4.3 Switch or stick? Date : ....

There are three cups and one token. Two students alternate between being player A (the host, quizmaster) and player B (the contestant) in the following game :

- At a turn player B closes his eyes while player A places the token under one of the cups and *remembers wich one*.
- Player B then opens their eyes, and points at one of the three cups.
- Player A lifts one of the *other* cups revealing it to be empty – this is always possible as at least one of the cups not chosen is bound to be empty, and if there are two empty cups, the host choses one at random.
- Player A then offers another opportunity for player B : either stick to his first choice, either switch cups.
- Having decided, player B either sticks or switches, lifts the chosen cup and records whether they got the token or not.

Should Player B switch or stick? Investigate.

You can start by playing a couple of games to understand the game.

Player hiding	**Player guessing**	**Switch or stick?**	**Win or lose?**

Stop after few games. Discuss your findings. Do you think it makes a difference?

Carry on with more games, and tally up at the end.

Solutions

Wrapping up A possible numberphile explanation. https://youtu.be/7u6kFlWZOWg

Explanation 2 If player B adopts the strategy of switching, they will only lose if Cup 1 actually covered the sweet. Since this happens with probbility $\frac{1}{3}$, a switching strategy will only lose $\frac{1}{3}$ of the time.

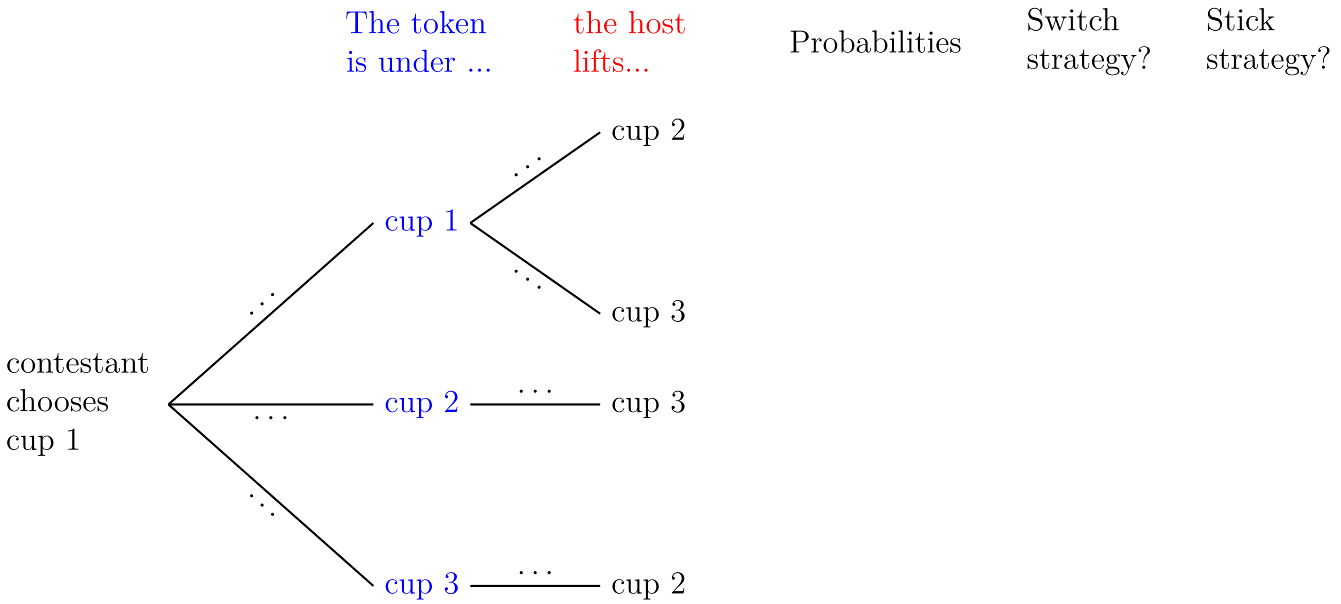

Figure 4.1 – Tree diagramm assuming the token is hidden under cup 1.

Explanation 1 If player B points at initially Cup 1. There is probability $\frac{1}{3}$ of picking the right cup to start with. When player A lifts an empty cup 2, this provides no additional information, since Player A will always reveal an empty cup regardless of whether cup 1 covered the sweet or not. The initial probability is unchanged, wich means that the probability that cup 3 covers the sweet is $\frac{2}{3}$.