

永中鍾定桐

DATE 2024

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ex. 8 下图中, 每列数字为上一列相邻两数的和

Def a_n 为每列最左边的数字, 求

1) a_n 递推式

$$a_1 = 1$$

$$a_2 = 3$$

$$a_3 = 8$$

$$a_4 = 20$$

$$a_5 = 48$$

$$> 2$$

$$> 5$$

$$> 12$$

$$> 28$$

1	2	3	...	9	10
3	5	7	9	19	
8	12	16			
20	28				
48					

$$\begin{cases} a_1 = 1 \\ a_n = 2a_{n-1} + 2^{n-2} \end{cases}$$

$$a_n = 2a_{n-1} + 2^{n-2}$$

$$\Rightarrow a_n = (n+1) \cdot 2^{n-2}$$

$$\begin{aligned} \cancel{a_1} &= \cancel{1} \\ \cancel{a_2} &= \cancel{2a_1} + \cancel{2^{1-2}} \\ \cancel{a_3} &= \cancel{2a_2} + \cancel{2^{2-2}} \end{aligned}$$

$$\begin{aligned} \cancel{a_{n-2}} &= \cancel{2a_{n-3}} + \cancel{2^{n-4}} \\ \cancel{a_{n-1}} &= \cancel{2a_{n-2}} + \cancel{2^{n-3}} \end{aligned}$$

$$+) \quad a_n = 2a_{n-1} + 2^{n-2}$$

$$\begin{aligned} a_n &= (n+1) 2^{n-2} + 2^{n-1} \\ &= (n+1) 2^{n-2} + 2 \cdot 2^{n-2} \\ &= (n+1) 2^{n-2} \end{aligned}$$

2) 图中最下方数字

$$a_{10} = (10+1) \cdot 2^{10-2}$$

$$= 11 \cdot 2^8 = 11 \cdot 256 = 2816$$