Analysis of Variance

Analysis of variance is a technique used to test equality of means, when more than two populations are considered. In z–test and t–test we considered only the equality of two population means. If there are more than two populations, for testing the equality of their means the Analysis of variance method is applied. This technique introduced by R.A.Fisher was originally used in agricultural experiment in which different types of fertilizers were applied to plots of land, different types of feeding methods to animals and so on. This technique is widely used in different fields for example, to study the pattern of average sales by using different sales techniques, the types of drugs manufactured by different companies to cure a particular disease.

Definition

ANOVA *ANOVA* is a procedure used to test the null hypothesis that the means of three or more populations are all equal.

Suppose that teachers at a school have devised three different methods to teach arithmetic. They want to find out if these three methods produce different mean scores. Let

μ_1, μ_2, and μ_3 be the mean scores of all students who are taught by Methods I, II, and III, respectively. To test if the three teaching methods produce different means, we test the null hypothesis

$$H_0: \mu_1 = \mu_2 = \mu_3 \quad \text{(All three population means are equal.)}$$

against the alternative hypothesis

$$H_1: \text{Not all three population means are equal.}$$

By using a **one-way ANOVA** test, we analyze only one factor or variable.

For instance, in the example of testing for the equality of mean arithmetic scores of students taught by each of the three different methods, we are considering only one factor, which is the effect of different teaching methods on the scores of students.

Sometimes we may analyze the effects of two factors.

For example, if different teachers teach arithmetic using these three methods, we can analyze the **effects of teachers** and **teaching methods** **on the scores of students**. This is done by using a **two-way ANOVA**.

The procedure under discussion in this chapter is called the analysis of variance (ANOVA) because the test is based on the analysis of variation in the data obtained from different samples.

One Way ANOVA- Completely Randomized Design

Two Way ANOVA- Randomized Blocked Design

Assumptions of One-Way ANOVA The following assumptions must hold true to use *one-way ANOVA*.

1. The populations from which the samples are drawn are (approximately) normally distributed.
2. The populations from which the samples are drawn have the same variance (or standard deviation).
3. The samples drawn from different populations are random and independent.

For instance, in the example about three methods of teaching arithmetic, we first assume that the scores of all students taught by each method are (approximately) normally distributed. Second, the means of the distributions of scores for the three teaching methods may or may not be the same, but all three distributions have the same variance, σ^2. Third, when we take samples to make an ANOVA test, these samples are drawn independently and randomly from three different populations.

We now consider two types of analysis of variance.

(1) One–way classification

(2) Two–way classification

In one–way classification observations are classified according to one factor. This is exhibited column–wise.

In two–way classification observations are classified according to two factors, one column–wise and the other row–wise.

One –Way Classification (Completely Randomized Design)

A set of $N = cr$ observations classified in one direction may be represented as follows:

	X_1	X_2	....	X_C
	x_{11}	x_{21}		x_{C_1}
	x_{12}	x_{22}		x_{C_3}
	x_{13}	x_{23}		x_{C_3}
			....	
			....	
	x_{1r}	x_{2r}		x_{Cr}
Mean:	$\overline{X}_1$	$\overline{X}_2$		$\overline{X}_C$

$$\overline{\overline{X}} = \frac{\overline{X}_1 + \overline{X}_2 + \ldots\ldots + \overline{X}_C}{C}$$

Then,

'Between column' sum of squares: $SSC = \sum_j (\overline{X}_j - \overline{\overline{X}})^2$

'Within column' sum of squares: $SSE = \sum_j \sum_i (x_{ij} - \overline{X}_j)^2$

Total Sum of squares $SST = \Sigma\Sigma(x_{ij} - \overline{\overline{X}})^2$

Also it can be shown that $SST = SSC + SSE$

If each of these sum of squares is divided by the corresponding number of degrees of freedom, we get mean sum of squares.

These informations are presented in the following table called **ANOVA Table**

Source of Variation	Sum of squares	Degrees of freedom	Mean sum of squares	Variance ratio
Between columns	SSC	C − 1	$MSC = \frac{SSC}{C-1}$	$F = \frac{MSC}{MSE}$ (or) $F = \frac{MSE}{MSC}$.
Within columns	SSE	N − C	$MSE = \frac{SSE}{N-C}$	
Total	TSS	N − 1		

The F ratio should be calculated in such a way that F > 1.

S.S.C - Sum of Squares between columns

T.S.S - Total Sum of Squares

S.S.E Error sum of squares (or) within sum of squares

MSC- Mean Sum of Squares (between Columns)
MSE – Mean Sum of Squares (Within Columns)

Short – Cut Method:

In order to make the working of F ratio easier, we adopt the short cu procedure.

STEP I : Find N, the total number of observations.

STEP II : Find T, the total of all observations.

STEP III : Find $\frac{T^2}{N}$, the correction factor.

STEP IV : Calculate the total sum of squares

$$SST = \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 + \ldots\ldots - \frac{T^2}{N}$$

STEP V : Calculate the column sum of squares

$$SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \ldots\ldots - \frac{T^2}{N}$$

Here N_1 is the number of elements in each column.

STEP VI : Prepare the ANOVA Table to calculate F – ratio.

Note : The null and alternative hypotheses are stated below.

$H_0 : \mu_1 = \mu_2 = \mu_3$ (Population means are equal).

$H_1 : \mu_1 \neq \mu_2 \neq \mu_3$ (Population means are not equal)

Example 1:

Set up **ANOVA** table for the following per hectare yield for three varieties of wheat, each grown in four plots:

Per hectare yield (in hundred kgs.)

Plots of land	Variety of wheat		
	A_1	A_2	A_3
1	6	5	5
2	7	5	4
3	3	3	3
4	8	7	4

Also work out F–ratio and test whether there is significant difference among the average yields in the 3 varieties of wheat.

Solution:

$H_0: \mu_1 = \mu_2 = \mu_3$ (i.e., the average yield in the 3 varieties of wheats are the same).

$H_1: \mu_1 \neq \mu_2 \neq \mu_3$

Let us apply F – test to test the above hypothesis.

X_1	X_2	X_3	X_1^2	X_2^2	X_3^2
6	5	5	36	25	25
7	5	4	49	25	16
3	3	3	9	9	9
8	7	4	64	49	16
24	20	16	158	108	66

N = Number of observations = 12

T = total of all the observations = 24 + 20 + 16 = 60

$$\frac{T^2}{N} = \text{Correction factor} = \frac{60^2}{12} = 300$$

SST = Total Sum of Squares

$$= \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 - \frac{T^2}{N}$$

$$= 158 + 108 + 66 - 300 = 32$$

SSC = Column Sum of squares

$$= \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{24^2}{4} + \frac{20^2}{4} + \frac{16^2}{4} - 300$$

$$= 144 + 100 + 64 - 300 = 8$$

$\therefore$ SSE = 32 − 8 = 24

ANOVA Table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Sum of Squares	Variation Ratio
Between columns	SSC= 8	2	$MSC = \frac{SSC}{2} = 4$	$F_C = \frac{4 \times 9}{24} = 1.5$
Error	SSE=24	9	$MSE = \frac{SSE}{9} = \frac{24}{9}$	
Total	SST=32	11		

The test statistic is $F_C = \frac{MSC}{MSE} = 1.5$

ndf = 2, 9

Table value of F at 5% level = 4.26

Conclusion:

H_0 is accepted at 5% level since the calculated value of $F <$ the table value of F.

$\therefore$ There is no significant difference among the means.

Example 4:

Three samples below have been obtained from normal populations with equal variances. Test the hypothesis at 5% level that the population means are equal.

8	7	12
10	5	9
7	10	13
14	9	12
11	9	14

The table value of F at 5% level for $\nu_1 = 7$ and $\nu_2 = 17$ is 3.88.

Solution:

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

$$H_1 : \mu_1 \neq \mu_2 \neq \mu_3$$

X_1	X_2	X_3	X_1^2	X_2^2	X_3^2
8	7	12	64	49	144
10	5	9	100	25	81
7	10	13	49	100	169
14	9	12	196	81	144
11	9	14	121	81	196
50	40	60	530	336	734

$N = 5 + 5 + 5 = 15$

$T = 50 + 40 + 60 = 150$

$$\frac{T^2}{N} = \frac{150^2}{15} = 1500$$

$$SST = \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 - \frac{T^2}{N}$$

$$= 530 + 336 + 734 - 1500 = 100$$

$$SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{50^2}{5} + \frac{40^2}{5} + \frac{60^2}{5} - 1500$$

$$= 500 + 320 + 720 - 1500 = 40$$

ANOVA Table

Source of Variation	SS	DF	MSS	VR
Between columns	40	2	20	$F = \frac{20}{5} = 4$
Error	60	12	5	
Total	100	14		

The test statistic is $F = \frac{MSC}{MSE} = 4$

ndf = 2, 12

Table value of F at 5% level = 3.88

Conclusion:

H_0 is rejected since the calculated value of F > the table value of F.

Hence the populations means are not equal at 5% level.

Different Method

A completely randomised design experiment with 10 plots and 3 treatments gave the following results:

Plot No.	:	1	2	3	4	5	6	7	8	9	10
Treatment	:	A	B	C	A	C	C	A	B	A	B
Yield	:	5	4	3	7	5	1	3	4	1	7

Analyse the results for treatment effects.

Rearranging the data according to the treatments, we have the following table:

Treatment	*Yield from plots* (x_{ij})				T_i	T_i^2	n_i	$\frac{T_i^2}{n_i}$
A	5	7	3	1	16	256	4	64
B	4	4	7	–	15	225	3	75
C	3	5	1	–	9	81	3	27
				Total	$T = 40$	–	$N = 10$	166

$$\sum\sum x_{ij}^2 = (25 + 49 + 9 + 1) + (16 + 16 + 49) + (9 + 25 + 1)$$

$$= 84 + 81 + 35 = 200$$

Table 10.1 *ANOVA table for one factor of classification*

Source of variation (S.V.)	*Sum of squares (S.S.)*	*Degree of freedom (d.f.)*	*Mean square (M.S.)*	*Variance ratio (F)*
Between classes	Q_1	$h-1$	$Q_1/(h-1)$	$\dfrac{Q_1/(h-1)}{Q_2/(N-h)}$ (OR)
Within classes classes	Q_2	$N-h$	$Q_2/(N-h)$	$\dfrac{Q_2/(N-h)}{Q_1/(h-1)}$
Total	Q	$N-1$	–	–

$$Q = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 200 - \frac{40^2}{10} = 200 - 160 = 40$$

$$Q_1 = \sum \frac{T_i^2}{n_i} - \frac{T^2}{N} = 166 - 160 = 6$$

$$Q_2 = Q - Q_1 = 40 - 6 = 34$$

S.V.	*S.S.*	*d.f.*	*M.S.*	F_0
Between classes (treatments)	$Q_1 = 6$	$h - 1 = 2$	3.0	$\frac{4.86}{3.0}$
Within classes	$Q_2 = 34$	$N - h = 7$	4.86	$= 1.62$
Total	$Q = 40$	$N - 1 = 9$	–	–

From the F-table, $F_{5\%}$ $(\nu_1 = 7, \nu_2 = 2) = 19.35$
We note that $F_0 < F_{5\%}$
Let H_0 : The treatments do not differ significantly.
$\therefore$ The null hypothesis is accepted.
i.e., the treatments are not significantly different.

In order to determine whether there is significant difference in the durability of : makes of computers, samples of size 5 are selected from each make and the frequency of repair during the first year of purchase is observed. The results are a follows :

Makes

A	B	C
5	8	7
6	10	3
8	11	5
9	12	4
7	4	1

In view of the above data, what conclusion can you draw?

Make	x_{ij}					T_i	n_i	T_i^2 / n_i	$\sum_j x_{ij}^2$
A	5	6	8	9	7	35	5	245	255
B	8	10	11	12	4	45	5	405	445
C	7	3	5	4	1	20	5	80	100
					Total	100	15	730	800

$$T = \sum T_i = 100 \,;\ \sum\sum x_{ij}^2 = 800;\ N = \sum n_i = 15$$

$$Q = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 800 - \frac{100^2}{15} = 133.33$$

$$Q_1 = \sum \frac{T_i^2}{n_i} - \frac{T^2}{N} = 730 - 666.67 = 63.33$$

$$Q_2 = Q - Q_1 = 70$$

ANOVA table

S.V.	S.S.	d.f.	M.S.	F_0
Between makes	$Q_1 = 63.33$	$h - 1 = 2$	31.67	$\frac{31.67}{5.83}$
Within makes	$Q_2 = 70$	$N - h = 12$	5.83	5.43
Total	$Q = 133.33$	$N - 1 = 14$	–	–

From the F-tables, $F_{5\%}\ (\nu_1 = 2,\ \nu_2 = 12) = 3.88$

$$F_0 > F_{5\%}$$

Hence the null hypothesis (H_0: the 3 makes of computers do not differ in the durability) is rejected.

viz., there is significant difference in the durability of the 3 makes of computers.

Two Way ANOVA (or) Randomised Block Design

Assumptions for the Two-Way ANOVA

1. The populations from which the samples were obtained must be normally or approximately normally distributed.
2. The samples must be independent.
3. The variances of the populations from which the samples were selected must be equal.
4. The groups must be equal in sample size.

In doing a study that involves a two-way analysis of variance, the researcher is able to test the effects of two independent variables or factors on one *dependent variable. In* addition, the interaction effect of the two variables can be tested.

For example, suppose a researcher wishes to test the effects of two different types of plant food and two different types of soil on the growth of certain plants. The two independent variables are the type of plant food and the type of soil, while the dependent variable is the plant growth. Other factors, such as water, temperature, and sunlight, are held constant.

Hypotheses

1. H_0: There is no difference in population means among the levels of the row factor.
   H_1: At least two population means are different among the levels of the row factor.

2. H_0: There is no difference in population means among the levels of the column factor.
   H_1: At least two population means are different among the levels of the column factor.

3. H_0: There is no interaction between the factors.
   H_1: There is an interaction between the factors.

Two – Way Classification:

In two–way classification of analysis of variance, we consider one classification along column–wise and the other along row–wise.

Procedure for Testing Means

H_0: There is no significant difference between column means as well as between row means.

H_1: There is significant difference between column means or between row means.

STEP I : Find N

STEP II : Find T

STEP III : Find $\dfrac{T^2}{N}$

STEP IV : Find SST

STEP V : Find SSC

STEP VI : Find SSR (Row Sum of squares)

Then, SST = SSC + SSR + SSE

Prepare the **ANOVA** Table.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean sum of Squares	Variance Ratio
Between columns	SSC	$c-1$	$MSC = \frac{SSC}{C-1}$	$F_C = \frac{MSC}{MSE}$
Between Rows	SSR	$r-1$	$MSR = \frac{SSR}{r-1}$	
Residual (Error)	SSE	$N-c-r+1$	$MSE = \frac{SSE}{N-c-r+1}$	$F_R = \frac{MSR}{MSE}$
Total	SST	$N-1$		

Note: The F–ratio F_C and F_R should be calculated in such a way that F_C and F_R are greater than 1. Then only it is possible to compare these ratios with the table values.

OR (c-1) (r-1)

Example 5:

Perform a Two–way **ANOVA** on the data given below:

		Treatment I		
		i	ii	iii
Treatment II	i	30	26	38
	ii	24	29	28
	iii	33	24	35
	iv	36	31	30
	v	27	35	33

Use the coding method, subtracting 30 from the given numbers.

Solution:

H_0 : There is no significant difference between column means as well as between row means.

H_1 : There is significant difference between column means as well as between row means.

The value of F is unaffected by subtracting 30 from all the numbers.

	X_1	X_2	X_3	Total	X_1^2	X_2^2	X_3^2
Y_1	0	-4	8	4	0	16	64
Y_2	-6	-1	-2	-9	36	1	4
Y_3	3	-6	5	2	9	36	25
Y_4	6	1	0	7	36	1	0
Y_5	-3	5	3	5	9	25	9
Total	0	-5	14	9	90	79	102

$N = 5 + 5 + 5 = 15$

$T = 0 - 5 + 14 = 9$

$$\frac{T^2}{N} = \frac{9^2}{15} = 5.4$$

$$\text{SST} = \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 - \frac{T^2}{N}$$

$$= 90 + 79 + 102 - 5.4 = 271 - 5.4 = 265.5$$

$$\text{SSC} = \frac{(\Sigma \text{X}_1)^2}{N_1} + \frac{(\Sigma \text{X}_2)^2}{N_1} + \frac{(\Sigma \text{X}_3)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{0^2}{5} + \frac{(-5)^2}{5} + \frac{14^2}{5} - 5.4$$

$$= 0 + 5 + 39.2 - 5.4 = 38.8$$

$$SSR = \frac{(\Sigma Y_1)^2}{3} + \frac{(\Sigma Y_2)^2}{3} + \frac{(\Sigma Y_3)^2}{3} + \frac{(\Sigma Y_4)^2}{3} + \frac{(\Sigma Y_5)^2}{3} - \frac{T^2}{N}$$

$$= \frac{4^2}{3} + \frac{(-9)^2}{3} + \frac{2^2}{3} + \frac{7^2}{3} + \frac{5^2}{3} - 5.4$$

$$= 5.33 + 27 + 1.33 + 16.33 + 8.33 - 5.4 = 52.92$$

ANOVA Table

Source of Variation	SS	DF	MSS	VR
Between columns	38.8	2	$\frac{38.2}{2} = 19.4$	$F_C = \frac{21.72}{19.4} = 1.12$
Between rows	52.92	4	$\frac{52.92}{4} = 13.23$	$F_R = \frac{21.72}{13.23} = 1.64$
Error	173.78	8	$\frac{173.78}{8} = 21.72$	
Total	265.5	14		

The test statistics are

$$F_C = \frac{MSE}{MSC} = 1.12$$

$$F_R = \frac{MSE}{MSR} = 1.64$$

ndf for "between columns" = 8, 2

ndf for "between rows" = 8, 4

Table value of F for (8, 2) df at 5% level = 19.4

Table value of F for (8, 4) df at 5% level = 6.04.

Conclusion:

In both the cases, calculated value of F < the table value of F.

$\therefore H_0$ is accepted at 5% level. Hence there is no significant difference between the means due to treatment I or treatment II.

Three breeds of cattle A, B and C were fed by 4 different rations P,Q, R and S. The following table gives the gains in weight. Test whether there is any significant difference between breeds and rations at 5% level of significance.

	Rations			
	P	**Q**	**R**	**S**
1	6	3	2	9
Breed 2	1	3	8	7
3	7	3	5	2

Null hypothesis: i) There is no significant between breeds.
ii) There is no significant difference between rations

Null hypothesis: i) There is no significant between breeds.
ii) There is no significant difference between rations

Workers	Rations				Total
	P	**Q**	**R**	**S**	
1	6	3	2	9	20
2	1	3	8	7	19
3	7	3	5	2	17
Total	14	9	15	18	56(T)

Step 1: Total T = 56

Step 2: Correction Factor CF $= \dfrac{T^2}{N} = \dfrac{(56)^2}{12} = 261.33$

Step 3: SSC = Sum of Squares between columns(Rations)

$$= \left(\frac{(\sum X_1)^2}{n} + \frac{(\sum X_2)^2}{n} + \ldots..\right) - \left(\frac{T^2}{N}\right)$$

$$= \left(\frac{(14)^2}{3} + \frac{(9)^2}{3} + .\frac{(15)^2}{3} + \frac{(18)^2}{3} \ldots.\right) - (261.33)$$

$= 14$

Step 4: SSR = Sum of Squares between rows (workers)

$$\frac{(\Sigma\, Y_1)^2}{N_2} + \frac{(\Sigma\, Y_2)^2}{N_2} + \frac{(\Sigma\, Y_3)^2}{N_2} - \frac{T^2}{N}$$

$$= \left(\frac{(20)^2}{4} + \frac{(19)^2}{4} + .\frac{(17)^2}{4}\right) - (261.33)$$

$$= 1.17$$

Step 5: Total Sum of Squares (TSS) = Sum of squares of each values – CF

$$= \left(\sum X_1^2 + \sum X_2^2 + \ldots\right) - \left(\frac{T^2}{N}\right)$$

$$= (6)^2 + (3)^2 + (2)^2 + (9)^2 + \ldots\ldots + (2)^2 - 261.33$$

$$= 78.67$$

Step 6: SSE = Residual

$= \text{TSS} - (\text{SSC} + \text{SSR})$

$= 78.67 - (14 + 1.17)$

$= 63.5$

Source of Variation	Sum of Squares	Degree of freedom	Mean Square	F – ratio
Between columns (k = Number of columns)	SSC = 14	k – 1 = 4 -1 = 3	$MSC = \frac{SSC}{k-1}$ = 4.67	$F_C = \frac{MSC}{MSE}$ = 2.26
Between rows (r = Number of rows)	SSR = 1.17	r -1 = 3 – 1 = 2	$MSR = \frac{SSR}{r-1}$ = 0.585	$F_R = \frac{MSR}{MSE}$ = 18.08
Residual (or) Error	SSE = 63.5	(k-1)(r-1) = 6	$MSE = \frac{SSE}{(r-1)(k-1)}$ = 10.58	

Tabulated value: i) (6,3) df at 5% level is 8.94
ii) (6,2) df at 5% level is 19.3

Conclusion : i) CV <TV
Accept H_0
ii) CV < TV
Accept H_0.

Four doctors each test four treatments for a certain disease and observe the number of days each patient takes to recover. The results are as follows (recovery time in days)

	Treatment			
Doctor	*1*	*2*	*3*	*4*
A	10	14	19	20
B	11	15	17	21
C	9	12	16	19
D	8	13	17	20

Discuss the difference between (a) doctors and (b) treatments.

We subtracted 15 from the given values and work out with the new values of x_{ij}.

S.V.	S.S.	d.f.	M.S.	F_0
Between rows (doctors)	$Q_1 = 11.19$	$h - 1 = 3$	3.73	$\frac{3.73}{0.62} = 6.02$
Between cols. (treatments)	$Q_2 = 250.19$	$k - 1 = 3$	83.40	$\frac{83.40}{0.62} = 134.52$
Residual	$Q_3 = 5.56$	$(h - 1)(k - 1) = 9$	0.62	–
Total	$Q = 266.94$	$hk - 1 = 15$	–	–

From the F-tables, $F_{5\%}$ $(\nu_1 = 3, \nu_2 = 9) = 3.86$

Since $F_0 > F_{5\%}$ with respect to rows and columns, the difference between the doctors is significant and that between the treatments is highly significant.

Example 6:

Perform two–way **ANOVA** for the data given below.

Plots of Land	Treatment			
	A	B	C	D
I	38	40	41	39
II	45	42	49	36
III	40	38	42	42

Use coding method, subtracting 40 from the given numbers.

Solution:

Subtract 40 from all the numbers. By doing so, the F ratio is unaffected. It reduces the numbers to smaller numbers.

	A	B	C	D
I	–2	0	1	–1
II	5	2	9	–4
III	0	–2	2	2

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	−2	0	1	−1	−2	4	0	1	1
Y_2	5	2	9	−4	12	25	4	81	16
Y_3	0	−2	2	2	2	0	4	4	4
Total	3	0	12	−3	12	29	8	86	21

$$N = 3 + 3 + 3 + 3 = 12$$

$$T = 3 + 0 + 12 - 3 = 12$$

$$\frac{T^2}{N} = \frac{12^2}{12} = 12$$

$$SST = 29 + 8 + 86 + 21 - 12 = 132$$

$$SSC = \frac{9}{3} + \frac{0}{3} + \frac{144}{3} + \frac{9}{3} - 12$$

$$= 3 + 0 + 48 + 3 - 12 = 42$$

$$SSR = \frac{4}{5} + \frac{144}{5} + \frac{4}{5} - 12 = 18.4$$

ANOVA Table

Source of Variation	SS	Df	MSS	VR
Between columns	42	3	14	$F_C = \frac{14}{11.77} = 1.19$
Between rows	18.4	2	9.2	$F_R = \frac{11.77}{9.2} = 1.28$
Error	70.6	6	11.77	
Total	131	11		

H_0 : There is no significant difference between column means as well as row means.

H_1 : There is significant difference between column means or the row means.

The test statistic $F_C = \frac{MSC}{MSR} = 1.19$

$$F_R = \frac{MSE}{MSR} = 1.28$$

ndf for column means (3, 6)

ndf for row means (6, 2)

The table value of F are $F\,(3, 6) = 4.76$, $F\,(6, 2) = 19.3$

Conclusion:

In both cases the calculated value of $F <$ the table value of F.

$\therefore H_0$ is accepted.

Hence there is no significant difference between column means as well as row means.