Cumulative normal distribution

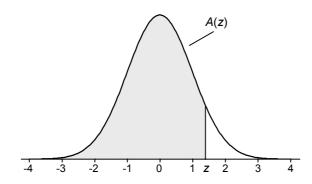
Critical values of the *t* distribution

Critical values of the *F* distribution

Critical values of the chi-squared distribution

Table A.1

Cumulative Standardized Normal Distribution



A(z) is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

| Z | A(z) | |
|-------|--------|---------------------------------|
| 1.645 | 0.9500 | Lower limit of right 5% tail |
| 1.960 | 0.9750 | Lower limit of right 2.5% tail |
| 2.326 | 0.9900 | Lower limit of right 1% tail |
| 2.576 | 0.9950 | Lower limit of right 0.5% tail |
| 3.090 | 0.9990 | Lower limit of right 0.1% tail |
| 3.291 | 0.9995 | Lower limit of right 0.05% tail |

| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | 3.,,,, | 0.,,,, | 3.,,,, | 5.,,,, | 3.,,,, | 0.,,,, | 0.,,,, |

TABLE A.2

t Distribution: Critical Values of t

| | | | | Significa | nce level | | |
|---------------|------------------|----------------|----------------|----------------|----------------|-----------------|------------------|
| Degrees of | Two-tailed test: | 10% | 5% | 2% 1% | 1% | 0.2% | 0.1% |
| freedom | One-tailed test: | 5% 6.314 | 2.5% 12.706 | 31.821 | 0.5% 63.657 | 0.1% 318.309 | 0.05% 636.619 |
| 1 2 | | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| 3 | | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| 4 | | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| 5 | | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| 6 | | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| 7 8 | | 1.894 1.860 | 2.365 2.306 | 2.998 2.896 | 3.499 3.355 | 4.785 4.501 | 5.408 5.041 |
| 9 | | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| 10 | | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| 11 | | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| 12 | | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| 13 | | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| 14 | | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| 15 | | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| 16 | | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| 17 | | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 |
| 18 19 | | 1.734 1.729 | 2.101 2.093 | 2.552 2.539 | 2.878 2.861 | 3.610 3.579 | 3.922 3.883 |
| 20 | | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 |
| 21 | | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| 22 | | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 |
| 23 | | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 |
| 24 | | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 |
| 25 | | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 |
| 26 | | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 |
| 27 | | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 |
| 28 | | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 |
| 29 30 | | 1.699 1.697 | 2.045 2.042 | 2.462 2.457 | 2.756 2.750 | 3.396 3.385 | 3.659 3.646 |
| 32 | | 1.694 | 2.037 | 2.449 | 2.738 | 3.365 | 3.622 |
| 34 | | 1.691 | 2.032 | 2.441 | 2.728 | 3.348 | 3.601 |
| 36 | | 1.688 | 2.028 | 2.434 | 2.719 | 3.333 | 3.582 |
| 38 | | 1.686 | 2.024 | 2.429 | 2.712 | 3.319 | 3.566 |
| 40 | | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| 42 | | 1.682 | 2.018 | 2.418 | 2.698 | 3.296 | 3.538 |
| 44 | | 1.680 | 2.015 | 2.414 | 2.692 | 3.286 | 3.526 |
| 46 48 | | 1.679 1.677 | 2.013 2.011 | 2.410 2.407 | 2.687 2.682 | 3.277 3.269 | 3.515 3.505 |
| 50 | | 1.676 | 2.009 | 2.407 | 2.678 | 3.261 | 3.496 |
| 60 | | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| 70 | | 1.667 | 1.994 | 2.381 | 2.648 | 3.211 | 3.435 |
| 80 | | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.416 |
| 90 | | 1.662 | 1.987 | 2.368 | 2.632 | 3.183 | 3.402 |
| 100 | | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 | 3.390 |
| 120 | | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 | 3.373 |
| 150 | | 1.655 | 1.976 | 2.351 | 2.609 | 3.145 | 3.357 |
| 200 300 | | 1.653 1.650 | 1.972 1.968 | 2.345 2.339 | 2.601 2.592 | 3.131 | 3.340 3.323 |
| 400 | | 1.630 | 1.966 | 2.339 | 2.588 | 3.118 3.111 | 3.323 |
| 500 | | 1.648 | 1.965 | 2.334 | 2.586 | 3.107 | 3.310 |
| 600 | | 1.648 | 1.963 | 2.334 | 2.584 | 3.107 | 3.310 |
| ∞ | | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |
| | | | | | | | |

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Table A.3

F Distribution: Critical Values of F (5% significance level)

| <i>v</i> ₁ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 |
|---------------------------------|--|---|--------------------------------------|--------------------------------------|--------------------------------------|---|--------------------------------------|--------------------------------------|---|--------------------------------------|--------------------------------------|-----------------------|--------------------------------------|---|--------------------------------------|
| v ₂ 1 2 3 4 5 | 161.45 18.51 10.13 7.71 6.61 | 199.50 19.00 9.55 6.94 5.79 | | 9.12 | | 233.99 19.33 8.94 6.16 4.95 | | | 240.54 19.38 8.81 6.00 4.77 | 19.40 8.79 | | 19.42 8.71 5.87 | | 247.32 19.44 8.67 5.82 4.58 | |
| 6 7 8 9 10 | 5.99 5.59 5.32 5.12 4.96 | 5.14 4.74 4.46 4.26 4.10 | 4.76 4.35 4.07 3.86 3.71 | 4.53 4.12 3.84 3.63 3.48 | 4.39 3.97 3.69 3.48 3.33 | 4.28 3.87 3.58 3.37 3.22 | 4.21 3.79 3.50 3.29 3.14 | 4.15 3.73 3.44 3.23 3.07 | 4.10 3.68 3.39 3.18 3.02 | 4.06 3.64 3.35 3.14 2.98 | 4.00 3.57 3.28 3.07 2.91 | 3.53 3.24 | 3.92 3.49 3.20 2.99 2.83 | 3.90 3.47 3.17 2.96 2.80 | 3.87 3.44 3.15 2.94 2.77 |
| 11 12 13 14 15 | 4.84 4.75 4.67 4.60 4.54 | 3.98 3.89 3.81 3.74 3.68 | 3.59 3.49 3.41 3.34 3.29 | 3.36 3.26 3.18 3.11 3.06 | 3.20 3.11 3.03 2.96 2.90 | 3.09 3.00 2.92 2.85 2.79 | 3.01 2.91 2.83 2.76 2.71 | 2.95 2.85 2.77 2.70 2.64 | 2.90 2.80 2.71 2.65 2.59 | 2.85 2.75 2.67 2.60 2.54 | 2.79 2.69 2.60 2.53 2.48 | 2.64 2.55 2.48 | 2.70 2.60 2.51 2.44 2.38 | 2.67 2.57 2.48 2.41 2.35 | 2.65 2.54 2.46 2.39 2.33 |
| 16 17 18 19 20 | 4.49 4.45 4.41 4.38 4.35 | 3.63 3.59 3.55 3.52 3.49 | 3.24 3.20 3.16 3.13 3.10 | 3.01 2.96 2.93 2.90 2.87 | 2.85 2.81 2.77 2.74 2.71 | 2.74 2.70 2.66 2.63 2.60 | 2.66 2.61 2.58 2.54 2.51 | 2.59 2.55 2.51 2.48 2.45 | 2.54 2.49 2.46 2.42 2.39 | 2.49 2.45 2.41 2.38 2.35 | 2.42 2.38 2.34 2.31 2.28 | | 2.33 2.29 2.25 2.21 2.18 | 2.30 2.26 2.22 2.18 2.15 | 2.28 2.23 2.19 2.16 2.12 |
| 21 22 23 24 25 | 4.32 4.30 4.28 4.26 4.24 | 3.47 3.44 3.42 3.40 3.39 | 3.03 | 2.84 2.82 2.80 2.78 2.76 | 2.68 2.66 2.64 2.62 2.60 | 2.57 2.55 2.53 2.51 2.49 | 2.49 2.46 2.44 2.42 2.40 | 2.42 2.40 2.37 2.36 2.34 | 2.37 2.34 2.32 2.30 2.28 | 2.27 | 2.25 2.23 2.20 2.18 2.16 | | 2.16 2.13 2.11 2.09 2.07 | 2.12 2.10 2.08 2.05 2.04 | 2.10 2.07 2.05 2.03 2.01 |
| 26 27 28 29 30 | 4.22 4.21 4.20 4.18 4.17 | 3.37 3.35 3.34 3.33 3.32 | 2.98 2.96 2.95 2.93 2.92 | 2.74 2.73 2.71 2.70 2.69 | 2.59 2.57 2.56 2.55 2.53 | 2.47 2.46 2.45 2.43 2.42 | 2.39 2.37 2.36 2.35 2.33 | 2.32 2.31 2.29 2.28 2.27 | 2.27 2.25 2.24 2.22 2.21 | 2.22 2.20 2.19 2.18 2.16 | 2.15 2.13 2.12 2.10 2.09 | | 2.05 2.04 2.02 2.01 1.99 | 2.02 2.00 1.99 1.97 1.96 | 1.99 1.97 1.96 1.94 1.93 |
| 35 40 50 60 70 | 4.12 4.08 4.03 4.00 3.98 | 3.27 3.23 3.18 3.15 3.13 | 2.87 2.84 2.79 2.76 2.74 | 2.64 2.61 2.56 2.53 2.50 | 2.49 2.45 2.40 2.37 2.35 | 2.37 2.34 2.29 2.25 2.23 | 2.29 2.25 2.20 2.17 2.14 | 2.22 2.18 2.13 2.10 2.07 | 2.16 2.12 2.07 2.04 2.02 | 2.03 | 2.04 2.00 1.95 1.92 1.89 | 1.95 1.89 1.86 | 1.94 1.90 1.85 1.82 1.79 | 1.91 1.87 1.81 1.78 1.75 | 1.88 1.84 1.78 1.75 1.72 |
| 80 90 100 120 150 | 3.96 3.95 3.94 3.92 3.90 | 3.11 3.10 3.09 3.07 3.06 | 2.72 2.71 2.70 2.68 2.66 | 2.49 2.47 2.46 2.45 2.43 | 2.33 2.32 2.31 2.29 2.27 | 2.21 2.20 2.19 2.18 2.16 | 2.13 2.11 2.10 2.09 2.07 | 2.06 2.04 2.03 2.02 2.00 | 2.00 1.99 1.97 1.96 1.94 | 1.95 1.94 1.93 1.91 1.89 | 1.88 1.86 1.85 1.83 1.82 | 1.79 1.78 | 1.77 1.76 1.75 1.73 1.71 | 1.73 1.72 1.71 1.69 1.67 | 1.70 1.69 1.68 1.66 1.64 |
| 200 250 300 400 500 | 3.89 3.88 3.87 3.86 3.86 | 3.04 3.03 3.03 3.02 3.01 | 2.65 2.64 2.63 2.63 2.62 | 2.42 2.41 2.40 2.39 2.39 | 2.26 2.25 2.24 2.24 2.23 | 2.14 2.13 2.13 2.12 2.12 | 2.06 2.05 2.04 2.03 2.03 | 1.98 1.98 1.97 1.96 1.96 | 1.93 1.92 1.91 1.90 1.90 | 1.88 1.87 1.86 1.85 1.85 | 1.80 1.79 1.78 1.78 1.77 | 1.73 1.72 | 1.69 1.68 1.67 1.66 | 1.66 1.65 1.64 1.63 1.62 | 1.62 1.61 1.61 1.60 1.59 |
| 600 750 1000 | 3.86 3.85 3.85 | 3.01 3.01 3.00 | 2.62 2.62 2.61 | 2.39 2.38 2.38 | 2.23 2.23 2.22 | 2.11 2.11 2.11 | 2.02 2.02 2.02 | 1.95 1.95 1.95 | 1.90 1.89 1.89 | 1.85 1.84 1.84 | 1.77 1.77 1.76 | 1.70 | 1.66 1.66 1.65 | 1.62 1.62 1.61 | 1.59 1.58 1.58 |

Table A.3 (continued)

F Distribution: Critical Values of F (5% significance level)

| <i>v</i> ₁ | 25 | 30 | 35 | 40 | 50 | 60 | 75 | 100 | 150 | 200 |
|---------------------------------|---|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|---|--------------------------------------|--------------------------------------|
| v ₂ 1 2 3 4 5 | 249.26 19.46 8.63 5.77 4.52 | 19.46 8.62 5.75 | 19.47 8.60 5.73 | 19.47 8.59 | | 19.48 8.57 | 8.56 | 253.04 19.49 8.55 5.66 4.41 | 19.49 8.54 | |
| 6 7 8 9 10 | 3.83 3.40 3.11 2.89 2.73 | 3.08 2.86 | 3.06 2.84 | | 3.75 3.32 3.02 2.80 2.64 | 3.74 3.30 3.01 2.79 2.62 | 2.99 | | 2.96 2.74 | |
| 11 12 13 14 15 | 2.60 2.50 2.41 2.34 2.28 | 2.38 2.31 | 2.36 2.28 | 2.34 2.27 | 2.51 2.40 2.31 2.24 2.18 | 2.49 2.38 2.30 2.22 2.16 | 2.21 | 2.46 2.35 2.26 2.19 2.12 | 2.24 2.17 | |
| 16 17 18 19 20 | 2.23 2.18 2.14 2.11 2.07 | 2.15 2.11 2.07 | 2.12 2.08 2.05 | | 2.12 2.08 2.04 2.00 1.97 | 2.11 2.06 2.02 1.98 1.95 | | 2.07 2.02 1.98 1.94 1.91 | 2.00 1.96 | 2.04 1.99 1.95 1.91 1.88 |
| 21 22 23 24 25 | 2.05 2.02 2.00 1.97 1.96 | 1.98 1.96 1.94 | 1.93 1.91 | 1.91 1.89 | 1.94 1.91 1.88 1.86 1.84 | 1.92 1.89 1.86 1.84 1.82 | 1.84 1.82 | 1.88 1.85 1.82 1.80 1.78 | 1.80 1.78 | 1.84 1.82 1.79 1.77 1.75 |
| 26 27 28 29 30 | 1.94 1.92 1.91 1.89 1.88 | 1.88 1.87 1.85 | 1.84 1.83 | 1.82 | 1.82 1.81 1.79 1.77 1.76 | 1.80 1.79 1.77 1.75 1.74 | 1.75 1.73 | 1.76 1.74 1.73 1.71 1.70 | | 1.73 1.71 1.69 1.67 1.66 |
| 35 40 50 60 70 | 1.82 1.78 1.73 1.69 1.66 | 1.74 1.69 1.65 | 1.72 1.66 | | 1.70 1.66 1.60 1.56 1.53 | 1.68 1.64 1.58 1.53 1.50 | 1.55 | 1.63 1.59 1.52 1.48 1.45 | | 1.48 |
| 80 90 100 120 150 | 1.64 1.63 1.62 1.60 1.58 | 1.59 1.57 1.55 | 1.57 1.55 1.54 1.52 1.50 | 1.54 1.53 1.52 1.50 1.48 | 1.51 1.49 1.48 1.46 1.44 | 1.48 1.46 1.45 1.43 1.41 | 1.45 1.44 1.42 1.40 1.38 | 1.43 1.41 1.39 1.37 1.34 | 1.39 1.38 1.36 1.33 1.31 | 1.38 1.36 1.34 1.32 1.29 |
| 200 250 300 400 500 | 1.56 1.55 1.54 1.53 1.53 | 1.52 1.50 1.50 1.49 1.48 | 1.48 1.47 1.46 1.45 1.45 | 1.46 1.44 1.43 1.42 1.42 | 1.41 1.40 1.39 1.38 1.38 | 1.39 1.37 1.36 1.35 1.35 | 1.35 1.34 1.33 1.32 1.31 | 1.32 1.31 1.30 1.28 1.28 | 1.28 1.27 1.26 1.24 1.23 | 1.26 1.25 1.23 1.22 1.21 |
| 600 750 1000 | 1.52 1.52 1.52 | 1.48 1.47 1.47 | 1.44 1.44 1.43 | 1.41 1.41 1.41 | 1.37 1.37 1.36 | 1.34 1.34 1.33 | 1.31 1.30 1.30 | 1.27 1.26 1.26 | 1.23 1.22 1.22 | 1.20 1.20 1.19 |

Table A.3 (continued)

F Distribution: Critical Values of F (1% significance level)

| <i>v</i> ₁ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 |
|--------------------------|---|---|---|---|---|---|---|---|---|----------------|--|--|--|--|------|
| v ₂ 1 2 3 4 5 | 4052.18 98.50 34.12 21.20 16.26 | 4999.50 99.00 30.82 18.00 13.27 | 5403.35 99.17 29.46 16.69 12.06 | 5624.58 99.25 28.71 15.98 11.39 | 5763.65 99.30 28.24 15.52 10.97 | 5858.99 99.33 27.91 15.21 10.67 | 5928.36 99.36 27.67 14.98 10.46 | 5981.07 99.37 27.49 14.80 10.29 | 6022.47 99.39 27.35 14.66 10.16 | 99.40 27.23 | 6106.32 99.42 27.05 14.37 9.89 | 6142.67 99.43 26.92 14.25 9.77 | 6170.10 99.44 26.83 14.15 9.68 | 6191.53 99.44 26.75 14.08 9.61 | |
| 6 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.72 | 7.60 | 7.52 | 7.45 | 7.40 |
| 7 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.47 | 6.36 | 6.28 | 6.21 | 6.16 |
| 8 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.67 | 5.56 | 5.48 | 5.41 | 5.36 |
| 9 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 5.11 | 5.01 | 4.92 | 4.86 | 4.81 |
| 10 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.71 | 4.60 | 4.52 | 4.46 | 4.41 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.40 | 4.29 | 4.21 | 4.15 | 4.10 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.16 | 4.05 | 3.97 | 3.91 | 3.86 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.96 | 3.86 | 3.78 | 3.72 | 3.66 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.80 | 3.70 | 3.62 | 3.56 | 3.51 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.67 | 3.56 | 3.49 | 3.42 | 3.37 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.55 | 3.45 | 3.37 | 3.31 | 3.26 |
| 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.46 | 3.35 | 3.27 | 3.21 | 3.16 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.37 | 3.27 | 3.19 | 3.13 | 3.08 |
| 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.30 | 3.19 | 3.12 | 3.05 | 3.00 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.23 | 3.13 | 3.05 | 2.99 | 2.94 |
| 21 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 | 3.31 | 3.17 | 3.07 | 2.99 | 2.93 | 2.88 |
| 22 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 3.12 | 3.02 | 2.94 | 2.88 | 2.83 |
| 23 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 | 3.21 | 3.07 | 2.97 | 2.89 | 2.83 | 2.78 |
| 24 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 3.03 | 2.93 | 2.85 | 2.79 | 2.74 |
| 25 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 | 2.99 | 2.89 | 2.81 | 2.75 | 2.70 |
| 26 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 2.96 | 2.86 | 2.78 | 2.72 | 2.66 |
| 27 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 | 3.06 | 2.93 | 2.82 | 2.75 | 2.68 | 2.63 |
| 28 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.90 | 2.79 | 2.72 | 2.65 | 2.60 |
| 29 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 | 3.00 | 2.87 | 2.77 | 2.69 | 2.63 | 2.57 |
| 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.84 | 2.74 | 2.66 | 2.60 | 2.55 |
| 35 | 7.42 | 5.27 | 4.40 | 3.91 | 3.59 | 3.37 | 3.20 | 3.07 | 2.96 | 2.88 | 2.74 | 2.64 | 2.56 | 2.50 | 2.44 |
| 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.66 | 2.56 | 2.48 | 2.42 | 2.37 |
| 50 | 7.17 | 5.06 | 4.20 | 3.72 | 3.41 | 3.19 | 3.02 | 2.89 | 2.78 | 2.70 | 2.56 | 2.46 | 2.38 | 2.32 | 2.27 |
| 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.50 | 2.39 | 2.31 | 2.25 | 2.20 |
| 70 | 7.01 | 4.92 | 4.07 | 3.60 | 3.29 | 3.07 | 2.91 | 2.78 | 2.67 | 2.59 | 2.45 | 2.35 | 2.27 | 2.20 | 2.15 |
| 80 | 6.96 | 4.88 | 4.04 | 3.56 | 3.26 | 3.04 | 2.87 | 2.74 | 2.64 | 2.55 | 2.42 | 2.31 | 2.23 | 2.17 | 2.12 |
| 90 | 6.93 | 4.85 | 4.01 | 3.53 | 3.23 | 3.01 | 2.84 | 2.72 | 2.61 | 2.52 | 2.39 | 2.29 | 2.21 | 2.14 | 2.09 |
| 100 | 6.90 | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 | 2.50 | 2.37 | 2.27 | 2.19 | 2.12 | 2.07 |
| 120 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 | 2.47 | 2.34 | 2.23 | 2.15 | 2.09 | 2.03 |
| 150 | 6.81 | 4.75 | 3.91 | 3.45 | 3.14 | 2.92 | 2.76 | 2.63 | 2.53 | 2.44 | 2.31 | 2.20 | 2.12 | 2.06 | 2.00 |
| 200 | 6.76 | 4.71 | 3.88 | 3.41 | 3.11 | 2.89 | 2.73 | 2.60 | 2.50 | 2.41 | 2.27 | 2.17 | 2.09 | 2.03 | 1.97 |
| 250 | 6.74 | 4.69 | 3.86 | 3.40 | 3.09 | 2.87 | 2.71 | 2.58 | 2.48 | 2.39 | 2.26 | 2.15 | 2.07 | 2.01 | 1.95 |
| 300 | 6.72 | 4.68 | 3.85 | 3.38 | 3.08 | 2.86 | 2.70 | 2.57 | 2.47 | 2.38 | 2.24 | 2.14 | 2.06 | 1.99 | 1.94 |
| 400 | 6.70 | 4.66 | 3.83 | 3.37 | 3.06 | 2.85 | 2.68 | 2.56 | 2.45 | 2.37 | 2.23 | 2.13 | 2.05 | 1.98 | 1.92 |
| 500 | 6.69 | 4.65 | 3.82 | 3.36 | 3.05 | 2.84 | 2.68 | 2.55 | 2.44 | 2.36 | 2.22 | 2.12 | 2.04 | 1.97 | 1.92 |
| 600 | 6.68 | 4.64 | 3.81 | 3.35 | 3.05 | 2.83 | 2.67 | 2.54 | 2.44 | 2.35 | 2.21 | 2.11 | 2.03 | 1.96 | 1.91 |
| 750 | 6.67 | 4.63 | 3.81 | 3.34 | 3.04 | 2.83 | 2.66 | 2.53 | 2.43 | 2.34 | 2.21 | 2.11 | 2.02 | 1.96 | 1.90 |
| 1000 | 6.66 | 4.63 | 3.80 | 3.34 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 | 2.34 | 2.20 | 2.10 | 2.02 | 1.95 | 1.90 |

Table A.3 (continued)

F Distribution: Critical Values of F (1% significance level)

| <i>v</i> ₁ | 25 | 30 | 35 | 40 | 50 | 60 | 75 | 100 | 150 | 200 |
|---------------------------------|--|--|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| v ₂ 1 2 3 4 5 | 6239.83 99.46 26.58 13.91 9.45 | 6260.65 99.47 26.50 13.84 9.38 | 99.47 26.45 13.79 | 99.47 26.41 | 99.48 26.35 13.69 | 99.48 26.32 | 26.28 | 99.49 26.24 13.58 | 99.49 26.20 | 99.49 26.18 13.52 |
| 6 7 8 9 10 | 7.30 6.06 5.26 4.71 4.31 | 7.23 5.99 5.20 4.65 4.25 | 5.15 4.60 | 5.12 | 5.86 5.07 | 7.06 5.82 5.03 4.48 4.08 | 5.00 | 4.96 4.41 | 5.72 | 4.91 |
| 11 12 13 14 15 | 4.01 3.76 3.57 3.41 3.28 | 3.94 3.70 3.51 3.35 3.21 | 3.65 3.46 | 3.43 3.27 | 3.81 3.57 3.38 3.22 3.08 | 3.78 3.54 3.34 3.18 3.05 | 3.31 3.15 | 3.47 3.27 | 3.24 3.08 | 3.06 |
| 16 17 18 19 20 | 3.16 3.07 2.98 2.91 2.84 | 3.10 3.00 2.92 2.84 2.78 | 2.96 2.87 2.80 | 2.84 2.76 | 2.71 | | | 2.76 2.68 2.60 | 2.64 | |
| 21 22 23 24 25 | 2.79 2.73 2.69 2.64 2.60 | 2.72 2.67 2.62 2.58 2.54 | 2.62 2.57 2.53 | 2.58 2.54 2.49 | 2.53 2.48 2.44 | | 2.46 2.41 2.37 | 2.42 2.37 2.33 | 2.38 2.34 2.29 | 2.36 2.32 |
| 26 27 28 29 30 | 2.57 2.54 2.51 2.48 2.45 | 2.50 2.47 2.44 2.41 2.39 | 2.42 2.39 2.36 | 2.35 | 2.36 2.33 2.30 2.27 2.25 | 2.26 | 2.23 | 2.22 2.19 2.16 | 2.15 | 2.19 2.16 2.13 2.10 2.07 |
| 35 40 50 60 70 | 2.35 2.27 2.17 2.10 2.05 | 2.28 2.20 2.10 2.03 1.98 | 2.15 2.05 | 2.01 | 2.06 1.95 1.88 | 2.10 2.02 1.91 1.84 1.78 | 1.98 1.87 | 1.94 1.82 1.75 | 1.90 | 1.76 |
| 80 90 100 120 150 | 2.01 1.99 1.97 1.93 1.90 | 1.94 1.92 1.89 1.86 1.83 | 1.89 1.86 1.84 1.81 1.77 | 1.85 1.82 1.80 1.76 1.73 | 1.79 1.76 1.74 1.70 1.66 | | 1.65 | 1.65 1.62 1.60 1.56 1.52 | 1.61 1.57 1.55 1.51 1.46 | 1.58 1.55 1.52 1.48 1.43 |
| 200 250 300 400 500 | 1.87 1.85 1.84 1.82 1.81 | 1.79 1.77 1.76 1.75 1.74 | 1.74 1.72 1.70 1.69 1.68 | 1.69 1.67 1.66 1.64 1.63 | 1.63 1.61 1.59 1.58 1.57 | 1.58 1.56 1.55 1.53 1.52 | 1.53 1.51 1.50 1.48 1.47 | 1.48 1.46 1.44 1.42 1.41 | 1.42 1.40 1.38 1.36 1.34 | 1.39 1.36 1.35 1.32 1.31 |
| 600 750 1000 | 1.80 1.80 1.79 | 1.73 1.72 1.72 | 1.67 1.66 1.66 | 1.63 1.62 1.61 | 1.56 1.55 1.54 | 1.51 1.50 1.50 | 1.46 1.45 1.44 | 1.40 1.39 1.38 | 1.34 1.33 1.32 | 1.30 1.29 1.28 |

Table A.3 (continued)

F Distribution: Critical Values of F (0.1% significance level)

| <i>v</i> ₁ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 |
|-----------------------|------------------|------------------|---------------------------|---------------------------|------------------|---------------------------|---------------------------|------------------|---------------------------|---------------------------|---|------------------|--------|------------------|------------------|
| | 998.50 167.03 | 999.00 148.50 | 999.17 141.11 56.18 | 999.25 137.10 53.44 | 999.30 134.58 | 999.33 132.85 50.53 | 999.36 131.58 49.66 | 999.37 130.62 | 999.39 129.86 48.47 | 999.40 129.25 48.05 | 6.11e05 999.42 128.32 47.41 26.42 | 999.43 127.64 | 999.44 | 999.44 126.74 | 999.45 126.42 |
| 6 | 35.51 | 27.00 | 23.70 | 21.92 | 20.80 | 20.03 | 19.46 | 19.03 | 18.69 | 18.41 | 17.99 | 17.68 | 17.45 | 17.27 | 17.12 |
| 7 | 29.25 | 21.69 | 18.77 | 17.20 | 16.21 | 15.52 | 15.02 | 14.63 | 14.33 | 14.08 | 13.71 | 13.43 | 13.23 | 13.06 | 12.93 |
| 8 | 25.41 | 18.49 | 15.83 | 14.39 | 13.48 | 12.86 | 12.40 | 12.05 | 11.77 | 11.54 | 11.19 | 10.94 | 10.75 | 10.60 | 10.48 |
| 9 | 22.86 | 16.39 | 13.90 | 12.56 | 11.71 | 11.13 | 10.70 | 10.37 | 10.11 | 9.89 | 9.57 | 9.33 | 9.15 | 9.01 | 8.90 |
| 10 | 21.04 | 14.91 | 12.55 | 11.28 | 10.48 | 9.93 | 9.52 | 9.20 | 8.96 | 8.75 | 8.45 | 8.22 | 8.05 | 7.91 | 7.80 |
| 11 | 19.69 | 13.81 | 11.56 | 10.35 | 9.58 | 9.05 | 8.66 | 8.35 | 8.12 | 7.92 | 7.63 | 7.41 | 7.24 | 7.11 | 7.01 |
| 12 | 18.64 | 12.97 | 10.80 | 9.63 | 8.89 | 8.38 | 8.00 | 7.71 | 7.48 | 7.29 | 7.00 | 6.79 | 6.63 | 6.51 | 6.40 |
| 13 | 17.82 | 12.31 | 10.21 | 9.07 | 8.35 | 7.86 | 7.49 | 7.21 | 6.98 | 6.80 | 6.52 | 6.31 | 6.16 | 6.03 | 5.93 |
| 14 | 17.14 | 11.78 | 9.73 | 8.62 | 7.92 | 7.44 | 7.08 | 6.80 | 6.58 | 6.40 | 6.13 | 5.93 | 5.78 | 5.66 | 5.56 |
| 15 | 16.59 | 11.34 | 9.34 | 8.25 | 7.57 | 7.09 | 6.74 | 6.47 | 6.26 | 6.08 | 5.81 | 5.62 | 5.46 | 5.35 | 5.25 |
| 16 | 16.12 | 10.97 | 9.01 | 7.94 | 7.27 | 6.80 | 6.46 | 6.19 | 5.98 | 5.81 | 5.55 | 5.35 | 5.20 | 5.09 | 4.99 |
| 17 | 15.72 | 10.66 | 8.73 | 7.68 | 7.02 | 6.56 | 6.22 | 5.96 | 5.75 | 5.58 | 5.32 | 5.13 | 4.99 | 4.87 | 4.78 |
| 18 | 15.38 | 10.39 | 8.49 | 7.46 | 6.81 | 6.35 | 6.02 | 5.76 | 5.56 | 5.39 | 5.13 | 4.94 | 4.80 | 4.68 | 4.59 |
| 19 | 15.08 | 10.16 | 8.28 | 7.27 | 6.62 | 6.18 | 5.85 | 5.59 | 5.39 | 5.22 | 4.97 | 4.78 | 4.64 | 4.52 | 4.43 |
| 20 | 14.82 | 9.95 | 8.10 | 7.10 | 6.46 | 6.02 | 5.69 | 5.44 | 5.24 | 5.08 | 4.82 | 4.64 | 4.49 | 4.38 | 4.29 |
| 21 | 14.59 | 9.77 | 7.94 | 6.95 | 6.32 | 5.88 | 5.56 | 5.31 | 5.11 | 4.95 | 4.70 | 4.51 | 4.37 | 4.26 | 4.17 |
| 22 | 14.38 | 9.61 | 7.80 | 6.81 | 6.19 | 5.76 | 5.44 | 5.19 | 4.99 | 4.83 | 4.58 | 4.40 | 4.26 | 4.15 | 4.06 |
| 23 | 14.20 | 9.47 | 7.67 | 6.70 | 6.08 | 5.65 | 5.33 | 5.09 | 4.89 | 4.73 | 4.48 | 4.30 | 4.16 | 4.05 | 3.96 |
| 24 | 14.03 | 9.34 | 7.55 | 6.59 | 5.98 | 5.55 | 5.23 | 4.99 | 4.80 | 4.64 | 4.39 | 4.21 | 4.07 | 3.96 | 3.87 |
| 25 | 13.88 | 9.22 | 7.45 | 6.49 | 5.89 | 5.46 | 5.15 | 4.91 | 4.71 | 4.56 | 4.31 | 4.13 | 3.99 | 3.88 | 3.79 |
| 26 | 13.74 | 9.12 | 7.36 | 6.41 | 5.80 | 5.38 | 5.07 | 4.83 | 4.64 | 4.48 | 4.24 | 4.06 | 3.92 | 3.81 | 3.72 |
| 27 | 13.61 | 9.02 | 7.27 | 6.33 | 5.73 | 5.31 | 5.00 | 4.76 | 4.57 | 4.41 | 4.17 | 3.99 | 3.86 | 3.75 | 3.66 |
| 28 | 13.50 | 8.93 | 7.19 | 6.25 | 5.66 | 5.24 | 4.93 | 4.69 | 4.50 | 4.35 | 4.11 | 3.93 | 3.80 | 3.69 | 3.60 |
| 29 | 13.39 | 8.85 | 7.12 | 6.19 | 5.59 | 5.18 | 4.87 | 4.64 | 4.45 | 4.29 | 4.05 | 3.88 | 3.74 | 3.63 | 3.54 |
| 30 | 13.29 | 8.77 | 7.05 | 6.12 | 5.53 | 5.12 | 4.82 | 4.58 | 4.39 | 4.24 | 4.00 | 3.82 | 3.69 | 3.58 | 3.49 |
| 35 | 12.90 | 8.47 | 6.79 | 5.88 | 5.30 | 4.89 | 4.59 | 4.36 | 4.18 | 4.03 | 3.79 | 3.62 | 3.48 | 3.38 | 3.29 |
| 40 | 12.61 | 8.25 | 6.59 | 5.70 | 5.13 | 4.73 | 4.44 | 4.21 | 4.02 | 3.87 | 3.64 | 3.47 | 3.34 | 3.23 | 3.14 |
| 50 | 12.22 | 7.96 | 6.34 | 5.46 | 4.90 | 4.51 | 4.22 | 4.00 | 3.82 | 3.67 | 3.44 | 3.27 | 3.41 | 3.04 | 2.95 |
| 60 | 11.97 | 7.77 | 6.17 | 5.31 | 4.76 | 4.37 | 4.09 | 3.86 | 3.69 | 3.54 | 3.32 | 3.15 | 3.02 | 2.91 | 2.83 |
| 70 | 11.80 | 7.64 | 6.06 | 5.20 | 4.66 | 4.28 | 3.99 | 3.77 | 3.60 | 3.45 | 3.23 | 3.06 | 2.93 | 2.83 | 2.74 |
| 80 | 11.67 | 7.54 | 5.97 | 5.12 | 4.58 | 4.20 | 3.92 | 3.70 | 3.53 | 3.39 | 3.16 | 3.00 | 2.87 | 2.76 | 2.68 |
| 90 | 11.57 | 7.47 | 5.91 | 5.06 | 4.53 | 4.15 | 3.87 | 3.65 | 3.48 | 3.34 | 3.11 | 2.95 | 2.82 | 2.71 | 2.63 |
| 100 | 11.50 | 7.41 | 5.86 | 5.02 | 4.48 | 4.11 | 3.83 | 3.61 | 3.44 | 3.30 | 3.07 | 2.91 | 2.78 | 2.68 | 2.59 |
| 120 | 11.38 | 7.32 | 5.78 | 4.95 | 4.42 | 4.04 | 3.77 | 3.55 | 3.38 | 3.24 | 3.02 | 2.85 | 2.72 | 2.62 | 2.53 |
| 150 | 11.27 | 7.24 | 5.71 | 4.88 | 4.35 | 3.98 | 3.71 | 3.49 | 3.32 | 3.18 | 2.96 | 2.80 | 2.67 | 2.56 | 2.48 |
| 200 | 11.15 | 7.15 | 5.63 | 4.81 | 4.29 | 3.92 | 3.65 | 3.43 | 3.26 | 3.12 | 2.90 | 2.74 | 2.61 | 2.51 | 2.42 |
| 250 | 11.09 | 7.10 | 5.59 | 4.77 | 4.25 | 3.88 | 3.61 | 3.40 | 3.23 | 3.09 | 2.87 | 2.71 | 2.58 | 2.48 | 2.39 |
| 300 | 11.04 | 7.07 | 5.56 | 4.75 | 4.22 | 3.86 | 3.59 | 3.38 | 3.21 | 3.07 | 2.85 | 2.69 | 2.56 | 2.46 | 2.37 |
| 400 | 10.99 | 7.03 | 5.53 | 4.71 | 4.19 | 3.83 | 3.56 | 3.35 | 3.18 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 | 2.34 |
| 500 | 10.96 | 7.00 | 5.51 | 4.69 | 4.18 | 3.81 | 3.54 | 3.33 | 3.16 | 3.02 | 2.81 | 2.64 | 2.52 | 2.41 | 2.33 |
| 600 | 10.94 | 6.99 | 5.49 | 4.68 | 4.16 | 3.80 | 3.53 | 3.32 | 3.15 | 3.01 | 2.80 | 2.63 | 2.51 | 2.40 | 2.32 |
| 750 | 10.91 | 6.97 | 5.48 | 4.67 | 4.15 | 3.79 | 3.52 | 3.31 | 3.14 | 3.00 | 2.78 | 2.62 | 2.49 | 2.39 | 2.31 |
| 1000 | 10.89 | 6.96 | 5.46 | 4.65 | 4.14 | 3.78 | 3.51 | 3.30 | 3.13 | 2.99 | 2.77 | 2.61 | 2.48 | 2.38 | 2.30 |

Table A.3 (continued)

F Distribution: Critical Values of F (0.1% significance level)

| | 25 | 30 | 35 | 40 | 50 | 60 | 75 | 100 | 150 | 200 |
|-----|-------|---|--------|--------|--------|------------------|--------|--------|------------------|-------|
| | | 6.26e05 999.47 125.45 45.43 24.87 | 999.47 | 124.96 | 999.48 | 999.48 124.47 | 999.49 | 999.49 | 999.49 123.87 | |
| 6 | 16.85 | 16.67 | 16.54 | 16.44 | 16.31 | 16.21 | 16.12 | 16.03 | 15.93 | 15.89 |
| 7 | 12.69 | 12.53 | 12.41 | 12.33 | 12.20 | 12.12 | 12.04 | 11.95 | 11.87 | 11.82 |
| 8 | 10.26 | 10.11 | 10.00 | 9.92 | 9.80 | 9.73 | 9.65 | 9.57 | 9.49 | 9.45 |
| 9 | 8.69 | 8.55 | 8.46 | 8.37 | 8.26 | 8.19 | 8.11 | 8.04 | 7.96 | 7.93 |
| 10 | 7.60 | 7.47 | 7.37 | 7.30 | 7.19 | 7.12 | 7.05 | 6.98 | 6.91 | 6.87 |
| 11 | 6.81 | 6.68 | 6.59 | 6.52 | 6.42 | 6.35 | 6.28 | | 6.14 | 6.10 |
| 12 | 6.22 | 6.09 | 6.00 | 5.93 | 5.83 | 5.76 | 5.70 | | 5.56 | 5.52 |
| 13 | 5.75 | 5.63 | 5.54 | 5.47 | 5.37 | 5.30 | 5.24 | | 5.10 | 5.07 |
| 14 | 5.38 | 5.25 | 5.17 | 5.10 | 5.00 | 4.94 | 4.87 | | 4.74 | 4.71 |
| 15 | 5.07 | 4.95 | 4.86 | 4.80 | 4.70 | 4.64 | 4.57 | | 4.44 | 4.41 |
| 16 | 4.82 | 4.70 | 4.61 | 4.54 | 4.45 | 4.39 | 4.32 | 4.26 | 4.19 | 4.16 |
| 17 | 4.60 | 4.48 | 4.40 | 4.33 | 4.24 | 4.18 | 4.11 | 4.05 | 3.98 | 3.95 |
| 18 | 4.42 | 4.30 | 4.22 | 4.15 | 4.06 | 4.00 | 3.93 | 3.87 | 3.80 | 3.77 |
| 19 | 4.26 | 4.14 | 4.06 | 3.99 | 3.90 | 3.84 | 3.78 | 3.71 | 3.65 | 3.61 |
| 20 | 4.12 | 4.00 | 3.92 | 3.86 | 3.77 | 3.70 | 3.64 | 3.58 | 3.51 | 3.48 |
| 21 | 4.00 | 3.88 | 3.80 | 3.74 | 3.64 | 3.58 | 3.52 | 3.46 | 3.39 | 3.36 |
| 22 | 3.89 | 3.78 | 3.70 | 3.63 | 3.54 | 3.48 | 3.41 | 3.35 | 3.28 | 3.25 |
| 23 | 3.79 | 3.68 | 3.60 | 3.53 | 3.44 | 3.38 | 3.32 | 3.25 | 3.19 | 3.16 |
| 24 | 3.71 | 3.59 | 3.51 | 3.45 | 3.36 | 3.29 | 3.23 | 3.17 | 3.10 | 3.07 |
| 25 | 3.63 | 3.52 | 3.43 | 3.37 | 3.28 | 3.22 | 3.15 | 3.09 | 3.03 | 2.99 |
| 26 | 3.56 | 3.44 | 3.36 | 3.30 | 3.21 | 3.15 | 3.08 | 3.02 | 2.95 | 2.92 |
| 27 | 3.49 | 3.38 | 3.30 | 3.23 | 3.14 | 3.08 | 3.02 | 2.96 | 2.89 | 2.86 |
| 28 | 3.43 | 3.32 | 3.24 | 3.18 | 3.09 | 3.02 | 2.96 | 2.90 | 2.83 | 2.80 |
| 29 | 3.38 | 3.27 | 3.18 | 3.12 | 3.03 | 2.97 | 2.91 | 2.84 | 2.78 | 2.74 |
| 30 | 3.33 | 3.22 | 3.13 | 3.07 | 2.98 | 2.92 | 2.86 | 2.79 | 2.73 | 2.69 |
| 35 | 3.13 | 3.02 | 2.93 | 2.87 | 2.78 | 2.72 | 2.66 | 2.59 | 2.52 | 2.49 |
| 40 | 2.98 | 2.87 | 2.79 | 2.73 | 2.64 | 2.57 | 2.51 | 2.44 | 2.38 | 2.34 |
| 50 | 2.79 | 2.68 | 2.60 | 2.53 | 2.44 | 2.38 | 2.31 | 2.25 | 2.18 | 2.14 |
| 60 | 2.67 | 2.55 | 2.47 | 2.41 | 2.32 | 2.25 | 2.19 | 2.12 | 2.05 | 2.01 |
| 70 | 2.58 | 2.47 | 2.39 | 2.32 | 2.23 | 2.16 | 2.10 | 2.03 | 1.95 | 1.92 |
| 80 | 2.52 | 2.41 | 2.32 | 2.26 | 2.16 | 2.10 | 2.03 | 1.96 | 1.89 | 1.85 |
| 90 | 2.47 | 2.36 | 2.27 | 2.21 | 2.11 | 2.05 | 1.98 | 1.91 | 1.83 | 1.79 |
| 100 | 2.43 | 2.32 | 2.24 | 2.17 | 2.08 | 2.01 | 1.94 | 1.87 | 1.79 | 1.75 |
| 120 | 2.37 | 2.26 | 2.18 | 2.11 | 2.02 | 1.95 | 1.88 | 1.81 | 1.73 | 1.68 |
| 150 | 2.32 | 2.21 | 2.12 | 2.06 | 1.96 | 1.89 | 1.82 | 1.74 | 1.66 | 1.62 |
| 200 | 2.26 | 2.15 | 2.07 | 2.00 | 1.90 | 1.83 | 1.76 | 1.68 | 1.60 | 1.55 |
| 250 | 2.23 | 2.12 | 2.03 | 1.97 | 1.87 | 1.80 | 1.72 | 1.65 | 1.56 | 1.51 |
| 300 | 2.21 | 2.10 | 2.01 | 1.94 | 1.85 | 1.78 | 1.70 | 1.62 | 1.53 | 1.48 |
| 400 | 2.18 | 2.07 | 1.98 | 1.92 | 1.82 | 1.75 | 1.67 | 1.59 | 1.50 | 1.45 |
| 500 | 2.17 | 2.05 | 1.97 | 1.90 | 1.80 | 1.73 | 1.65 | 1.57 | 1.48 | 1.43 |
| 600 | 2.16 | 2.04 | 1.96 | 1.89 | 1.79 | 1.72 | 1.64 | 1.56 | 1.46 | 1.41 |
| 750 | 2.15 | 2.03 | 1.95 | 1.88 | 1.78 | 1.71 | 1.63 | 1.55 | 1.45 | 1.40 |
| 000 | 2.14 | 2.02 | 1.94 | 1.87 | 1.77 | 1.69 | 1.62 | 1.53 | 1.44 | 1.38 |

Table A.4 $\chi^{2} \mbox{ (Chi-Squared) Distribution: Critical Values of } \chi^{2}$

| | | Significance lev | el |
|-----------------------|--------|------------------|--------|
| Degrees of freedom | 5% | 1% | 0.1% |
| 1 | 3.841 | 6.635 | 10.828 |
| 2 | 5.991 | 9.210 | 13.816 |
| 3 | 7.815 | 11.345 | 16.266 |
| 4 | 9.488 | 13.277 | 18.467 |
| 5 | 11.070 | 15.086 | 20.515 |
| 6 | 12.592 | 16.812 | 22.458 |
| 7 | 14.067 | 18.475 | 24.322 |
| 8 | 15.507 | 20.090 | 26.124 |
| 9 | 16.919 | 21.666 | 27.877 |
| 10 | 18.307 | 23.209 | 29.588 |

| | | | | FABLE | OF AF | REAS | | | | |
|-----|-------|-------|-------|--------------|-------|-------|--------|-------|-------|-------|
| ↓z→ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 . | 8 | 9 |
| -0 | -0000 | -0040 | -0080 | -0120 | -0160 | -0199 | -0239- | -0279 | -0319 | -0359 |
| -1 | -0398 | -0438 | -0478 | -0517 | -0557 | -0596 | -0636 | -0675 | -0714 | -0759 |
| .2 | -0793 | -0832 | -0871 | -0916 | -0948 | -0987 | -1026 | -1064 | -1103 | -1141 |
| .3 | -1179 | 1217 | -1255 | -1293 | -1331 | -1368 | -1406 | -1443 | -1480 | -1517 |
| .4 | -1554 | -1591 | -1628 | -1664 | -1700 | -1736 | 1772 | -1808 | -1844 | -1879 |
| .5 | -1915 | -1950 | -1985 | -2019 | -2054 | -2088 | -2123 | -2157 | -2190 | -2224 |
| -6 | -2257 | -2291 | -2324 | -2357 | -2389 | -2422 | -2454 | -2486 | -2517 | -2549 |
| .7 | -2580 | -2611 | -2642 | -2673 | -2703 | -2734 | -2764 | -2794 | -2823 | -2852 |
| -8 | -2881 | -2910 | -2939 | -2967 | -2995 | -3023 | -3051 | *3078 | -3106 | -3133 |
| .9 | -3159 | -3186 | -3212 | -3238 | -3264 | -3289 | -3315 | -3340 | -3365 | -3389 |
| 1.0 | -3413 | -3438 | -3461 | -3485 | -3508 | -3531 | -3554 | -3577 | -3599 | *3621 |
| 1-1 | -3643 | -3655 | -3686 | -3708 | -3729 | -3749 | -3770 | -3790 | -3810 | -3830 |
| 1.2 | -3849 | -3869 | -3888 | -3907 | -3925 | -3944 | -3962 | -3980 | -3997 | 4015 |
| 1.3 | 4032 | 4049 | -4066 | 4082 | -4099 | 4115 | 4131 | 4147 | -4162 | 4177 |
| 1:4 | 4192 | -4207 | 4222 | -4236 | 4251 | 4265 | 4279 | 4292 | 4306 | 4319 |

| 1.5 | 4332 | 4345 | 4357 | -4370 | 4382 - | 4394 | 4406 | -4418 | -4429 | 4441 |
|-----|-------|-------|-------|-------|--------|-------|-------|-------|-------|-------|
| 1.6 | 4452 | 4463 | 4474 | -4484 | -4495 | 4505 | 4515 | 4525 | 4535 | 4545 |
| 1.7 | -4554 | 4564 | 4573 | -4582 | 4591 | 4599 | 4608 | 4616 | -4625 | 4633 |
| 1.8 | 4641 | 4649 | -4656 | -4664 | -4671 | 4678 | 4686 | 4693 | -4699 | 470€ |
| 1.9 | -4713 | -4719 | 4726 | -4732 | -4738 | 4744 | 4750 | 4756 | 4761 | 4767 |
| 2.0 | -4772 | -4778 | 4783 | -4788 | -4793 | 4798 | 4803 | 4808 | -4812 | 4817 |
| 2-1 | 4821 | 4826 | -4830 | -4834 | -4838 | 4842 | 4846 | 4850 | -4854 | 4857 |
| 2.2 | -4861 | -4864 | 4868 | -4871 | -4875 | 4678 | 4881 | 4884 | -4887 | 4890 |
| 2.3 | 4893 | 4896 | -4898 | -4901 | 4904 | 4906 | 4909 | 4911 | -4913 | 4916 |
| 2-4 | -4918 | 4920 | 4922 | -4925 | -4927 | 4929 | 4931 | 4932 | 4934 | 4936 |
| 2.5 | 4938 | 4940 | 4941 | -4943 | 4945 | 4946 | 4948 | 4959 | -4951 | 4952 |
| 2.6 | -4953 | 4955 | 4956 | -4957 | -4959 | -1960 | 4961 | 4962 | -4963 | 4964 |
| 2.7 | -4965 | 4966 | -4967 | -4968 | -4969 | 4970° | 4971 | 4972 | -4973 | 4974 |
| 2:8 | -4974 | -4975 | 4976 | -4977 | -4977 | 4978 | 4979 | 4979 | -4980 | 4981 |
| 2.9 | 4981 | 4982 | -4982 | 4983 | -4984 | 4984 | 4985 | 4985 | -4986 | 4986 |
| 3.0 | 4987 | -4987 | 4987 | -4988 | -4988 | 4989 | 4989 | 4989 | -4990 | 4990 |
| 3-1 | 4990 | 4991 | -4991 | 4991 | -4992 | 4992 | 4992 | 4992 | 4993 | 4993 |
| 3.2 | -4993 | -4993 | 4994 | -4994 | -4994 | 4994 | 4994 | 4995 | -4995 | 4995 |
| 3.3 | 4995 | 4995 | -4995 | -4996 | -4996 | 4996 | 4996 | 4996 | -4996 | 4997 |
| 3-4 | -4997 | -4997 | 4997 | -4997 | -4997 | 4997 | 4997 | 4997 | 4997 | 4998 |
| 3-5 | 4998 | 4998 | -4998 | 4998 | -4998 | 4998 | 4998 | 4998 | -4998 | 4998 |
| 3.6 | -4998 | 4998 | 4999 | -4999 | -4999 | 4999 | 4999 | 49991 | -4999 | 4999 |
| 3.7 | 4999 | 4999 | -4999 | 4999 | 4999 | 4999 | 4999 | 4999 | 4999 | 4999 |
| 3.9 | -5000 | -5000 | -5000 | -5000 | -5000 | -5000 | -5000 | -5000 | -5000 | -5000 |

| | Level of | significance | |
|-----------------|----------------------|-----------------------|----------------------|
| | 1% (0.01) | 5% (0.05) | 10% (0.1) |
| Two-tailed test | $ z_{\alpha} =2.58$ | z = 1.966 | z = 0.645 |
| Right-tailed | $z_{\alpha} = 2.33$ | $z_{\alpha} = 1.645$ | $z_{\alpha} = 1.28$ |
| Left-tailed | $z_{\alpha} = -2.33$ | $z_{\alpha} = -1.645$ | $z_{\alpha} = -1.28$ |