2.8 Switch or stick?

There are three cups and one token. Two students alternate between being player A (the host, quizmaster) and player B (the contestant) in the following game :

- At a turn player B closes his eyes while player A places the token under one of the cups and *remembers wich one*.
- Player B then opens their eyes, and points at one of the three cups.
- Player A lifts one of the *other* cups revealing it to be empty – this is always possible as at least one of the cups not chosen is bound to be empty, and if there are two empty cups, the host choses one at random.
- Player A then offers another opportunity for player B : either stick to his first choice, either switch cups.
- Having decided, player B either sticks or switches, lifts the chosen cup and records whether they got the token or not.

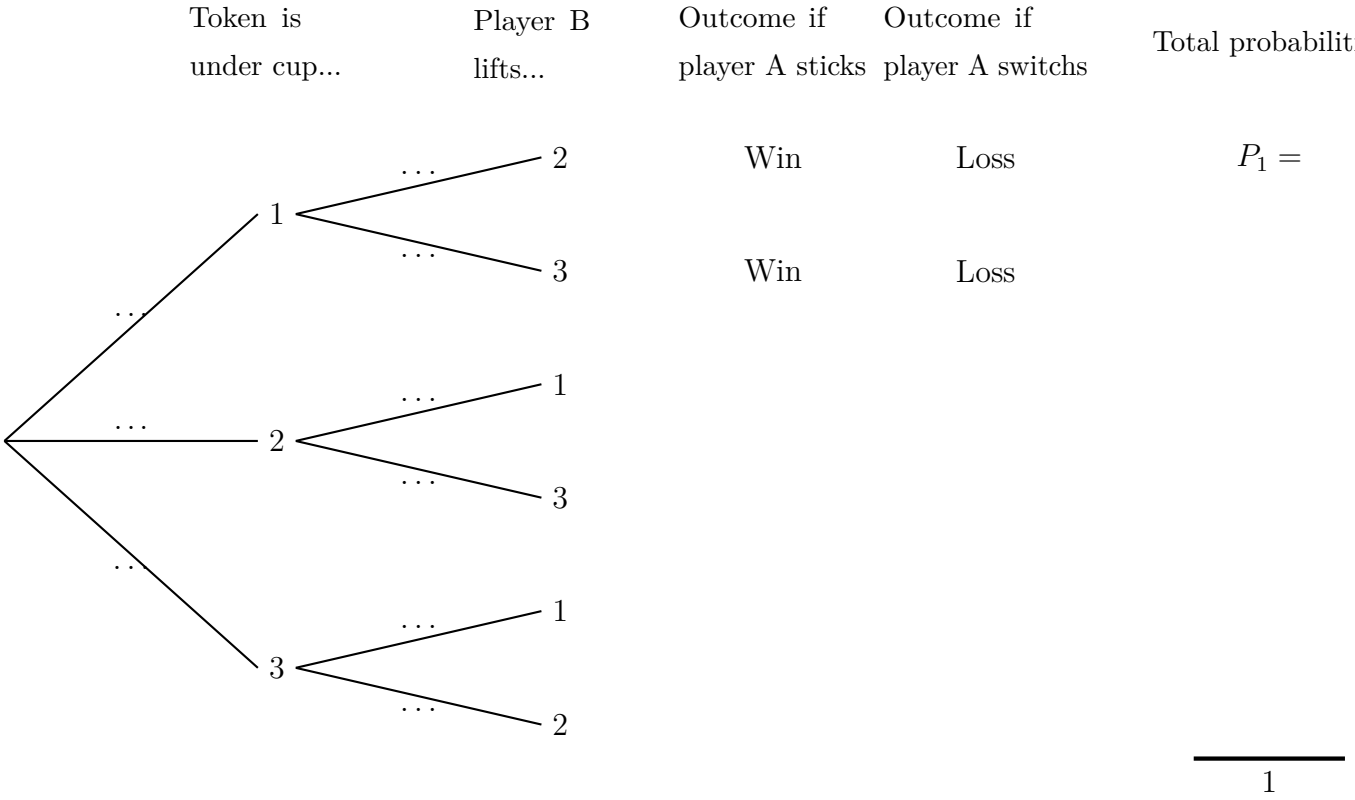
Should Player B switch or stick? Investigate.

You can start by playing a couple of games to understand the game.

Player hiding	**Player guessing**	**Switch or stick?**	**Win or lose?**

Stop after few games. Discuss your findings. Do you think it makes a difference?
Carry on with more games, and tally up at the end.

The probability tree shows the possible outcomes once player A made his first choice. For simplicity, we suppose player A chose cup 1.



a) Complete the tree to show the probabilities of all outcomes.

b) If player A always sticks to his first choice, what is the probability that he wins the token?

c) If player A always switches his choice after player B reveals an empty cup, what is the probability that he wins the token?

Wrapping up A possible numberphile explanation. https://youtu.be/7u6kFlWZOWg