DATE 2024 .12 17.29 NO 三校数136

三校数作98 永中鍾定楓

ex.1 設 $f(x) = x^3 - x^2 + x - 1$，求 $f(x^4)$，$r(x)$ of $f(x^5) \div f(x)$

$f(x^4) = x^{12} - x^8 + x^4 - 1$ # $r(x)$ of $f(x^4) \div f(x)$

$f(x^5) = x^{15} - x^{10} + x^5 - 1$

$$
\begin{array}{l}
\qquad\qquad\qquad x^{12} + x^{11} + x^8 - x^6 + x^4 + x + 1 \\
x^3 - x^2 + x - 1 \,\big)\, x^{15} + 0x^{14} + 0x^{13} + 0x^{12} + 0x^{11} - x^{10} + 0x^9 + 0x^8 + 0x^7 + 0x^6 + x^5 + 0x^4 + 0x^3 + 0x^2 + 0x - 1 \\
\qquad x^{15} - x^{14} + x^{13} - x^{12} \\
\hline
\qquad x^{14} - x^{13} + x^{12} + 0x^{11} \\
\qquad x^{14} - x^{13} + x^{12} - x^{11} \\
\hline
\qquad x^{11} - x^{10} + 0x^9 + 0x^8 \\
\qquad x^{11} - x^{10} + x^9 - x^8 \\
\hline
\qquad -x^9 + x^8 + 0x^7 + 0x^6 \\
\qquad -x^9 + x^8 - x^7 + x^6 \\
\hline
\qquad x^7 - x^6 + x^5 + 0x^4 \\
\qquad x^7 - x^6 + x^5 - x^4 \\
\hline
\qquad x^4 + 0x^3 + 0x^2 + 0x \\
\qquad x^4 - x^3 + x^2 - x \\
\hline
\qquad x^3 - x^2 + x - 1 \\
\qquad x^3 - x^2 + x - 1 \\
\hline
\end{array}
$$

$r(x) = 0$ #

☆ $x^4 - 1 = (x+1)(x^3 - x^2 + x - 1)$

$\Rightarrow x^4 \equiv 1 \pmod{f(x)}$

$f(x^4) \equiv (x^4)^3 - (x^4)^2 + x^4 - 1$

$\equiv 1 - 1 + 1 - 1$

$\equiv 0$ # $\pmod{f(x)}$