1. details about procruste transformation in maximizing likelihood:
Given matrix X and Y, say, both in $R^{p\times n}$. We want to minimize $\|X-\Gamma(Y-\mu 1^\tau)\|$, for any $\mu\in R^p$, and orthonormal Γ in $R^{p\times p}$. It can be shown easily that $\tau=\frac{1}{n}(Y-X)1$ minimizes it. Now it remains to find the rotation matrix, and we assume both X and Y are centered at 0 in the following.

$$\begin{aligned}&\|X-\Gamma Y\|^2\\ =&\operatorname{tr}((X-\Gamma Y)^\tau(X-\Gamma Y))\\ =&\operatorname{tr}(X^\tau X-Y^\tau Y-2X^\tau\Gamma Y)\\ =&const-2\operatorname{tr}(\Gamma Y X^\tau)\end{aligned}$$

Now we seek to maximize $\operatorname{tr}(\Gamma YX^\tau)$. Let $YX^\tau=u\Sigma v^\tau$ be the sigular value decomposition, it becomes:

$$\begin{aligned}&\operatorname{tr}(\Gamma YX^\tau)\\ =&\operatorname{tr}(v^\tau\Gamma u\Sigma)\end{aligned}$$

so $\Gamma=vu^\tau$ maximize above expression.
In the dynamic latent space model, when we translate or rotate all points at a fixed timepoint t, we do not change the pairwise distance, hence the likelihood based on X_t does not change. However, the transformation reduces $\|X_t-X_{t-1}\|$, and hence increases the likelihood based on random walk.

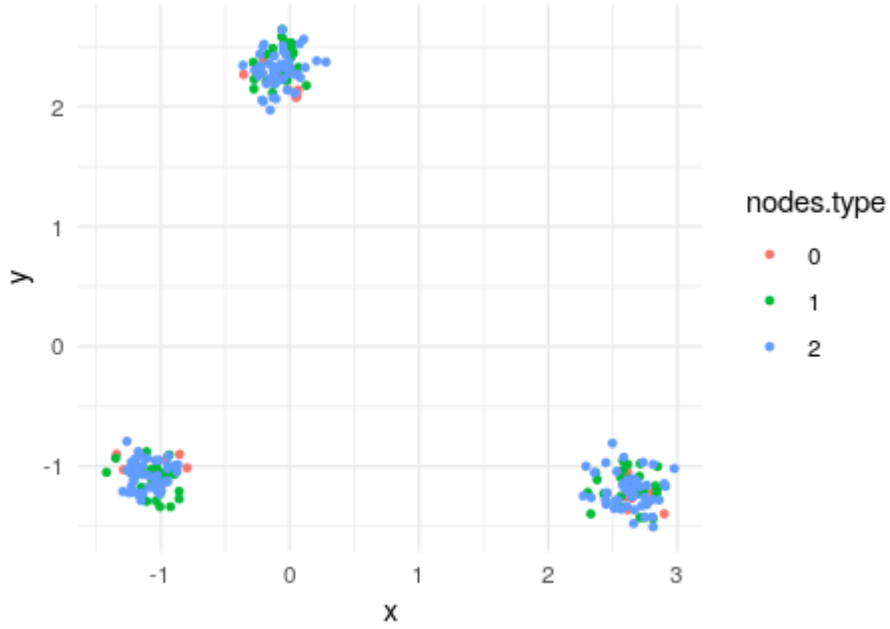
2. a simulation involving weighted links:

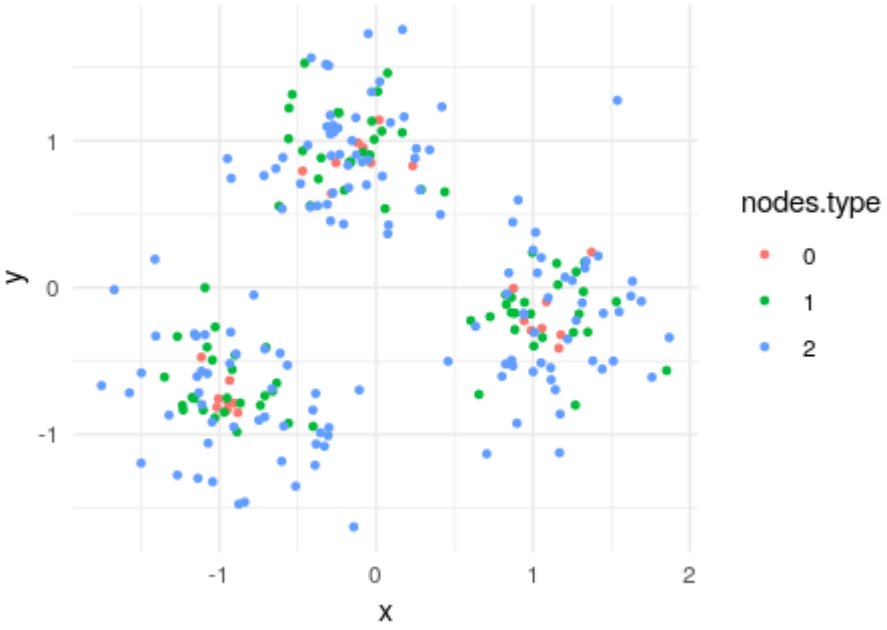
We consider 3 types of nodes. For simplicity, we call them "author", "paper", and "word" respectively.
We generate 3 "communities", each contains 8 authors, 25 papers, and 47 words. We only consider two types of links: the binary link between author and paper, and the weighted nonnegative integer valued link between paper and word.
The regression model is not simply log-linear. While η is calculated in the same way, we set $\mu=\exp(\eta)$ when $\eta<0$ and $\mu=\eta+1$ otherwise, to prevent it from increasing exponentially when two points get close.
In one simulation, true value of parameters:

```
model betas:
   1.0000   1.0000   1.0000
   1.0000   1.0000   0.1000
   1.0000   1.0000   1.0000

model radius:
        0   0.4000        0
        0        0   0.6000
        0        0        0
```

Simulated value after $10,000$ steps:

```
model betas:
        0   1.4211        0
        0        0   0.2147
        0        0        0

model radius:
        0   0.4014        0
        0        0   0.5986
        0        0        0
```





3. histogram of pairwise distance from the previous simulation: