

Harmonic, Geometric Mean and Mode by using the method of Grouping

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Harmonic Mean

For Ungrouped Data	For Grouped Data
$H.M \text{ of } X = \bar{X} = \frac{n}{\sum\left(\frac{1}{x}\right)}$	$H.M \text{ of } X = \bar{X} = \frac{\sum f}{\sum\left(\frac{f}{x}\right)}$

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Example:

Calculate the harmonic mean of the numbers 13.5, 14.5, 14.8, 15.2 and 16.1

Solution:

The harmonic mean is calculated as below:

x	$\frac{1}{x}$
13.2	0.0758
14.2	0.0704
14.8	0.0676
15.2	0.0658
16.1	0.0621
Total	$\sum \left(\frac{1}{x}\right) = 0.3417$

$$H. M \text{ of } X = \bar{X} = \frac{n}{\sum \left(\frac{1}{x}\right)}$$

$$H. M \text{ of } X = \bar{X} = \frac{5}{0.3417} = 14.63$$

Example:

Given the following frequency distribution of first year students of a particular college, calculate the harmonic mean.

Age (Years)	13	14	15	16	17
Number of Students	2	5	13	7	3

Solution:

Ages (Years) x	Number of Students f	$\frac{f}{x}$
13	2	0.1538
14	5	0.3571
15	13	0.8667
16	7	0.4375
17	3	0.1765
Total	$\sum f = 30$	$\sum \left(\frac{f}{x}\right) = 1.9916$

Now we will find the harmonic mean as

$$\bar{X} = \frac{\sum f}{\sum \left(\frac{f}{x}\right)} = \frac{30}{1.9916} = 15.0631 \approx 15 \text{ years.}$$

Calculate the harmonic mean for the data given below:

Marks	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89	90 – 99
f	2	3	11	20	32	25	7

Solution:

The necessary calculations are given below:

Marks	x	f	$\frac{f}{x}$
30 – 39	34.5	2	0.0580
40 – 49	44.5	3	0.0674
50 – 59	54.5	11	0.2018
60 – 69	64.5	20	0.3101

70 – 79	74.5	32	0.4295
80 – 89	84.5	25	0.2959
90 – 99	94.5	7	0.0741
Total		$\sum f = 100$	$\sum \left(\frac{f}{x}\right) = 1.4368$

Now we will find the harmonic mean as

$$\bar{X} = \frac{\sum f}{\sum \left(\frac{f}{x}\right)} = \frac{100}{1.4368} = 69.60$$

Geometric Mean

the geometric mean for a value X containing n values such as $x_1, x_2, x_3, \dots, x_n$ is denoted by $G.M$ of X and given as:

$$G.M \text{ of } X = \bar{X} = \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdot \dots \cdot x_n}$$

(for ungrouped data)

If we have a series of n positive values with repeated values such as $x_1, x_2, x_3, \dots, x_k$ which are repeated $f_1, f_2, f_3, \dots, f_k$ times respectively, then the geometric mean will become:

$$G.M \text{ of } X = \bar{X} = \sqrt[n]{x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3} \cdot \dots \cdot x_k^{f_k}}$$

(For Grouped Data)

Where $n = f_1 + f_2 + f_3 + \dots + f_k$

Example:

Find the geometric mean of the values 10, 5, 15, 8, 12.

Solution:

Here $x_1 = 10, x_2 = 5, x_3 = 15, x_4 = 8, x_5 = 12$ and $n = 5$

$$G.M \text{ of } X = \overline{X} = \sqrt[5]{10 \times 5 \times 15 \times 8 \times 12}$$

$$\overline{X} = \sqrt[5]{72000} = (72000)^{\frac{1}{5}} = 9.36$$

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Find the geometric mean of the following data:

X	13	14	15	16	17
f	2	5	13	7	3

Solution:

$$\overline{X} = \sqrt[30]{(13)^2 \cdot (14)^5 \cdot (15)^{13} \cdot (16)^7 \cdot (17)^3}$$

$$\overline{X} = \sqrt[30]{2.33292 \times 10^{35}} = (2.33292 \times 10^{35})^{\frac{1}{30}}$$

$$\overline{X} = 15.0984 \approx 15.10$$

The method explained above to calculate the geometric mean is useful when the values in the given data are small in number and an electronic calculator is available. When a set of data contains a large number of values then we need an alternate way to compute the geometric mean. The modified or alternative way of computing the geometric mean is given as:

For Ungrouped Data	For Grouped Data
$G.M \text{ of } X = \bar{X} = \text{Anti log} \left(\frac{\sum \log x}{n} \right)$	$G.M \text{ of } X = \bar{X} = \text{Anti log} \left(\frac{\sum f \log x}{\sum f} \right)$

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Example:

Find the geometric mean of the values 10, 5, 15, 8, 12

x	$\log x$
10	1.0000
5	0.6990
15	1.1761
8	0.9031
12	1.0792
Total	$\sum \log x$ $= 4.8573$

$$G.M \text{ of } X = \bar{X} = \text{Anti log} \left(\frac{\sum \log x}{n} \right)$$

$$\bar{X} = \text{Anti log} \left(\frac{4.8573}{5} \right)$$

$$\bar{X} = \text{Anti log}(0.9715)$$

$$\bar{X} = 9.36$$

Example:

Find the geometric mean for the following distribution of students' marks:

Marks	0 – 30	30 – 50	50 – 80	80 – 100
No. of Students	20	30	40	10

Marks	No. of Students f	Mid Points x	$f \log x$
0 – 30	20	15	$20 \log 15$ $= 23.5218$
30 – 50	30	40	$30 \log 40$ $= 48.0168$
50 – 80	40	65	$40 \log 65$ $= 72.5165$
80 – 100	10	90	$10 \log 90$ $= 19.5424$
Total	$\sum f = 100$		$\sum f \log x$ $= 163.6425$

$$G.M \text{ of } X = \overline{X} = \text{Anti log} \left(\frac{\sum f \log x}{\sum f} \right)$$

$$\overline{X} = \text{Anti log} \left(\frac{163.6425}{100} \right)$$

$$\overline{X} = \text{Anti log}(1.6364)$$

$$\overline{X} = 43.29$$