DATE 2024 .9 .20,21 NO 三校数56

三校数作41 永中鍾克栩

ex.5 计算

1) $\binom{7}{0}+\binom{7}{1}\times 2+\binom{7}{2}\times 2^2+\dots+\binom{7}{7}\times 2^7$

$=\sum_{k=0}^{7}\binom{7}{k}\cdot 2^k=\sum_{k=0}^{7}\binom{7}{k}\cdot 2^k\cdot 1^{7-k}$

$=(2+1)^7$

$=3^7$

$=2187$ #

2) $\binom{10}{0}(\frac{1}{2})^{10}+\binom{10}{1}(\frac{1}{2})^9(\frac{3}{2})+\binom{10}{2}(\frac{1}{2})^8(\frac{3}{2})^2+\dots+\binom{10}{10}(\frac{3}{2})^{10}$

$=\sum_{k=0}^{10}\binom{10}{k}(\frac{1}{2})^k(\frac{3}{2})^{10-k}$

$=(\frac{3}{2}+\frac{1}{2})^{10}$

$=2^{10}$

$=1024$ #

3) $\binom{10}{0}+3\binom{10}{1}+5\binom{10}{2}+7\binom{10}{3}+\dots+19\binom{10}{9}+21\binom{10}{10}$

$=\sum_{k=0}^{10}(2k+1)\binom{10}{k}$

$=\sum_{k=0}^{10}2k\binom{10}{k}+\sum_{k=0}^{10}\binom{10}{k}$

$\sum_{k=0}^{10}\binom{10}{k}=2^{10}=1024$

$\sum_{k=0}^{10}k\binom{10}{k}=10\sum_{k=1}^{10}\binom{9}{k-1}$

$=10\cdot 2^9$

$=10\cdot 512$

$=5120$

$2\cdot 5120+1024=10240+1024$

$=11264$ #