= 1 - \left[(1-0.92)(1-0.95)(1-0.96)\right]$$

$$= 0.9998$$

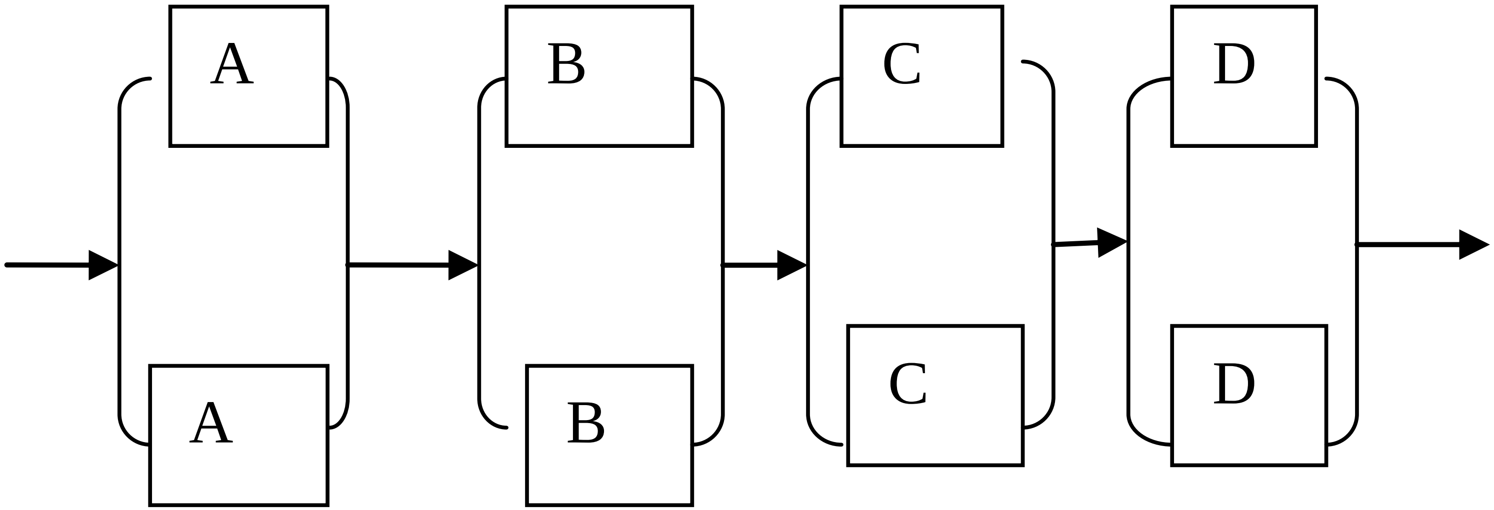
For series system:

$$R_s(t) = \prod_{i=1}^{3} R_i = (0.92)(0.95)(0.96) = 0.8390.$$

P3: Compute the reliability of the system for the connection given in the following figures (fig-i and fig-ii), if the reliability of A, B, C and D are 0.95, 0.99, 0.90 and 0.96 respectively.



Fig(i)



Fig(ii)

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Solution:

(i)

$$R_1 = R(\text{first sub system})$$
$$= 0.95 \times 0.99 \times 0.90 \times 0.96$$
$$= 0.81$$

$$R_2 = R(\text{second sub system})$$
$$= 0.95 \times 0.99 \times 0.90 \times 0.96$$
$$= 0.81$$

Therefore,

$$R_s(t) = 1 - \prod_{i=1}^{2} (1 - R_i)$$
$$= 1 - \left[(1 - 0.81)(1 - 0.81)\right]$$
$$= 0.9639$$

(ii)

$$R_1 = R(\text{first sub system})$$
$$= 1-(1-0.95)^2 = 0.9975.$$
$$R_2 = R(\text{second sub system})$$
$$= 1-(1-0.99)^2 = 0.9999.$$
$$R_3 = R(\text{third sub system})$$
$$= 1-(1-0.90)^2 = 0.99.$$
$$R_4 = R(\text{fourth sub system})$$
$$= 1-(1-0.96)^2 = 0.9984$$

Therefore,

$$R_s(t) = R_1 R_2 R_3 R_4 = 0.9859$$

Example 1 *An electronic circuit consists of 5 silicon transistors, 3 silicon diodes, 10 composition resistors and 2 ceramic capacitors connected in series configuration. The hourly failure rate of each component is given below:*

Silicon transistor	$: \lambda t = 4 \times 10^{-5}$
Silicon diode	$: \lambda_d = 3 \times 10^{-5}$
Composition resistor	$: \lambda_r = 2 \times 10^{-4}$
Ceramic capacitor	$: \lambda_c = 2 \times 10^{-4}$

Calculate the reliability of the circuit for 10 hours, when the components follow exponential distribution.

Solution

Since the components are connected in series, the system (circuit) reliability is given by

$$R_s(t) = R_1(t) \,.\, R_2(t) \,.\, R_3(t) \,.\, R_4(t)$$

$$= e^{-\lambda_1 t} . e^{-\lambda_2 t} . e^{-\lambda_3 t} . e^{-\lambda_4 t}$$

$$= e^{-(5\lambda_t + 3\lambda_d + 10\lambda_r + 2\lambda_c)t}$$

$$\therefore \qquad R_s(10) = e^{-(20\times10^{-5} + 9\times10^{-5} + 20\times10^{-4} + 4\times10^{-4})\times10}$$

$$= e^{-(20+9+200+40)\times10^{-4}}$$

$$= e^{-0.0269} = 0.9735.$$

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