ex-3 已知 $\begin{cases} f(x) \equiv 2x+3 \pmod{x^2+1} \\ f(x) \equiv -7x+15 \pmod{x^2-2} \end{cases}$,

求 $r(x)$ of $f(x) \div (x^2+1)(x^2-2)$

$f(x) \equiv \left(-\frac{2}{3}x-1\right)(x^2-2) + \left(-\frac{2}{3}x+5\right)(x^2+1)$

$\equiv -\frac{2}{3}x^3 - x^2 + \frac{4}{3}x + 1 - \frac{2}{3}x^3 + 5x^2 - \frac{2}{3}x + 5$

$\equiv -3x^3 + 4x^2 - x + 6 \text{ \#} \pmod{x^4 - x^2 - 2}$

$(x^2+1)(x^2-2) = x^4 - x^2 - 2$

$f(x) = (x^2+1)q_1(x) + 2x+3$

$f(x) = (x^2-2)q_2(x) + (-7x+15)$

$f(x) = (x^2+1)(x^2-2)q_3(x) + \underline{ax^3+bx^2+cx+d}$

$= (x^2+1)(mx+n) + 2x+3$

$= (x^2-2+3)(mx+n) + 2x+3$

$= (x^2-2)(mx+n) + 3(mx+n) + 2x+3 \quad \stackrel{\|}{-7x+15}$

$mx^3 + (m+n)x^2 + (m+2)x + (n+3)$

$= -3x^3 + x^2 - x + 7 \text{ \#}$

$\begin{cases} 3m+2 = -7 \Rightarrow m = -3 \\ 3n+3 = 15 \Rightarrow n = 4 \end{cases}$