Maintainability

Maintenance of a system is one of the effective ways of increasing the reliability of the system. Systems are designed to operate with or without maintenance.

Usually two kinds of maintenance are adopted. They are

(i) **Preventive maintenance :**

It is a maintenance done periodically before the failure of the system.

(ii) **Repair maintenance:**

It is a maintenance done after the failure of the system.

A measure of the length of a repairing time for a failure component / system is known as maintainability.

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Reliability under Preventive Maintenance

Let $R(t)$ and $R_M(t)$ be the reliability of a system without maintenance and with maintenance.

Let the preventive maintenance be performed on the system at intervals of T .

Since $R_M(t) = P\{\text{the maintained system does not fail before } t\}$, we have

$$R_M(t) = R(t), \text{ for } 0 \le t < T$$
$$= R(T), \text{for } t = T$$

After performing the first maintenance operation at T , the system becomes as good as new.

Hence if $T \le t < 2T$,

$$R_M(t) = P\{\text{the system does not fail upto} T \text{and it survives for a time } (t - T) \text{ without failure}\}$$

$$R_M(t) = R(T).R(t-T) \text{ for } T \le t < 2T$$

Similarly, after two maintenance operations,

$$R_M(t) = [R(T)]^2.R(t-2T) \text{ for } 2T \le t < 3T$$

Proceeding like this, we get in general,

$$R_M(t) = [R(T)]^n . R(t - nT) \text{ for } nT \le t < (n+1)T$$

MTTF of the system with preventive maintenance is given by

$$\text{MTTF} = \int_0^\infty R_M(t)dt = \sum_{n=0}^{\infty} \int_{nT}^{(n+1)T} R_M(t)dt$$

(by dividing the range into intervals of length T)

$$= \sum_{n=0}^{\infty} \int_{nT}^{(n+1)T} \{R(T)\}^n R(t - nT)dt$$

$$\{\text{put } t - nT = t'\}$$

$$= \sum_{n=0}^{\infty} \{R(T)\}^n \int_0^T R(t')dt'$$

This implies that, $\text{MTTF} = \dfrac{\int_0^T R(t)dt}{1 - R(T)}$

$R(t - kT) = R(t)$ if PM restores system to "as good as new".

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Reliability under Repair Maintenance

A measure of how fast a component (system) may be repaired following failure is known as *maintainability*.

Repairs require different lengths of time and even the time to perform a given repair is uncertain (random), because circumstances, skill level, experience of maintenance personnel and such other factors vary. Hence the time T required to repair a failed component (system) is a continuous R.V.

Maintainability, M(t) is mathematically defined as the cumulative distribution function of the random variable T representing the time to repair.

That is, $M(t) = P(T \le t) = \int_0^t m(t)dt$ (1)

where $m(t)$ is the probability density function of T .

The expected value of repair time T us called the mean time to repair (MTTR) and is given by $\text{MTTR} = E(T) = \int_0^\infty tm(t)dt$ (2)

If the conditional probability that the (component) system will be repaired (made operational) between t and $t + \Delta t$, given that it has failed at t and the repair starts immediately, is $\mu(t)\,\Delta t$, then $\mu(t)$ is called the *instantaneous repair rate* or simply *the repair rate* and denotes the number of repairs in unit time.

i.e.,
$$\mu(t)\,\Delta t = \frac{P\{t \le T \le t + \Delta t\}}{P(T > t)}$$

Probability that system will survive at least until time t

$$= \frac{m(t)\Delta t}{1 - M(t)}$$

$$\therefore \qquad \mu(t) = \frac{m(t)}{1 - M(t)} \tag{3}$$

From (1), on differentiation, we get

$$m(t) = \frac{d}{dt} M(t) \tag{4}$$

Using (4) in (3), we have

$$\mu(t) = \frac{M'(t)}{1 - M(t)} \tag{5}$$

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Integrating both sides of (5) with respect to t between 0 and t, we get

$$\int_0^t \mu(t)dt = \int_0^t \frac{M^{'}(t)}{1-M(t)}dt$$

That is, $[-\log(1-M(t))]_0^t = \int_0^t \mu(t)dt$

That is, $1-M(t) = e^{-\int_0^t \mu(t)dt}$

This implies that, $M(t) = 1-e^{-\int_0^t \mu(t)dt}$ (6)

Using (5) and (6) in (4), we get

$$m(t) = \mu(t).e^{-\int_0^t \mu(t)dt} \tag{7}$$

Availability

Availability $A(t)$ is defined as the probability that a component (or system) is performing its planned function at a given time t on the assumption that it is operated and maintained as per the prescribed conditions. It is also called the point availability.

If $A(t)$ is the point availability of a component (or system), then

$$A(t_2 - t_1) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} A(t)dt$$

is called the interval availability (or) mission availability

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In particular, the interval availability over the interval $(0, T)$ is given by $A(T) = \frac{1}{T} \int_0^T A(t)dt$.

Now, A (or) $A(\infty) = \lim_{T \to \infty} A(t)$ is called the steady-state (or) long-run availability.

Availability Function of a Single Component (or System)

In the case of non-repairable system, the steady-state availability, $A(\infty) = \dfrac{\text{MTTF}}{\text{MTTF+MTTR}} = \dfrac{1/\lambda}{1/\lambda + 1/\mu}$,

where mu is repair rate and lambda is failure rate.

The above equation can also be written as follows

In the case of non-repairable system, the steady-state availability $A(\infty) = \dfrac{\text{MTBF}}{\text{MTBF+MTTR}}$

System Availability

If we consider a system consisting of n independent components connected in series, then the system reliability $A_s(t)$ is given by

$$A_s(t) = A_1(t).A_2(t).....A_n(t)$$

{since all the components must be available for the system to be available}

For a system consisting of n independent components connected in parallel, then the system reliability $A_s(t)$ is given by

$$A_s(t) = 1 - \{1 - A_1(t)\}\{1 - A_2(t)\}...\{1 - A_n(t)\}$$

{since all the components must be unavailable for the system to be unavailable}.where $A_i(t)$ is the availability of the i^{th} component.

Example 4 *The time to repair a power generator is best described by its pdf*

$$m(t) = \frac{t^2}{333},\ 1 \le t \le 10 \text{ hours}$$

(a) *Find the probability that a repair will be completed in 6 hours.*
(b) *What is the MTTR?*
(c) *Find the repair rate.*

Solution

(a) $P(T < 6) = P(1 \le T < 6)$, where T is the time to repair

$$= \int_1^6 m(t)dt$$

$$= \int_1^6 \frac{t^2}{333} dt = \left(\frac{t^3}{999}\right)_1^6 = 0.2152$$

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(b) $$\text{MTTR} = \int_0^{\infty} t m(t) dt = \int_1^{10} \frac{t^3}{333} dt = \left(\frac{t^4}{4 \times 333} \right)_1^6$$
$$= 7.5 \text{ hours}$$

(c) $$\text{Repair rate} = \mu(t) = \frac{m(t)}{1 - M(t)}$$

$$= \frac{t^2/333}{\int_t^{10} \frac{t^2}{333} dt} = \frac{t^2/333}{\frac{1}{999}(10^3 - t^3)}$$

$$= \frac{3t^2}{1000 - t^3} \text{ per hour.}$$

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P2: In a repairable system, the mean time between failures is 100 hr and a mean time to repair is 15 hr. What is the availability (inherent availability) of the system?

Solution: Given that MTBF = 100 and MTTR = 15

Inherent availability = $A_i = \dfrac{MTBF}{MTBF + MTTR} = \dfrac{100}{100+15} = 0.8695 \approx 87\%.$

P3: Equipment is repaired to meet an inherent availability requirement 0.985 and a mean time between failures of 100 hr. What is the permissible mean time to repair?

Solution: Given that $A_i = 0.985$, MTBF =100hr, MTTR = ?

We know that, $A_i = \dfrac{MTBF}{MTBF + MTTR}$

This implies that, $0.985 = \dfrac{100}{100 + MTTR}$

Therefore, MTTR = $\dfrac{100(1-0.985)}{0.985} = 1.5228$ hour

The **design life** of a component or product is the period of time during which the item is expected by its designers to work within its specified parameters; in other words, the life expectancy of the item.

Example 1 *If a device has a failure rate*

$\lambda(t) = (0.015 + 0.02t)$/*year, where t is in years,*

(a) Calculate the reliability for a 5 year design life, assuming that no maintenance is performed.

(b) Calculate the reliability for a 5 year design life, assuming that annual preventive maintenance restores the device to an as-good-as new condition.

(c) Repeat part (b) assuming that there is a 5% chance that the preventive maintenance will cause immediate failure.

Solution

(a) $R(t) = e^{-\int_0^t \lambda(t)dt}$

$$\therefore \qquad R(5) = e^{-\int_0^5 (0.015+0.02t)dt} \tag{1}$$

$$= e^{-(0.015 \times 5 + 0.01 \times 25)}$$

$$= e^{-0.325} = 0.7225$$

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(b) Since annual preventive maintenance is performed, there will be 4 preventive maintenances in the first 5 years.

$$R_M(t) = \{R(T)\}^n \times R(t - nT), \text{ after } n \text{ maintenances}$$

Here $\quad t = 5, T = 1$ and $n = 4$

$$\therefore \quad R_M(5) = \{R(1)\}^4 \times R(5 - 4)$$
$$= \{R(1)\}^5$$
$$= \{e^{-0.025}\}^5, \text{ using (1)}$$
$$= 0.8825$$

(c) $P\{\text{preventive maintenance causes immediate failure}\} = 0.05$

$\therefore P\{\text{the device survives after each preventive maintenance}\} = 0.95$

As there are 4 maintenances,

$$R_M(5) = R_M(5) \text{ without breakdown} \times \text{probability of no breakdown in}$$

5 years

$$= 0.8825 \times (0.95)4$$
$$= 0.7188.$$

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