VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Department of Mathematics
School of Advanced Sciences
Fall Semester 2018 – 19
Continuous Assessment Test 1

Course: Calculus for Engineers | **Course Code:** MAT1011
Slot: | **Date:** _____
Max. Marks: 50 | **Time:** 90 minutes

Answer all the Questions.
Each question carries 10 *marks*

1. Find the values of a, m and b such that function $f(x)$ satisfies the hypotheses of the mean value theorem on [0,2], and hence an appropriate constant c, where
$$f(x)=\begin{cases}3, & x=0\\ a+3x-x^2, & 0<x<1\\ mx+b, & 1\le x\le 2.\end{cases}$$

2. Find the area of the region $\mathcal{R}$, bounded by the curve $y=4-x^2$, the y-axis and the line $y = 1$. If $\mathcal{R}$ is revolved about the y-axis, find the volume of the solid so generated from $x = 0$ to $y=\sqrt{3}$. Is this volume the same as the volume of the solid of revolution of $\mathcal{R}$ about the x-axis?

3. (a) Find the Laplace transform of $e^{-2t}t\sin^2 7t$.
   (b) Find the Laplace transform of $\frac{1-\cosh 3t}{t}$, and hence evaluate
$$\int_0^\infty\left(\frac{e^{-5t}-e^{-2t}-e^{-8t}}{t}\right)dt.$$

4. Show that $\mathcal{L}\{f(t)\}=\frac{1}{s}\tanh\left(\frac{\pi s}{4}\right)$, where f is a periodic signal with period π, whose definition in one period is given by
$$f(t)=\begin{cases}-1, & 0<x<\pi/2\\ 1, & \frac{\pi}{2}<x<\pi.\end{cases}$$

5. Obtain the inverse of the Laplace transform $F(s)=\frac{s^2+6}{(s^2+1)(s^2+4)}$ by using the convolution theorem, and verify your result with the method of partial fractions.