T test

The t test (also called Student’s T Test) **compares two averages (means) and tells you if they are different from each other**. The t test also tells you how significant the differences are; In other words it lets you know if those differences could have happened by chance.

Definition: The **Degrees of Freedom** refers to the number of values involved in the calculations that have the freedom to vary.

Test for a Specified Mean

Given a random sample of size n (n<30) with sample mean $\bar{x}$, and the population standard deviation is not known and we want to test whether the population mean has a specified value, then we apply the same procedure as in the case of a large sample.

Case 1 : Two tail test

Null hypothesis $H_0 : \mu = \mu_0$

Alternative hypothesis $H_1 : \mu \neq \mu_0$

On the assumption H_0 is true, the test statistic is

$t = \dfrac{\bar{x} - \mu}{S / \sqrt{n}}$ where $S^2 = \dfrac{n}{n-1} s^2$, s being the s.d of the sample.

This t – statistic follows the t – distribution with number of degrees of freedom $\nu = n -$

We compare the calculated value of t with the table value of t for $(n-1) d.f.$ for a given significance level.

If the calculated value of $t <$ the table value of t, H_0 is accepted.

If the calculated value of $t >$ the table value t, H_0 is rejected.

Here the table value t is got for a given α from the column $t_{\frac{\alpha}{2}}$ (i.e. $t_{.025}$ for 5% level and $t_{.005}$ at 1% level).

Case 2: One – tail test

In the case of one tail test the null and alternative hypothesis may be classified in one of the following forms:

$$H_0 : \mu = \mu_0; \quad H_1 : \mu > \mu_0$$

(OR)

$$H_0 : \mu = \mu_0; \quad H_1 : \mu < \mu_0$$

However, in this case, the table value of t for a given significance level α is t_α ($t_{.05}$, $t_{.01}$ respectively for 5% and 1% level).

The conclusion is drawn on the same lines as in case 1.

Example 1

A sample of ten house owners is drawn and the following values of their incomes are obtained. Mean Rs. 6,000; standard deviation Rs.650. Test the hyphothesis that the average income of house owners of the town is Rs. 5,500.

Solution:

$n = 10 \quad s = 650$

$\bar{x} = 6,000 \quad \mu_0 = 5,500$

Since the sample size $n = 10 < 30$, the sample is a small sample. Therefore we have to apply t – test for testing the mean.

$H_0: \mu = 5,500$ (i.e. the average income of the house owners of the town is Rs. 5,500)

$H_1: \mu \neq$ Rs. 5,500

The test statistic is $t = \dfrac{\bar{x} - \mu}{S / \sqrt{n}}$ where S is the estimated s.d. of the population

given by $S = \sqrt{\dfrac{ns^2}{n-1}} = \sqrt{\dfrac{10}{9}} \times 650 = 685.16$

$$t = \frac{6,000 - 5,500}{685.16/\sqrt{10}} = \frac{500 \times \sqrt{10}}{685.16} = 2.31$$

Number of degrees of freedom $= n - 1 = 9$

The table value of t for 9 d.f. at 5% level = 2.262

Conclusion:

H_0 is rejected since the calculated value of $t >$ the table value of t. Hence the averge income of house owners in that town is not Rs. 5,500/-

Example 5:

A soap manufacturing company was distributing a particular brand of soap through a large number of retail shops. Before a heavy advertisement compaign,

the mean sales per week per shop was 140 dozens. After the campaign a sample of 26 shops was taken and the mean sales was found to be 147 dozens with a standard deviation of 16 dozens. Can you consider the advertisement effective?.

Solution:

$\mu = 140$ dozens.

$n = 26, \bar{x} = 147$ dozens, $s = 16$ dozens.

Since $n<30$, the sample is a small sample, let us therefore apply t–test for testing the mean.

$H_0 : \mu = 140$ dozens

$H_1 : \mu > 140$ dozens, (one tail test)

The test statistic is given by

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n-1}}$$

$$= \frac{147 - 140}{16 / \sqrt{25}} = \frac{7 \times 5}{16} = 2.1875$$

ndf = $n-1$ = 25

Table value t for 25 df at 5% level = 1.708.

Conclusion:

H_0 is rejected since the calculated value of t > the table value of t. Hence $\mu > 140$. This means the sales have increased after advertisement and hence the advertisement is effective.

Example 9:

The heights of 10 males of a given locality are found to be 175, 168, 155, 170, 152, 170, 175, 160, 160 and 165 cms. Based on this sample of 10 items test the hypothesis that the mean height of males is 170 cms.

Solution:

This is a small sample, since n<30. Let us first find the mean and the standard deviation of the sample.

x	d	d^2
175	10	100
168	3	9
155	−10	100
170	5	25
152	−13	169
170	5	25
175	10	100
160	−5	25
160	−5	25
165	0	0
1650	0	578

Let $d = x - 165$

$\bar{x} = 165$

$$s = \sqrt{\frac{578}{10} - 0} = \sqrt{57.8} = 7.6$$

$$s_1^2 = \frac{1}{n_1}\sum d_1^2 - \left(\frac{1}{n_1}\sum d_1\right)^2$$

$H_0 : \mu = 170$

$H_1 : \mu \neq 170$

The test statistic is $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}} = \dfrac{165 - 170}{7.6/\sqrt{9}} = \dfrac{-5 \times 3}{7.6}$

$$= \frac{-15}{7.6} = -1.97$$

$|t| = 1.97$

ndf = 9. ; table value at 5% level = 2.76

Conclusion:

H_0 is accepted at 5% level since the calculated value of $|t|$ < the table value.

This means that the mean height of males can be regarded as 170 cms.

Test of significance for the difference between two population means when population standard deviations are not known.

Given Data:

		Mean	S.D	size
Sample	1	$\bar{x}_1$	s_1	n_1
	2	$\bar{x}_2$	s_2	n_2

$(n_1 < 30, n_2 < 30)$

We want to test whether the means μ_1 and μ_2 of the two populations are equal or not. (i.e., do they differ significantly ?).

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

It has been proved mathematically that the standard error of $(\bar{x}_1 - \bar{x}_2)$ is

$$S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \text{ where } S^2 = \frac{n_1 s_1^{\,2} + n_2 s^2}{n_1 + n_2 - 2}$$

$n_1 + n_2 - 2$ is the number of degrees of freedom of the statistic.

The test statistic for testing the significance of the difference between the population means based on sample information is given by

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

From t–table, we can determine, the table value of t, for $(n_1 + n_2 - 2)$ df.

If the calculated value of $|t| <$ the table value, we accept the hypothesis H_0.

If the calculated value of $|t| >$ the table value, we reject the hypothesis H_0.

Note: The above procedure is only for a two–tail test. **In the case of one–tail test, we state the hypothesis as follows:**

$H_0: \mu_1 = \mu_2; \quad H_1: \mu_1 > \mu_2$

(OR)

$H_0: \mu_1 = \mu_2; \quad H_1: \mu_1 < \mu_2$

In this case, the table value of t for the significance level α is got referring to the column 0.05 probability for 5% and 0.01 probability for 1% level. The conclusion is drawn on the same lines as given earlier.

Example 1:

Two salesmen A and B are working in a certain district. From a sample survey conducted by the Head office, the following results were obtained. State whether there is any significant difference in the average sales between the two salesmen:

	A	B
No. of sales	20	18
Average sales (in Rs.)	170	205
Standard Deviation (in Rs.)	20	25

Solution:

The two given samples are small samples. Let us apply t–test for testing the means.

$H_0: \mu_1 = \mu_2$ (there is no significant difference in the average sales of the two salesmen).

$H_1: \mu_1 \neq \mu_2$ (difference in the average sales of two salesmen is significant)

$\bar{x}_1 = 170 \quad \bar{x}_2 = 205$

$s_1 = 20 \quad s_2 = 25$

$n_1 = 20 \quad n_2 = 18$

The test statistic is

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad \text{where } S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{20 \times 20^2 + 18 \times 25^2}{20 + 18 - 2} = \frac{8000 + 11250}{36} = \frac{19250}{36}$$

$S^2 = 534.72. \therefore S = 23.12$

$$\therefore t = \frac{170-205}{23.12\sqrt{\frac{1}{20}+\frac{1}{18}}} = \frac{-35}{23.12 \times 0.325} = \frac{-35}{7.52} = -4.65$$

$\text{ndf} = n_1 + n_2 - 2 = 36.$

Table value of t for 36 df at 1% level = 2.58.

Conclusion:

H_0 is rejected at 1% level since the calculated value of $|t|$ > the table value. Hence the two salesmen differ significantly with regard to their average sales.

Example 13:

The lives of 12 cars manufactured by two companies A and B are given below in years.

A	14	15	18	12	18	17	19	21	19	16	12	11
B	21	18	14	22	23	19	20	16	16	13	20	14

Which company will you choose to purchase a car? Give reason.

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Solution:

The two given samples are small samples. Let us apply t–test for testing the mean.

$H_0: \mu_1 = \mu_2$ (average life of cars manufactured by companies A & B is the same)

$H_1: \mu_2 > \mu_1$ (B has more life than A)

x	d	d^2	y	d	d^2
14	−2	4	21	3	9
15	−1	1	18	0	0
18	2	4	14	−4	16
12	−4	16	22	4	16
18	2	4	23	5	25
17	1	1	19	1	1
19	3	9	20	2	4
21	5	25	16	−2	4
19	3	9	16	−2	4
16	0	0	13	−5	25
12	−4	16	20	2	4
11	−5	25	14	−4	16
192	0	114	216	0	124

$$\overline{X} = \frac{192}{12} = 16 \quad ; \quad \overline{Y} = \frac{216}{12} = 18$$

$$d = x - 16 \qquad d = x - 18$$

$$s_1^{\,2} = \frac{114}{12}$$

$$s_1^2 = \frac{1}{n_1}\sum d_1^2 - \left(\frac{1}{n_1}\sum d_1\right)^2$$

$$s_2^{\,2} = \frac{124}{12}$$

$$S^2 = \frac{n_1 s_1^{\,2} + n_2 s_2^{\,2}}{n_1 + n_2 - 2}$$

$$S^2 = \frac{114 + 124}{22} = \frac{238}{22} = 10.818$$

$$\therefore S = 3.29$$

The test statistic is

$$t = \frac{\overline{X} - \overline{Y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{16 - 18}{3.29 \times \sqrt{\left(\frac{1}{12} + \frac{1}{12}\right)}}$$

$$= \frac{-2}{3.29 \times .41} = \frac{-2}{1.35} = -1.48$$

$|t| = 1.48$

ndf = 22.

Table value of t for 22df at 5% level for a one tail test = 1.717

Conclusion:

H_0 is accepted since the calculated value of $|t| <$ the table value of $|t|$.

$\therefore$ The average life of the two companies A and B do not differ significantly.

$\therefore$ We can purchase the car of either of the two companies.

Example 9:

A group of 5 patients treated with medicine A weigh 42, 39, 48, 60 and 41 kgs; A second group of 7 patients from the same hospital treated with medicine B weigh 38, 42, 56, 64, 68, 69 and 62 kgs. Do you agree with the claim that medicine B increases weight significantly. (The value of t at 5% level of significance for 10df is 2.228).

Solution:

The two given samples are small samples. We have to apply t–test for the data given.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 < \mu_2$

(Medicine B increases weight significantly) (one tail test).

x	d $(x-46)$	d^2	y	d $(y-5)$	d^2
42	–4	16	38	–19	361
39	–7	49	42	–15	225
48	2	4	56	–1	1
60	14	196	64	7	49
41	–5	25	68	11	121
			69	12	144
			62	5	25
230	0	290	399	0	926

$$\overline{X} = \frac{230}{5} = 46 \quad ; \quad \overline{Y} = \frac{399}{7} = 57$$

$$d = x - 46 \quad ; d = y - 57$$

$$s_1^{\,2} = \frac{290}{5} - 0 = 58$$

$$s_2^{\,2} = \frac{926}{7} - 0 = \frac{926}{7}$$

$$S^2 = \frac{n_1 s_1^{\,2} + n_2 s_2^{\,2}}{n_1 + n_2 - 2}$$

$$= \frac{5 \times 58 + 10 \times \dfrac{926}{7}}{5 + 7 - 2} = \frac{290 + 926}{10} = \frac{1216}{10} = 121.6$$

$$\therefore \; S = \sqrt{121.6} = 11.03$$

The test statistic is

$$t = \frac{\overline{X}_1 - \overline{X}_2}{S\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}} = \frac{46 - 57}{11.03 \times \sqrt{\left(\dfrac{1}{5} + \dfrac{1}{7}\right)}}$$

$$= \frac{-11}{11.03 \times 0.59} = \frac{-11}{6.51} = -1.69 = -1.7$$

$$|t| = 1.7$$

ndf$= n_1 + n_2 - 2 = 10$

Table value of t for 10df at 5% level for one tail test = 1.812

Conclusion:

H_0 is accepted at 5% level.

$\therefore$ Medicine B does not increase weight significantly.

(ie) Medicines A and B do not differ significantly w.r.to increase in weights.

t–TEST FOR PAIRED OBSERVATIONS

In the sample test we considered earlier, the two samples were independent; that is the values of items in one sample had to be independent of the values of the other. But situations may arise in practice that the condition of independence may not hold. For example we choose 10 students from a college and give an IQ–TEST. Suppose that these students are given intensive training and after that, their IQ are calculated. Here the same sample of students is considered before and after training. The sample values are no longer independent. In fact they are correlated pairs. In this situation we want to test whether the training has any effect on the IQ of those students. We can apply t–test after converting this into a single sample type by considering the difference in IQ in all these pairs.

The test procedure is given below.

Define a vailable $d = X_1 - X_2$ where (X_1, X_2) are the IQ's of each pair before and after the training.

Suppose μ_1 and μ_2 be the population means before and after the training.

Then take

$H_0: \mu_2 - \mu_1 = 0$

$H_1: \mu_2 - \mu_1 \neq 0$

The test statistic is

$$t = \frac{\bar{d}}{s_d / \sqrt{n-1}}$$

where $\bar{d}$ = mean value of the differences and s_d is the S.D of these differences.

Number of df $= n-1$

Conclusion:

If the calculated value of $t <$ the table value ,accept H_0 and if the calculated value of $t >$ the table value reject H_o.

Note: The same procedure is adopted for one tail test as in other problems. Here for example the hypothesis can be

$H_0 : \mu_2 - \mu_1 = 0$

$H_1 : \mu_2 - \mu_1 > 0$

Suitable table value has to be obtained for comparison with calculated value of t.

Example 2:

A company arranged an intensive training course for its team of salesmen. A random sample of 10 salesmen was selected and the value (in '000) of their sales made in the weeks immediately before and after the course are shown in the following table.

Salesman	1	2	3	4	5	6	7	8	9	10
Sales before	12	23	5	18	10	21	19	15	8	14
Sales After	18	22	15	21	13	22	17	19	12	16

Test whether there is evidence of an increase in mean sales.

Solution:

The same salesmen are tested for their sales potential before and after the training programme. So we apply the t–test for correlated pairs.

$H_0 : \mu = 0$ (there is no evidence of increase in mean sales).

$H_1 : \mu > 0$ (there is evidence of increase in mean sales) (one–tail test).

d = difference in sales = $y - x$.

Sales men	x	y	d	d^2
1	12	18	6	36
2	23	22	−1	1
3	5	15	10	100
4	18	21	3	9
5	10	13	3	9
6	21	22	1	1
7	19	17	−2	4
8	15	19	4	16
9	8	12	4	16
10	14	16	2	4
			30	196

$$\bar{d} = \frac{\Sigma d}{n} = \frac{30}{10} = 3$$

$$s^2 = \frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2 = \frac{196}{10} - (3)^2 = 19.6 - 9 = 10.6$$

$$\therefore s = \sqrt{10.6} = 3.26$$

The test statistic is

$$t = \frac{\bar{d}}{s / \sqrt{n-1}} = \frac{3}{3.26 / \sqrt{9}} = \frac{3}{1.09} = 2.76$$

$\text{ndf} = n - 1 = 9$

The table value of t for 9df at 5% level for one tail test = 1.833.

Conclusion:

H_0 is rejected and hence there is evidence of increase in sales, after the training programme.

Example 3:

A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure. (BP):

5, 2, 8, –1, 3, 0, –2, 1, 5, 0, 4, 6.

Can it be concluded that the stimulus will in general be accompanied by an increase in blood pressure.

Solution:

This is a sample of correlated pairs. We apply t–test for testing the increase in blood pressure.

$H_0 : \mu = 0$

$H_1 : \mu > 0$ (one tail–test)

Stimulus: A thing or event that evokes a specific functional reaction in an organ or tissue

Patient No.	Increase in BP (d)	d^2
1	5	25
2	2	4
3	8	64
4	–1	1
5	3	9
6	0	0
7	–2	4
8	1	1
9	5	25
10	0	0
11	4	16
12	6	36
	31	185

Let d = increase in blood pressure.

$$\bar{d} = \frac{31}{12} = 2.58$$

$$s = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} = \sqrt{\frac{185}{12} - (2.58)^2}$$

$$= \sqrt{15.42 - 6.66} = \sqrt{8.76} = 2.96$$

The test statistic is

$$t = \frac{\bar{d}}{s / \sqrt{n-1}} = \frac{2.58}{2.96 / \sqrt{11}}$$

$$= \frac{2.58}{2.96 / 3.32} = \frac{2.58}{0.89} = 2.89$$

$\text{ndf} = n - 1 = 11$

Table value of t for 11df at 5% level for one tail test = 1.796

Conclusion:

H_0 is rejected at 5% level since the calculated value of t > the table value of t.

Hence the stimulus generally increases the blood pressure.

Example 4:

An IQ test was administered to 5 persons before and after they were traine[d]. The results are given below:

Candidates :	I	II	III	IV	V
IQ before training :	110	120	123	132	125
IQ after training :	120	118	125	136	121

Test whether there is any change in IQ after the training programme.

Solution:

The sample values are correlated pair. So we apply **difference test.**

$H_0: \mu = 0$ (there is no change in IQ due to the training programme)

$H_1: \mu \neq 0$ (there is a change in IQ due to the training programme).

Candidate	IQ Before x	IQ After y	$(y-x)$ d	d^2
I	110	120	10	100
II	120	118	-2	4
III	123	125	2	4
IV	132	136	4	16
V	125	121	-4	16
			10	140

$$\bar{d} = \frac{10}{5} = 2$$

$$s^2 = \frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2 = \frac{140}{5} - 4$$

$$= 28 - 4 = 24$$

$$\therefore s = 4.899$$

The test statistic is

$$t = \frac{|\bar{d}|}{s/\sqrt{n-1}} = \frac{2}{4.899/\sqrt{4}}$$

$$= \frac{4}{4.899} = 0.816$$

$\text{ndf} = 5 - 1 = 4$

Table value of t for 4df at 1% level $= 4.6$

Conclusion:

H_0 is accepted at 1% level since the calculated value of t < the table value of t.

$\therefore$ There is no significant change in IQ due to the training programme.

An F-Test is a statistical test that **compares the variances of two samples** so as to test the hypothesis that **the samples have been taken from populations with different variances.**

Procedure for test of Equality of Two Population Variances

The null hypothesis is $H_0: \sigma_1^2 = \sigma_2^2$

The alternative hypothesis is $H_1: \sigma_1^2 \neq \sigma_2^2$

The test static is $F = \dfrac{S_1^2}{S_2^2}$ where $S_1^2 = \dfrac{n_1}{n_1 - 1} s_1^2$, $S_2^2 = \dfrac{n_2}{n_2 - 1} s_2^2$ where n_1 and n_2 are the sizes of the samples drawn from the two populations and s_1^2 and s_2^2 are the sample variances.

The number of degrees of freedom $= n_1 - 1,\ n_2 - 1$.

Example 2:

In a sample of 8 observations, the sum of the squared deviations of items from the mean was 94.5. In another sample of 10 observations, the value was found to be 101.7. Test whether the difference in the variances is significant at 5% level.

Solution:

$$n_1 = 8, \ \ \Sigma(x-\bar{x})^2 = 94.5$$

$$n_2 = 10, \ \ \Sigma(y-\bar{y})^2 = 101.7$$

$\therefore$ The sample variances are

$$s_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1} = \frac{94.5}{8}$$

$$s_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2} = \frac{101.7}{10}$$

$H_0 : \sigma_1^2 = \sigma_2^2$ (the samples are taken from populations whose variance are equal.)

$H_1 : \sigma_1^2 \neq \sigma_2^2$ (the samples are taken from populations whose variances are not equal)

The estimated variances of the populations are given by

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{8}{7} \times \frac{94.5}{8} = 13.5$$

$$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{10}{9} \times \frac{101.7}{10} = 11.3$$

Here $S_1^2 > S_2^2$

The test statistic is given by

$$F = \frac{S_1^2}{S_2^2} = \frac{13.5}{11.3} = 1.174$$

$\text{ndf} = (n_1 - 1,\ n_2 - 1) = (7,\ 9)$

Table value of F for $(7, 9)$ df at 5% level $= 3.29$

Coclusion :

H_0 is accepted at 5% level of significance since the calculated value of $F <$ the table value of F.

$\therefore$ The samples belong to populations with equal variance.

Example 5:

Time taken by workers in performing a job are given below

Method I :	20	16	26	27	23	22	
Method II :	27	33	42	35	32	34	38

Test whether there is any significant difference between the variances of time distribution.

Solution:

Let us first calculate the variance of the samples.

Sample I $x - 22$			Sample II $y - 34$		
x	d	d^2	y	d	d^2
20	–2	4	27	–7	49
16	–6	36	33	–1	1
26	4	16	42	8	64
27	5	25	35	1	1
23	1	1	32	–2	4
22	0	0	34	0	0
			38	4	16
134	2	82	241	3	135

$$\overline{X} = \frac{134}{6} = 22.33 \quad ; \quad \overline{Y} = \frac{241}{7} = 34.43$$

$$s_1^2 = \frac{82}{6} - \left(\frac{2}{6}\right)^2 = 13.67 - 0.44 = 13.23$$

$$s_2^2 = \frac{135}{7} - \left(\frac{3}{7}\right)^2 = 19.29 - 0.18 = 19.11$$

$$n_1 = 6 \quad ; \quad n_2 = 7$$

$$s_1^2 = 13.23 \quad s_2^2 = 19.11$$

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{6 \times 13.23}{5} = 15.88$$

$$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{7 \times 19.11}{6} = 22.30$$

$S_2^2 > S_1^2$

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

The test statistic is

$$F = \frac{S_2^2}{S_1^2} = \frac{22.30}{15.88} = 1.40$$

ndf = 6, 5.

Table value of F at 5% level = 4.28

Conclusion:

H_0 is accepted since the calculated value of $F <$ the table value of F

$\therefore$ There is no significant difference between the variances of time distribution.

Example 7:

Two random samples were drawn from two normal populations and their values are:

A	66	67	75	76	82	84	88	90	92		
B	64	66	74	78	82	85	87	92	93	95	97

Test whether the two populations have the same variance at 5% level of significance.

Solution:

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

First let us calculate the variances of the two samples.

Sample I $(x - 80)$			Sample II $(y - 83)$		
x	d	d^2	y	d	d^2
66	−14	196	64	−19	361
67	−13	169	66	−17	289
75	−5	25	74	−9	81
			78	−5	25
76	−4	16	82	−1	1
82	2	4	85	2	4
84	4	16	87	4	16
88	8	64	92	9	81
90	10	100	93	10	100
92	12	144	95	12	144
			97	14	196
720	0	734	913	0	1298

$$\overline{X} = \frac{720}{9} = 80 \quad ; \quad \overline{Y} = \frac{913}{11} = 83$$

$$S_1^2 = \frac{9}{8} \times \frac{734}{9} = 91.75$$

$$S_2^2 = \frac{11}{10} \times \frac{1298}{11} = 129.8$$

$$S_2^2 > S_1^2$$

The test statistic F is $F = \frac{S_2^2}{S_1^2} = \frac{129.8}{91.75} = 1.41$

ndf = 10, 8

Table value of $F = 4.301$ at 5% level.

Conclusion:

H_0 is accepted.

$\therefore$ There is no significant difference between the variances of the two populations.