HOMEWORK 2: SUPERVISED LEARNING – CLASSIFICATION

(Duration: .. sessions)

Exercise 1: The Sigmoid Function

Let the sigmoid function be defined as:

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

1. Plot the function $\sigma(z)$ for $z \in [-10, 10]$. What are the asymptotes?
2. Compute $\sigma(0), \sigma(2), \sigma(-2)$. Interpret the results.
3. Show that $\frac{d\sigma(z)}{dz} = \sigma(z)(1 - \sigma(z))$

Exercise 2: Manual Classification

Given the dataset:

$$X = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 3 & 3 \\ 1 & 4 & 5 \end{bmatrix}, \quad y = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

- The first column is the bias term ($x_0 = 1$).
- The second and third columns are feature values.

Let the initial parameter vector be:

$$\theta = \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$$

Questions

1. For each training example, calculate the value : $z = \theta^\top \mathbf{x}^{(i)}$
2. Calculate the predictions $h_\theta(\mathbf{x}^{(i)})$ using the sigmoid function $\sigma(z)$
3. What is the decision boundary equation for the given dataset?
4. Calculate the cross-entropy cost for the current θ
5. We want to perform Gradient Descent Algorithm. So:
   a) Calculate $h_\theta(X)$ for all examples
   b) Compute the gradient of the cost function
   c) Compute the new value of θ after one step of gradient descent, where alpha = 0.1

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Exercise 3: Performance Metrics of Logistic Regression

Questions:

1. Define accuracy, precision, recall, and F1-score.
2. Given the following confusion matrix, compute these metrics:

	Predicted 1	**Predicted 0**
Actual 1	45	5
Actual 0	10	40

3. Explain when accuracy serves no purpose.
4. Explain when precision is more important than recall.
5. Suppose we want to detect a rare disease. Which metric is most crucial?

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