Rational Algebraic Expressions Reviewer

Grade 8 Mathematics

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1. What Are Rational Algebraic Expressions?

A rational algebraic expression is a fraction where the numerator and/or denominator contain variables. Think of it like a regular fraction, but with algebra instead of just numbers.

Examples of Rational Expressions

- $\frac{3}{x}$ — numerator is a number, denominator has variable
- $\frac{x+2}{5}$ — numerator has variable, denominator is a number
- $2\frac{x}{x+3}$ — both numerator and denominator have variables
- $\frac{x^2-1}{x^2+4x+4}$ — polynomials in both

1.1. Important: Domain Restrictions

We can NEVER divide by zero. This means we must identify what values make the denominator equal to zero—these are not allowed.

Finding Domain Restrictions

For $\frac{5}{x-2}$:

The denominator is zero when $x - 2 = 0$, so $x = 2$.

Domain restriction: $x \neq 2$

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For $\frac{x+1}{(x-3)(x+5)}$:

The denominator is zero when $x = 3$ or $x = -5$.

Domain restrictions: $x \neq 3$ and $x \neq -5$

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2. Simplifying Rational Expressions

Before we do any operations, we should simplify rational expressions by canceling common factors.

2.1. Steps to Simplify

1. Factor the numerator completely
2. Factor the denominator completely
3. Cancel common factors
4. Write the simplified expression

Simplifying Rational Expressions

Simplify $\frac{x^2-1}{x+1}$

Step 1: Factor the numerator: $x^2 - 1 = (x + 1)(x - 1)$

Step 2: Denominator is already factored: $(x + 1)$

Step 3: Cancel the common factor $(x+1)$:

$\frac{(x+1)(x-1)}{x+1} = x - 1$

Step 4: Simplified expression: $x - 1$ (where $x \neq -1$)

Note: We still need to remember $x \neq -1$ even though it cancels!

More Complex Simplification

Simplify $\frac{6x^2+12x}{2x}$

Step 1: Factor numerator: $6x^2 + 12x = 6x(x+2)$

Step 2: Factor denominator: $2x = 2x$

Step 3: Cancel common factors:

$\frac{6x(x+2)}{2x} = \frac{3(x+2)}{1} = 3(x+2) = 3x + 6$

Step 4: Simplified: $3x + 6$ (where $x \neq 0$)

Practice Problem

Simplify each rational expression. State domain restrictions.

a) $\frac{x+3}{x^2+6x+9}$

b) $\frac{2x^2-8}{2x-4}$

c) $\frac{x^2-4}{x^2-5x+6}$

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3. Multiplying Rational Expressions

Multiplying rational expressions works just like multiplying fractions: multiply numerators together and denominators together.

3.1. Steps to Multiply

1. Factor all numerators and denominators completely
2. Write as one fraction (numerators × numerators, denominators × denominators)
3. Cancel common factors
4. Simplify

Formula: $\frac{A}{B} \times \frac{C}{D} = \frac{A \times C}{B \times D}$ (where $B \neq 0$ and $D \neq 0$)

Simple Multiplication

Multiply $\frac{3}{x} \times \frac{x^2}{6}$

Step 1: Already factored

Step 2: $\frac{3 \times x^2}{x \times 6} = \frac{3x^2}{6x}$

Step 3: Cancel common factors (3 and x):

$\frac{3x^2}{6x} = \frac{x}{2}$

Step 4: Answer: $\frac{x}{2}$ (where $x \neq 0$)

Multiplication with Polynomials

Multiply $\left(\frac{x+2}{x-3}\right) \times \left(\frac{x^2-9}{x^2-4}\right)$

Step 1: Factor everything:

- Numerator 1: $(x+2)$ ✓
- Denominator 1: $(x-3)$ ✓
- Numerator 2: $(x-3)(x+3)$ [DOTS]
- Denominator 2: $(x-2)(x+2)$ [QT]

Step 2: Write as one fraction:

$\frac{(x+2)(x-3)(x+3)}{(x-3)(x-2)(x+2)}$

Step 3: Cancel $(x+2)$ and $(x-3)$:

$\frac{x+3}{x-2}$

Step 4: Answer: $\frac{x+3}{x-2}$ (where $x \neq 2, -2, 3$)

Practice Problem

Multiply each pair of rational expressions. Simplify completely.

a) $\frac{5}{a^2} \times \frac{a^3}{10}$

b) $\left(\frac{x-1}{x+4}\right) \times \left(\frac{x+4}{x-1}\right)$

c) $\left(\frac{x^2-1}{x+3}\right) \times \left(\frac{x+3}{x-1}\right)$

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4. Dividing Rational Expressions

Dividing rational expressions uses a special trick: **flip the second fraction and multiply instead**.

4.1. Steps to Divide

1. Keep the first fraction as is
2. Change ÷ to ×
3. Flip (take the reciprocal of) the second fraction
4. Follow the multiplication steps: factor, combine, cancel, simplify

Formula: $\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C}$ (where $B \neq 0$, $C \neq 0$, $D \neq 0$)

Simple Division

Divide $\frac{4}{x} \div \frac{2}{x^3}$

Step 1-2: Keep first, flip second:

$\frac{4}{x} \times \frac{x^3}{2}$

Step 3: Combine:

$\frac{4 \times x^3}{x \times 2} = \frac{4x^3}{2x}$

Step 4: Simplify (cancel 2 and x):

$2x^2$ (where $x \neq 0$)

Division with Polynomials

Divide $\left(\frac{x^2-1}{x+2}\right) \div \left(\frac{x-1}{3x+6}\right)$

Step 1-2: Flip and change to multiply:

$\left(\frac{x^2-1}{x+2}\right) \times \left(\frac{3x+6}{x-1}\right)$

Step 3: Factor everything:
- $x^2 - 1 = (x-1)(x+1)$ [DOTS]
- $x + 2 = (x+2)$
- $3x + 6 = 3(x+2)$ [GCF]
- $x - 1 = (x-1)$

$\frac{(x-1)(x+1)}{x+2} \times \frac{3(x+2)}{x-1}$

Step 4: Cancel $(x-1)$ and $(x+2)$:

$3(x+1) = 3x + 3$ (where $x \neq -2, 1$)

Practice Problem

Divide each pair of rational expressions. Simplify completely.

a) $\frac{6}{a} \div \frac{3}{a^2}$

b) $\left(\frac{x+5}{x-2}\right) \div \left(\frac{x+5}{x+3}\right)$

c) $\left(\frac{x^2-4}{x+1}\right) \div \left(\frac{x-2}{2x+2}\right)$

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5. Adding Rational Expressions

Adding fractions with variables works the same as adding numeric fractions. We need a common denominator.

5.1. Case 1: Same Denominator

When denominators are identical, just add the numerators.

$\frac{A}{C} + \frac{B}{C} = \frac{A+B}{C}$

Adding with Same Denominator

Add $\frac{x}{x+1} + \frac{3}{x+1}$

Since both have denominator $(x+1)$:

$\frac{x+3}{x+1}$ (where $x \neq -1$)

5.2. Case 2: Different Denominators

When denominators are different, find the least common denominator (LCD).

Steps:
1. Factor all denominators
2. Find the LCD (use all factors that appear, using highest powers)
3. Rewrite each fraction with the LCD
4. Add the numerators
5. Simplify if possible

Different Denominators - Simple

Add $\frac{1}{2x} + \frac{3}{4x}$

Step 1-2: Factor denominators and find LCD:

- First denominator: $2x = 2 \cdot x$
- Second denominator: $4x = 4 \cdot x$
- LCD = $4x$ (higher power of each factor)

Step 3: Rewrite with LCD:

- First fraction: $\frac{1}{2x} = \frac{2}{4x}$
- Second fraction: Already $\frac{3}{4x}$

Step 4: Add:

$\frac{2}{4x} + \frac{3}{4x} = \frac{5}{4x}$ (where $x \neq 0$)

Different Denominators - Polynomial

Add $\frac{2}{x-1} + \frac{3}{x+2}$

Step 1: Denominators already factored

Step 2: LCD = $(x-1)(x+2)$

Step 3: Rewrite each fraction:

- First: $\frac{2(x+2)}{(x-1)(x+2)} = \frac{2x+4}{(x-1)(x+2)}$
- Second: $\frac{3(x-1)}{(x-1)(x+2)} = \frac{3x-3}{(x-1)(x+2)}$

Step 4: Add numerators:

$\frac{(2x+4)+(3x-3)}{(x-1)(x+2)} = \frac{5x+1}{(x-1)(x+2)}$

Step 5: Cannot simplify further

Answer: $\frac{5x+1}{(x-1)(x+2)}$ (where $x \neq 1, -2$)

Practice Problem

Add each pair of rational expressions. Simplify completely.

a) $\frac{2}{x} + \frac{5}{x}$

b) $\frac{1}{3x} + \frac{1}{6x}$

c) $\frac{2}{x+1} + \frac{1}{x-1}$

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6. Subtracting Rational Expressions

Subtracting rational expressions works the same as adding, except we subtract the numerators instead.

6.1. Key Point: Distribute the Negative

When subtracting, be careful to distribute the negative sign to all terms in the second numerator.

Formula: $\frac{A}{C} - \frac{B}{C} = \frac{A-B}{C}$

Subtracting - Same Denominator

Subtract $\frac{(x+2)}{x} - \frac{3}{x}$

$\frac{(x+2)-3}{x} = \frac{x-1}{x}$ (where $x \neq 0$)

Subtracting - Different Denominators

Subtract $\frac{x}{x+1} - \frac{2}{x+2}$

Step 1-2: LCD = $(x+1)(x+2)$

Step 3: Rewrite:

- First: $\frac{x(x+2)}{(x+1)(x+2)} = \frac{x^2+2x}{(x+1)(x+2)}$
- Second: $\frac{2(x+1)}{(x+1)(x+2)} = \frac{2x+2}{(x+1)(x+2)}$

Step 4: Subtract (distribute the negative!):

$\frac{(x^2+2x)-(2x+2)}{(x+1)(x+2)} = \frac{x^2-2}{(x+1)(x+2)}$

Answer: $\frac{x^2-2}{(x+1)(x+2)}$ (where $x \neq -1, -2$)

Practice Problem

Subtract each pair of rational expressions. Simplify completely.

a) $\frac{5}{x} - \frac{2}{x}$

b) $\frac{3}{2a} - \frac{1}{4a}$

c) $\frac{x}{x+3} - \frac{1}{x}$

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7. Complete Operations: Strategy Summary

When working with rational expressions:

1. **Always simplify first** if possible
2. **Identify what operation** you're doing
3. **For multiplication/division:** Factor everything, multiply/divide, cancel common factors
4. **For addition/subtraction:** Find LCD, rewrite fractions, combine, simplify
5. **Always state domain restrictions** (values that make denominator zero)

Complex Mixed Operations

Simplify $\left(\frac{(x^2-1)}{x+2}\right) \times \left(\frac{x+2}{x-1}\right) + \frac{1}{x+1}$

Step 1: Do multiplication first (order of operations):

Factor: $\frac{(x-1)(x+1)}{x+2} \times \frac{x+2}{x-1}$

Cancel $(x-1)$ and $(x+2)$: $(x+1)$

Step 2: Now add: $(x+1) + \frac{1}{x+1}$

Rewrite $x+1 = \frac{(x+1)^2}{x+1}$

$\frac{((x+1)^2+1)}{x+1} = \frac{x^2+2x+2}{x+1}$

Answer: $\frac{x^2+2x+2}{x+1}$ (where $x \neq -1, -2$)

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8. Review Problems

Directions: Simplify each expression. State domain restrictions and show all work.

1. Simplify: $\frac{x^2-4}{x+2}$
2. Multiply: $3\frac{x}{4} \times \frac{8}{x^2}$
3. Divide: $\left(\frac{a+1}{a-2}\right) \div \left(\frac{a+1}{a+3}\right)$
4. Add: $\frac{2}{x} + \frac{3}{x}$
5. Subtract: $\frac{5}{a+1} - \frac{2}{a+1}$
6. Multiply: $\left(\frac{x^2-9}{x+2}\right) \times \left(\frac{x+2}{x-3}\right)$
7. Divide: $\left(\frac{x^2-1}{2x}\right) \div \left(\frac{x+1}{4}\right)$
8. Add: $\frac{1}{2x} + \frac{3}{4x}$
9. Subtract: $\frac{x}{x+1} - \frac{1}{x}$
10. Multiply then Add: $\frac{x}{2} \times \frac{4}{x} + \frac{1}{x}$

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9. Answer Key

1. $x-2$ (where $x \neq -2$)
2. $\frac{6}{x}$ (where $x \neq 0$)
3. $\frac{a+3}{a-2}$ (where $a \neq -1, 2, -3$)
4. $\frac{5}{x}$ (where $x \neq 0$)
5. $\frac{3}{a+1}$ (where $a \neq -1$)
6. $(x+3)$ (where $x \neq -2, 3$)
7. $2(x-1)$ (where $x \neq 0, -1$)
8. $\frac{5}{4x}$ (where $x \neq 0$)
9. $\frac{x-1}{x}$ (where $x \neq 0, -1$)
10. $2 + \frac{1}{x} = \frac{2x+1}{x}$ (where $x \neq 0$)