

1.1.4 Solving puzzles : a guided example

Ten pets' puzzle

Fifty-six biscuits are to be fed to ten pets; each pet is either a cat or a dog. Each dog is to get six biscuits, and each cat is to get five. How many dogs and how many cats are there?

As a familiar student with algebra, you can :

select unknowns : x is y is

write down the problem into equations :

solve for x and y :

Sometimes, solving the equations isn't straightforward. One might try to use naive **trial and error** method and test out all eleven possibilities for the number of dogs (from zero to ten).

By looking at this problem in the right light, one can also figure out a simple.

Common sense solution

First feed five biscuits to each of the ten pets; this leaves six biscuits.

All cats have already had their portion! The six remaining biscuits are for the dogs.

Since each dog is to get one more biscuit, there must be six dogs.

1.1.5 Puzzle frenzy

Work in pairs for the next puzzles. Try using algebra if possible. Look for alternative solutions by trial-and-error or common-sense. Write down your reasoning in english.

Puzzle 1 Sarah is x years old. Thomas is 3 years older than Sarah. David is twice as old as Sarah. The total of their ages is 51. Work out Sarah's age.

Puzzle 2 In 1871 British mathematician Augustus De Morgan died. When asked about his age, De Morgan said that he was x years old in the year x^2 . Which year was De Morgan born in?

Do you think anyone living at this time can say the same thing about their age and the year they were born?

Puzzle 3 If you add the square of Tom's age to Lauren's age the total is 62. If you add the square of Lauren's age to Thom's age the total is 176. How old are Tom and Lauren?

In math we like to use some connections :

so means “and for that reason”

thus, hence means “consequently”

therefore means “as a logical consequence”

Puzzle 4

There are 30 rabbits (4 legs) and ducks (2 legs) both together in a field. Counting their feet, there are 94 altogether. How many of each are there?

Puzzle 5 Two people took part in an election. One got 311 votes more than the other. The total votes for the two candidates was 13,689. How many votes did each get?

Puzzle 6 Twenty chimps : male, female and young eat 200 bananas a day between them. Each male eats 30 a day. Each female eats 15 a day. Each young chimp eats 5 a day. How many of each chimps are there ? We suppose there are some of each.

1.1.6 Expected type of written answers

solution of puzzle 1. Equation solving. If Sarah is x years old, then Thomas is $x + 3$ years old, and David is $2x$ years old. The total of their ages is 51, so the equation for x is $x + x + 3 + 2x = 51$. Solving for x we get $x = 12$. Sarah is 12 years old. ■

solution du puzzle 2. Expected trial and error approach for both questions.

1. De Morgan's age is a whole number. He died in 1871, so x^2 is less than 1871.

$$40 \times 40 = 1600$$

so De Morgan died at 300 years

$$45 \times 45 = 2025$$

not possible, he died in 1871

$$44 \times 44 = 1936$$

not possible

$$43 \times 43 = 1849$$

reasonable

If De Morgan was 43 years old in 1849, he was born in 1806 and was 65 when he died. This is reasonable.

2. A person born in 1980 would be 45 by year $45^2 = 2025$.

A person can say he is 44 by year $44^2 = 1936$ if he was born in $1936 - 44 = 1892$. Today, he would be 128 years old. The oldest person we know is 95 years old. ■

solution of puzzle 3. The square of Tom's age is at most 62, so Tom is at most 7. The square of Tom's age is therefore at most 49, so Lauren is at least 13.

The square of Lauren's age is at most 176, so Lauren is at most 13. Therefore, Lauren must be 13, and Thom must be 7. ■

solution of puzzle 5. Common sense approach. Besides the 311 extra votes that went to the winning candidate. The remaining $13689 - 311 = 13378$ votes were equally shared by both candidates.

The losing candidate got only $13378 \div 2 = 6689$ votes. The winning candidate got 7000. ■

solution of puzzle 6. Common sense approach. If we give all 20 chimps 5 bananas, we are left with $200 - 20 \times 5 = 100$ bananas. The young chimps had their share, and from the remaining 100 bananas each females will receive 10, and each male will receive 25.