MAT2001
Statistics for Engineers

Module 7
Reliability

Reliability

Definition

The definitions given below with respect to a component hold good for a system or a device also.

If a component is put into operation at some specified time, say $t = 0$, and if T is the time until it fails or ceases to function properly, T is called the *life length* or *time to failure* of the component. Obviously $T (\leq 0)$ is a continuous random variable with some probability density function $f(t)$. Then the *reliability* or *reliability function* of the component at time 't', denoted by $R(t)$, is defined as

$$R(t) = P(T > t) \text{ or } 1 - P(T \leq t) \tag{1}$$

$$= 1 - F(t),$$

where $F(t)$ is the cumulative distribution function of T, given by

$$F(t) = \int_0^t f(t)\, dt$$

Thus

$$R(t) = 1 - \int_0^t f(t)\, dt = \int_t^\infty f(t)\, dt \tag{2}$$

Reliability
Definition

T – CRV represents the life-time of a system/Comp.
$0 \leq T < \infty$ with pdf $f(t)$.

Reliability (Reliability Function)

$$R(t) = P(T > t) = \int_t^{\infty} f(t)\,dt$$

$$R(t) = 1 - P(T \leq t)$$

$$R(t) = 1 - \int_0^t f(t)\,dt.$$

$$R(t) = 1 - F(t)$$

$$\frac{d(R(t))}{dt} = R'(t) = 0 - f(t)$$

$$\Rightarrow \boxed{f(t) = -R'(t)}$$

Note:

Since $F(0) = 0$ and $F(\infty) = 1$ by the property of *cdf*, $R(0) = 1$ and $R(\infty) = 0$ *i.e.*, $0 \leq R(t) \leq 1$. Also since $\frac{d}{dt} F(t) = f(t)$,

we get $$f(t) = -\frac{dR(t)}{dt} \tag{3}$$

Hazard Function
(Instantaneous Failure Rate Function)

the *instantaneous failure rate* or *hazard function* of the component, denoted by $\lambda(t)$.

Thus $$\boxed{\lambda(t) = \frac{f(t)}{R(t)}} \quad (4)$$

$$\boxed{f(t) = -R'(t)}$$

Now, using (3) in (4), we have

$$\boxed{-\frac{R'(t)}{R(t)} = \lambda(t)} \quad (5)$$

Note:

$$R(t) = e^{-\int_0^t \lambda(t)\,dt}$$

$$f(t) = \lambda(t)\, e^{-\int_0^t \lambda(t)\,dt}$$

Mean and Variance

Mean Time To Failure (MTTF)

$$\text{MTTF} = E(T) = \int_0^{\infty} t f(t)\, dt$$

$$\text{MTTF} = \int_0^{\infty} R(t)\, dt$$

Variance

$$\text{Var}(T) = \sigma_T^2 = E\{T - E(T)\}^2 \text{ or } E(T^2) - \{E(T)\}^2$$

$$= \int_0^{\infty} t^2 f(t) dt - (MTTF)^2$$

Conditional Reliability of a System or Component

For a Wear-in Period / Burn-in Period / After to Warranty Period:

It is defined as

$$R(t/T_0) = P\{T > T_0 + t / T > T_0\}$$

$$= \frac{P\{T > T_0 + t\}}{P\{T > T_0\}} = \frac{R(T_0 + t)}{R(T_0)}$$

$$= \frac{e^{-\int_0^{T_0+t} \lambda(t)\,dt}}{e^{-\int_0^{T_0} \lambda(t)\,dt}} = e^{-\left[\int_0^{T_0+t} \lambda(t)\,dt - \int_0^{T_0} \lambda(t)\,dt\right]}$$

$$= e^{-\int_{T_0}^{T_0+t} \lambda(t)\,dt} = e^{-\int_{T_0}^{T_0+t} \lambda(t)\,dt}$$

Special Failure Probability Distributions

1. Exponential Distribution

If the time to failure T follows an exponential distribution with parameter λ, then its *pdf* is given by

$$f(t) = \lambda e^{-\lambda t},\ t \geq 0$$

$$R(t) = \int_t^{\infty} \lambda e^{-\lambda t}\, dt = [-e^{-\lambda t}]_t^{\infty} = e^{-\lambda t}$$

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

This means that when the failure distribution is an exponential distribution with parameter λ, the failure rate at any time is a constant, equal to λ. Conversely when $\lambda(t)$ = a constant λ, we get

$$f(t) = \lambda \cdot e^{-\int_0^t \lambda\, dt} = \lambda e^{-\lambda t},\ t \geq 0$$

Due to this property, the exponential distribution is often referred to as *constant failure rate distribution* in reliability contexts.

1. Exponential Distribution

$$\text{MTTF} = E(T) = \frac{1}{\lambda}$$

$$\text{Var}(T) = \sigma^2_T = \frac{1}{\lambda^2}$$

$$R(t/T_0) = \frac{R(T_0 + t)}{R(T_0)} = \frac{e^{-\lambda(T_0 + t)}}{e^{-\lambda T_0}}$$

$$= e^{-\lambda t} = e^{-\lambda t}$$

$$R(t) = e^{-\lambda t}$$

This means that the time to failure of a component is not dependent on how long the component has been functioning. In other words the reliability of the component for the next 1000 hours, say, is the same regardless of whether the component is brand new or has been operating for several hours. This property is known as the *memoryless property* of the constant failure rate distribution.

2. Weibull Distribution

The *pdf* of the Weibull distribution was defined as

$$f(t) = \alpha\beta t^{\beta-1}\, e^{-\alpha t^{\beta}}\ ,\ t \geq 0$$

An alternative form of Weibull's *pdf* is

$$f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right], \theta > 0,\ \beta > 0,\ t \geq 0$$

by putting $\boxed{\alpha = \frac{1}{\theta^{\beta}}}$ β is called the *shape parameter* and θ is called the *characteristic life* or *scale parameter* of the Weibull's distribution. If T follows Weibull's distribution

$$R(t) = \int_t^{\infty} \frac{\beta}{\theta}\cdot\left(\frac{t}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right] dt$$

$$= \int_x^{\infty} e^{-x}\, dx, \text{ on putting } \left(\frac{t}{\theta}\right)^{\beta} = x$$

$$= e^{-x} = \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right]$$

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\beta}{\theta}\cdot\left(\frac{t}{\theta}\right)^{\beta-1}$$

2. Weibull Distribution

$$\text{MTTF} = E(T) = \theta \Gamma\left(1 + \frac{1}{\beta}\right)$$

$$\text{Var}(T) = \sigma^2_T = \theta^2 \left\{\Gamma\left(1 + \frac{2}{\beta}\right) - \left[\Gamma\left(1 + \frac{1}{\beta}\right)\right]^2\right\}$$

$$R(t/T_0) = \frac{R(t + T_0)}{R(T_0)}$$

$$= \frac{\exp\left[-\left(\frac{t + T_0}{\theta}\right)^\beta\right]}{\exp\left[-\left(\frac{T_0}{\theta}\right)^\beta\right]}$$

$$= \exp\left[-\left(\frac{t + T_0}{\theta}\right)^\beta + \left(\frac{T_0}{\theta}\right)^\beta\right]$$

3. Normal Distribution

$N(\mu, \sigma)$

If the time to failure T follows a normal distribution $N(\mu, \sigma)$ its *pdf* is given by

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(t-\mu)^2}{2\sigma^2}\right], -\infty < t < \infty$$

In this case, $MTTF = E(T) = \mu$ and

$$\text{Var}\,(T) = \sigma_T^2 = \sigma^2.$$

$R(t) = \int_t^\infty f(t)\, dt$ is found out by expressing the integral in terms of standard normal integral and using the normal tables.

then found out $\lambda(t)$

4. Log-Normal Distribution L-N (t_M, S)

If $X = \log T$ follows a normal distribution $N(\mu, \sigma)$, then T follows a lognormal distribution whose *pdf* is given by

$$f(t) = \frac{1}{st\sqrt{2\pi}} \exp\left[-\frac{1}{2s^2}\left\{\log\left(\frac{t}{t_M}\right)\right\}^2\right], t \geq 0$$

where $s = \sigma$ is a *shape parameter* and t_M, the *median time to failure* is the location parameter, given by $\log t_M = \mu$.

It can be proved that

$$\text{MTTF} = E(T) = t_M \exp\left(\frac{s^2}{2}\right)$$

$$\text{Var}(T) = \sigma^2_T = t^2_M \exp(s^2)\,[\exp(s^2) - 1]$$

Example:

The density function of the time to failure in years of the gizmos (for use
widgets) manufactured by a certain company is given by $f(t) = \frac{200}{(t+10)^3}, t \geq$

(a) Derive the reliability function and determine the reliability for the fi
year of operation.

(b) Compute the MTTF.

(c) What is the design life for a reliability 0.95?

(d) Will a one-year burn-in period improve the reliability in part (*a*)? If
what is the new reliability?

Solution:

(a) $$f(t) = \frac{200}{(t+10)^3},\ t \geq 0$$

$$R(t) = \int_t^{\infty} f(t)\,dt = \left[\frac{-100}{(t+10)^2}\right]_t^{\infty} = \frac{100}{(t+10)^2}$$

$$R(1) = \frac{100}{(1+10)^2} = 0.8264.$$

(b) $$\text{MTTF} = \int_0^{\infty} R(t)\,dt = \int_0^{\infty} \frac{100}{(t+10)^2}\,dt$$

$$= \left(\frac{-100}{t+10}\right)_0^{\infty} = 10 \text{ years.}$$

Solution (Continued):

(c) *Design life* is the time to failure (t_D) that corresponds to a specified reliability. Now it is required to find t_D corresponding to $R = 0.95$

$$\frac{100}{(t_D + 10)^2} = 0.95$$

i.e., $(t_D + 10)^2 = 100.2632$

$$t_D = 0.2598 \text{ year or } 95 \text{ days}$$

(d) $$R(t/1) = \frac{R(t+1)}{R(1)} = \frac{100}{(t+11)^2} \div \frac{100}{11^2} = \frac{121}{(t+11)^2}$$

Now $R(t/1) > R(t)$, if $\dfrac{121}{(t+11)^2} > \dfrac{100}{(t+10)^2}$

if $$\frac{(t+10)^2}{(t+11)^2} > \frac{100}{121}$$

if $$\frac{t+10}{t+11} > \frac{10}{11}$$

i.e., $11t > 10t$, which is true, as $t \geq 0$

$\therefore$ One year burn-in period will improve the reliability.

Now $R(1/1) = \dfrac{121}{(1+11)^2} = 0.8403 > 0.8264.$

Example:

The time to failure in operating hours of a critical solid-state power unit has the hazard rate function $\lambda(t) = 0.003\left(\frac{t}{500}\right)^{0.5}$, for $t \geq 0$.

(a) What is the reliability if the power unit must operate continuously for 50 hours?

(b) Determine the design life if a reliability of 0.90 is desired.

(c) Compute the MTTF.

(d) Given that the unit has operated for 50 hours, what is the probability that it will survive a second 50 hours of operation?

Solution:

(a) $$R(t) = \exp\left[-\int_0^t \lambda(t)\,dt\right]$$

$$R(50) = \exp\left[-\int_0^{50} 0.003\left(\frac{t}{500}\right)^{0.5} dt\right]$$

$$= \exp\left[-\frac{0.003}{\sqrt{500}}\cdot\frac{2}{3}t^{3/2}\right]_0^{50}$$

$$= \exp\left[-\frac{0.003}{\sqrt{500}}\times\frac{2}{3}\times 50\sqrt{50}\right]$$

$$= \exp\,[-0.03162]$$

$$= 0.9689.$$

Solution (Continued):

(b) $R(t_D) = 0.90$

$$\exp\left[-\int_0^{t_D} 0.003\left(\frac{t}{500}\right)^{0.5} dt\right] = 0.90$$

$$-\int_0^{t_D} \frac{0.003}{\sqrt{500}}\, t^{1/2}\, dt = -\, 0.10536$$

$$\frac{0.003}{\sqrt{500}} \times \frac{2}{3}\, t_D^{3/2} = 0.10536$$

$$t_D = \left\{\frac{3 \times \sqrt{500} \times 0.10536}{2 \times 0.003}\right\}^{2/3} = 111.54 \text{ hours.}$$

Solution (Continued):

(c) $$\text{MTTF} = \int_0^\infty R(t)\, dt$$

$$= \int_0^\infty e^{-\left(\frac{0.003}{\sqrt{500}} \times \frac{2}{3} \times t^{3/2}\right)}\, dt$$

$$= \int_0^\infty e^{-at^{3/2}}\, dt, \text{ where } a = \frac{0.003 \times 2}{3 \times \sqrt{500}}$$

$$= \int_0^\infty e^{-x} \cdot \frac{2}{3a^{2/3}} x^{-1/3}\, dx, \text{ on putting } x = at^{3/2}$$

$$= \frac{2}{3a^{2/3}} \Gamma(2/3) = \frac{2}{3a^{2/3}} \cdot \frac{3}{2} \Gamma(5/3)$$

$$= \frac{0.9033}{a^{2/3}}, \text{ from the table of values of Gamma function.}$$

$$= 451.65 \text{ hours.}$$

Solution (Continued):

(d) $P(T \geq 100 / T \geq 50) = \dfrac{P(T \geq 100)}{P(T \geq 50)} = \dfrac{R(100)}{R(50)}$

$$= \exp\left[-\int_{50}^{100} \lambda(t)\,dt\right]$$

$$= \exp\left[\left\{-\frac{0.002}{\sqrt{500}} \times 100^{3/2}\right\} - \left\{\frac{-0.002}{\sqrt{500}} \times 50^{3/2}\right\}\right]$$

$$= \exp\left[\{-0.08944\} - \{-0.03162\}\right]$$

$$= 0.9438$$

Exercise:

The reliability of a turbine blade is given by $R(t) = \left(1 - \frac{t}{t_0}\right)^2$, $0 \leq t \leq t_0$, where t_0 is the maximum life of the blade.

(a) Show that the blades are experiencing wear out.

(b) Compute MTTF as a function of the maximum life.

(c) If the maximum life is 2000 operating hours, what is the design life for a reliability of 0.90?

Example:

A manufacturer determines that, on the average, a television set is used 1.8 hours per day. A one-year warranty is offered on the picture tube having a MTTF of 2000 hours. If the distribution is exponential, what percentage of the tubes will fail during the warranty period?

Solution:

Since the distribution of the time to failure of the picture tube is exponential;

$$R(t) = e^{-\lambda t}, \text{ where } \lambda \text{ is the failure rate}$$

Given that MTTF = 2000 hours

i.e.; $\int_0^{\infty} e^{-\lambda t}\, dt = 2000$

i.e. $\frac{1}{\lambda} = 2000$ or $\lambda = 0.0005/\text{hour}$

$P(T \le 1 \text{ year}) = P(T \le 365 \times 1.8 \text{ hours})$ [$\because$ the T.V. is operated for 1.8 hours/day]

$$= 1 - P\{T > 657\}$$

$$= 1 - R(657)$$

$$= 1 - e^{-0.0005 \times 657}$$

$$= 0.28$$

i.e., 28% of the tubes will fail during the warranty period.

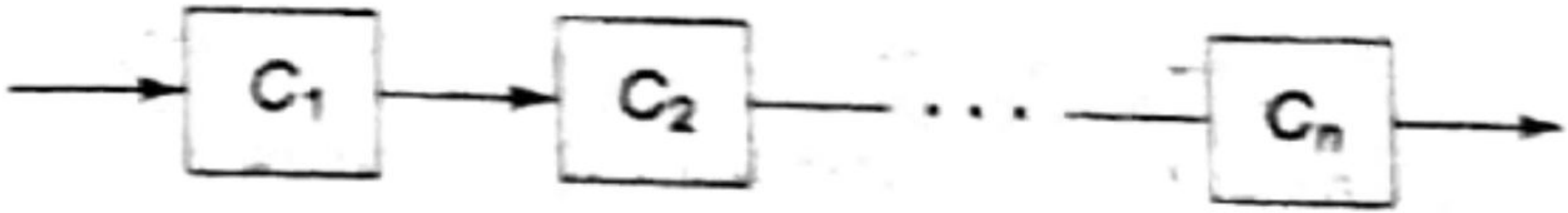
Exercise:

A one-year guarantee is given based on the assumption that no more than 10% of the items will be returned. Assuming an exponential distribution, what is the maximum failure rate that can be tolerated?

System Reliability

Serial or Non-redundant Configuration

Series or *nonredundant* configuration is one in which the components of the system are connected in series (or serially) as shown in the following reliability block diagram . Each block represents a component.



Serial or Non-redundant Configuration

In series configuration, all components must function for the system to function. In other words the failure of any component causes system failure.

Let $R_1(t)$, $R_2(t)$ and $R_s(t)$ be the reliabilities of the components C_1 and C_2 and the system (assuming that there are only 2 components in series),

Then $R_1 = P(C_1)$ = probability that C_1 functions

and $R_2 = P(C_2)$ = probability that C_2 functions

Now R_s = probability that both C_1 and C_2 function

$= P(C_1 \cap C_2) = P(C_1) \cdot P(C_2)$, assuming that C_1 and C_2 function independently.

$= R_1 \times R_2$

Serial or Non-redundant Configuration

This result may be extended. If $C_1, C_2, \cdots, C_n$ be a set of n independent components in series with reliabilities $R_1(t), R_2(t), \cdots, R_n(t)$, then

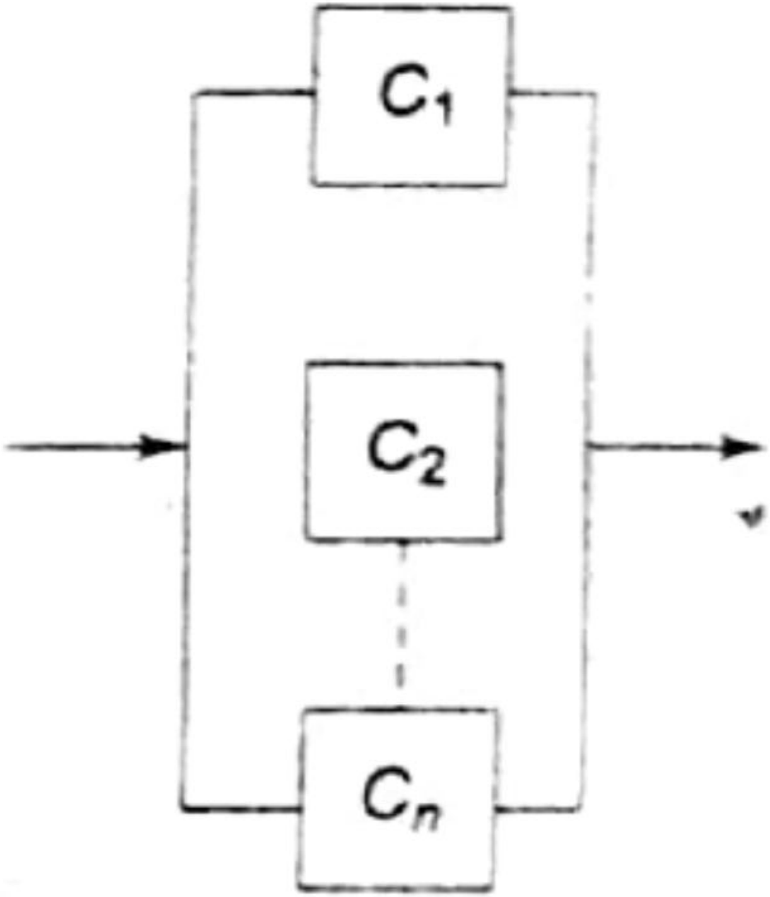
$$R_s(t) = R_1(t) \times R_2(t) \times \cdots \times R_n(t)$$

$$\leq \min \{R_1(t), R_2(t), \cdots, R_n(t)\} \qquad [\because 0 < R_i(t) < 1]$$

i.e., the system reliability will not be greater than the smallest of the component reliabilities.

Parallel or Redundant Configuration

Parallel or *redundant* configuration is one in which the components of the system are connected in parallel as shown in the following reliability block diagram.



Parallel or Redundant Configuration

In parallel configuration, all components must fail for the system to fail. This means that if one or more components function, the system continues to function.

Taking $n = 2$ and denoting the system reliability by R_p ('p' for parallel configuration), we have

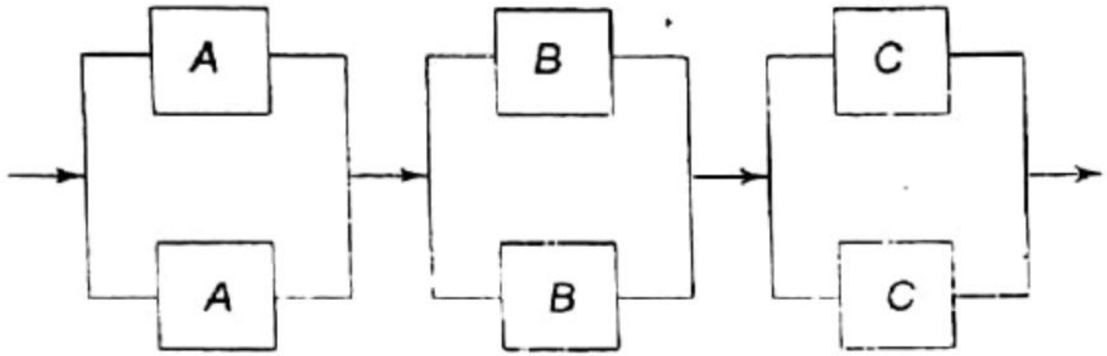
$$
\begin{aligned}
R_p &= P(C_1 \text{ or } C_2 \text{ or both function}) \\
&= P(C_1 \cup C_2) \\
&= P(C_1) + P(C_2) - P(C_1 \cap C_2) \\
&= P(C_1) + P(C_2) - P(C_1)\,P(C_2), \text{ since } C_1 \text{ and } C_2 \text{ are independent} \\
&= R_1 + R_2 - R_1 R_2 = 1 - (1 - R_1)(1 - R_2)
\end{aligned}
$$

Extending to n components, we have

$$
\begin{aligned}
R_p &= 1 - (1 - R_1)(1 - R_2) \cdots (1 - R_n) \\
&\geq \text{Max}\,\{R_1, R_2, \cdots, R_n\}
\end{aligned}
$$

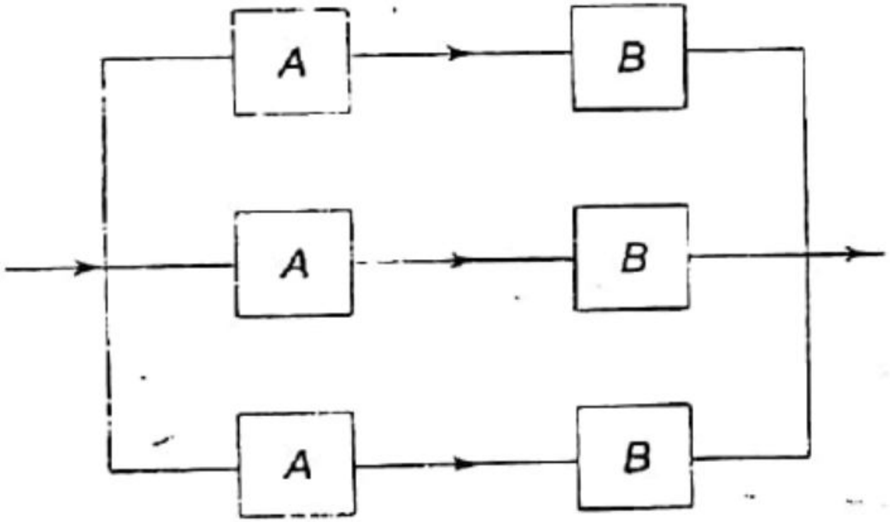
Parallel Series Configuration

A system, in which m subsystems are connected in series where each subsystem has n components connected in parallel as in shown Fig. is said to be in parallel series configuration or *low-level redundancy*.



Series Parallel Configuration

A system, in which m subsystems are connected in parallel where each subsystem has n components connected in series as in Fig. , is said to be in series-parallel configuration or *high level redundancy*.



Example:

There are 16 components in a non-redundant system. The average reliability of each component is 0.99. In order to achieve at least this system reliability using a redundant system with 4 identical new components, what should be the least reliability of each now component?

Solution:

For the non-redundant system,

$$R_s = R^{16} = (0.99)^{16} = 0.85$$

Let the new components have a reliability of R' each.

Then for the redundant system with 4 components, $R_p \geq 0.85$

i.e., $$1 - (1 - R')^4 \geq 0.85$$

i.e., $$(1 - R')^4 \leq 0.15$$

$$\therefore \quad 1 - R' \leq (0.15)^{\frac{1}{4}} \text{ or } 0.62$$

$$\therefore \quad R' \geq 0.38$$

i.e., the reliability of each of the new components should be at least 0.38.

Exercise:

Thermocouples of a particular design have a failure rate of 0.008 per hour. How many thermocouples must be placed in parallel if the system is to run for 100 hours with a system failure probability of no more than 0.05? Assume that all failures are independent.

Maintainability

No equipment (system) can be perfectly reliable in spite of the utmost care and best effort on the part of the designer and manufacturer. In fact, very few systems are designed to operate without maintenance of any kind. For a large number of systems, maintenance is a must, as it is one of the effective ways of increasing the reliability of the system.

Usually, two kinds of maintenance are adopted. They are *preventive maintenance* and *corrective or repair maintenance*. Preventive maintenance is maintenance done periodically before the failure of the system, so as to increase the reliability of the system by removing the ageing effects of wear, corrosion, fatigue and related phenomena. On the other hand, repair maintenance is performed after the failure has occurred so as to return the system to operation as soon as possible.

The amount and type of maintenance that is used depends on the respective costs and safety consideration of system failure. It is generally assumed that a preventive maintenance action is less costly than a repair maintenance action.

Reliability Under Preventive Maintenance

Let $R(t)$ and $R_M(t)$ be the reliability of a system without maintenance and with maintenance.

Let the preventive maintenance be performed on the system at intervals of T.

Since $R_M(t) = P\{$the maintained system does not fail before $t\}$, we have

$$R_M(t) = R(t), \text{ for } 0 \le t < T$$

$$= \boxed{R(T), \text{ for } t = T.}$$

After performing the first maintenance operation at T, the system becomes as good as new.

Hence, if $T \le t < 2T$,

$R_M(t) = P\{$the system does not fail upto T and it survives for a time $(t - T)$ without failure$\}$

$$= R(T) \cdot R(t - T), \text{ for } T \le t < 2T$$

Similarly after two maintenance operations,

$$R_M(t) = \{R(T)\}^2 \cdot R(t - 2T), \text{ for } 2T \le t < 3T$$

Proceeding like this, we get in general,

$$R_M(t) = \{R(T)\}^n \cdot R(t - nT), \text{ for } nT \le t < (n + 1)T \qquad (n = 0, 1, 2$$

Reliability Under Preventive Maintenance

$$\boxed{R_M(t) = \{R(T)\}^n \times R(t-nT)}$$

for $T \rightarrow$ Period

n = No. of Maintenance

$\underline{nT < t < (n+1)T}$, $n = 0, 1, 2, \ldots$

$\downarrow$

Period wise

MTTF of a System with Preventive Maintenance

$$\boxed{\text{MTTF} = \int_0^{\infty} R_M(t)\, dt}$$

$$= \sum_{n=0}^{\infty} \int_{nT}^{(n+1)T} R_M(t)\, dt, \text{ by dividing the range into intervals of length } T$$

$$= \sum_{n=0}^{\infty} \int_{nT}^{(n+1)T} \{R(T)\}^n R(t - nT)\, dt$$

$$= \sum_{n=0}^{\infty} \{R(T)\}^n \int_0^{T} R(t')\, dt', \text{ on putting } t - nT = t'$$

$$\boxed{\text{MTTF} = \frac{\int_0^T R(t)\, dt}{1 - R(T)}}$$

Reliability Under Repair or Corrective Maintenance

Maintainability

A measure of how fast a component (system) may be repaired following failure is known as *maintainability*. Repairs require different lengths of time and even the time to perform a given repair is uncertain (random), because circumstances, skill level, experience of maintenance personnel and such other factors vary. Hence the time T required to repair a failed component (system) is a continuous R.V.

Maintainability is mathematically defined as the cumulative distribution function (*cdf*) of the R.V.T. representing the time to repair and denoted as $M(t)$

i.e.,
$$\boxed{M(t) = P\{T \leq t\} = \int_0^t m(t)\, dt}$$

where $m(t)$ is the *pdf* of T.

pdf of T

CRV – time to Repair

MTTR and Repair Rate Function

The expected value of repair time T is called *the mean time to repair* (MTTR) and is given by

$$\boxed{\text{MTTR} = E(T) = \int_0^t t \times m(t)\, dt}$$

If the conditional probability that the (component) system will be repaired (made operational) between t and $t + \Delta t$, given that it has failed at t and the repair starts immediately, is $\mu(t)\,\Delta t$, then $\mu(t)$ is called the *instantaneous repair rate* or *simply the repair rate* and denotes the number of repairs in unit time.

i.e., $$\mu(t)\,\Delta t = \frac{P\{t \le T \le t + \Delta t\}}{P(T > t)}$$

$$\boxed{\mu(t) = \frac{m(t)}{1 - M(t)}}$$

$$m(t) = \frac{d}{dt} M(t)$$

$$\mu(t) = \frac{M'(t)}{1 - M(t)}$$

$$M(t) = 1 - e^{-\int_0^t \mu(t)\, dt}$$

$$m'(t) = \mu(t) \cdot e^{-\int_0^t \mu(t)\, dt}$$

Example:

If a device has a failure rate of

$$\lambda(t) = (0.015 + 0.02t)/\text{year}, \text{ where } t \text{ is in years,}$$

(a) Calculate the reliability for a 5 year design life, assuming that no maintenance is performed.

(b) Calculate the reliability for a 5 year design life, assuming that annual preventive maintenance restores the device to an as-good as new condition.

(c) Repeat part (b) assuming that there is a 5% chance that the preventive maintenance will cause immediate failure.

Solution:

(a)

$$R(t) = e^{-\int_0^t \lambda(t)\,dt}$$

$$R(5) = e^{-\int_0^5 (0.015+0.02t)\,dt}$$

$$= e^{-(0.015 \times 5 + 0.01 \times 25)}$$

$$= e^{-0.325} = 0.7225$$

(b) Since annual preventive maintenance is performed, there will be 4 preventive maintenances in the first 5 years.

$$R_M(t) = \{R(T)\}^n \times R(t - nT), \text{ after } n \text{ maintenances}$$

Here $t = 5,\ T = 1$ and $n = 4$

$\therefore$ $$R_M(5) = \{R(1)\}^4 \times R(5 - 4)$$

$$= \{R(1)\}^5$$

$$= \{e^{-0.025}\}^5, \text{ using (1)}$$

$$= 0.8825.$$

Solution (Continued):

(c) P {preventive maintenance causes immediate failure} $= 0.05$

$\therefore P$ {the device survives after each preventive maintenance} $= 0.95$

As there are 4 maintenances,

$$R_M(5) = R_M(5) \text{ without breakdown} \times \text{probability of no breakdown in 5 years}$$

$$= 0.8825 \times (0.95)^4$$

$$= 0.7188.$$

Example:

The time to repair a power generator is best described by its *pdf*

$$m(t) = \frac{t^2}{333}, \quad 1 \le t \le 10 \text{ hours}$$

(a) Find the probability that a repair will be completed in 6 hours.
(b) What is the MTTR?
(c) Find the repair rate.

(a). $P(T \le 6) = ?$

$M(6) = ?$

$M(t) = P(T \le t)$

$= \int_0^t m(t)\,dt$

Solution:

(a) $P(T<6) = P(1 \leq T < 6)$, where T is the time to repair

$$= \int_1^6 m(t)\, dt$$

$$= \int_1^6 \frac{t^2}{333}\, dt = \left(\frac{t^3}{999}\right)_1^6 = 0.2152$$

$\parallel$

$M(6)$

(b) $$\text{MTTR} = \int_0^\infty tm(t)\, dt = \int_1^{10} \frac{t^3}{333} dt = \left(\frac{t^4}{4 \times 333}\right)_1^{10}$$

$$= 7.5 \text{ hours}$$

(c) $$\text{Repair rate} = \mu(t) = \frac{m(t)}{1 - M(t)}$$

$$= \frac{t^2/333}{\int_t^{10} \frac{t^2}{333} dt} = \frac{t^2/333}{\frac{1}{999}\left(10^3 - t^3\right)}$$

$$= \frac{3t^2}{1000 - t^3} \text{ per hour.}$$

$$\boxed{1 - M(t) = 1 - \int_0^t m(t)\,dt = \int_t^\infty m(t)\,dt}$$

Exercise:

A reliability engineer has determined that the hazard rate function for a milling machine is $\lambda(t) = 0.0004521t^{0.8}$, $t \geq 0$, where t is measured in years. Determine which of the following options will provide the greatest reliability over the machine's 20 years operating life.

Option A : Do nothing-operate the machine until in fails.

Option B: An annual preventive maintenance program (with no maintenance-induced failures)

Option C: Operate a second machine in parallel with the first (active redundant).

Availability

Closely associated with the reliability of repairable (maintained) systems is concept of *availability*. Like reliability and maintainability, availability is also a probability.

Availability is defined as the probability that a component (or system) is performing its intended function at a given time 't' on the assumption that it is operated and maintained as per the prescribed conditions. This is referred to as *point availability* and denoted by $A(t)$

It is to be observed that reliability is concerned with failure-free operation up to time t, whereas availability is concerned with the capability to operate at the point of time t.

If $A(t)$ is the point availability of a component (or system), then

$$\boxed{A(t_2 - t_1) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} A(t)\, dt}$$

is called the *interval availability* or *mission availability*.

In particular, the interval availability over the interval $(0, T)$ is

$$\boxed{A(T) = \frac{1}{T} \int_0^T A(t) \cdot dt}$$

Now $\lim_{T \to \infty} A(t)$ is called the *steady-state* or *asymptotic* or *long-run availability* and denoted by A or $A(\infty)$.

Availability Function of a Single Component (or System)

Constants $\left\{ \begin{array}{l} \lambda \rightarrow \text{Harzard Function} \\ \mu \rightarrow \text{Repair Rate Function} \end{array} \right.$

Point Availability

$$A(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

Interval Availability Over (0,*T*)

$$A(T) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{(\lambda + \mu)^2 \times T} \{1 - e^{-(\lambda + \mu)T}\}$$

Steady-State Availability

$$A(\infty) = \frac{1/\lambda}{1/\lambda + 1/\mu} = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$$

Example:

Reliability testing has indicated that a voltage inverter has a 6 month reliability of 0.87 without repair facility. If repair facility is made available with an MTTR of 2.2 months, compute the availability over the 6-month period. (Assume constant failure and repair rates)

Soln/.

E.D (λ or μ)

$$R(6) = 0.87 = e^{-\lambda t} = e^{-\lambda 6} \Rightarrow \lambda = ?$$

$$MTTR = 1/\mu = 2.2 \Rightarrow \mu = 1/2.2$$

$$A(0,6) = A(T=6) = ?$$

Solution:

For constant failure rate λ, reliability is given by $R(t) = e^{-\lambda t}$.

As $\quad R(6) = 0.87,\ e^{-6\lambda} = 0.87$

$\therefore \quad \lambda = 0.0232/\text{month}$

$$\text{MTTR} = \frac{1}{\mu} = 2.2 \ \therefore\ \mu = 0.4545/\text{month}$$

Interval availability over $(0, T)$ is given by

$$A(T) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{(\lambda + \mu)^2 T}\{1 - e^{-(\lambda + \mu)T}\}$$

$$A(6) = \frac{0.4545}{0.4777} + \frac{0.0232}{(0.4777)^2 \times 6}\{1 - e^{-0.4777 \times 6}\}$$

$$= 0.967$$

Example:

A new computer has a constant failure rate of 0.02 per day (assuming continuous use) and a constant repair rate of 0.1 per day.

Compute the interval availability for the first 30 days and the steady-state availability.

Solution:

$\lambda = 0.02, \mu = 0.1$ & $T = 30$

$$A_I(T) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{(\lambda + \mu)^2 \times T}\{1 - e^{-(\lambda + \mu)T}\}$$

$$A_I(30) = \frac{0.1}{0.12} + \frac{0.02}{(0.12)^2 \times 30}\{1 - e^{-0.12 \times 30}\}$$

$$= 0.8784$$

$$A(\infty) = \frac{\mu}{\lambda + \mu} = \frac{0.1}{0.12} = 0.8333$$

Exercise:

The distribution of the time to failure of a component is Weibull with $\beta = 2.4$ and $\theta = 400$ hours and the repair distribution is lognormal with $t_M = 4.8$ hours and $s = 1.2$. Find the steady-state availability.