

DATE 2024. 8

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NO. 三校教42
三校教作30

2024 中金鍾定本

ex. 5 求滿足以下條件之正整數 n , 共多少種?

$$\text{條件一} - \gcd(n, 2^3 \times 3^2 \times 5) = 2 \times 3^2 \times 5$$

$$\text{條件二} - \text{lcm}(n, 3 \times 5^3 \times 7^2) = 2 \times 3^4 \times 5^3 \times 7^2$$

$$\left\{ \begin{array}{l} \text{條件一} \Rightarrow n = 2 \times 3^{k_1} \times 5^{k_2} \times \boxed{\dots} \times 7^{k_3}, k_1 \geq 2, k_2 \geq 1, k_1, k_2 \in \mathbb{Z} \\ \text{條件二} \Rightarrow n = 2 \times 3^4 \times 5^{k_2} \times 7^{k_3}, 3 \geq k_2 \geq 0, 2 \geq k_3 \geq 0, k_2, k_3 \in \mathbb{Z} \end{array} \right.$$

$$\Rightarrow n = 2 \times 3^4 \times 5^{k_2} \times 7^{k_3}, 1 \leq k_2 \leq 3, 0 \leq k_3 \leq 2, k_2, k_3 \in \mathbb{Z}$$

$$\Rightarrow \begin{cases} k_2 = 3 \text{ 種} \\ k_3 = 3 \text{ 種} \end{cases} \Rightarrow 3 \times 3 = 9 \text{ (種)}^\#$$