VIT UNIVERSITY

Final Assessment Test (FAT) - May 2017

Course: MAT1014 - Discrete Mathematics and Graph Theory

Class NBR(s):4419 / 4422 / 5327

Time: Three Hours

Max. Marks: 100

Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. a) Obtain the pdnf and pcnf of the following formula and hence conclude whether it is a tautology $(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$ [10]

   b) Test the consistency of the following Statements. [10]

      i. If Jack studies well he will pass in exams.

      ii. If Jack studies well he will get a job.

      iii. Succeeding in exam and getting a job simultaneously are not possible for him.

      iv. Jack either enjoys or studies well.

      v. Finally, Jack enjoys. [10]

2. a) Show that the conclusion $(x)(F(x) \to \neg S(x))$ follows from

      (i) $(x)(F(x) \wedge S(x)) \to (y)(M(y) \to W(y))$ and

      (ii) $(\exists y)(M(y) \wedge \neg W(y))$ [10]

   b) Show that

      (i) $\neg(P \wedge Q) \to (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$

      (ii) $(P \vee Q) \wedge (\neg P \wedge (\neg P \wedge Q)) \Leftrightarrow (\neg P \wedge Q)$ [4]

3. a) (i) Prove that for any monoid $\langle M, * \rangle$, no two rows or columns of the composition table are identical. [6]

      (ii) Establish the isomorphism between the following two algebraic systems:

      I. $\langle F, \circ \rangle$ where $F = \{f^0, f^1, f^2, f^3\}$ with $f = f^1 = \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 3,4\rangle, \langle 4,1\rangle\}$ and the composite functions are formed from the equation $f^k = f^{k-1} \circ f, k \geq 2$. Further, $f^0 = f^4$ is the identity map.

      II. $\langle Z_4, +_4 \rangle$, the algebraic system of equivalence classes generated by congruence modulo 4 under addition modulo 4. [14]

   b) State and prove Lagrange's theorem. [10]

4. a) What is the condition for a code to correct 'k' or fewer errors. Generate a single error correcting code with m = 4 and n = 7. [10]

   b) Obtain the Hasse diagrams of the lattices $\langle S_n, D \rangle$ when $n = 30, 45$. Which of these are complemented? Are these lattices distributive? Explain. [5]

5. a) (i) State and prove the isotonicity property of a lattice $\langle L, \leq \rangle$. [5]

      (ii) Obtain the simplified Boolean expression which is equivalent to the expression $m_0 + m_1 + m_2 + m_3$ [10]

   b) Obtain the Karnaugh map for the Boolean function $f = x_1 \circ [x_2 + (x_3 \circ x_4)]$.

6. a) (i) Prove that any simple graph with n vertices has at most $\frac{n(n-1)}{2}$ edges.

      (ii) Prove that the graph K_5 is nonplanar.