Algorithmic Studies on Variations of the Vehicle Routing Problem

Research Proposal

Author: Ritaraj Pal
Under the supervision of Prof. Priya Ranjan Sinha Mahapatra
Institution: University of Kalyani
Date: September 12, 2025

Introduction

Vehicle Routing Problem (VRP) is a combinatorial optimization problem that deals with optimal delivery of goods by a set of vehicles that is dispatched from a depot to serve customers to a set of geographically separated locations.

VRP is extensively applied across industries to optimize the efficient allocation of vehicles for tasks such as delivery, transportation, and service provision etc. By determining optimal routes for vehicles to visit multiple locations, VRP helps minimize costs, reduce fuel consumption, and enhance overall operational efficiency. Logistics companies use VRP to streamline package delivery, public transportation agencies optimize bus routes, and waste management services improve the effectiveness of garbage collection. Beyond transportation, VRP is employed in fields like healthcare, retail inventory replenishment, and electric vehicle fleet management, contributing to resource utilization and service quality improvements across diverse sectors.

VRP and its variants

The Vehicle Routing Problem initially started from the "Truck Dispatching Problem" introduced by Dantzig and Ramser (1959) [9] where a fleet of homogeneous trucks were tasked to supply oil to a number of gas stations from a central hub with minimal travel distance. Now VRP and its numerous variants have become one of the most widely studied topics in the field of Operations Research.

CVRP This is the simplest form of a VRP [9]. All other complex VRP variants can be derived from this problem type by updating the optimization function and constraints.

A Capacitated VRP problem consists of a group of customers that are geographically located in different locations and each of them have a specific amount of demands that need to be fulfilled. There is a central depot that can dispatch a finite number of vehicles that services those customers by delivering goods to their location. Each of those vehicles have a finite capacity that is the maximum amount of goods that they can load. These vehicles cannot carry more goods than their capacity allows and when they start serving a customer their demands must be fully serviced by that vehicle (no

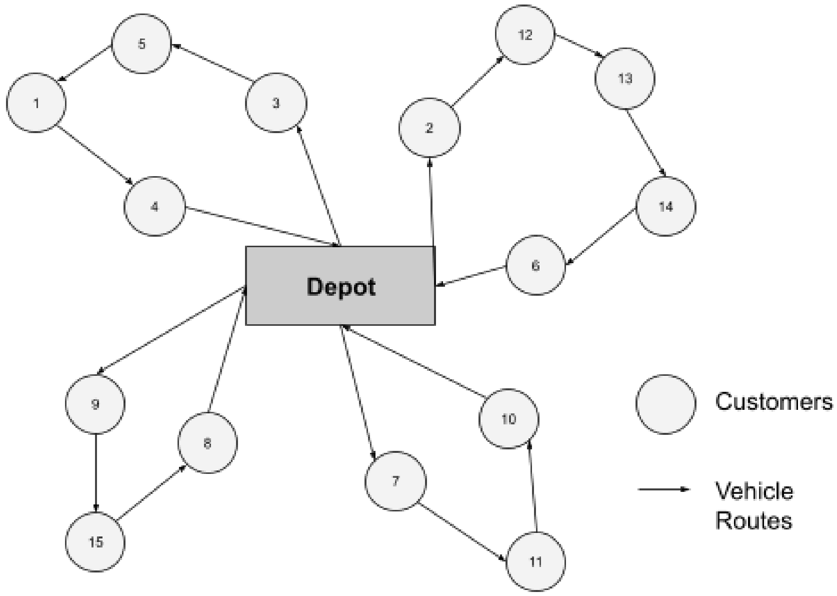

Figure 1: Graphical representation of a VRP solution

partial delivery allowed). Each vehicle starts their journey from the central depot at full capacity, they can travel to different customers and service them and at the end of the journey they must return back to the depot. This path is called the Route of that vehicle. Each route has a travel cost associated with them which is determined by the distance they traveled.

The objective of the CVRP is to allocate routes to each vehicle so that the total travel cost is minimized. A feasible solution must ensure that:

- Each customer has been serviced by exactly one vehicle.
- Each of the customers have their demand fulfilled.
- The capacity of each of the vehicles is taken into account, that is the total demand serviced by each vehicle must not exceed their capacity.
- Each route must start and end on the depot.

VRPTW Stands for Vehicle Routing Problem with Time Windows [3]. This problem is an extension of the CVRP where each customer is associated with a time interval that mentions a start and end time. This interval is called a time window for that customer. Just as before the customer must be serviced by one vehicle but now there are time restrictions associated with it where the service must start within the time window and the vehicle must wait at the location of that customer for a specific time which is called the service time.

The optimization problem boils down to finding a collection of vehicle routes that minimizes travel cost. The constraints that has to be satisfied are mentioned in the CVRP section, on top of that more constraints are added:

- The service for each customer should start within their time window.
- Each vehicle must stop at a customer for a set amount of time which is that customer's Service Time.

VRPPC In the case of the VRP with a private fleet and common carrier [5], on the other hand, an organization can request third-party common carrier services to serve one or more customers instead of using its private fleet. Common carrier handles the delivery for a fixed amount of cost that is associated with each customer.

The objective function then becomes concerned with minimizing both the total private fleet and common carrier cost. The constraints of the CVRP are modified to take the common carrier into account:

- Each customer's demand must be fulfilled by either a private fleet or the common carrier.

VRPPD VRP with Pickup and Delivery [16] models the problem where a quantity of homogeneous goods must be picked up from a pickup location and delivered to a delivery location. Each request in this problem has a pickup and delivery location associated with them, where a vehicle must pick up the goods from the associated pickup location at first and then the same vehicle must deliver those goods to the delivery location associated with the request all the while ensuring that the capacity constraint of the vehicle is not violated.

Same as CVRP, the VRPPD also has the objective to minimize the total travel cost for all the routes, but the constraints need to take into account the pickup and delivery restrictions.

- Each for each delivery request the pickup location and the delivery location must be visited by the same vehicle.
- The pickup location must be visited before the corresponding delivery location for a request.
- During this loading and unloading process the total capacity of the vehicle must not exceed their maximum capacity (which also should be non negative).

VRPB Vehicle Routing Problem with Backhauls [12] is a variation of the CVRP where the vehicles are utilized for both delivering goods to customers and picking up additional goods (backhauls) from certain locations. The customers are separated into two subsets: one subset represents linehaul customers that demand a quantity of product to be delivered and the other are backhaul customers where a quantity of products must be picked up from them instead.

Just as before the objective is to minimize the travel cost for all the routes while satisfying the constraints:

- The total demands of the linehaul and backhaul customers visited by a vehicle do not exceed, separately, the vehicle capacity.
- In each route all the linehaul customers precede the backhaul customers

These are the common variations of VRP while there are many more and these individual types can be combined with other types (Such as VRPPD with Time Windows) to model more real world problems. More details on VRP variations and taxonomy can be found in the papers by Braekers, et al [4].

Solution Strategies

Since the Vehicle Routing Problem is an NP-hard problem the complexity of the problem and therefore the runtime of an exact solution algorithm increases exponentially. Because of this reason heuristic and metaheuristic solutions are given priority that can provide a good enough solution in a reasonable amount of time.

The solution strategies can be broadly categorized into the following categories:

Exact Solutions

The exact solution aims to find the optimal solution to a problem by exhaustively exploring the entire solution space. As we have previously mentioned VRP is an NP-hard problem and because of this reason even though the exact solutions can provide the best solutions the solver cannot finish in a reasonable amount of time for a problem instance that reflects a real world complex problem in this category.

These exact solutions include: the Integer Linear Programming method used by Dantzig and Ramser [9] to solve CVRP, Branch and Bound strategy was used by Theurich, Fischer and Scheithauer [17] to solve VRP with Customer Costs, Set partitioning formulation with additional cuts [1] and Exact Branch-Price-and-Cut Algorithm [7] to solve CVRP and many more.

Heuristic Solutions

The heuristic solutions on the other hand aims to address the issues with the exact solutions by quickly exploring the search space and providing a good enough solution in a very reasonable amount of time. Heuristic solutions use a rule of thumb approach to solution search and therefore do not explore the entire solution space and as a result they cannot guarantee an optimal solution. Even with these setbacks they are way more useful in real world applications where timing is essential and a good enough solution would suffice. These heuristics can be further categorized into Constructive Heuristics, Improvement Heuristics and Metaheuristics [13].

These heuristic solutions include: Nearest Neighbor method which was used by Potvin as a construction heuristic to generate initial solution for solving Travelling Salesperson Problem [15], Clarke and Wright's Savings Method [6] which is very effective at solving simple instances of CVRP. Improvement heuristics like k-opt method, swap, relocate, exchange, GENI (GENeralized Insertion) [10] etc. are very commonly used to generate neighborhoods and optimize routes.

Metaheuristic Solutions

Metaheuristics are a part of heuristic solutions but they are worth discussing separately because of their ability to form solutions by extrapolating the features and structures of the problem without being too dependent on the problem itself. They typically take inspiration from natural phenomena or physical processes to arrive at a heuristic solution to the problem at hand.

These metaheuristics include:

Tabu Search Tabu Search is an optimization algorithm that employs a memory-based strategy known as the tabu list to enhance exploration efficiency and prevent the revisitation of recently explored solutions. This method has become widely popular in solving VRP. Osman utilized Tabu-Search with simulated annealing to solve CVRP [14] while Gendreau et al used it to solve VRP with a heterogeneous fleet of vehicles [11]. Tabu search was also used with a unified neighborhood search strategy to solve VRPPC problems by Côté et al [8].

Simulated Annealing This method is inspired by the annealing process in metallurgy. A temperature is associated with the solution search process. When the temperature is high the algorithm has a higher probability to choose a worse solution which gradually decreases as the temperature is decreased. The first application of Simulated Annealing was a hybrid approach with Tabu Search to solve CVRP by Osman [14]. Simulated annealing was further utilized by Chiang and Russell to solve VRP with time windows (VRPTW).

Ant Colony Optimization Drawing inspiration from the foraging behavior of ants, which efficiently discover the least costly routes to their destination and communicate these paths to their fellow ants through the use of pheromones. This approach has been utilized to solve VRP by Bell, John E., and Patrick R. McMullen [2].

Genetic Algorithm This approach is inspired by the natural selection process which chooses a set of fittest candidates from a population that are used to create a new generation. A Genetic Algorithm approach by Vidal, Thibaut, et al [18] proved to work effectively on a large number of VRPTW problems and its variants.

Effective Solutions are devised by combining one or more heuristic and metaheuristic solutions together and adapting them to the problem at hand.

Proposed work

In the proposed research work different variants of VRP such as CVRP, VRPTW, VRPPD will be thoroughly studied, different heuristic and metaheuristic approaches that have shown promising results for solving those problems will also be studied and analyzed. Alongside the literature review, concrete implementation of these algorithms will also take priority, and detailed examinations will be carried out using standardized benchmark datasets. The results of these experiments will be compared with the historical results mentioned in the literature and significant effort will be dedicated toward improving the quality of the solutions and also the runtime of the solution generation process.

This research work will evolve in the following ways:

Improving on Existing Variants

This work will consider VRP problems that are primarily concerned with practical logistical operations that models real world situations. Problems like VRPTW, VRPPD and VRPPC will take priority. Metaheuristics like Tabu-search approach with a unified neighborhood search [9] has already proven to be a great choice for VRPPC problem while Large Neighborhood Search with a combination of Simulated Annealing heuristic has shown great promise with VRP problems that are constrained by time windows [6], therefore they will be considered when solving the aforementioned problems. The general framework for the solution methodology will work in 2 phases. Starting with a deterministic heuristic to generate the initial solution and then improving on that solution using a metaheuristic approach. The improvement process will be an iterative one and will mostly consist of generating a set of different solutions from the current solution, then evaluating their candidacy based on some acceptance criterion and choosing one solution to select for the next iteration. One or more metaheuristic approaches will be considered so that the optimization process can effectively explore the solution space and reach global optima. Several improvement heuristics will also be considered to improve the candidate solutions as a whole.

Solving new VRP variants

As evident in the VRP literature the practical application based nature of the domain ensures that modeling of new VRP variants, their study and solution will always be a priority. Thus, our research will also focus on studying new emerging VRP types. In order to solve them existing solution strategies will need to be adapted to these new problems and also development of brand new heuristic and metaheuristic approaches to solve those said problems will be necessary. Therefore, a significant portion of our work will also focus on these emerging problems and considering new solution techniques.

Bibliography

[1] Roberto Baldacci, Nicos Christofides, and Aristide Mingozzi. "An exact algorithm for the vehicle routing problem based on the set partitioning formulation with additional cuts". In: *Mathematical Programming* 115 (2008), pp. 351–385.

[2] John E Bell and Patrick R McMullen. "Ant colony optimization techniques for the vehicle routing problem". In: *Advanced engineering informatics* 18.1 (2004), pp. 41–48.

[3] Aida Mauziah Benjamin and John E Beasley. "Metaheuristics for the waste collection vehicle routing problem with time windows, driver rest period and multiple disposal facilities". In: *Computers & Operations Research* 37.12 (2010), pp. 2270–2280.

[4] Kris Braekers, Katrien Ramaekers, and Inneke Van Nieuwenhuyse. "The vehicle routing problem: State of the art classification and review". In: *Computers & industrial engineering* 99 (2016), pp. 300–313.

[5] Ching-Wu Chu. "A heuristic algorithm for the truckload and less-than-truckload problem". In: *European Journal of Operational Research* 165.3 (2005), pp. 657–667.

[6] Geoff Clarke and John W Wright. "Scheduling of vehicles from a central depot to a number of delivery points". In: *Operations research* 12.4 (1964), pp. 568–581.

[7] Luciano Costa, Claudio Contardo, and Guy Desaulniers. "Exact branch-price-and-cut algorithms for vehicle routing". In: *Transportation Science* 53.4 (2019), pp. 946–985.

[8] Jean-François Côté and Jean-Yves Potvin. "A tabu search heuristic for the vehicle routing problem with private fleet and common carrier". In: *European Journal of Operational Research* 198.2 (2009), pp. 464–469.

[9] George B Dantzig and John H Ramser. "The truck dispatching problem". In: *Management science* 6.1 (1959), pp. 80–91.

[10] Michel Gendreau, Alain Hertz, and Gilbert Laporte. "New insertion and postoptimization procedures for the traveling salesman problem". In: *Operations Research* 40.6 (1992), pp. 1086–1094.

[11] Michel Gendreau et al. "A tabu search heuristic for the heterogeneous fleet vehicle routing problem". In: *Computers & Operations Research* 26.12 (1999), pp. 1153–1173.

[12] Marc Goetschalckx and Charlotte Jacobs-Blecha. "The vehicle routing problem with backhauls". In: *European Journal of Operational Research* 42.1 (1989), pp. 39–51.

[13] Fei Liu et al. "Heuristics for vehicle routing problem: A survey and recent advances". In: *arXiv preprint arXiv:2303.04147* (2023).

[14] Ibrahim Hassan Osman. "Metastrategy simulated annealing and tabu search algorithms for the vehicle routing problem". In: *Annals of operations research* 41 (1993), pp. 421–451.

[15] Jean-Yves Potvin. "Genetic algorithms for the traveling salesman problem". In: *Annals of operations Research* 63 (1996), pp. 337–370.

[16] Martin WP Savelsbergh and Marc Sol. "The general pickup and delivery problem". In: *Transportation science* 29.1 (1995), pp. 17–29.

[17] Franziska Theurich, Andreas Fischer, and Guntram Scheithauer. "A branch-and-bound approach for a Vehicle Routing Problem with Customer Costs". In: *EURO Journal on Computational Optimization* 9 (2021), p. 100003.

[18] Thibaut Vidal et al. "A hybrid genetic algorithm with adaptive diversity management for a large class of vehicle routing problems with time-windows". In: *Computers & operations research* 40.1 (2013), pp. 475–489.