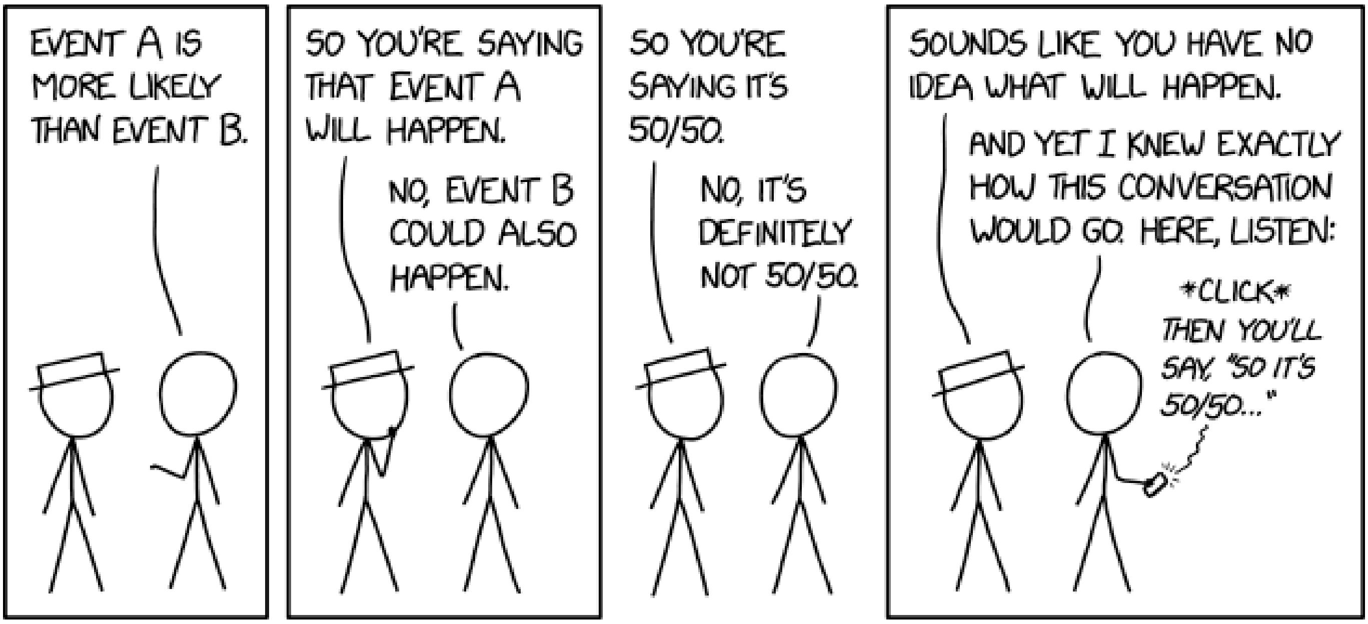
Unit

Probability 3

Notes

All taken from Jenny Gage and David Spiegelhalter's Teaching Probability[2]. These activities are indented to show how difficult it is to assign numerical probabilities to uncertainty terms, and that the interpretation of these termes varies widely between individuals and contexts.

Many organisations tried to standardize verbal expressions of uncertainty, and some of these efforts are described here for comment and criticism.



3.1 Probability vocabulary

A trial is the individual repetition of a random experiment.
An outcome is the result of a single execution of a trial.
The **sample space** Ω is the **set** of all possible **outcomes** ω.
An **event** A is a subset of the sample space Ω.
$P(A)$ is the assigned probability of the event A.
The relative frequency of an event is the number of times that the event occurs during experimental trials (absolute frequency), divided by the total number of trials conducted.

$$\text{relativefrequency}(A) = \frac{\text{number of favorable outcomes}}{\text{total number of trials}}$$

As number of trials increases, the relative frequency settles down to give an estimate of the true probability.

english	**french**
random	aléatoire
a trial	essai (en maths, d'une expérience aléatoire)
an outcome	un résultat, une issue
an event	un événement
a set	un ensemble
a subset	un sous ensemble
to assign	affecter , attribuer
likely	vraisemblable, probable
equally likely	aussi probable, équiprobable
sample	échantillon
sample space	l'univers, l'ensemble des issues possibles
sample space diagram	tableau à double entrée
tree diagram	arbre de probabilité
to roll a die	lancer un dé, faire rouler un dé
to toss	lancer, jeter en l'air
to pick out	prélever, choisir
a coin	une pièce
head	face
tail	pile
fair	(juste, équitable) en maths : équilibré
faire dice	des dés équilibrés
(absolute) frequency	effectif
relative frequency	fréquence
lowercase	minuscule
uppercase	majuscule

3.2 Lesson 1 Relative frequency with Grim dice

1) Pick red and blue die. Why are these dice different to normal? Which is better? Why? Which one would beat the other? Why? How would you find out which is better? How will you keep score?

2) Battle de dice 10 times and keep score. What results did we all get? Why do you have different numbers of wins? Which do you think is better now ? How can we be sure?

3) If your dice carry on behaving the same way, how many Reds would you expect out of 20 rolls? Out of 100 rolls? Out of 1000 rolls?

4) Continue rolling to 20 or so trials. Which do you think is better now ? How can we be sure?

5) The relative frequency of an event is the number of times that the event occurs during experimental trials (absolute frequency), divided by the total number of trials conducted. What are your relative frequencies for blue and red dice.

6) Roll the dice for 3 more minutes and record your results. What is your relative frequency now? Which dice do you think is better? How many Reds would you expect out of 1000 rolls?

7) Repeat with Red and olive dice.
   a) Which do you think is better? (Before rolling)
   b) Experiment
   c) Find relative frequency
   d) Compare with others
   e) How many Olives would you expect out of 1000?
   f) Write a conclusion, explaining your decision

8) Repeat with the Blue and Olive dice.

9) Conclusion

3.3 Lesson 2 Sample space diagrams with Grim dice

1) A normal coin is flipped twice. What could happen? What is the chance of each outcome?

2) What about if it were flipped three times?

3) What if the coin were unfair and had $P(\text{Heads}) = 0.4$.

4) Pick red and olive dice. Which dice was better? What could happen when you roll the dice?

5) Sometimes it is better to use a grid to show all the possible outcomes. How many Red wins? How many Olive wins? What is the probability of Olive winning?

6) Repeat with Red and Blue dice
7) **Comparing methods** Is it better to roll the dice many times or create a sample space diagram? What advantages does a sample space diagram have? What disadvantages does it have?

   hints accuracy , no assumptions, fair dice, experimental, fun.
8) **Investigation** Look at the yellow and magenta dice. Which do you think is better? Why? Prove it using a method of your choice.
9) **Investigation** Look at the yellow and olive dice. Which do you think is better? Why? Prove it using the method you think is fastest
10) **Conclusion** Draw a diagram showing which dice colours beat the others. Can you devise a strategy to beat 2 other players when rolling the dice?

		Red die					
Olive die							

On average which dice wins?

What is the probability of that dice wimming?

		Olive die					
Blue die							

On average which dice wins?

What is the probability of that dice wimming?

		Red die					
Blue die							

On average which dice wins?

What is the probability of that dice wimming?

		Magenta die					
Yellow die							

On average which dice wins?

What is the probability of that dice wimming?

		Yellow die					
Olive die							

On average which dice wins?

What is the probability of that dice wimming?

		Magenta die					
Red die							

On average which dice wins?

What is the probability of that dice wimming?

3.4 Lesson 3 Tree diagrams with Double Grim dice

1) Recall what we did last lesson. Why did we do it? What did we find?

2) We roll 2 blue dice. What are the possible outcomes? What are the possible totals of the two dice?

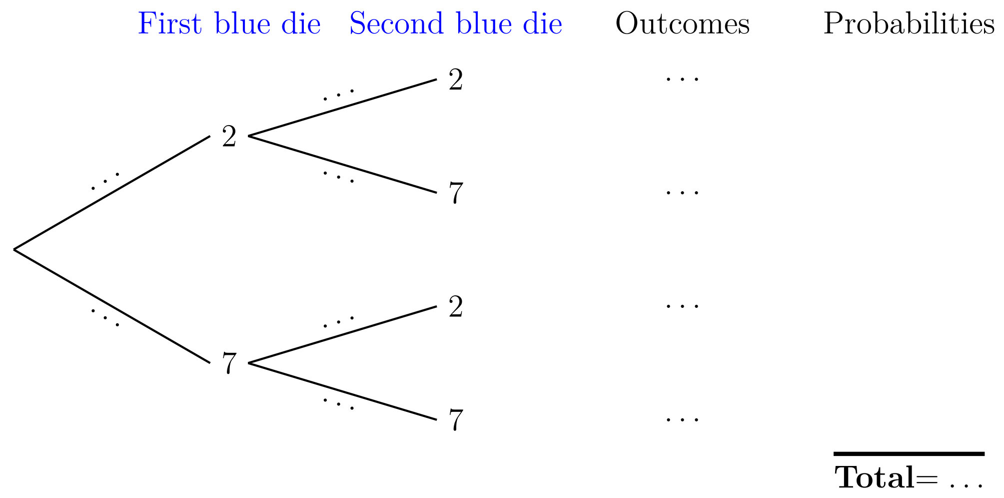
3) How can we find the chance of each outcome happening?

		Blue die					
Blue die							

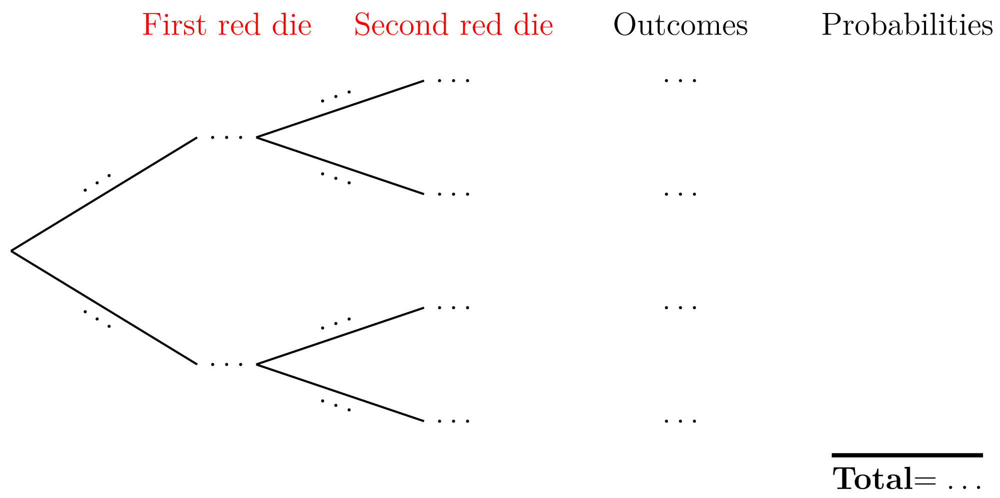
4) Do you know any other methods? Recall advantages of each.

5) Does it matter whether we roll one dice and then the other OR roll them both at once? Why?

> **Tree diagram** is a method of solving probability questions by listing all the outcomes of an event. Probabilities are calculated by multiplying down the branches.



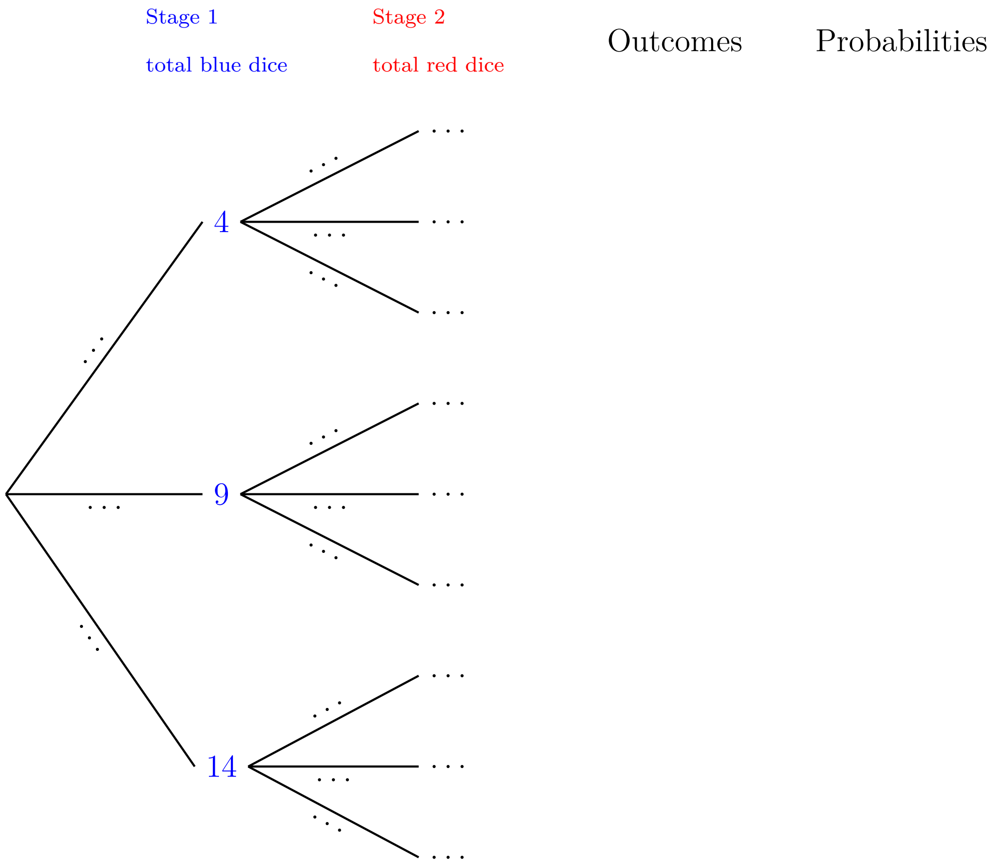
6) Using a tree diagram answer the following questions :
   a) What are the possible outcomes if we roll 2 red dice?
   b) What are the possible totals of the two dice?
   c) Does it matter whether we roll one dice and then the other OR roll them both at once? Why?
   d) Why might it be better to keep all the denominators the same?



7) In a previous lesson we found that single red dice beats blue dice with a chance of $\dfrac{7}{12}$.
Let us add a new twist to the game :

> Each players picks two dice of the same color.
> The one that rolls the highest total wins.

a) Which do you think is better - a pair of blue dice, or a pair of red dice?
With roughly what probability?
b) Find the exact probability of double red dice beating double blue dice.
c) Does it matter whether you consider the red dice or blue dice first?



8) Repeat for a double dice pair of your choice :
   a) Hypothesis
   b) Working out, shown clearly and logically.
   c) Answer at end with a comment.

9) Using the table answers on the board, can you draw a diagram showing which dice colours beat the others? Compare it to the diagram we had for a single die.

BLUE	**BLUE**
RED **OLIVE**	**RED** **OLIVE**
MAGENTA **YELLOW**	**MAGENTA** **YELLOW**
SINGLE	**DOUBLE**

10) Why are Red and Olive joined with a line instead of an arrow?
11) Can you devise a strategy to beat 2 other players when rolling double or single dice?

3.5 How probable is probable?

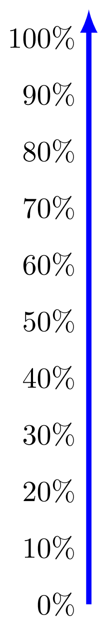
We all use a wide variety of terms to indicate uncertainty -'could', 'maybe', 'possible'- and so on. Can these be interpreted as numerical probabilities?

Interpretation of words Read the following paragraph.

> Arthur was worried. It was almost certain there would be a maths test today, and he hadn't been paying much attention recently. Sally would probably get more marks than him, but there was a distinct possibility that Zak would mess up. The weather forecast said it might rain, so he took a coat, and as he walked to school he thought he was likely to meet Zak, who always played around, and could make him late. If he were late, he was certain to get into trouble. Perhaps there would be a fire drill to disrupt the test? But really there was little chance of that, and it was also extremely unlikely an asteroid would hit the school. It was going to be a bothering sort of day.

Underline the words or phrases that express uncertainty, such as 'could', 'likely' etc. Make a list of these words, and rank them in terms of highest to lowest probability. Put each word on the vertical probability scale below, for example if you think that 'almost certain' is near 50%, write it next to 50%.

Collect together the responses and discuss the ranges in opinions.



In the 60s, CIA analyst **Sherman Kent** recognized the problem of using imprecise statements of uncertainty; as actions can be taken or not based on overestimations or underestimations of the content of intelligence reports.

To deal with this, Kent proposed different graduations of probability. To each he assigned a percentage range. For example 'Probable' would mean 93% (give or take about 6%). In a later experiment, 23 Nato officers were asked about the different perception of terms expressing probability. The results are shown in figure 3.1

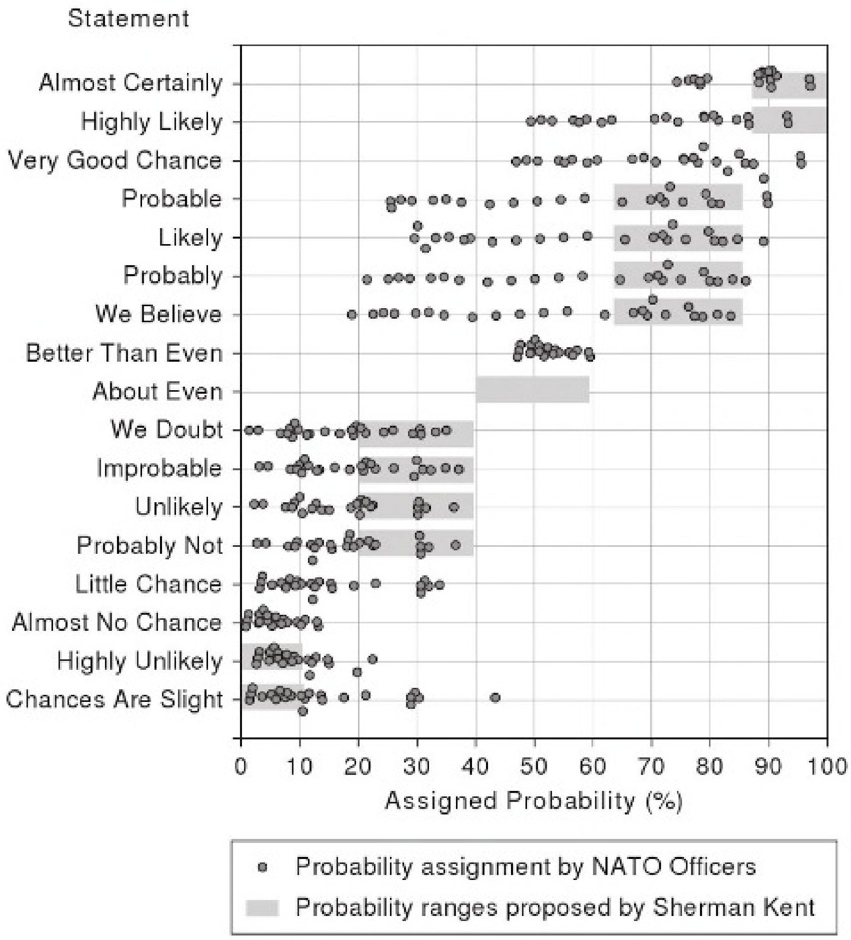


Figure 3.1 – Measuring perceptions of Uncertainty

Compare your group results with some of the terms. Can you find new terms to express probabilities?

Drug side effects Most medicines have occasional side effects - the drugs may make some people drowsy, make their muscles ache and so on. If you were told a side effect, say headache, was ‘common’, how frequently do you think it would occur, as a percentage of people taking the medicine? What is the headaches were described as ‘very common’?
The official European Medicines Agency scale defines ‘common’ as frequency between 1% and 10%, and ‘very common’ as anything above 10% (figure 3.2).
What do you think of this? [1]

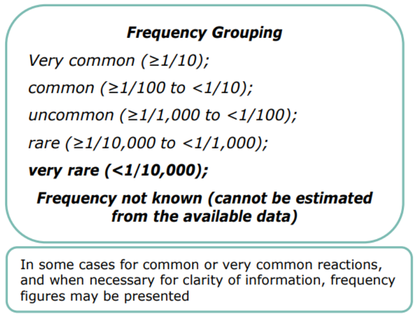


Figure 3.2 – Frequency Grouping according to the European Medicines Agency[1]

[1] to lead someone to, to make someone think something

Amniocentesis Pregnant women usually have a screening test for possible problems with their foetus. A test result that shows any probability above 1 in 150 (0.6%) [5] of having a baby with Down’s syndrome is called a ‘higher rist’on the NHS Choices website. Such women are offered an amniocentesis to confirm or rule out the diagnosis, but this procedure carries some risk of causing a miscarriage - this risk is estimated to be about 1%, and is described as a ‘small associated risk’ by NHS choices[4].
What do you think of this? Why do you think this wording has been used? [2]

[2] to encourage someone to

IPCC scale Many organisations have tried to standardise terms expressing uncertainty. Table 3.1 shows the scale used by the *Intergovernmental Panel on Climate Change* (IPCC)[3].

[3] You might be more familiar with the french name 'GIEC' **Groupe d'experts intergouvernemental sur l'évolution du climat**

Table 3.1 – IPCC scale

Verbal expression	Numerical probability range
virtually certain	99-100%
extremely likely	95-100%
very likely	90-100%
likely	66-100%
more likely than not	50-100%
about as likely as not	33-66%
unlikely	0-33%
very unlikely	0-10%
extremely unlikely	0-5%
exceptionnally unlikely	0-1%

For example in their recent report, they claimed that 'It is extremely likely that human influence has been the dominant cause of the observed warming since the mid-20th century'.
What do you think of this scale? How does it fit with the numbers assessed by the class at the start of this chapter?

[4]

[4] consistency=cohérence
to set guidelines(établir des directives)
to ignore guidelines
to provide numerical translation alongside text
to improve the scale we should/can

Possible answers and ideas

Interpretation of words These should be reasonably good agreement for terms like 'extremely unlikely', but 'could' could mean almost anything. [3]

more terms to convey probablity probable (85%), expected (75%), uncertain (25%),improbable (15%), tossup, seldom (not often, rarely). Using adverbs adds even more nuance :

1. somewhat likely
2. likely
3. very likely
4. rather likely
5. quite likely
6. fairly likely
1. good chance
2. fighting chance
3. little chance
4. pretty good chance
5. slight chance
6. strong chance
7. not much chance
8. almost no chance
9. fair chance

There is also problem of preception of numbers (couple, few, some, several, a lot, many, dozens, scores of)!

Drug side effects This is an example where the 'official' definition of a word has little resemblance to its use in everyday language, and could lead to patients believing side effects occur far more often than intended.

Amniocentesis Higher risk is numerically smaller than small risk. The wording may have been chosen to encourage women to have an amniocentesiss if they have a positive screening test, and not worry too muchabout the risk of miscarriage.

IPCC scale Too many verbal expression of probability can be confusing to follow. We can improve the scale by using less terms.

Providing numerical translation alongside reports or text containing verbal probailities should encourage consistency.

Giving numerical expressions of probabiliyy can mislead people. Some think that a statement like probability is 70% means it is inevitable.

References

[1] European Medicines Agency. *Section 4.8: Undesirable effects*. 2015. URL: https://www.ema.europa.eu/en/documents/presentation/presentation-section-48-undesirable-effects_en.pdf (see p. 15).

[2] Jenny Gage Gage and Spiegelhalter David. *Teaching Probability*. 1st edition. Cambridge mathematics teaching. City: Publisher, Aug. 2016, pages 164–149 (see p. 1).

[3] The Guardian. *How probable is 'probable'?* 2015. URL: https://www.theguardian.com/news/datablog/2015/aug/14/how-probable-is-probable (see p. 17).

[4] National Health Service. *Amniocentesis*. URL: https://www.nhs.uk/conditions/amniocentesis/ (see p. 15).

[5] National Health Service. *Screening for Down's syndrome, Edwards' syndrome and Patau's syndrome*. URL: https://www.nhs.uk/pregnancy/your-pregnancy-care/screening-for-downs-edwards-pataus-syndrome/ (see p. 15).