Representation theory of finite groups

Formalization project

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Outline

1 Representation Theory

2 Finite abelian groups

3 Formalization

4 Mathlib

5 Future work

Representations of finite groups

Definition

For a group G and a field k, a **representation** of G over k is a pair (V, ρ) where V is a vector space over k and $\rho : G \to \mathrm{GL}(V)$ is an action of G on V.

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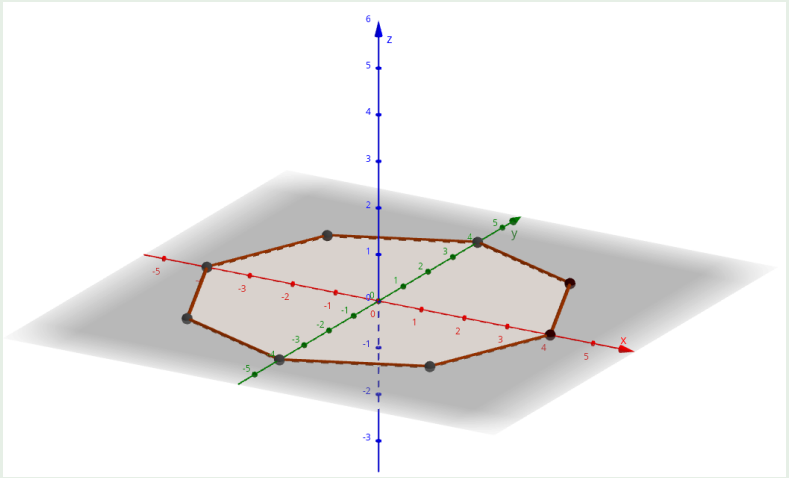
Convention: V has finite dimension, unless explicitly stated otherwise.

Definition

$\dim(V)$ is the **dimension** or **degree** of (V, ρ).

Example

$$D_{2n} = \langle a, b | a^n = b^2 = 1, bab = a^{-1} \rangle$$



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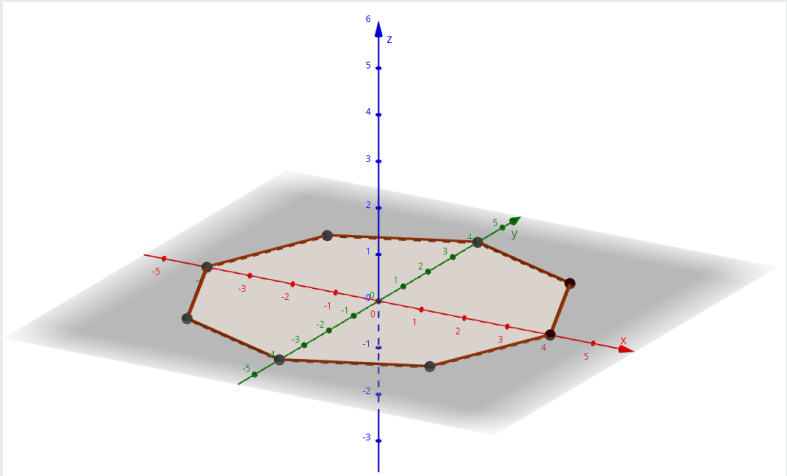

Representation $\rho : D_{2n} \to \mathrm{GL}(\mathbb{R}^3)$ with

- $\rho(a)$ as rotation about the Z-axis
- $\rho(b)$ as a rotation about a suitable axis in the XY-plane

Invariant subspaces, Irreducibility

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Let V be a representation and $U \subseteq V$ a subspace. U is an **invariant subspace** if $gu \in U$ for $\forall u \in U, g \in G$.

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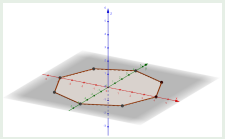

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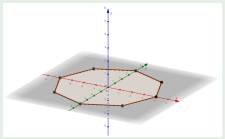

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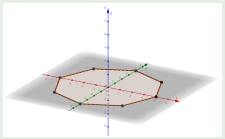

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A representation V is **irreducible** provided $V \neq 0$ and the only invariant subspaces are 0 and V.