Systems of Linear Equations Reviewer

Grade 8 Mathematics

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1. Understanding Systems of Linear Equations

A **system of linear equations** is when we have two or more equations with the same variables, and we need to find values that satisfy ALL equations at the same time.

A **solution** to a system is a point (or pair of values) where both equations are true.

What is a System?

System of equations:

$y = 2x + 1$

$y = -x + 4$

We need to find the values of x and y that make BOTH equations true at the same time.

The solution is $(x, y) = (1, 3)$ because:
- First equation: $3 = 2(1) + 1$ ✓ True
- Second equation: $3 = -(1) + 4$ ✓ True

1.1. Types of Solutions

A system of equations can have:
- **One solution:** The lines intersect at exactly one point
- **No solution:** The lines are parallel (never intersect)
- **Infinite solutions:** The lines are the same (identical equations)

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2. Method 1: Substitution Method

The substitution method works by solving one equation for one variable, then substituting that expression into the other equation.

2.1. Steps for Substitution

1. Choose an equation and solve for one variable
2. Substitute that expression into the other equation
3. Solve for the remaining variable
4. Substitute back to find the other variable
5. Write the solution as an ordered pair (x, y)
6. Check your solution in both original equations

Substitution Example 1

Solve the system:

$y = x + 2$

$2x + y = 8$

Step 1: The first equation is already solved for y: $y = x + 2$

Step 2: Substitute into the second equation:

$2x + (x + 2) = 8$

Step 3: Solve for x:

$2x + x + 2 = 8$

$3x + 2 = 8$

$3x = 6$

$x = 2$

Step 4: Find y by substituting $x = 2$ into the first equation:

$y = 2 + 2 = 4$

Step 5: Solution: $(2, 4)$

Step 6: Check:
- Equation 1: $4 = 2 + 2$ ✓
- Equation 2: $2(2) + 4 = 4 + 4 = 8$ ✓

Substitution Example 2

Solve the system:

$x + y = 5$

$2x - y = 4$

Step 1: Solve the first equation for y:

$x + y = 5$

$y = 5 - x$

Step 2: Substitute into the second equation:

$2x - (5 - x) = 4$

Step 3: Solve for x:

$2x - 5 + x = 4$

$3x - 5 = 4$

$3x = 9$

$x = 3$

Step 4: Find y:

$y = 5 - 3 = 2$

Step 5: Solution: $(3, 2)$

Step 6: Check:
- Equation 1: $3 + 2 = 5$ ✓
- Equation 2: $2(3) - 2 = 6 - 2 = 4$ ✓

Practice Problem

Solve each system using substitution:

a) $y = 2x$ and $x + y = 9$

b) $x = y - 3$ and $2x + y = 12$

c) $y = -x + 5$ and $3x + y = 11$

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3. Method 2: Elimination Method

The elimination method works by adding or subtracting equations to eliminate one variable, making it easier to solve.

3.1. Steps for Elimination

1. Arrange both equations in standard form ($ax + by = c$)
2. Look for a way to make one variable's coefficients opposites
3. Multiply one or both equations by a number if needed
4. Add the equations together to eliminate one variable
5. Solve for the remaining variable
6. Substitute back to find the other variable
7. Check your solution in both original equations

Elimination Example 1 - Variables Already Opposites

Solve the system:

$2x + y = 7$

$2x - y = 5$

Step 1: Already in standard form ✓

Step 2-3: Notice that y and $-y$ are opposites! No multiplication needed.

Step 4: Add the equations:

$(2x + y) + (2x - y) = 7 + 5$

$4x = 12$

Step 5: Solve for x:

$x = 3$

Step 6: Substitute into the first equation:

$2(3) + y = 7$

$6 + y = 7$

$y = 1$

Solution: $(3, 1)$

Step 7: Check:
- Equation 1: $2(3) + 1 = 6 + 1 = 7$ ✓
- Equation 2: $2(3) - 1 = 6 - 1 = 5$ ✓

Elimination Example 2 - Need to Multiply

Solve the system:

$3x + 2y = 11$

$x + 2y = 7$

Step 1: Already in standard form ✓

Step 2: To eliminate x, multiply the second equation by -3 to make x coefficients opposites:

Equation 1: $3x + 2y = 11$

Equation 2 (multiplied by -3): $-3x - 6y = -21$

Step 3: Multiply done ✓

Step 4: Add the equations:

$(3x + 2y) + (-3x - 6y) = 11 + (-21)$

$-4y = -10$

Step 5: Solve for y:

$y = \frac{-10}{-4} = \frac{10}{4} = \frac{5}{2} = 2.5$

Step 6: Substitute into equation 2:

$x + 2(2.5) = 7$

$x + 5 = 7$

$x = 2$

Solution: $(2, 2.5)$

Step 7: Check:
- Equation 1: $3(2) + 2(2.5) = 6 + 5 = 11$ ✓
- Equation 2: $2 + 2(2.5) = 2 + 5 = 7$ ✓

Elimination Example 3 - Multiply Both Equations

Solve the system:

$2x + 3y = 8$

$3x + 2y = 7$

Step 1: Already in standard form ✓

Step 2-3: To eliminate x:
- Multiply first equation by 3: $6x + 9y = 24$
- Multiply second equation by -2: $-6x - 4y = -14$

Step 4: Add the equations:

$(6x + 9y) + (-6x - 4y) = 24 + (-14)$

$5y = 10$

Step 5: Solve for y:

$y = 2$

Step 6: Substitute into the first equation:

$2x + 3(2) = 8$

$2x + 6 = 8$

$2x = 2$

$x = 1$

Solution: $(1, 2)$

Step 7: Check:
- Equation 1: $2(1) + 3(2) = 2 + 6 = 8$ ✓
- Equation 2: $3(1) + 2(2) = 3 + 4 = 7$ ✓

Practice Problem

Solve each system using elimination:

a) $x + y = 10$ and $x - y = 4$

b) $2x + y = 7$ and $x - y = 2$

c) $2x + 3y = 13$ and $x + 2y = 8$

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4. Method 3: Graphing Method

The graphing method involves plotting both equations on a coordinate plane. The point(s) where the lines intersect is the solution.

4.1. Steps for Graphing

1. Rewrite both equations in slope-intercept form: $y = mx + b$
2. Identify the slope (m) and y-intercept (b) for each line
3. Plot both lines on the same graph
4. Find the intersection point(s)
5. The intersection point (x, y) is your solution

Graphing Example

Solve the system by graphing:

$y = 2x - 1$

$y = -x + 2$

Step 1: Both equations are already in slope-intercept form ✓

Step 2:
- First line: $m = 2$, $b = -1$ (slope is 2, y-intercept is −1)
- Second line: $m = -1$, $b = 2$ (slope is −1, y-intercept is 2)

Step 3: Plot the lines:
- First line passes through $(0, -1)$ and $(1, 1)$
- Second line passes through $(0, 2)$ and $(1, 1)$

Step 4-5: The lines intersect at the point $(1, 1)$

Solution: $(1, 1)$

Verify:
- Equation 1: $1 = 2(1) - 1 = 1$ ✓
- Equation 2: $1 = -(1) + 2 = 1$ ✓

4.2. Converting to Slope-Intercept Form

If your equations aren't in $y = mx + b$ form, solve for y first.

Converting to Slope-Intercept Form

Convert $2x + y = 5$ to slope-intercept form:

$2x + y = 5$

$y = -2x + 5$

Now we can see $m = -2$ and $b = 5$

Practice Problem

Solve each system by graphing (describe the intersection):

a) $y = x$ and $y = -x + 2$

b) $y = 2x + 1$ and $y = x - 1$

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5. Word Problems Using Systems

Many real-world situations involve systems of equations. The key is translating English into mathematical equations.

5.1. Steps to Solve Word Problems

1. Define your variables (say what x and y represent)
2. Write two equations based on the problem
3. Solve the system using substitution or elimination
4. Check that your answer makes sense in the context
5. Write your answer in a complete sentence

Word Problem 1: Purchase Problem

Maria bought 3 notebooks and 2 pens for dollar sign 13. James bought 2 notebooks and 4 pens for dollar sign 16. Find the price of each item.

Step 1: Define variables
- Let x = price of one notebook
- Let y = price of one pen

Step 2: Write equations
- Maria's purchase: $3x + 2y = 13$
- James's purchase: $2x + 4y = 16$

Step 3: Solve using elimination

Multiply first equation by -2 and second by 3:
- $-6x - 4y = -26$
- $6x + 12y = 48$

Add them: $8y = 22$, so $y = 2.75$

Substitute: $3x + 2(2.75) = 13$

$3x + 5.5 = 13$

$3x = 7.5$

$x = 2.5$

Step 4: Check: Does this make sense?
- 3 notebooks ($3 \times 2.50 = 7.50$) plus 2 pens ($2 \times 2.75 = 5.50$) = dollar sign 13 ✓

Step 5: Answer: **A notebook costs dollar sign 2.50 and a pen costs dollar sign 2.75.**

Word Problem 2: Age Problem

The sum of two numbers is 24. The difference between them is 6. Find the numbers.

Step 1: Define variables
- Let x = the larger number
- Let y = the smaller number

Step 2: Write equations
- Sum: $x + y = 24$
- Difference: $x - y = 6$

Step 3: Solve using elimination

Add the equations:

$(x + y) + (x - y) = 24 + 6$

$2x = 30$

$x = 15$

Substitute: $15 + y = 24$, so $y = 9$

Step 4: Check:
- Sum: $15 + 9 = 24$ ✓
- Difference: $15 - 9 = 6$ ✓

Step 5: Answer: **The two numbers are 15 and 9.**

Word Problem 3: Distance/Rate/Time

A fast train and a slow train leave the same station. The fast train travels at 80 mph and the slow train at 60 mph. How long before they are 150 miles apart?

Step 1: Define variables
- Let t = time (in hours)
- Let d_f = distance traveled by fast train
- Let d_s = distance traveled by slow train

Step 2: Write equations
- Fast train distance: $d_f = 80t$
- Slow train distance: $d_s = 60t$
- Difference in distance: $d_f - d_s = 150$

Step 3: Substitute and solve

$80t - 60t = 150$

$20t = 150$

$t = 7.5$

Step 4: Check: Fast train goes $80 \times 7.5 = 600$ miles. Slow train goes $60 \times 7.5 = 450$ miles. Difference = 150 ✓

Step 5: Answer: **After 7.5 hours, the trains will be 150 miles apart.**

Practice Problem

Solve these word problems:

a) The sum of two numbers is 20. One number is 4 more than the other. Find the numbers.

b) Tickets cost dollar sign 5 for adults and dollar sign 3 for children. If 40 tickets were sold for a total of dollar sign 160, how many adult tickets and how many children's tickets were sold?

c) A rectangle has a perimeter of 36 inches. The length is 2 inches more than the width. Find the dimensions.

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6. Choosing the Best Method

- **Substitution:** Best when one equation is already solved for a variable or easily solved
- **Elimination:** Best when coefficients are easy to work with or already opposites
- **Graphing:** Best for visual understanding or when checking solutions

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7. Review Problems

Solve each system using the indicated method.

Substitution Method:

1. $y = 2x - 3$ and $x + y = 6$
2. $x = y + 2$ and $2x - y = 7$

Elimination Method:

1. $2x + y = 5$ and $x - y = 1$
2. $3x + 2y = 12$ and $x - 2y = 4$

Graphing Method:

1. $y = x + 1$ and $y = -x + 3$
2. $y = 2x - 2$ and $y = -x + 4$

Word Problems:

1. Two numbers have a sum of 30 and a difference of 8. Find the numbers.
2. A store sells apples for dollar sign 2 each and oranges for dollar sign 3 each. If you buy 8 pieces of fruit for dollar sign 20, how many apples and how many oranges did you buy?
3. The length of a rectangle is 5 inches more than its width. If the perimeter is 50 inches, find the length and width.

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8. Answer Key

Substitution:

1. $(3, 3)$ — Substitute $y = 2x - 3$ into $x + y = 6$ to get $x + 2x - 3 = 6$, so $3x = 9$, $x = 3$, $y = 3$
2. $(5, 3)$ — Substitute $x = y + 2$ into $2x - y = 7$ to get $2(y + 2) - y = 7 \Rightarrow 2y + 4 - y = 7 \Rightarrow y = 3$, then $x = 5$

Elimination:

1. $(2, 1)$ — Add equations: $3x = 6$, so $x = 2$. Substitute: $2 + y = 5$, $y = 1$
2. $(4, 0)$ — Add equations: $4x = 16$, so $x = 4$. Substitute: $3(4) + 2y = 12$, so $y = 0$

Graphing:

1. $(1, 2)$ — Lines intersect where $x + 1 = -x + 3$, so $2x = 2$, $x = 1$, $y = 2$
2. $(2, 2)$ — Lines intersect where $2x - 2 = -x + 4$, so $3x = 6$, $x = 2$, $y = 2$

Word Problems:

1. Answer: **The numbers are 19 and 11.**
2. Answer: **4 apples and 4 oranges**
3. Answer: **Length is 15 inches, width is 10 inches**