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Optimal capacitor placement in distribution systems for power loss reduction and voltage profile improvement

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Abstract: This study presents a two-stage procedure to identify the optimal locations and sizes of capacitors in radial distribution systems. In first stage, the loss sensitivity analysis using two loss sensitivity indices (LSIs) is employed to select the most candidate capacitors locations. In second stage, the ant colony optimisation algorithm is investigated to find the optimal locations and sizes of capacitors considering the minimisation of energy loss and capacitor costs as objective functions while system constraints are fully achieved. The fixed, practical switched and the combination of fixed and switched capacitors are considered to find the optimal solution. The backward/forward sweep algorithm is developed for the load flow calculations. The proposed procedure is applied to different standard test systems as 34-bus and 85-bus radial distribution systems. In addition, the application of the proposed procedure on a real distribution system of the East Delta Network as a part of the Unified Egyptian Network is used as a test system. Numerical results show the capability of the proposed procedure to find the optimal solution for significant saving in the total cost with more accurate and efficient, competitive compared with other methods in the literature especially with increasing the distribution system sizing.

1 Introduction

The main parts of the interconnected system are the generation, transmission and distribution systems. The major loads such as industrial, commercial and domestic are connected to the network through the distribution systems. Therefore, the quality of the service is based on the continuity of power and maintaining the supply voltage within certain limits with specified frequency. Due to the rapid spread in the loads, the long distance of radial structure and the high R/X ratio of transmission lines, the power loss reduction and the improvement of the voltage profile, the system reliability and the power factor are the challenge in the distribution system. To solve these problems with saving in energy, reduced in cost, increased in reliability and power quality, the shunt capacitors are installed on the radial feeders for reactive power injection. Therefore, the optimal locations and sizes of capacitors in distribution systems can be formulated as a constrained optimisation problem. To solve this problem, the optimisation techniques are applied.

Many optimisation techniques were applied to solve the optimal capacitor placement problem. Lee *et al.* [1] presented the optimal capacitors placement using the particle swarm optimisation algorithm with operators based on Gaussian and Cauchy probability distribution functions. In [2, 3], a two stage method was used to solve the optimal capacitor placement problem based on the loss sensitivity factors (LSF) to determine the optimal locations and the plant growth simulation algorithm (PGSA) to estimate the optimal sizes of capacitors. However, the optimal solution may not be obtained because the optimisation technique is restricted only to find the sizes of capacitors. Bhattacharya and Goswami [4] used the fuzzy based method for identification of probable capacitor nodes of radial distribution system, while the simulated annealing (SA) technique was utilised for final selection of the capacitor sizes. Etemadi and Fotuhi-Firuzabad [5] used the PSO algorithm to find the optimal capacitor placement with separate objective functions. Prakash and Sydulu [6] used the LSF and the PSO algorithm to find the optimal locations and sizing of capacitors, respectively. Reddy and Sydulu [7] presented a power loss index (PLI) based location and an index and genetic algorithm (GA) based sizing for the capacitor placement problem. Raju *et al.* [8] presented the direct search algorithm (DSA) to find the optimal locations and sizes of fixed and switched capacitors to maximise the net savings and minimise the active power loss.

Tabatabaei and Vahidi [9] introduced the optimal locations and sizes of capacitors using the fuzzy decision making and the bacteria foraging algorithm (BFA), respectively. Reddy and Veera [10] presented two stage methodology based on the fuzzy approach and GA to find the optimal locations and sizing of capacitors, respectively. Sultana and Roy [11] used the teaching learning based optimisation (TLBO) approach to find the optimal placement of capacitors in radial distribution systems. In [12, 13], the authors presented PLI to determine the high potential buses for capacitor placement. Then, the optimal sizing and placement of capacitors were obtained using accelerated PSO in [12] and cuckoo search algorithm in [13]. Nojavan *et al.* [14] presented a mixed integer non-linear programming approach for capacitor placement in radial/mesh distribution systems. Xu *et al.* [15] used the mixed-integer programming and the net present value criterion to find the optimal placement of capacitor banks and to evaluate the cost benefit of the capacitor installation project. El-Fergany [16] presented PLI and/or LSF to determine the high potential buses for capacitor placement. Then, the differential evolution and pattern search (DE-PS) algorithm was used to find the optimal locations and sizes of capacitors.

Ramadan *et al.* [17] used a fuzzy-based approach for optimal capacitor allocation and sizing in radial distribution systems. Rani *et al.* [18] used the LSF and the self-adaptive harmony search algorithm to solve the optimal capacitor placement problem in order to minimise the power loss. Kavousi-Fard and Niknam [19] presented the honey bee mating optimisation algorithm to find the optimal locations and sizes of capacitors. Farahani *et al.* [20] presented the discrete GA to find the optimal locations and sizes of capacitors, while the SA was applied to compare the performance of convergence. Kavousi-Fard and Niknam [21] used the firefly algorithm to find the optimal stochastic capacitor placement for reducing the system losses.

Recently, the ant colony optimisation (ACO) algorithm has been applied successfully in different power system applications such as optimal reactive power dispatch problem [22], voltage collapse problem [23] and the optimal phasor measurement unit placement problem [24]. Many researches are introduced the optimal locations of capacitors based on LSIs and the optimal sizes of capacitors are obtained from the optimisation technique. However, this paper presents LSIs as a tool to obtain the candidate buses for the capacitors placement, while the optimal locations and sizes of fixed, switched and the combination of fixed and switched capacitors are obtained from the optimisation technique. However, one drawback of the switched capacitors is the transient over voltages when switched in the distribution systems. Common problems such as adjustable speed-drive trips and malfunctions of other electronically controlled load equipment are occurred due to the switched capacitors [25–27].

2 Problem formulation

The optimal capacitor placement problem in radial distribution systems for minimising the total energy cost and maximising the saving can be formulated as:

$$\begin{aligned} \text{Min} \quad S &= K_P P_{\text{Loss}}^{\text{Total}} + K_C Q_C^{\text{Total}} \\ &= K_P \sum_{i=1}^{N_b - 1} P_{\text{Loss}i} + K_C \sum_{j=1}^{N_C} Q_{Cj} \end{aligned} \tag{1}$$

where, S is the total costs (\$/year), K_P is the annual cost per unit of power loss (\$/kW-year), K_C is the total capacitor purchase and installation cost (\$/kVAR), $P_{\text{Loss}}^{\text{Total}}$ and Q_C^{Total} are the total power loss and capacitors reactive power, respectively. $P_{\text{Loss}i}$ is the power loss in line i, Q_{Cj} is the total reactive power injected at location j, N_b is the total number of buses and N_C is the optimal number of capacitors placement. Therefore, the annual total cost of capacitors can be calculated as:

$$\text{Total capacitors cost} = \frac{K_C * Q_C^{\text{Total}}}{\text{Life expectancy}} \ \$/\text{year} \tag{2}$$

The objective function (1) is subjected to the following constraints:

(a) Operational constraints
- Bus voltage constraint

The voltage at each bus (V_i) must be within their permissible minimum and maximum limits as:

$$V_i^{\min} \le V_i \le V_i^{\max} \tag{3}$$

- *Power flow constraint*

The power flow in each line (PF_k) must be less than the maximum limit of power flow in this line ($PF_k^{\max}$) as:

$$|PF_k| < PF_k^{\max} \tag{4}$$

- *Overall power factor constraint*

The overall system power factor (pf_{overall}) must be greater than or equal to the minimum limit of power factor ($pf^{\min}$) as:

$$|pf_{\text{overall}}| \ge pf_{\text{overall}}^{\min} \tag{5}$$

(b) Capacitor constraints

- *Number of capacitors constraint*

This constraint aims to reduce the number of capacitors placement. Therefore, the optimal number of capacitors (N_C) must be less than or equal to the maximum number of possible locations ($N_C^{\max}$) as:

$$N_C \le N_C^{\max} \tag{6}$$

- *Capacitor size constraint*

The reactive power injection must be within their feasible minimum and maximum limits as:

$$Q_{Cj}^{\min} \le Q_{Cj} \le Q_{Cj}^{\max} \tag{7}$$

where, Q_{Cj} is the reactive power injection at location j.

- *Total reactive power constraint*

The total reactive power injection (Q_C^{Total}) must be less than or equal the total load reactive power (Q_L^{Total}) as:

$$Q_C^{\text{Total}} \le Q_L^{\text{Total}} \tag{8}$$

3 Sensitivity analysis and loss sensitivity indices (LSIs)

Two LSIs are presented in this paper to rank the load buses according to their severity for efficient detecting the candidate load buses for the installation of capacitors. The objective of the candidate load buses is to reduce the search space in the optimisation procedure. Consider 2 nodes connected by a branch as apart in a radial distribution system shown in Fig. 1, where the buses p and q are the sending and receiving end buses, respectively.

The active power P_p and reactive power Q_p flows through a branch k from node p to node q can be calculated as:

$$P_p = P_{\text{eff}/q} + P_{\text{Loss}k} \tag{9}$$

$$Q_p = Q_{\text{eff}}/q + Q_{\text{Loss}k} \tag{10}$$

where, P_p and Q_p are the power flows through branch k, $P_{\text{eff}/q}$ and $Q_{\text{eff}/q}$ are the total effective active and reactive power loads beyond the node q, respectively. $P_{\text{Loss}k}$ and $Q_{\text{Loss}k}$ are the active and reactive power losses through branch k, respectively.

The current flowing through branch k from the node p to the node q can be calculated as:

$$I_k = \frac{P_p - jQ_p}{V_p \angle -\delta_p} \tag{11}$$

Also

$$I_k = \frac{V_p \angle \delta_p - V_q \angle \delta_q}{R_k + jX_k} \tag{12}$$

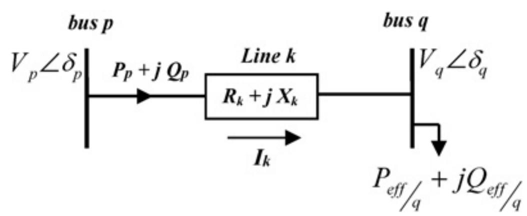


Fig. 1 *Representation of two nodes in a distribution system*

where, V_p and V_q are the voltage magnitudes at nodes p and q, respectively. δ_p and δ_q are the voltage angles at nodes p and q, respectively. R_k and X_k are the resistance and reactance of branch k, respectively.

From (11) and (12), it can found that:

$$V_p^2 - V_p V_q \angle \delta_q - \delta_p = \left(P_p - jQ_p\right)\left(R_k + jX_k\right) \quad (13)$$

By equating the real and imaginary parts on both sides in (13), it can obtain the follows:

$$V_p V_q * \cos\left(\delta_q - \delta_p\right) = V_p^2 - \left(P_p R_k + Q_p X_k\right) \quad (14)$$

$$V_p V_q * \sin\left(\delta_q - \delta_p\right) = Q_p R_k - P_p X_k \quad (15)$$

After squaring and adding (14) and (15), (16) is obtained as:

$$V_q^2 = V_p^2 - 2 * \left(P_p R_k + Q_p X_k\right) + \left(R_k^2 + X_k^2\right)\left(\frac{P_p^2 + Q_p^2}{V_p^2}\right) \quad (16)$$

The active and reactive power losses of branch k can be calculated as:

$$P_{\text{Loss}k} = I_k^2 * R_k = \left(\frac{P_{\text{eff}/q}^2 + Q_{\text{eff}/q}^2}{\left|V_q\right|^2}\right) * R_k \quad (17)$$

$$Q_{\text{Loss}\,k} = I_k^2 * X_k = \left(\frac{P_{\text{eff}/q}^2 + Q_{\text{eff}/q}^2}{\left|V_q\right|^2}\right) * X_k \quad (18)$$

The total active and reactive power losses of a radial distribution system can be written as:

$$\text{Total } P_{\text{Loss}} = \sum_{q=2}^{N_b} \sum_{k=1}^{N_{b-1}} \left(\frac{P_{\text{eff}/q}^2 + Q_{\text{eff}/q}^2}{V_q^2}\right) * R_k \quad (19)$$

$$\text{Total } Q_{\text{Loss}} = \sum_{q=2}^{N_b} \sum_{k=1}^{N_{b-1}} \left(\frac{P_{\text{eff}/q}^2 + Q_{\text{eff}/q}^2}{V_q^2}\right) * X_k \quad (20)$$

where, $P_{\text{eff}/q}$ and $Q_{\text{eff}/q}$ are the total effective active and reactive power loads beyond the node q, respectively. N_b is the total number of system buses, while N_{b-1} refers to the total number of system branches.

Now, the first loss sensitivity index (LSI_1) can be obtained by making the first derivative of $P_{\text{Loss}k}$ in (17) with respect to V_q, where $P_{\text{eff}/q}$, $Q_{\text{eff}/q}$ and R_k are constants. Therefore, the LSI_1 can be written as:

$$LSI_1 = \frac{\partial P_{Loss\,k}}{\partial V_q} = -2 * R_k * \left(\frac{P_{\text{eff}/q}^2 + Q_{\text{eff}/q}^2}{V_q^3}\right) \quad (21)$$

The values LSI_1 indicate the largest and the smallest performance index. Hence, the buses that have the largest negative values of LSI_1 are considered the candidate buses for the installation of capacitors and these buses will be indexed at the top of LSI_1 list, while the buses that have the smallest negative values of LSI_1 will be indexed at the bottom of this list, which are less sensitivity to install the capacitors.

The second loss sensitivity index (LSI_2) can be obtained by making the first derivative of P_{Loss} in (17) with respect to $Q_{\text{eff}/q}$ that gives:

$$LSI_2 = \frac{\partial P_{\text{Loss}\,k}}{\partial Q_{\text{eff}/q}} = \frac{2 * Q_{\text{eff}/q} * R_k}{V_q^2} \quad (22)$$

Similarly, the buses that have the largest positive values of LSI_2 are considered the candidate buses for the installation of capacitors and these buses will be indexed at the top of LSI_2 list, while the buses that have the smallest positive values of LSI_2 will be indexed at the bottom of this list, which are less sensitivity to install the capacitors.

To give the same opportunity (chance) for fixed and switched capacitors (optimal locations and sizes), the candidate buses based on the proposed sensitivity indices to each of them should be the same. Therefore, the candidate buses are chosen from the LSI_1 and LSI_2 listing starting from the top of these lists up to approximately 50% to 55% of the total number of system buses. The candidate buses for the installation of capacitors based on the combinations of LSI_1 and LSI_2 are inserted as control variables in the optimisation algorithm as:

$$C = \{x : x \in (A \cup B)\} \quad (23)$$

where, C is the control variables set, A and B are the sets of candidate buses based on LSI_1 and LSI_2, respectively.

4 Backward/forward sweep (BFS) algorithm

The BFS algorithm is one of the most common ways used for load flow distribution system because it is simple, fast and robust convergence and low memory requirement for processing with efficiencies and solution accuracies computational. The BFS algorithm involves mainly an iterative three basic steps based on Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL). The three steps are named as the nodal current calculation, the backward sweep and the forward sweep and they are repeated until the convergence is achieved. The BFS utilises as a simple and flexible radial distribution system numbering scheme in order to numbering each branch in the feeder, lateral and sub-lateral.

The BFS algorithm can be applied to find the load flow results using the following steps:

Step 1: Initialisation

Insert the follows:

- The distribution system line and load data.
- The base power and base voltage.
- Calculate the base impedance.
- Calculate the per unit values of line and load data.
- Take the voltage for all buses flat voltage (1 p.u.).
- Set convergence tolerance $\epsilon = 0.0001$ and $\Delta V_{\max} = 0$.

Step 2: Radial distribution system numbering scheme

The numbering scheme aims to give a number to each section in the distribution system, where a section is part of a feeder, lateral or sub-lateral that connects two buses in the distribution system. The total number of sections ($N_{\text{Sec}}^{\text{Total}}$) of a distribution system can be calculated as:

$$N_{\text{Sec}}^{\text{Total}} = N_{\text{bus}}^{\text{Total}} - 1 \quad (24)$$

where, $N_{\text{bus}}^{\text{Total}}$ is the total number of buses. Each section will carry a number which is one less than its receiving end bus number, for example, the number of section that connects the sending end p

and the receiving end q in Fig. 1 can be calculated as:

$$N_{\mathrm{Sec}/p\text{–}q} = N_{\mathrm{bus}/q} - 1 \tag{25}$$

where, $N_{\mathrm{Sec}/p\text{-}q}$ is the section number between buses p and q, $N_{\mathrm{bus}/q}$ is the number of bus q.

Now, the radial distribution system numbering scheme should be applied on the distribution system to give a number to each section in the system.

Step 3: Nodal current calculation

At iteration k, the nodal current injection at node i due to loads and any other shunt elements can be calculated as:

$$I_i^{(k)} = \left(\frac{S_i}{V_i^{(k-1)}}\right)^* - (Y_i)\left(V_i^{(k-1)}\right) \tag{26}$$

where, $I_i^{(k)}$ is the current injection at node i, S_i is the specified power injection at node i, $V_i^{(k-1)}$ is the voltage at node i at iteration $k-1$, Y_i is the sum of all shunt elements at node i.

Step 4: Backward sweep

At iteration k, start from the branches at the end nodes and moving towards the branches connected to the substation. Hence, all branch currents can be calculated by applying the KCL and then the powers through these branches can be determined as:

$$I_L^{(k)} = -I_j^{(k)} - \sum_{m=1}^{M}\left(\frac{S_m}{V_j^{(k)}}\right)^* \tag{27}$$

$$S_L^{(k)} = \left(V_j^k + Z_L * I_L^k\right)\left(I_L^k\right)^* \tag{28}$$

where, $I_L^{(k)}$ is the current flow in branch L at iteration k, $I_j^{(k)}$ is the current injected due to shunt elements at bus j, M is the number of branches connected to bus j, S_m is the complex power at the sending end of branch m, $V_j^{(k)}$ is the voltage at bus j, $S_L^{(k)}$ is the power flow in branch L and Z_L is the impedance of branch L.

Step 5: Forward sweep

At iteration k, the nodal voltages are updated in a forward sweep starting from the branches in the first section toward those in the last by applying the KVL. For a branch L connected sending end p and receiving end q, the voltage at receiving end at iteration k can be calculated as:

$$V_q^{(k)} = V_p^{(k)} - Z_L * I_L^{(k)} \tag{29}$$

where, $V_p^{(k)}$ and $V_q^{(k)}$ are the voltages at sending and receiving ends, respectively.

Step 6: Check the voltage mismatches

After the previous steps are computed, the voltage mismatches for all nodes are calculated, for example, the voltage mismatch at bus i at iteration k can be calculated as:

$$\Delta V_i^{(k)} = \left|\left|V_i^{(k)}\right| - \left|V_i^{(k-1)}\right|\right| \tag{30}$$

After calculating the voltage mismatches, check the convergence of the voltage as:

- If $\Delta V_i^{(k)} > \Delta V_{\max}$, then make $\Delta V_{\max} = \Delta V_i^{(k)}$.

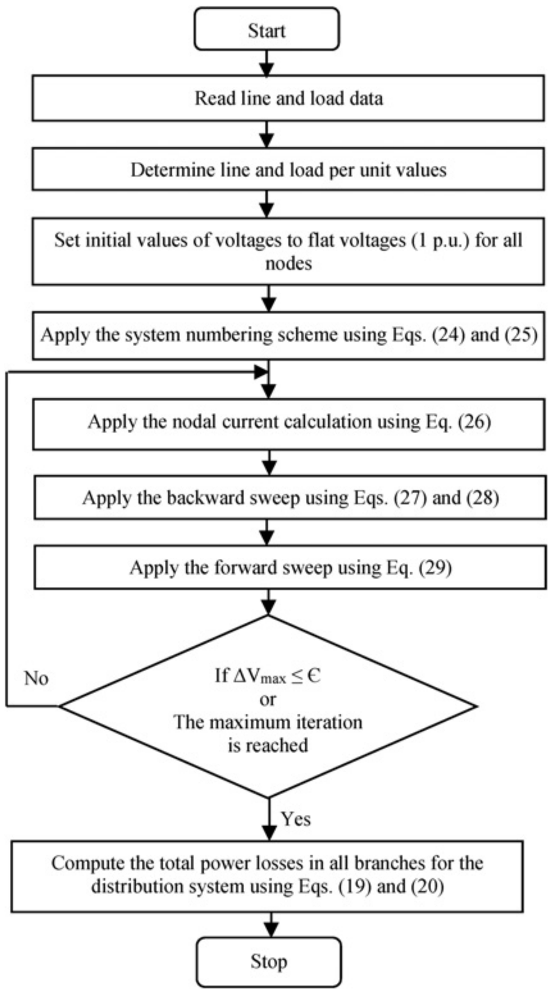


Fig. 2 *Flow chart of BFS load flow*

- If $\Delta V_{\max} \leq \epsilon$, go to step 8, otherwise increment the iteration number and go to step 3.

Step 7: Check stopping criterion

The program will be terminated when the maximum iteration is reached or the convergence from the voltage mismatches is verified.

Step 8: Power loss calculation

After computing the node voltages and branch currents using the BFS algorithm, the total active and reactive power losses in the distribution system are calculated from (19) and (20).

The steps of the BFS algorithm to find the radial distribution system load flow are shown in Fig. 2.

5. ACO algorithm

5.1 Description of real ants

ACO algorithms were first proposed by Dorigo [28, 29]. ACO algorithms are based on the behaviour of real ants that are members of a family of social insects [29].

5.2 Mathematical model of ACO algorithm

Each ant attracts to the shortest route according to the probabilistic transition rule that depends on the amount of pheromone deposited

IET Gener. Transm. Distrib., 2016, Vol. 10, Iss. 5, pp. 1209–1221

and a heuristic guide function. Therefore, the probabilistic transition rule of ant k to go from city i to city j can be expressed as in traveling salesman problem [29] as:

$$P_{ij}^{k}(t) = \frac{\left[\tau_{ij}(t)\right]^{\alpha}\left[\eta_{ij}(t)\right]^{\beta}}{\sum_{q}\left[\tau_{iq}(t)\right]^{\alpha}\left[\eta_{iq}(t)\right]^{\beta}}; \quad j, q \in N_i^k \tag{31}$$

where, τ_{ij} is the pheromone trail deposited between city i and j by ant k, η_{ij} is the visibility or sight and equal to the inverse of the distance or the transition cost between city i and j ($\eta_{ij} = 1/d_{ij}$). α and β are two parameters that influence the relative weight of pheromone trail and heuristic guide function, respectively. q is the cities that will be visited after city i, while N_i^k is a tabu list in the memory of ant that recodes the cities visited to avoid stagnations. After each tour is completed, a *local pheromone update* is determined by each ant depending on the route of each ant as:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \rho\tau_{\circ} \tag{32}$$

After all ants attractive to the shortest route, a *global pheromone update* is considered to show the influence of the new addition deposits by the other ants that attractive to the best tour as:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \varepsilon\Delta\tau_{ij}(t) \tag{33}$$

where, $\tau_{ij}(t+1)$ is the pheromone after one tour or iteration, ρ is the pheromone evaporation constant, ε is the elite path weighting constant, $\tau_o = 1/\ d_{ij}$ is the incremental value of pheromone of each ant. While, $\Delta\tau_{ij}$ is the amount of pheromone for elite path as:

$$\Delta\tau_{ij}(t) = w/d_{\text{best}} \tag{34}$$

where, w is a large positive constant and d_{best} is the shortest tour distance.

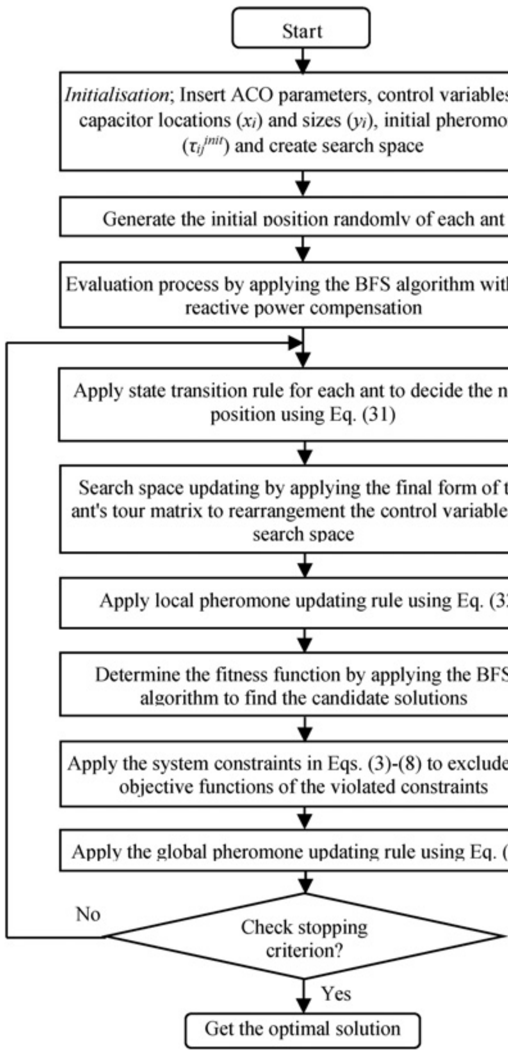


Fig. 3 *Flow chart of the ACO algorithm to get the optimal solution*

6 Optimal capacitor placement using ACO algorithm

The heuristic guide function of the problem is the inverse of the objective function in (1) at iteration $t+1$ as:

$$\eta(t+1) = 1/\sum_{i=1}^{N_b} F(x) \tag{35}$$

The ACO parameters are adjusted to find their optimal values. The ACO algorithm can be applied to find the optimal capacitor placement using the following steps:

Step 1: Initialisation

Insert the follows

- Insert the control variables that represent the capacitor locations randomly between 0 and 1 in the cases of fixed, switched and the combination of fixed and switched capacitors.
- Insert the control variables that represent the capacitor sizes between the minimum and maximum limits (150 and 1200 kVAR) in the cases of fixed, switched and the combination of them. In the case of fixed capacitors, these values are distributed randomly in the search space. In the case of switched capacitors, these values are distributed increasingly from the minimum limit to the maximum limit with fixed step (150 kVAR). In the combination of fixed and switched capacitors, a two-search space is used in parallel for fixed and switched capacitors.
- Insert the ACO parameters, number of ants (N_{ants}), α, β, ρ and ε.
- Create a search space with dimensions ($N_{\text{ants}} \times 2N_{\text{can}}$), where $N_{\text{can.}}$ is the number of candidate buses for capacitor placement. Thus, the control variable x_i initially can be represented randomly as:

$$x_i^{\text{initial}} = [x_1, x_2, x_3, \ldots, x_{\text{can.}}, y_1, y_2, y_3, \ldots, y_{\text{can.}}] \tag{36}$$

where, x_i refers to the states of capacitors locations and y_i refers to the sizes of capacitors.
- Create the initial pheromone with the same dimensions of search space which contains the elements with very small values to give the same chance of searching for all ants.

Step 2: Provide first position

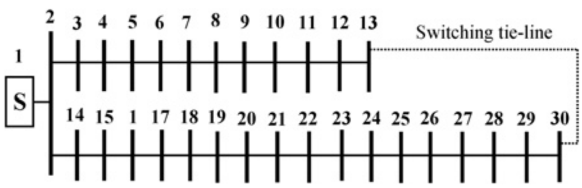


Fig. 4 *Single line diagram of the EDN distribution system*

Table 1 Bus and line data for EDN distribution system

Line no.	Sending bus	Receiving bus	*R*, Ω	*X*, Ω	Load at receiving bus	
					P, kW	*Q*, kVAR
1	1	2	0.05630	0.031500	2875	1814
2	2	3	0.07155	0.025974	1100	695
3	3	4	0.01855	0.006734	1058	669
4	4	5	0.05565	0.020202	899	568
5	5	6	0.05300	0.019240	770	486
6	6	7	0.05300	0.019240	668	423
7	7	8	0.02120	0.007696	598	378
8	8	9	0.10070	0.036556	546	345
9	9	10	0.04505	0.016354	380	240
10	10	11	0.03975	0.014430	210	132
11	11	12	0.11130	0.040404	94.586	59.368
12	12	13	0.01325	0.004810	34.423	21.518
13	2	14	0.06360	0.023088	1772	1118
14	14	15	0.07155	0.025974	1640	1035
15	15	16	0.02650	0.009620	1452	915
16	16	17	0.01060	0.003848	1434	904
17	17	18	0.09275	0.033670	1212	765
18	18	19	0.01060	0.003848	1086	685
19	19	20	0.02650	0.009620	953	602
20	20	21	0.04505	0.016354	827	521
21	21	22	0.05300	0.019240	716	452
22	22	23	0.05300	0.019240	550	347
23	23	24	0.0663	0.0240	434	273
24	24	25	0.2253	0.0818	346	218
25	25	26	0.0265	0.0096	316	199
26	26	27	0.0265	0.0096	184	116
27	27	28	0.0133	0.0048	139	87.911
28	28	29	0.1723	0.0625	113	71.734
29	29	30	0.0080	0.0029	34.25	21.734

Each ant is positioned on the initial state randomly within the reasonable range of each control variable in a search space with one ant at each position in the length of randomly distributed values.

Step 3: Evaluation

The initial value of objective function is obtained by applying the BFS algorithm without reactive power compensation.

Step 4: Transition rule

Each ant decides to visit a next position in the range of other control variables according to the probability transition rule in (31).
The next positions are decided as the following steps:

1. Calculate the probability using (31) for each ant.
2. Calculate a cumulative probability q_i for each ant as:

$$q_i = \sum_{j=1}^{N_b} P_j \tag{37}$$

3. Generate a random value q where, $0 \le q \le 1$.
4. If $q < q_i$, select the next position that has the highest probability, otherwise, the next node will be selected randomly from the list of unvisited positions.
5. Repeat the previous steps for *ant*-times.
6. Get the final form of the ant's tour matrix.

Step 5: Search space updating

The search space is updated by applying the final form of the ant's tour matrix on the search space in order to rearrangement the positions of control variables.

Table 2 Ranking of load buses based on LSIs for 34-bus distribution system

LSI_1				LSI_2			
Order	Value	Order	Value	Order	Value	Order	Value
4	−0.00069325	10	-1.9017×10^{-5}	4	0.0072978	10	0.0013679
2	−0.00058565	26	-4.8323×10^{-6}	5	0.0063444	26	0.00055534
5	−0.00057299	11	-4.5467×10^{-6}	6	0.0060446	11	0.00053175
6	−0.00051708	31	-2.0828×10^{-6}	2	0.0056228	28	0.00040076
3	−0.00049271	28	-2.0554×10^{-6}	3	0.0049517	32	0.00038896
17	−0.00027041	13	-1.962×10^{-6}	17	0.0047988	31	0.00038868
19	−0.00021132	32	-1.5638×10^{-6}	19	0.0045854	13	0.00037879
18	−0.00020572	14	-1.2312×10^{-6}	7	0.0044136	14	0.00034577
20	−0.00015157	29	-9.1415×10^{-7}	18	0.0040158	29	0.0002673
7	−0.00013052	27	-5.3921×10^{-7}	22	0.0038371	33	0.00019455
22	−0.00011626	33	-5.2154×10^{-7}	20	0.0037091	27	0.00016604
21	−0.00011532	12	-5.0527×10^{-7}	21	0.0032402	12	0.00015781
23	-7.9089×10^{-5}	30	-2.2862×10^{-7}	23	0.0031694	30	0.00013368
8	-5.927×10^{-5}	15	-1.8064×10^{-7}	24	0.0029918	15	9.3109×10^{-5}
24	-5.8567×10^{-5}	34	-8.6939×10^{-8}	9	0.0028469	34	6.4857×10^{-5}
9	-5.4732×10^{-5}	16	-2.14×10^{-9}	8	0.002419	16	6.6507×10^{-6}
25	-2.0427×10^{-5}			25	0.0014437		

Table 3 Ranking of load buses based on LSIs for 85-bus distribution system

LSI_1						LSI_2					
Order	Value	Order	Value	Order	Value	Order	Value	Order	Value	Order	Value
8	−0.0021508	12	-1.7638×10^{-5}	55	-8.7152×10^{-7}	8	0.048681	12	0.0029888	78	0.00072643
6	−0.00072046	33	-1.5869×10^{-5}	76	-8.2599×10^{-7}	6	0.016565	44	0.0026265	54	0.00067917
4	−0.00045512	52	-1.506×10^{-5}	39	-8.1234×10^{-7}	58	0.010766	61	0.002553	55	0.0006783
7	−0.00044762	63	-1.4452×10^{-5}	51	-7.2111×10^{-7}	7	0.010373	19	0.0024733	76	0.00065446
3	−0.00036498	69	-1.139×10^{-5}	22	-7.1052×10^{-7}	4	0.0092119	80	0.0024231	39	0.00064723
2	−0.00024383	19	-1.108×10^{-5}	14	-6.1799×10^{-7}	27	0.0087127	9	0.0023393	85	0.00059609
5	−0.00021882	61	-9.5461×10^{-6}	75	-5.9922×10^{-7}	25	0.0084677	73	0.0023047	24	0.00059543
25	−0.00017016	80	-8.6485×10^{-6}	37	-5.1832×10^{-7}	29	0.0075644	32	0.0020701	51	0.00056257
58	−0.00015185	44	-7.1711×10^{-6}	85	-4.6531×10^{-7}	34	0.0072943	45	0.0016889	16	0.00043714
27	−0.00014849	49	-5.8974×10^{-6}	24	-4.4069×10^{-7}	3	0.00712	33	0.0016594	37	0.00041905
26	−0.00012761	73	-4.7324×10^{-6}	65	-4.1908×10^{-7}	30	0.0069863	63	0.0015701	71	0.00041217
29	−0.00011101	45	-3.2576×10^{-6}	74	-4.1097×10^{-7}	60	0.006608	41	0.001527	79	0.00040706
30	$-9.4225*10^{-5}$	13	-3.054×10^{-6}	71	-3.2767×10^{-7}	26	0.0065744	13	0.0013046	14	0.00039587
60	$-8.542*10^{-5}$	40	-2.4842×10^{-6}	79	-3.2159×10^{-7}	64	0.0061203	62	0.0011732	43	0.00034687
34	$-6.7791*10^{-5}$	41	-2.4392×10^{-6}	16	-3.0994×10^{-7}	10	0.0050413	38	0.0011692	74	0.00032616
28	$-6.4357*10^{-5}$	17	-1.9924×10^{-6}	43	-2.7711×10^{-7}	2	0.0047217	83	0.0011136	84	0.0002904
57	$-5.8547*10^{-5}$	53	-1.9306×10^{-6}	72	-2.6997×10^{-7}	68	0.0046145	40	0.0010379	65	0.00026797
9	$-5.4935*10^{-5}$	20	-1.8717×10^{-6}	66	-2.6833×10^{-7}	5	0.0045154	21	0.0010327	15	0.00023733
64	$-5.4662*10^{-5}$	81	-1.8457×10^{-6}	59	-2.6023×10^{-7}	52	0.0044589	49	0.001017	72	0.00021532
10	$-4.5902*10^{-5}$	70	-1.8263×10^{-6}	23	-2.0929×10^{-7}	28	0.0040617	46	0.00098564	66	0.00021444
31	$-4.2019*10^{-5}$	50	-1.5647×10^{-6}	15	-1.8528×10^{-7}	35	0.0040324	22	0.0009773	59	0.0002101
35	$-2.6562*10^{-5}$	21	-1.5006×10^{-6}	42	-1.6622×10^{-7}	18	0.0039885	53	0.00092167	42	0.00020808
11	$-2.5617*10^{-5}$	62	-1.4635×10^{-6}	82	-1.3058×10^{-7}	57	0.0038448	70	0.00088816	23	0.0001817
32	$-2.4725*10^{-5}$	38	-1.4559×10^{-6}	36	-1.1326×10^{-7}	11	0.0035975	17	0.00087918	56	0.0001685
68	$-2.4724*10^{-5}$	83	-1.216×10^{-6}	84	-9.0109×10^{-8}	48	0.0035575	20	0.00085936	47	0.00016786
67	-2.3404×10^{-5}	46	-1.1087×10^{-6}	56	-5.3955×10^{-8}	69	0.0034351	81	0.00079176	36	0.00014075
18	-2.2423×10^{-5}	78	-8.9426×10^{-7}	47	-5.3647×10^{-8}	31	0.0033028	75	0.00075463	82	0.0001053
48	-2.0623×10^{-5}	54	-8.7321×10^{-7}	77	-8.382×10^{-9}	67	0.0031696	50	0.00074104	77	$2.6798*10^{-5}$

Table 4 Ranking of load buses based on LSIs for EDN system

LSI_1				LSI_2			
Order	Value	Order	Value	Order	Value	Order	Value
2	−0.0068485	25	-7.8591×10^{-5}	2	0.013572	24	0.0011989
14	−0.0027531	23	-6.3796×10^{-5}	14	0.0091847	4	0.0010613
15	−0.002385	24	-4.3829×10^{-5}	15	0.0091074	17	0.00099207
18	−0.0011665	9	-4.0513×10^{-5}	18	0.0072927	19	0.00068812
3	−0.00070925	8	-1.843×10^{-5}	3	0.0049397	8	0.00043472
16	−0.00065368	10	-5.8567×10^{-6}	25	0.0029662	10	0.00035701
5	−0.00024325	26	-4.4632×10^{-6}	16	0.0029057	29	0.0002972
17	−0.00019028	27	-1.5988×10^{-6}	5	0.0025553	26	0.00024259
21	−0.0001607	11	-1.1473×10^{-6}	6	0.0019198	12	0.00015763
20	−0.00014938	29	-1.0221×10^{-6}	21	0.0018895	11	0.00014812
6	−0.0001439	12	-4.6503×10^{-7}	22	0.0017268	27	0.00014538
4	−0.00012615	28	-2.9658×10^{-7}	7	0.0014765	28	4.4338×10^{-5}
22	−0.00011385	13	-3.9349×10^{-9}	9	0.001405	13	4.9921×10^{-6}
19	$-9.0832*10^{-5}$	30	-2.5519×10^{-9}	20	0.0013963	30	3.1896×10^{-6}
7	$-8.4975*10^{-5}$			23	0.0012931		

Table 5 Comparison between the optimal capacitor placement with other methods for 34-bus distribution system

Items	Un-compensated	Compensated					
		PGSA [2]	BFA [9]	GA [10]	APSO [12]	Proposed method	
						Case1	Case 2
total losses (kW)	221.67	169.167	169.07	168.955	168.023	162.680	164.508
loss reduction (%)	–	23.69	23.73	23.78	24.20	26.61	25.79
minimum bus voltage(p.u.)	0.9417 (#27)	0.9492 (#27)	0.9503 (#27)	0.9491 (#27)	0.9416 (#27)	0.9501 (#27)	0.9501 (#27)
maximum bus voltage(p.u.)	0.9941 (#2)	0.995 (#2)	0.9948 (#2)	0.9948 (#2)	0.9949 (#2)	0.995 (#2)	0.9949 (#2)
maximum power flow (kW)	4858.3 (#$L_{1\text{-}2}$)	4805.7 (#$L_{1\text{-}2}$)	4806.5 (#$L_{1\text{-}2}$)	4805.5 (#$L_{1\text{-}2}$)	4804.5 (#$L_{1\text{-}2}$)	4799.2 (#$L_{1\text{-}2}$)	4801 (#$L_{1\text{-}2}$)
overall power factor	0.85	0.9842	0.9588	0.9658	0.9738	0.984	0.981
optimal locations and sizes in kVAR	–	19 1200 20 200 22 639	9 600 22 900 – –	7 buses 1629	19 1050 25 750 – –	9 645 22 719 25 665	9 450 19 450 25 1050
total capacitors power (kVAR)	–	2039	1500	1629	1800	2029	1950
Annual cost ($/year)	37,241	28,420	28,404	28,384	28,228	27,330	27,637
total capacitors cost ($/year)	–	1019.5	750	814.5	900	1014.5	975
net savings ($/year)	–	8821	8837	8857	9013	9911	9604

Table 6 Comparison between the optimal capacitor placement with other methods for 85-bus distribution system

Items	Un-compensated	Compensated													
		PGSA [2]		PSO [6]		GA [7]		DSA [8]		TLBO [11]		Proposed method			
												Case 1		Case 2	
Total losses (kW)	315.714	174.912		163.32		146.061		144.01		143.18		143.347		143.874	
loss reduction (%)	–	44.60		48.27		53.74		54.39		54.65		54.60		54.43	
minimum bus voltage(p.u.)	0.8713 (#54)	0.9090 (#54)		0.9156 (#54)		0.9246 (#55)		0.9185 (#54)		0.9242 (#54)		0.9244 (#54)		0.9220 (#54)	
maximum bus voltage(p.u.)	0.9957 (#2)	0.9973 (#2)		0.9974 (#2)		0.9973 (#2)		0.9975 (#2)		0.9976 (#2)		0.9976 (#2)		0.9975 (#2)	
maximum power flow (kW)	2886.7 (#$L_{1\text{-}2}$)	2745.2 (#$L_{1\text{-}2}$)		2734.1 (#$L_{1\text{-}2}$)		2716.6 (#$L_{1\text{-}2}$)		2715.7 (#$L_{1\text{-}2}$)		2713.5 (#$L_{1\text{-}2}$)		2713.6 (#$L_{1\text{-}2}$)		2714.2 (#$L_{1\text{-}2}$)	
overall power factor	0.7	0.9926		0.9983		0.9872		0.9996		0.9995		0.9992		0.9996	
optimal locations and sizes in kVAR	–	7	200	7	324	26	48.437	6	150	4	300	4	185	7	150
		8	1200	8	796	28	214.062	8	150	7	150	7	150	8	300
		58	908	27	901	37	103.125	14	150	9	300	9	210	19	300
		–	–	58	453	38	120.312	17	150	21	150	13	150	27	300
		–	–	–	–	39	178.125	18	150	26	150	18	280	32	300
		–	–	–	–	51	100	20	150	31	300	26	320	48	300
		–	–	–	–	54	212.5	26	150	45	150	31	250	61	300
		–	–	–	–	55	101.562	30	150	49	150	35	205	68	300
		–	–	–	–	59	4.6875	36	450	55	150	53	200	80	300
		–	–	–	–	60	157.812	57	150	61	300	61	250	–	–
		–	–	–	–	61	112.5	61	150	68	300	68	330	–	–
		–	–	–	–	62	104.688	66	150	83	150	80	196	–	–
		–	–	–	–	66	9.375	69	300	85	150	–	–	–	–
		–	–	–	–	69	100	80	150	–	–	–	–	–	–
		–	–	–	–	72	67.187	–	–	–	–	–	–	–	–
		–	–	–	–	74	112.5	–	–	–	–	–	–	–	–
		–	–	–	–	76	71.875	–	–	–	–	–	–	–	–
		–	–	–	–	80	356.25	–	–	–	–	–	–	–	–
		–	–	–	–	82	31.25	–	–	–	–	–	–	–	–
total capacitors power (kVAR)	–	2308		2474		2206.25		2550		2700		2726		2550	
annual cost ($/year)	53,040	29,385		27,438		24,538		24,194		24,054		24,082		24,171	
total capacitors cost ($/year)	–	1154		1237		1103.1		1275		1350		1363		1275	
net savings ($/year)	–	23,655		25,602		28,502		28,846		28,986		28,958		28,869	

Step 6: Local pheromone updating

The initial pheromone of each ant is locally updated as in (32).

Step 7: Fitness function

After all ants are attracted to the shortest path, the control variables that represent the capacitor locations are rounded to unity. Then, the final form of control variables can be obtained as:

$$x_i^1 = x_i \cdot y_i \tag{38}$$

After that, the BFS algorithm is applied to find the candidate solutions of the objective function as:

$$F_i^1 = [F_1, F_2, F_3, \ldots, F_{N\text{ants}}] \tag{39}$$

Step 8: Check the constraints

For each ant, check the system constraints in (3)–(8) based on the values of control variables. Therefore, the objective function F_i can be written as (see (40))

Step 9: Global pheromone updating

After all ants attractive to the minimum objective function, compare the current global best with the previous global best to find the optimal solution (F_{gbest}). Therefore, the global pheromone is updated using (33).

Step 10: Check stopping criterion

The program will be terminated when the maximum iteration is reached or the best solution is obtained without the ants stagnations.

The flow chart of the ACO algorithm to find the optimal solution is shown in Fig. 3.

7 Applications

7.1 Test systems

The proposed methodology using the multi-stage method is applied on two standard radial distribution systems in order to solve the optimal capacitor placement problem. These test systems are 34-bus [2, 4] and 85-bus [11, 30]. The results are compared with those obtained using other reported methods. Moreover, a real distribution system of the East Delta Network (EDN) as a part of the Unified Egyptian Network (UEN) is used as a test system. The single line diagram of EDN is shown in Fig. 4. Table 1 shows the bus and line data of the EDN system, where the rated line voltage is 11 kV and the rated KVA is 27,221 with 0.854 power factor. The ACO parameters are adjusted to be, $\alpha = 1$, $\beta = 5$, $\rho = 0.7$, $\varepsilon = 5$ and $w = 100$.

K_p is assumed to be 168 \$/(kw-year) and K_C is assumed to be 5 \$/kVAR, with a life expectancy of 10 years, where the maintenance and running costs are neglected. The limit of voltage magnitude is taken between 0.95 and 1.05 p.u. for 34-bus and EDN systems, while the voltage limit is 0.90 and 1.1 p.u. for 85-bus system. The maximum number of possible locations ($N_C^{\max}$) is assumed to be 4 for 34-bus and EDN test systems. $N_C^{\max}$ is 15 for 85-bus test system, because the voltage magnitude at 46 buses without compensation is lower than the minimum voltage limit, means that there is a violation of the minimum voltage limit at 54.12% of the total number of system buses. The maximum limits

$$F_i = \begin{cases} \text{Excluded} & \text{for the index of ant corresponding to the violated constraints} \\ \text{Keep the same value} & \text{otherwise} \end{cases} \tag{40}$$

of power flow (PF^{max}) are 5, 3 and 25 MW for 34-bus, 85-bus and EDN test systems, respectively. The minimum limit of overall power factor ($pf^{min}_{overall}$) is 0.9 lagging for all test systems. The substation is located at bus number 1 for all test systems with constant voltage (1 p.u.).

Two different types of capacitors are considered to find the optimal solution, which are:

- Fixed capacitors with minimum and maximum limits are 150 and 1200 kVAR.
- Switched capacitors with standard commercial available sizes are from 150 to 1200 by step 150 kVAR.

Three case studies are employed to check the capability of the proposed method as:

Case 1: Optimal locations and sizes of fixed capacitors.
Case 2: Optimal locations and sizes of switched capacitors.
Case 3: Optimal locations and sizes of the combination of fixed and switched capacitors.

7.2 Results and comments

The results of the proposed method are obtained using the MATLAB code version 7.0.1 applied on a Pentium 4, 3.0 GHz PC, 1 GB of RAM memory.

7.2.1 Results of LSIs: Tables 2–4 show the results of the LSIs for all radial distribution systems. The buses that are located at the top of LSIs list are considered the candidate buses for the capacitors placement, while the buses have the less sensitivity to the installation of capacitors are indexed at the bottom of this list.

Table 2 shows the ordering of load buses based on LSI_1 and LSI_2 for 34-bus distribution system. The load buses in the first 17 rows from LSI_1 and LSI_2 are considered the candidate buses for the capacitors placement.

Table 3 shows the ordering of load buses based on LSI_1 and LSI_2 for 85-bus distribution system. The load buses in the first 45 rows from LSI_1 and LSI_2 are considered the candidate buses for the capacitors placement.

Table 4 shows the ordering of load buses based on LSI_1 and LSI_2 for EDN system. The load buses in the first 15 rows from LSI_1 and LSI_2 are considered the candidate buses for the capacitors placement.

Table 7 Optimal locations and sizes of capacitors for cases 1 and 2 from the proposed method for EDN system

Items	Un-compensated	Compensated	
		Case 1	Case 2
Total losses (kW)	805.730	648.148	646.383
loss reduction (%)	–	19.56	19.78
minimum bus voltage(p.u.)	0.9463 (#29,30)	0.9539 (#29,30)	0.9529 (#29,30)
maximum bus voltage(p.u.)	0.9854 (#2)	0.9867 (#2)	0.9868 (#2)
maximum power flow (kW)	23,247 (#$L_{1\text{-}2}$)	23,089 (#$L_{1\text{-}2}$)	23,088 (#L1-2)
overall power factor	0.8457	0.9211	0.9229
optimal locations and sizes in kVAR	–	18 1147	9 1200
		20 1180	18 1200
		21 1167	21 1200
		25 1180	25 1200
total capacitors power (kVAR)	–	4674	4800
annual cost ($/year)	135,360	108,890	108,590
total capacitors cost ($/year)	–	2337	2400
net savings ($/year)	–	26,470	26,770

7.2.2 Results of optimal locations and sizes of capacitors: Tables 5–7 show the optimal locations and sizes of fixed (case 1) and switched (case 2) capacitors required to reduce the total active power loss and voltage profile improvement for all radial distribution systems. Moreover, a comparison between the optimal capacitor placement that is obtained using the proposed method and the other techniques is presented. However, Table 8 shows the optimal locations and sizes of the combination of fixed and switched capacitors (case 3) for all radial distribution systems.

Table 5 shows the optimal results for 34-bus system. It is clear that, the initial power loss without compensation is reduced from 221.67 kW to 162.680 kW and 164.508 kW for cases 1 and 2, respectively. The optimal locations of capacitors are at buses {9,22,25} with total rating 2029 kVAR for case 1 and {9,19,25} with total rating 1950 kVAR for case 2. Moreover, the minimum and maximum voltage magnitudes are improved in cases 1 and 2.

Table 6 presents the optimal results for 85-bus system. It can be observed that, the initial power loss without compensation is reduced from 315.714 kW to 143.347 kW and 143.874 kW for cases 1 and 2, respectively. The optimal locations of capacitors are at buses {4,7,9,13,18,26,31,35,53,61,68,80} with total rating 2726 kVAR for case 1 and {7,8,19,27,32,48,61,68,80} with total rating 2550 kVAR for case 2. The total power loss obtained using

Table 8 Optimal locations and sizes of capacitors for case 3 from the proposed method for all radial distribution systems

	34-bus system				85-bus system				EDN system			
	Case 3				Case 3				Case 3			
	Fixed capacitors		Switched capacitors		Fixed capacitors		Switched capacitors		Fixed capacitors		Switched capacitors	
Total losses (kW)	163.454				143.99				646.92			
loss reduction (%)	26.26				54.39				19.71			
minimum bus voltage(p.u.)	0.9502 (#27)				0.9239 (#54)				0.9529 (#29,30)			
maximum bus voltage(p.u.)	0.9950 (#2)				0.9976 (#2)				0.9868 (#2)			
maximum power flow (kW)	4800 (#$L_{1\text{-}2}$)				2714.3 (#$L_{1\text{-}2}$)				23,088 (#$L_{1\text{-}2}$)			
overall power factor	0.9833				0.9996				0.9239			
optimal locations and sizes in kVAR	22	494	9	600	4	155	19	300	18	937	18	1050
	24	175	24	450	7	212	27	300	21	1081	22	900
	25	296	–	–	9	249	–	–	22	900	–	–
	–	–	–	–	13	298	–	–	–	–	–	–
	–	–	–	–	32	343	–	–	–	–	–	–
	–	–	–	–	35	150	–	–	–	–	–	–
	–	–	–	–	53	185	–	–	–	–	–	–
	–	–	–	–	61	155	–	–	–	–	–	–
	–	–	–	–	68	346	–	–	–	–	–	–
total capacitors power (kVAR)	2015				2693				4868			
annual cost ($/year)	27,460				24,190				108,680			
total capacitors cost ($/year)	1007.5				1346.5				2434			
net savings ($/year)	9781				28,850				26,680			

the proposed method is lower than those obtained using PGSA [2], PSO [6], GA [7] and DSA [8], while the total power loss obtained using TLBO [11] is lower than obtained using the other methods because all buses are considered candidate buses for capacitors installation in [11], while some of the system buses are considered candidate buses based on LSI_1 and LSI_2 using the proposed

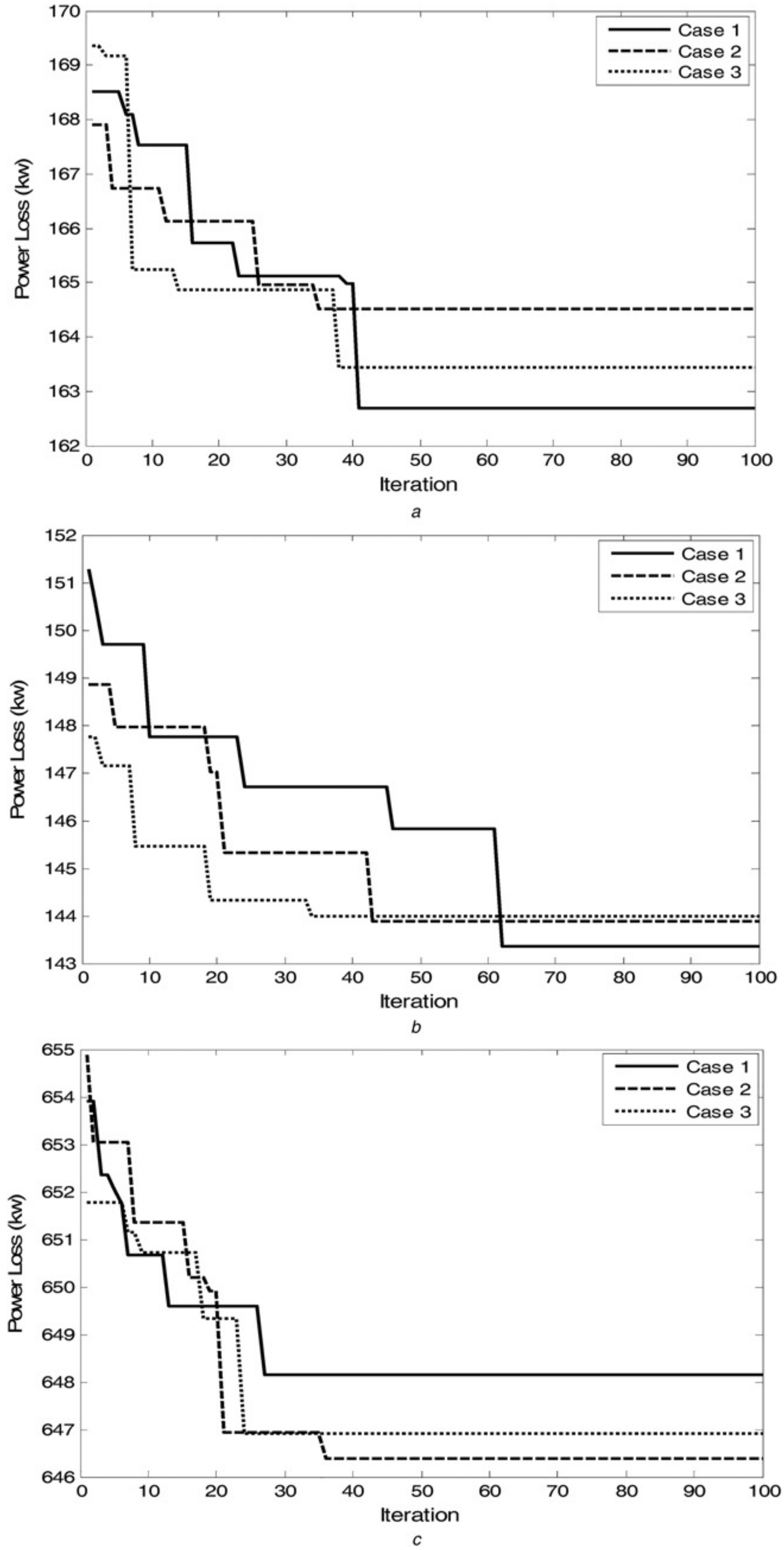
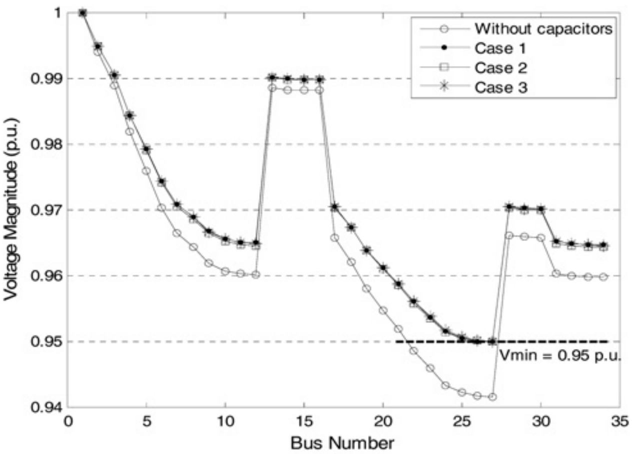
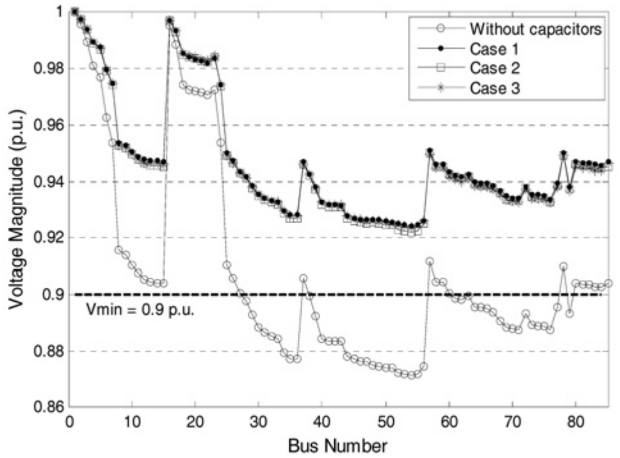


Fig. 5 *Convergence curves of the ACO algorithm for different test distribution systems*

a 34-bus system
b 85-bus system
c EDN system





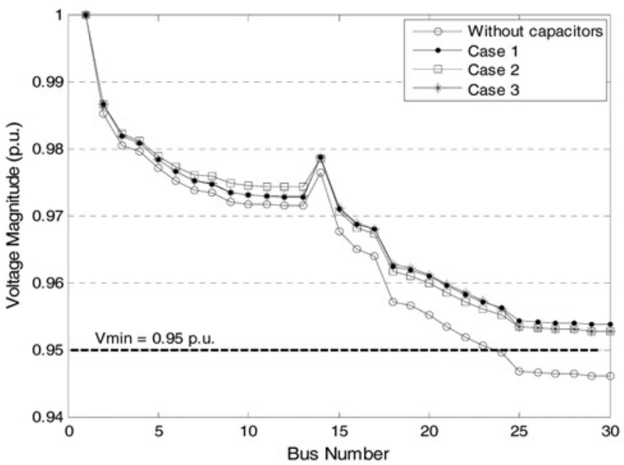


Fig. 6 *Voltage profile for different test distribution systems*

a 34-bus system
b 85-bus system
c EDN system

method to reduce the search space. Moreover, the number of constraints in this paper is more than considered in the other methods. However, the minimum and maximum voltage magnitudes using the proposed method are better than other methods.

Table 7 presents the optimal results for EDN system. It is clear that, the initial power loss without compensation is reduced from 805.730 kW to 648.148 kW and 646.383 kW for cases 1 and 2, respectively. The optimal locations of capacitors are at buses {18,20,21,25} with total rating 4674 kVAR for case 1 and {9,18,21,25} with total rating 4800 kVAR for case 2. Moreover, the minimum and maximum voltage magnitudes are improved in cases 1 and 2.

Table 8 shows the optimal results for case 3 for all test systems. For 34-bus system, the initial power loss without compensation is reduced from 221.67 kW to 163.454 kW, with optimal locations of capacitors are at buses {22,24,25} and {9,24} for fixed and switched capacitors, respectively, with total rating 2015 kVAR. For 85-bus system, the initial power loss without compensation is reduced from 315.714 kW to 143.99 kW, with optimal locations of capacitors are at buses {4,7,9,13,32,35,53,61,68} and {19,27} for fixed and switched capacitors, respectively, with total rating 2693 kVAR. For EDN system, the initial power loss without compensation is reduced from 805.730 kW to 646.92 kW, with optimal locations of capacitors are at buses {18,21,22} and {18,22} for fixed and switched capacitors, respectively, with total rating 4868 kVAR. Moreover, the minimum and maximum voltage magnitudes are improved in case 3.

From these tables, the optimal number of capacitors placement is the same with some other methods but the total power loss and the total kVAR injected by the proposed procedure are lower than those obtained using the other methods. In addition, the optimal results obtained from case 1 are better than those obtained from cases 2 and 3 because the fixed capacitors give the flexibility of selecting the values of control variables between the minimum and maximum limits, while the values of control variables in the switched capacitors are restricted to specified values between the minimum and maximum limits. Moreover, the overall power factor is improved in cases 1–3. In addition, the maximum power flow in all branches and the overall power factor are within permissible limits. Therefore, this comparison reflects to the great capability of the proposed method in finding the optimal locations and sizes of capacitors in order to reduce the total costs and improve the system reliability.

7.2.3 Convergence curves of ACO algorithm: Fig. 5 shows the convergence curves of the ACO algorithm to reduce the total power loss for cases 1–3 for all radial distribution systems. It is clear that, the ACO algorithm leads to obtain the optimal solution with fast convergence curve.

7.2.4 Voltage profile: Fig. 6 shows the voltage profiles of four cases (without capacitors, cases 1, 2 and 3) for all radial distribution systems. The voltage profiles are improved at cases 1–3. The voltage profile improvement from case 1 is better than those obtained from cases 2 and 3 for all the test systems, where the average values of voltages for cases 1, 2 and 3 are 0.9704, 0.9702 and 0.9704 for 34-bus, 0.9472, 0.9464 and 0.9766 for 85-bus, 0.9683, 0.9682 and 0.9682 for EDN system. Moreover, the violation of minimum voltage limit at some buses is existed in the case of without capacitors for all distribution systems.

7.2.5 Lines power flow: Fig. 7 shows the lines power flow of three cases (without capacitors, cases 1 and 2) for all radial distribution systems. It can be observed that, the lines power flow from cases 1 and 2 are lower than those obtained in the case of without capacitors. Moreover, the power flow in all branches is within permissible limits.

8 Conclusions

This paper presented an efficient multi-stage procedure based on two LSIs and the ACO algorithm to find the optimal locations and sizes of capacitors placement for power loss reduction and voltage profile improvement in radial distribution systems. First, the LSIs have been used to select the candidate locations for the capacitors to reduce the search space for the optimisation process. Then, the ACO algorithm has been used to find the optimal locations and sizes of capacitors at three case studies based on fixed, switched and combination of fixed and switched capacitors. The BFS load flow algorithm has been used for the load flow calculations. The proposed procedure has been tested on small and large scale distribution systems. Moreover, a

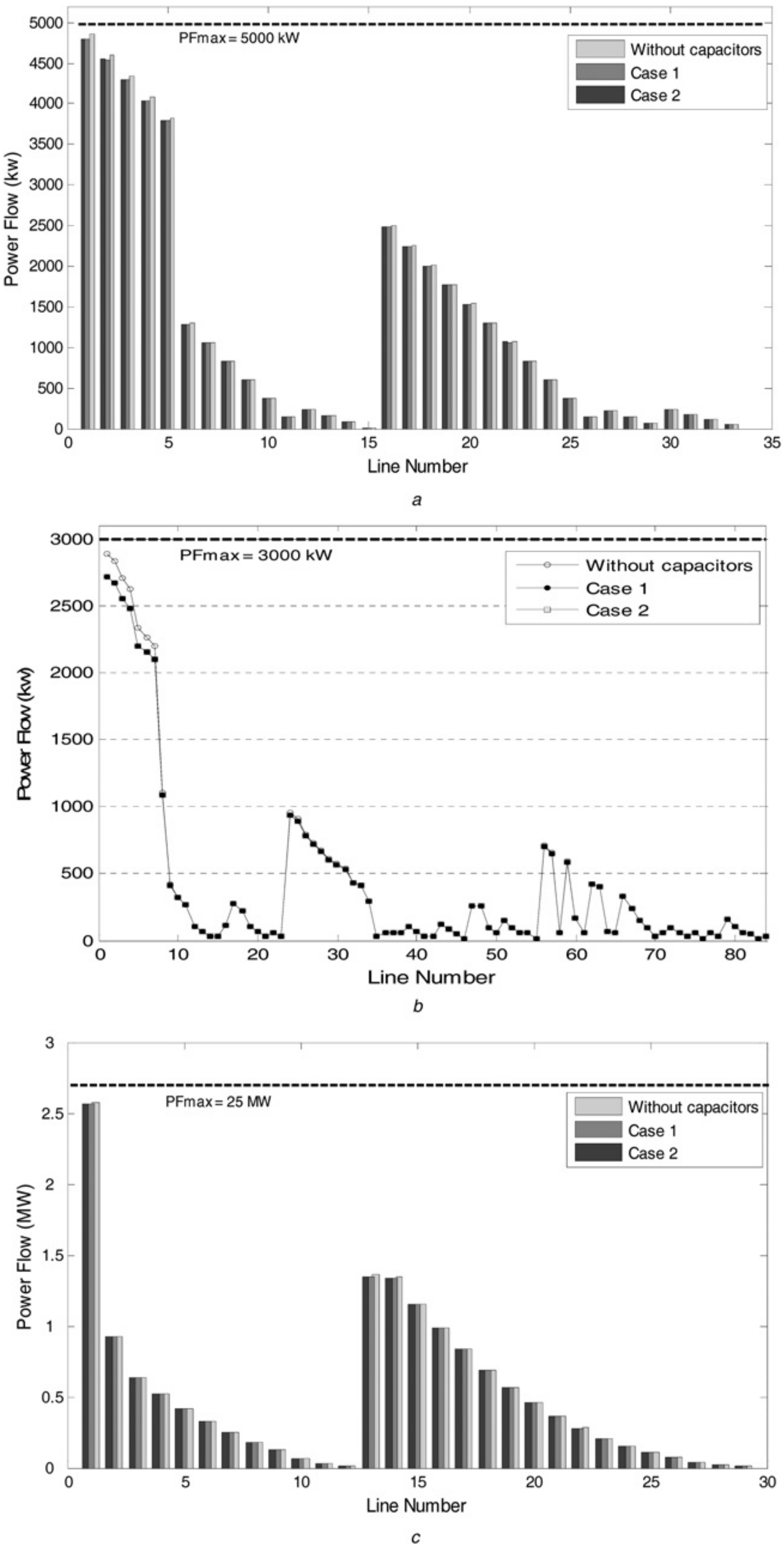


Fig. 7 *Lines power flow for different test distribution systems*

a 34-bus system
b 85-bus system
c EDN system

real distribution system as a part of the UEN has been used to show the capability of proposed methodology. In addition, the ACO algorithm gives fast convergence with more accurate and efficient optimal solution. The optimal results using the proposed procedure have been compared with other methods and have been proved that the capability of the proposed procedure to find the optimal solutions with minimum power loss and voltage profile improvement. Therefore, the proposed procedure represents a potential tool to reduce the system losses and helps their operators in smart grid environment.

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