Factoring Polynomials Reviewer

Grade 8 Mathematics

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1. Understanding Polynomials Before We Factor

Before we factor, we need to understand what a polynomial is. A polynomial is an expression made up of terms added or subtracted together. Each term has a coefficient (number) and variables with exponents.

Example: $3x^2 + 7x - 5$

- First term: $3x^2$ (coefficient is 3)
- Second term: $7x$ (coefficient is 7)
- Third term: -5 (this is a constant)

When we factor, we're finding what we can pull out of all terms. Think of it like finding common ingredients in a recipe.

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2. Method 1: Greatest Common Factor (GCF)

2.1. What is GCF?

The Greatest Common Factor is the largest number or variable that divides evenly into all terms. It's like finding the biggest piece that all terms share.

Understanding GCF

Look at $12x^2 + 8x$

- What divides into 12? 1, 2, 3, 4, 6, 12
- What divides into 8? 1, 2, 4, 8
- What divides into both? 1, 2, 4 → **Greatest is 4**

Both terms also have at least one x, so our GCF is $4x$.

2.2. Steps for GCF Factoring

1. Find the GCF of all coefficients
2. Find the lowest power of each variable that appears in every term
3. Factor out the GCF
4. Write what's left in parentheses

Detailed Example

Factor $6x^3 + 9x^2 - 12x$

Step 1: GCF of numbers (6, 9, 12)
- 6 = 2 × 3
- 9 = 3 × 3
- 12 = 2 × 2 × 3
- Common factor: 3

Step 2: Each term has at least one $x \rightarrow x^1$

Step 3: GCF = $3x$

Step 4: $6x^3 + 9x^2 - 12x = 3x(2x^2 + 3x - 4)$

Practice Problem

Factor each polynomial using GCF:

a) $5a + 10b$

b) $4x^2 + 8x$

c) $15m^3 + 10m^2 - 5m$

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3. Method 2: Difference of Two Squares (DOTS)

3.1. What is DOTS?

DOTS stands for **Difference OF Two Squares**. This is when you have two perfect squares being subtracted.

The pattern: $a^2 - b^2 = (a + b)(a - b)$

3.2. Recognizing Perfect Squares

A perfect square is a number or expression that can be written as something times itself.

Perfect Square Recognition

- $x^2 = x \times x \to$ perfect square
- $4 = 2 \times 2 = 2^2 \to$ perfect square
- $9x^2 = 3x \times 3x = (3x)^2 \to$ perfect square
- $x = x^1 \to$ NOT a perfect square
- $3x^2 \to$ NOT a perfect square (3 isn't a perfect square)

3.3. How to Factor Using DOTS

1. Confirm both terms are perfect squares
2. Confirm there's a minus sign between them
3. Take the square root of each term
4. Write as (square root 1 + square root 2)(square root 1 - square root 2)

Detailed Example

Factor $x^2 - 25$

Step 1: Is x^2 a perfect square? Yes, square root is x

Step 2: Is 25 a perfect square? Yes, square root is 5

Step 3: Do we have a minus sign? Yes

Step 4: $x^2 - 25 = (x + 5)(x - 5)$

More Complex Example

Factor $9a^4 - 16b^2$

Step 1: square root of $9a^4 = 3a^2$

Step 2: square root of $16b^2 = 4b$

Step 3: Both are perfect squares with minus sign ✓

Step 4: $9a^4 - 16b^2 = (3a^2 + 4b)(3a^2 - 4b)$

Practice Problem

Factor each polynomial using DOTS:

a) $x^2 - 16$

b) $4a^2 - 9$

c) $m^2 - 100n^2$

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4. Method 3: Sum and Difference of Cubes (SDOTC)

4.1. What is SDOTC?

SDOTC handles **Sum** and **Difference** of cubes. These are similar to DOTS but with cubes instead of squares.

- **Sum of Cubes:** $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- **Difference of Cubes:** $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

4.2. Recognizing Perfect Cubes

A perfect cube is something multiplied by itself three times.

Perfect Cube Recognition

- $x^3 = x \times x \times x \to$ perfect cube
- $8 = 2 \times 2 \times 2 = 2^3 \to$ perfect cube
- $27a^3 = 3a \times 3a \times 3a = (3a)^3 \to$ perfect cube
- $64b^6 = 4b^2 \times 4b^2 \times 4b^2 = (4b^2)^3 \to$ perfect cube

4.3. How to Factor Using Sum of Cubes

For $a^3 + b^3$:

1. Find the cube roots: find a and b
2. Write the first binomial: $(a + b)$
3. For the trinomial: $(a^2 - ab + b^2)$
4. Answer: $(a + b)(a^2 - ab + b^2)$

Sum of Cubes Example

Factor $x^3 + 8$

Step 1: cube root of $x^3 = x$ and cube root of $8 = 2$

Step 2: First binomial: $(x + 2)$

Step 3: Trinomial: $(x^2 - 2x + 4)$
- $a^2 = x^2$
- $ab = x \times 2 = 2x$ (we put minus here)
- $b^2 = 2^2 = 4$

Step 4: $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$

4.4. How to Factor Using Difference of Cubes

For $a^3 - b^3$:

1. Find the cube roots: find a and b
2. Write the first binomial: $(a - b)$
3. For the trinomial: $(a^2 + ab + b^2)$ ← Note: plus signs here
4. Answer: $(a - b)(a^2 + ab + b^2)$

Difference of Cubes Example

Factor $27m^3 - 64$

Step 1: cube root of $27m^3 = 3m$ and cube root of $64 = 4$

Step 2: First binomial: $(3m - 4)$

Step 3: Trinomial: $(9m^2 + 12m + 16)$
- $a^2 = (3m)^2 = 9m^2$
- $ab = 3m \times 4 = 12m$ (plus here!)
- $b^2 = 4^2 = 16$

Step 4: $27m^3 - 64 = (3m - 4)(9m^2 + 12m + 16)$

Practice Problem

Factor each polynomial using SDOTC:

a) $a^3 + 27$

b) $b^3 - 125$

c) $8x^3 + 1$

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5. Method 4: Perfect Square Trinomials (PST)

5.1. What is a Perfect Square Trinomial?

A perfect square trinomial is when a binomial is squared. It follows a pattern:
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$

When factoring, we reverse this process to find the original binomial.

5.2. Recognizing PST

Look for:
1. First term is a perfect square
2. Last term is a perfect square
3. Middle term equals 2 × square root of first × square root of last

Recognizing PST

Is $x^2 + 6x + 9$ a perfect square trinomial?

- First term: $x^2 \to$ square root is x ✓
- Last term: $9 \to$ square root is 3 ✓
- Middle check: $2 \times x \times 3 = 6x \to$ matches! ✓

This IS a perfect square trinomial!

5.3. How to Factor PST

1. Take the square root of the first term
2. Take the square root of the last term
3. Determine the sign from the middle term
4. Write as $(a \pm b)^2$

PST Factoring Examples

Example 1: $x^2 + 8x + 16$

- square root of $x^2 = x$
- square root of $16 = 4$
- Middle term is positive: $+8x$
- Answer: $(x + 4)^2$

Example 2: $4a^2 - 12a + 9$

- square root of $4a^2 = 2a$
- square root of $9 = 3$
- Middle term is negative: $-12a$
- Check: $2 \times 2a \times 3 = 12a$ ✓
- Answer: $(2a - 3)^2$

Practice Problem

Factor each polynomial if it's a perfect square trinomial:

a) $m^2 + 10m + 25$

b) $4b^2 - 4b + 1$

c) $x^2 + 5x + 4$ (Is this PST?)

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6. Method 5: Quadratic Trinomials (QT)

6.1. What are Quadratic Trinomials?

A quadratic trinomial is a polynomial with three terms in the form $ax^2 + bx + c$ where a, b, and c are numbers.

When we can't use the other methods, we use the quadratic trinomial method.

6.2. The AC Method (Most Reliable)

Step 1: Multiply $a \times c$

Step 2: Find two numbers that multiply to give ac and add to give b

Step 3: Rewrite the middle term using these two numbers

Step 4: Factor by grouping

AC Method - Simple Example

Factor $x^2 + 5x + 6$

Step 1: $a = 1$, $c = 6 \to a \times c = 6$

Step 2: We need two numbers that:
- Multiply to give 6: (1,6), (2,3)
- Add to give 5 (the b value): 2 + 3 = 5 ✓

Step 3: Rewrite: $x^2 + 2x + 3x + 6$

Step 4: Factor by grouping:
- Group 1: $x^2 + 2x = x(x + 2)$
- Group 2: $3x + 6 = 3(x + 2)$
- Combined: $(x + 2)(x + 3)$

AC Method - More Complex

Factor $2x^2 + 7x + 3$

Step 1: $a = 2$, $c = 3 \to a \times c = 6$

Step 2: Two numbers that multiply to 6 and add to 7:
- Pairs: (1,6), (2,3)
- 1 + 6 = 7 ✓

Step 3: Rewrite: $2x^2 + 1x + 6x + 3$

Step 4: Factor by grouping:
- Group 1: $2x^2 + 1x = x(2x + 1)$
- Group 2: $6x + 3 = 3(2x + 1)$
- Combined: $(2x + 1)(x + 3)$

Check: $(2x + 1)(x + 3) = 2x^2 + 6x + x + 3 = 2x^2 + 7x + 3$ ✓

6.3. When the Leading Coefficient is 1

When $a = 1$ (so we have $x^2 + bx + c$), we can use a simpler method:

Find two numbers that multiply to c and add to b.

Simple Trinomial Factoring

Factor $x^2 - 7x + 10$

We need two numbers that:
- Multiply to 10: (1,10), (2,5)
- Add to −7: $-2 + (-5) = -7$ ✓

Answer: $(x - 2)(x - 5)$

Practice Problem

Factor each quadratic trinomial:

a) $x^2 + 7x + 12$

b) $2x^2 + 11x + 5$

c) $x^2 - 6x + 8$

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7. Putting It All Together: Factoring Strategy

When you see a polynomial to factor, follow these steps:

1. **Always check for GCF first!** If there's a common factor, pull it out.
2. **Count the terms:**
   - 2 terms? Check for DOTS or SDOTC
   - 3 terms? Check for PST or use QT method
   - 4+ terms? Try factoring by grouping
3. **After factoring, check if you can factor again.** Sometimes a factor can be factored further!

Complete Factoring Example

Factor completely: $2x^3 - 50x$

Step 1: GCF $\rightarrow 2x(x^2 - 25)$

Step 2: We have $(x^2 - 25)$ with 2 terms. Is this DOTS?
- x^2 is a perfect square ✓
- 25 is a perfect square ✓
- Minus sign ✓

Step 3: Factor DOTS: $(x - 5)(x + 5)$

Final Answer: $2x(x - 5)(x + 5)$

Practice Problem

Factor each polynomial completely:

a) $3a^2 - 27$

b) $x^3 + 6x^2 + 9x$

c) $4b^4 - 16b^2$

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8. Practice Review Problems

Directions: Factor each polynomial completely. State which method(s) you used.

1. $6x + 12$
2. $a^2 - 49$
3. $x^2 + 10x + 25$
4. $2x^2 + 5x + 2$
5. $m^3 - 27$
6. $5x^2 - 20x$
7. $9x^2 - 1$

8. $x^2 - 8x + 12$
9. $a^3 + 125$
10. $6x^3 + 12x^2 + 6x$

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9. Answer Key

1. $6x + 12 = 6(x + 2)$ [GCF]
2. $a^2 - 49 = (a + 7)(a - 7)$ [DOTS]
3. $x^2 + 10x + 25 = (x + 5)^2$ [PST]
4. $2x^2 + 5x + 2 = (2x + 1)(x + 2)$ [QT]
5. $m^3 - 27 = (m - 3)\big(m^2 + 3m + 9\big)$ [SDOTC]
6. $5x^2 - 20x = 5x(x - 4)$ [GCF]
7. $9x^2 - 1 = (3x + 1)(3x - 1)$ [DOTS]
8. $x^2 - 8x + 12 = (x - 2)(x - 6)$ [QT]
9. $a^3 + 125 = (a + 5)\big(a^2 - 5a + 25\big)$ [SDOTC]
10. $6x^3 + 12x^2 + 6x = 6x\big(x^2 + 2x + 1\big) = 6x(x + 1)^2$ [GCF, then PST]