American Economic Journal: Economic Policy 2020, 12(2): 340–373
https://doi.org/10.1257/pol.20160541

Optimal Climate Policy When Damages are Unknown†

By Ivan Rudik*

Integrated assessment models (IAMs) are economists' primary tool for analyzing the optimal carbon tax. Damage functions, which link temperature to economic impacts, have come under fire because of their assumptions that may be incorrect in significant but a priori unknowable ways. Here I develop recursive IAM frameworks to model uncertainty, learning, and concern for misspecification about damages. I decompose the carbon tax into channels capturing state uncertainty, insurance motives, and precautionary saving. Damage learning improves ex ante welfare by $750 billion. If damage functions are misspecified and omit the potential for catastrophic damages, robust control may be beneficial ex post. (*JEL* H23, Q54, Q58)

Integrated assessment models (IAMs) are economists' primary tool for analyzing the optimal carbon tax. IAMs are macroeconomic models linked to a climate module by the "damage function," which translates rising temperature into economic losses.[1] The world's true underlying damage function is complex and characterized by deep uncertainties stemming from a lack of knowledge of how warming will affect natural and economic systems. In order to develop tractable models in the face of these unknowns, integrated assessment modelers have historically pinned down the damage function with strong structural and parametric assumptions. Alongside the discount rate, the damage function is one of the most contentious feature of IAMs. In fact, some economists have suggested abandoning quantitative integrated assessment because of the alleged arbitrariness of the damage assumptions underpinning it (Pindyck 2013, 2017).

Rather than abandoning the quantitative integrated assessment agenda that has proved useful for analysis and policymaking, I integrate uncertainty and even skepticism about damage functions into an IAM. I advance methodology in two ways. First, I incorporate parametric uncertainty about the parameters of the damage function and Bayesian learning about a parameter that maps into the temperature elasticity of damages. Second, I implement robust control, a technique to account for

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† Go to https://doi.org/10.1257/pol.20160541 to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

[1] This is specific to the benchmark DICE model. Other IAMs may have explicit impacts of CO_2. Despite varying levels of detail across IAMs, damage functional forms are largely uncertain and some economists have claimed they are implemented in IAMs in an ad hoc fashion (e.g., Pindyck 2013).

concerns from economists and scientists about structural damage function uncertainty, namely that it is misspecified in unknown ways. Using these methodological advances, I aim to ascertain the policy and welfare implications of (i) accounting for parametric uncertainty over the damage function, (ii) including endogenous damage learning over a parameter that has been a central focus of the climate economics literature, (iii) allowing the policymaker to distrust the structure of her damage model via the use of robust control, and (iv) interacting learning with the policymaker's distrust of her own model.

Similar to approaches in the saving literature (e.g., Gollier 2001), I decompose the optimal carbon tax into different channels representing the policy implications of uncertainty and learning, and I present a novel channel representing concerns about model misspecification. The other uncertainty channels are analogous to precautionary saving and insurance motives; they may increase or decrease the optimal carbon tax depending on both the current state of the world and whether the policymaker updates her damage beliefs. Without learning, the total effect of these uncertainty channels initially decreases the carbon tax by a small amount; however, over time it increases the carbon tax by up to 1 percent. With learning, the total effect always increases the carbon tax by up to 5 percent. Insurance against model misspecification increases the optimal carbon tax by up to 5 percent when not learning, and slightly decreases the optimal carbon tax when learning.

Allowing for updating the estimate of the temperature elasticity of damages leads to ex ante present value welfare gains of about \$750 billion. Using robust control to guard against model misspecification increases the carbon tax over the next century, but if the model is correctly specified, these policy changes lead to welfare losses of \$150 billion. Since the purpose of robust control is to guard against unknown misspecifications I also compute ex post welfare gains when the damage function in the model is misspecified and the true damage function has an extra catastrophic term similar to Weitzman (2012) and Dietz and Stern (2015). Here I find that robust control can generate ex post present value welfare gains if catastrophe sets in at sufficiently low temperatures or ex post welfare is calculated over a sufficiently long time horizon.

In this paper, I develop an IAM by combining an annual version of the DICE model's economic system, a generalized version of the DICE damage function, and a reduced-form climate module that takes advantage of recent findings in climate science. Damage functions have been a central focus of analysis by researchers and have garnered a range of criticisms that may lead a policymaker to distrust IAMs.[2] Broadly, economists have voiced their concerns about the calibration and parameterization of damage functions. Some researchers have suggested that the data used to calibrate damage functions are missing sectoral damage estimates such as diminished ecosystem services or increased healthcare costs (Howard 2014), while others claim that the studies historically used in the calibration of damage functions underestimate the actual impact of warming (Hanemann 2008, Howard and Sterner 2017). The sharpest critiques focus on the functional form, saying that it is "… completely ad hoc, with no theoretical or empirical foundation …" (Pindyck 2013), and that

[2] See Diaz and Moore (2017) for a review of damage function critiques, one of which is that there has not been sufficient attention paid toward parametric uncertainty and stochasticity. Both are accounted for here.

there is "… no rationale, whether empirical or theoretical, for adopting [the DICE model's] quadratic form … although the practice is endemic in IAMs …" (Stanton, Ackerman, and Kartha 2009). Scientists and economists have said the damage function is far removed from evidence in economics and natural sciences (Ackerman and Stanton 2012, Pindyck 2013).

Damages functions in DICE and other IAMs often take a power functional form:

$$D = d_1[T]^{d_2},$$

where temperature (T) is defined as the temperature increase over preindustrial levels, damages (D) are the percent loss of GDP, and where d_2, the temperature elasticity of damages, is assumed to be equal to 2 in DICE. The remaining parameter d_1 is calibrated by fitting the quadratic function to a set of the existing damage estimates. Some kind of aggregate damage functional form assumption is necessary for several reasons. First, optimizing climate policy requires us to be able to quantify the benefits of abating emissions—the avoided damages. Second, there has historically been a lack of theory and data to tell us what this damage functional form should be (Pindyck 2013). Last, the ideal approach to capturing damages would be to develop a detailed competitive economy with a disaggregated set of sector-specific and location-specific damage functions, and with substitution and trade responses to climate damages. This is highly burdensome computationally, particularly if a modeler were to account for damage uncertainties.

Uncertainty and criticism about these key damage function parameters have led to a proliferation of research aimed at understanding the sensitivity of policy prescriptions to parametric assumptions and the implications of parametric uncertainty. Recent work analyzing the sensitivity of optimal policy to both the functional form and calibration of the damage function has demonstrated that changing the damage function parameters may have nontrivial policy impacts (Stanton, Ackerman, and Kartha 2009; Kopp et al. 2012; Weitzman 2012), and that several alternative polynomial functions can fit real world data just as well as the quadratic form (Stern 2006, Hsiang et al. 2017). Because of the limitations of existing damage functions, some economists have developed alternative polynomial damage functions that better fit stylized facts about how we expect damage functions to act at high temperatures (Weitzman 2010, 2012; Pindyck 2012; Dietz and Stern 2015), or explored alternative polynomial forms or coefficients as a sensitivity analysis (e.g., Dietz and Stern 2015, Dietz and Venmans 2019, Hsiang et al. 2017).[3] Addressing these concerns about the damage function is critical since IAMs have become a key component of determining environmental regulation. For instance, a suite of IAMs has been used by the US government to price greenhouse gas emissions in cost-benefit analyses of federal policies (Greenstone, Kopits, and Wolverton 2013).

The weaknesses of current damage functions and uncertainty about the economic consequences of additional warming indicate that damage uncertainty should be explicitly captured in modeling. Damage uncertainty has been included only

[3] Some of these papers use an exponential damage function, however, the exponential function maps into an infinite monomial series.

recently in IAMs (Crost and Traeger 2013, 2014), because the dynamic stochastic IAM literature is relatively new. The literature has heavily focused on policy and learning when the sensitivity of the climate to CO_2 is uncertain (Kelly and Kolstad 1999; Leach 2007; Kelly and Tan 2015; Fitzpatrick and Kelly 2017; Hwang, Reynès, and Tol 2017; Lemoine and Rudik 2017), even though there is evidence that damage uncertainty has more significant policy implications (Lemoine and McJeon 2013). Dynamic stochastic IAMs have also recently been used to explore tipping points or irreversibilities (Lemoine and Traeger 2014; Cai et al. 2015; Lontzek et al. 2015; Lemoine and Traeger 2016a, 2016b; Cai and Lontzek 2019), and to examine the implications of geoengineering (Heutel, Moreno-Cruz, and Shayegh 2016, 2018).[4]

Our current level of uncertainty in estimates of climate damages is not permanent. Uncertainty can be resolved through research, and in recent years, there has been an explosion of effort aimed at estimating sectoral climate damages.[5] Moreover, there have also been recent surveys and meta-analyses aimed at updating estimates of aggregate global damage functions (Howard and Sylvan 2016, Howard and Sterner 2017). The updating of damage functions, and anticipation of future damage function updates by a forward-looking policymaker, has yet to be incorporated into an IAM.

Robust control and other techniques to account for model uncertainty have been recently applied environmental and integrated assessment contexts (Roseta-Palma and Xepapadeas 2004; Athanassoglou and Xepapadeas 2012; Anderson et al. 2014; Li, Narajabad, and Temzelides 2014; Berger, Emmerling, and Tavoni 2017).[6] I build upon these strands of existing work by realistically capturing parametric uncertainty, endogenous learning and estimation of damages, and structural misspecification concerns in a recursive IAM where a policymaker trades off near-term costs of emissions reductions with the future benefits of lower temperature via reduced damages.

This paper makes several contributions to the broader discussion about the use of damage functions. First, if the standard damage functions in IAMs are misspecified and the true damage function follows one of the recently proposed catastrophic functional forms,[7] protecting against model misspecification via robust control has limited welfare effects over the next 200 years relative to simply using a carbon tax optimized for a noncatastrophic damage function. This suggests that if real world climate policy eventually follows IAM policy prescriptions, guarding against these specific catastrophic misspecifications by using an even more aggressive carbon tax may not be of first-order importance.[8] Second, when the true damage function has one

[4] The dynamic stochastic IAM literature expands on previous work that approximates uncertainty using Monte Carlo methods (Hope 2006; Stern 2006; Nordhaus 2008; Ackerman, Stanton, and Bueno 2010; Kopp et al. 2012), however Monte Carlo analyses do not correctly capture decision-making under uncertainty (Lemoine and Rudik 2017). There have been recent calls for improved treatment of risk, uncertainty, and ambiguity in pricing carbon (Stoerk, Wagner, and Ward 2018).

[5] See Hsiang (2016); Hsiang, Oliva, and Walker (2019); and National Academy of Sciences, Engineering, and Medicine (2017) for recent reviews of this literature. Hsiang et al. (2017) aggregate sectoral damage estimates in the United States and estimate a US-specific damage function. They also provide a framework for easily updating this damage function in response to new research.

[6] Robust control has also been applied to a policymaker learning about the dynamics of inflation and unemployment (Cogley et al. 2008).

[7] These catastrophic forms are informed by scientific arguments. For example, Weitzman (2012) and Dietz and Stern (2015) reference heat stress (Sherwood and Huber 2010) and tipping points (Kriegler et al. 2009).

[8] If climate sensitivity was uncertain as in Kelly and Kolstad (1999), Kelly and Tan (2015), or Lemoine and Rudik (2017), then a particularly sensitive climate may result in extreme warming and catastrophic damages. In this case

of these catastrophic forms, updating a misspecified damage function can increase or decrease welfare by trillions of dollars depending on the true damage functional form and the time horizon for evaluating ex post welfare. This indicates that the realized social benefits of incorporating new damage estimates into IAMs will be highly dependent upon the damage functional form assumptions of these IAMs.

The paper is organized as follows. Section I gives more background on the construction of the damage function, and provides an overview of the dynamic stochastic IAM and all four frameworks. Section II describes how to decompose the optimal carbon tax under uncertainty into different channels. Section III reports results on how the different frameworks affect optimal policy and welfare. Section IV concludes. The online Appendix fully describes the IAM in this paper, details the computational methodology, provides a numerical error analysis, and provides a full derivation of the optimal carbon tax and its decomposition.

I. The Damage Function and the Integrated Assessment Modeling Framework

I begin by giving a brief overview of the DICE damage function, which is the foundation for the damage function in this paper. Next, I outline the IAM as a whole and then describe the four frameworks I use to investigate damage uncertainty, learning, and misspecification concerns through the use of robust control.

A. *The DICE Damage Function*

In the benchmark DICE model, time $t+1$ damages, $D(T^s_{t+1})$, multiplicatively reduce the level of time $t+1$ output, Y^g_{t+1}, as a function of the time $t+1$ surface temperature, T^s_{t+1}.[9] The damage function specific to the DICE model is given by

$$D\left(T^s_{t+1}\right) \;=\; d_1 \left[T^s_{t+1}\right]^{d_2},$$

where d_2 is assumed to be 2 and d_1 is calibrated to estimates of damages from specific levels of warming over preindustrial levels.

The most recent version of the DICE damage function is directly calibrated to a set of monetized damage estimates with an upward adjustment of 25 percent to account for impacts that are more difficult to estimate such as human conflict (Nordhaus and Sztorc 2013). Even with this change in the damage calibration procedure, it does not include damage estimates from a burgeoning strand of literature that has estimated climate damages in a variety of sectors.[10]

there may be large ex post benefits from robust control. Here I do not explore uncertainty about the climate sensitivity but interactions between these two uncertainties are important and should be studied in future work.

[9] Although not investigated here, there are models that incorporate damages directly on capital (Kopp et al. 2012, Dietz and Stern 2015) or utility (Sterner and Persson 2008, Barrage 2018). Sterner and Persson (2008) explicitly include nonmarket environmental services in the utility function with limited substitutability, which can be degraded by warming. There is also empirical evidence that climate damages affect the growth rate of output, not just the level (Dell, Jones, and Olken 2012), and this may have substantial policy impacts (Moore and Diaz 2015).

[10] See Hsiang (2016) for references to several recent examples.

B. *Four Frameworks for Damage Uncertainty and the Dynamic Programming DICE Model*

In light of critiques about the damage function, I develop four different frameworks for investigating the policy and welfare implications of damage uncertainty. The *uncertainty framework* has the policymaker applying distributions over d_1 and d_2 and treating these parameters as uncertain. The *learning framework* allows the policymaker to learn and refine her beliefs about d_2 over time. The *robust control framework* has the policymaker being uncertain about both parameters, but the framework also applies robust control to capture concerns that the damage function in the model is misspecified. Finally, the *robust control and learning* (*RC+L*) *framework* has the policymaker combine updating her beliefs about d_2 with the use of robust control.

The IAM I use to study damage uncertainty is a finite-horizon, annual timestep Ramsey-Cass-Koopmans growth model coupled to a cumulative carbon framework representation of the joint carbon-climate system.[11] The state space is composed of capital; the cumulative emissions of CO_2; a state to capture the realized amount of damages; and states for the location and scale parameters governing the distribution of the policymaker's beliefs about d_2.

Figure 1 displays a schematic of the model. The full model description can be found in the online Appendix. The model begins in 2005 and ends in 2605 with a given terminal value function.[12] Each period, the economy begins with an existing level of capital, labor, and technology. These are combined in a Cobb-Douglas production function to generate output.

The production process generates CO_2 emissions, and the cumulative amount of CO_2 emitted is stored as a state variable. IAMs like DICE often use a complex multistate carbon-climate systems to model climate dynamics. However, recent advances in climate science have indicated that surface temperature T^s_{t+1} is proportional to the cumulative amount of CO_2 emissions E_{t+1}, both in the historical data and over different global climate model simulations spanning multiple climate scenarios and timeframes (Matthews et al. 2009; IPCC 2013; Knutti, Rugenstein, and Hegerl 2017).[13] Exploiting this reduced-form relationship, surface temperature is then given by

$$T^s_{t+1} = \zeta E_{t+1},$$

where ζ is called the transient climate response to emissions. Higher surface temperature causes more damage and reduces output through the damage function.

The remaining output after damages can be used in three ways: investment to increase the future stock of capital, abatement to reduce industrial CO_2 emissions

[11] This representation of the climate system has recently been gaining traction in climate economics (Anderson et al. 2014, Berger and Marinacci 2017, Brock and Xepapadeas 2017, Dietz and Venmans 2019).

[12] The same terminal value function is used across all four frameworks since it is sufficiently far in the future to have a negligible impact on policy. For example, given the growth-adjusted discount factor in the model, \$1 at the terminal time 2605 is discounted to far less than \$0.01 in 2105.

[13] The online Appendix contains more details on why this relationship arises.

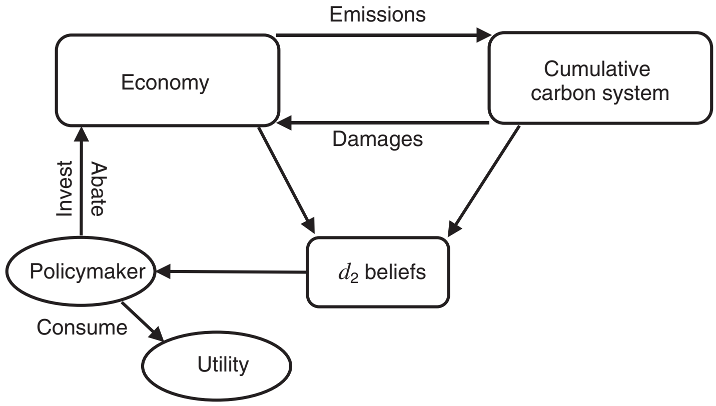

Figure 1. A Schematic of This Paper's IAM with Learning

Notes: Capital, labor, and technology are combined to produce output. Emissions from factor production flow into the cumulative carbon framework representation of the joint carbon-climate system. Greater cumulative emissions raise surface temperature, which damages output. A learning policymaker uses her observations of temperature and realized damages to refine her beliefs. Her beliefs influence her policy choices.

caused by factor production, or consumption to increase flow utility. The policymaker's objective is to maximize her discounted expected stream of utility from consumption. When deciding on the optimal way to divide output between abatement, consumption, and investment, the policymaker uses her current beliefs about future damages. These beliefs are given by probability distributions over the damage function parameters d_1 and d_2, and the probability distribution over a multiplicative damage shock ω_{t+1} that inhibits her learning about d_2. If the policymaker learns over time, she uses observations of surface temperature (or equivalently cumulative emissions) and damages in a given period to update her beliefs about d_2.

The timing of the model is shown in Figure 2. The period begins and then the policymaker observes the state of the world. Damages are unknown prior to the beginning of the period because the policymaker does not know the true value of the damage parameters and because of objective stochasticity in the realization of the damage shock. After the policymaker observes the current state, she uses Bayes' Law to update her distribution over d_2. Next, with her new beliefs, she selects levels of abatement, consumption, and investment that maximize her expected welfare. Finally, the world transitions to the next period.

The policymaker's problem can be represented by a Bellman equation:

$$V_t(\mathbf{S}_t) = \max_{c_t,\alpha_t} \left\{ U(c_t) + \beta_t \mathbb{E}_{d_2,d_1,\omega_{t+1}} \left[V_{t+1}(\mathbf{S}_{t+1}) \right] \right\} \tag{1}$$

subject to

$$\mathbf{S}_{t+1} = f(\mathbf{S}_t, c_t, \alpha_t),$$

where $\mathbf{S}_t$ is the time t state vector, α_t is abatement, c_t is consumption, $U(c_t)$ is her flow utility function, $V_t(\mathbf{S}_t)$ is the policymaker's time t value function, f is the set

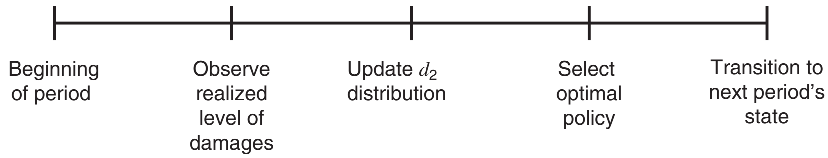

FIGURE 2. THE TIMELINE FOR EACH PERIOD t

Notes: The policymaker begins by observing the realization of damages, which was unknown prior to the beginning of the period due to uncertainty about the damage function parameters and damage shock. She then uses the damage observation to update her beliefs about d_2. Finally, she optimizes her policy conditional on the current state of the world, which includes capital, CO_2, the amount of damages, and the parameters governing her updated beliefs.

of state transition equations, β_t is a growth-adjusted discount factor, and $\mathbb{E}$ is the expectation operator. Expectations are always taken using the time t information set. In each period, the policymaker maximizes the sum of her current flow utility and her discounted expected continuation value where she takes expectations over the two damage function parameters and the damage shock using her time t beliefs. The online Appendix contains the full representation of the policymaker's problem.

C. *The Damage Function Calibration and Learning*

In all four frameworks the policymaker takes both parameters of the damage function as uncertain. At time t the policymaker assigns the damage coefficient a lognormal prior distribution, $d_1 \sim \log\mathcal{N}(\mu_c, \sigma_c^2)$ and she assigns the damage exponent a normal distribution, $d_2 \sim \mathcal{N}(\mu_t^e, \Sigma_t^e)$. In the frameworks with learning, the policymaker updates her distribution over d_2 each period. There are no time subscripts on the d_1 prior parameters since the policymaker is not learning about d_1. I select these distributional families since they admit Markovian updating rules, which makes modeling learning tractable.[14] The policymaker's immediate learning of the value of d_2 is hindered by a sequence of independent and identically distributed lognormal random damage shocks. These shocks capture random variation in how warming affects factor production through, for example, random variation in the frequency and magnitude of droughts or cyclones. The shock ω_{t+1} enters the damage function multiplicatively:

$$D\left(T_{t+1}^s, \omega_{t+1}\right) = d_1\left(T_{t+1}^s\right)^{d_2} \omega_{t+1}. \tag{2}$$

Here I focus on time $t+1$ damages so the timing matches the carbon tax derivation in Section II. The variable Y_{t+1}^g is the gross output from factor production prior to any damages, and Y_{t+1}^n is net output after damages. Damages reduce the level of gross output in the following fashion (Nordhaus 2008):

$$Y_{t+1}^n = \frac{Y_{t+1}^g}{1 + d_1\left(T_{t+1}^s\right)^{d_2} \omega_{t+1}}.$$

[14]Tractable learning has played a role in distributional family selection in the prior literature (e.g., Kelly and Kolstad 1999, Kelly and Tan 2015, Lemoine and Rudik 2017).

Rearranging the equation and taking the natural logarithm of both sides yields an equation such that observed variables are on the left-hand side, and all unobserved variables are on the right-hand side:

$$\log\left(\frac{Y^g_{t+1}}{Y^n_{t+1}} - 1\right) = \log(d_1) + d_2 \log(T^s_{t+1}) + \log(\omega_{t+1}). \tag{3}$$

When the policymaker begins time period $t+1$, she observes the stochastically evolving level of output net of damages, Y^n_{t+1}, and she infers gross output before damages, Y^g_{t+1}, using the production function, the level of technology, and the capital and labor stocks.[15] This comprises all the variables on the left-hand side. On the other side of the equation, $\log(d_1)$, d_2, and $\log(\omega_{t+1})$ are normally distributed random variables that are not directly observed, while $\log(T^s_{t+1})$ is observed by the policymaker.

Define $Q_{t+1} \equiv \log((Y^g_{t+1}/Y^n_{t+1}) - 1)$. Since the random variables on the right-hand side of equation (3) are linearly separable, we have that

$$Q_{t+1} \sim \mathcal{N}\Big(\mu_c + \mu^e_{t+1}\log(T^s_{t+1}) + \mu_\omega, \sigma^2_c + \Sigma^e_t\left[\log(T^s_{t+1})\right]^2 + \sigma^2_\omega\Big), \tag{4}$$

where μ_ω and σ^2_ω are the location and scale parameters of the damage shock ω_{t+1}.[16] With each observation of the random variable Q_{t+1} equal to some realized value q_{t+1} at time $t+1$, the learning policymaker updates the parameters of her prior over d_2 according to Bayes' Law:

$$\mu^e_{t+1} = \frac{(\sigma^2_\omega + \sigma^2_c)\mu^e_t + \log(T^s_{t+1})\Sigma^e_t\left[q_{t+1} - (\mu_c + \mu_\omega)\right]}{(\sigma^2_\omega + \sigma^2_c) + \left[\log(T^s_{t+1})\right]^2\Sigma^e_t}, \tag{5}$$

$$\Sigma^e_{t+1} = \frac{\Sigma^e_t(\sigma^2_\omega + \sigma^2_c)}{(\sigma^2_\omega + \sigma^2_c) + \left[\log(T^s_{t+1})\right]^2\Sigma^e_t}. \tag{6}$$

The term μ^e_{t+1}, the location parameter and mean of the policymaker's beliefs about d_2, is a weighted average of its previous value, and the realization of $q_{t+1} - (\mu_c + \mu_\omega)$, a noisy signal of the true value of d_2. In expectation it is equal to μ^e_t. The term Σ^e_{t+1} is the scale parameter and variance of the policymaker's beliefs about d_2.

I estimate the distributions over d_1, d_2, and ω_{t+1} using data on temperature and damages from the recent meta-analysis by Howard and Sterner (2017).[17] Specifically I estimate the logarithm of the damage function definition in equation (2):

$$\log(D_i) = \log(d_1) + d_2\log(T_i) + \varepsilon_i,$$

[15] This is an approximation to the real world learning process. In reality, we observe net output, but we may not know the global production function nor stock of capital that we required to back out gross output. However, researchers have been estimating damages over time in different sectors using observations of temperature fluctuations and measures of production. See Hsiang (2016) for examples of recent research.

[16] Since d_1 is not being learned and the parameters of its distribution are constant, $d_1\omega_{t+1}$ could be combined into a single random variable $d_1\omega_{t+1} \equiv \tilde{\omega}_{t+1} \sim \log\mathcal{N}(\mu_c + \mu_\omega, \sigma^2_c + \sigma^2_\omega)$.

[17] The dataset includes estimates of severe and catastrophic damages (Weitzman 2010; Burke, Hsiang, and Miguel 2015). The temperature data are in terms of anomalies and the damage data are in percentage terms.

Table 1—Estimates for the Location and Scale Parameters of the Lognormally Distributed Damage Coefficient, Normally Distributed Damage Exponent, and Lognormally Distributed Damage Shock

	Coefficient		Exponent		Shock	
Parameter	μ_c	σ_c^2	μ_{2005}^e	Σ_{2005}^e	μ_ω	σ_ω^2
Estimated value	−5.31	0.23	1.88	0.203	−0.59	1.18

using ordinary least squares, where i indicates a separate estimate within the meta-analysis, D_i is the reported damages, and T_i is the reported temperatures.[18] The estimate on the log temperature coefficient $\hat{d}_2$ is distributed $\mathcal{N}(\mu_0^e, \Sigma_0^e)$ where μ_{2005}^e and Σ_{2005}^e are the point estimate and squared standard error, and the subscript 2005 indicates these will be the values for the policymaker's initial beliefs in 2005. Note that $\hat{d}_2$ has the conventional regression interpretation as the estimate of the temperature elasticity of damages. Similarly, $\log(\hat{d}_1) \sim \mathcal{N}(\mu_c, \sigma_c^2)$ yields the estimated distribution of $\log(d_1)$, or if this term is exponentiated we have that $d_1 \sim \log\mathcal{N}(\mu_c, \sigma_c^2)$. Finally, the estimated variance for the disturbance term ε_i yields the variance of $\log(\omega_{t+1})$.

Table 1 shows the estimates for the location and scale parameters of the three distributions. The location (μ_c) and scale (σ_c^2) parameters for d_1 imply an expectation of 0.00556 and a standard deviation of 0.00379. The expected coefficient is approximately double that of the standard DICE calibration. The mean estimate for d_2 is approximately equal to the selected value of 2 in DICE. The damage shock ω_{t+1} has mean of 1 and a standard deviation of 1.49.

The year 2005 distribution for d_1 is shown in the left panel of Figure 3, and the year 2005 distribution for d_2 is shown in the center panel. The right panel of Figure 3 displays the distribution of damages at 1°C, 2°C, and 3°C using the year 2005 information set. The 1°C distribution is highly peaked at damages of less than 1 percent of output, and the right tail decays rapidly. The 2°C and 3°C distributions peak at slightly higher damages, but are characterized by right tails that decay more slowly and assign greater weight to higher damage outcomes.

D. *Robust Control*

Ideally, skepticism about damage functions should be built into an IAM by modeling a policymaker who does not completely trust that the damage function in her model is a precise representation of reality. In this case, the policymaker would believe that her damage model is only an approximation to the real world, and she would recognize that the optimal policy she obtains from her approximating model will almost surely not be optimal if taken and applied in real world policy. Instead of striving to develop an optimal policy using a model that is likely incorrect, the policymaker can instead try to find policy rules that are robust to unknown, and in the short-term unlearnable, errors in her model.[19]

[18] Taking the logarithm allows me to estimate the parameters with a linear regression.

[19] Implicitly the four frameworks are taking different approaches to estimating an unknown data generating process for damages. In the uncertainty framework a monomial damage function is estimated once before policy-

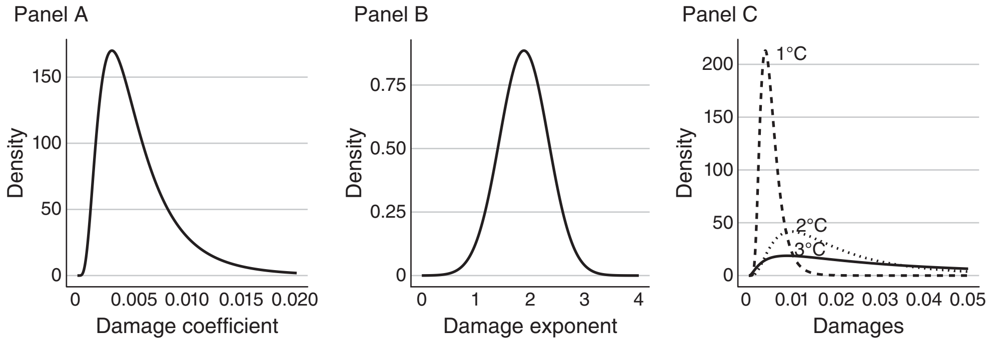

Figure 3. The Probability Density Function for the Damage Function Parameters

Note: The probability density function of the coefficient's distribution (left), the probability density function of the exponent's prior distribution in 2005 (middle), and the probability density function of damages given 2005 beliefs at 1° C, 2° C, and 3° C of warming (right).

To capture concerns about damage function misspecifications, I model a policymaker who uses robust control techniques in order to find policies that perform well even when the damage function inside her IAM may be incorrect (Hansen and Sargent 2007, 2008). The policymaker begins with her approximating model in equation (2) and the associated parameter distributions. She then optimizes her policy, but she does not optimize policy conditional on only her approximating model. She instead optimizes over a set of potential models that are close to her approximating model in terms of Kullback-Leibler divergence. The policymaker believes the true model likely resides in this set, but because the models in this set are all relatively similar to her own, they are difficult to statistically distinguish from one another over the time frame of her policymaking. During policy optimization, alternative damage models more similar to her approximating model are given greater weight, while those that are more different get less weight. By taking this approach, the policymaker is recognizing that her approximating damage function is not exactly correct, and she wishes to guard against unknown errors.[20]

Omitting the transition equations, the policymaker's objective under robust control can be formulated as a min-max problem (Hansen and Sargent 2008):

$$(7)\quad V_t(\mathbf{S}_t) \;=\; \min_{q_{t+1}}\left\{\max_{c_t,\alpha_t}\left[u(c_t) + \beta_t\Big(\int \theta D_{t+1}(p_{t+1}||q_{t+1}) + V_{t+1}(\mathbf{S}_{t+1})\, dq_{t+1}\Big)\right]\right\},$$

where $\theta \in (0,\infty]$ is called the penalty parameter and will be described below, $D_{t+1}(p_{t+1}||q_{t+1})$ is the Kullback-Leibler divergence function,

$$D_{t+1}(p_{t+1}||q_{t+1}) \;=\; \int \log\left(\frac{dp_{t+1}}{dq_{t+1}}\right) dp_{t+1},$$

making begins. The learning framework allows for periodic updates to the model; the robust control framework considers the possibility of other approximations to the damage function; and the framework combining robust control and learning allows for the possibility of other damage functions with continual re-estimation of the exponent on the approximating damage function.

[20]This is in contrast to maxmin expected utility, which maximizes welfare subject to the worst-case model in some set (Gilboa and Schmeidler 1989).

and p_{t+1} is her approximating model's probability measure for taking expectations. Further, q_{t+1} is a distorted probability measure that she acts as though is the true model when she selects her controls, however p_{t+1} still governs the actual state transitions. In other words, q_{t+1} only affects the policymaker's decision rule, not the actual state transitions.

Hansen and Sargent (2008) provide an intuitive interpretation of the robust control formulation in equation (7). Now q_{t+1} is the optimal selection of a probability measure by an evil agent whose objective is to minimize the payoff of the policymaker. The evil agent selects a new q_{t+1} at each time t. The evil agent's selection of q_{t+1} is penalized proportionally to the Kullback-Leibler divergence of its chosen distorted probability measure q_{t+1} relative to the approximating model's probability measure p_{t+1}, where the coefficient of proportionality of the penalty is θ, the penalty parameter. Large values of θ significantly penalize the evil agent for selecting probability measures q_{t+1} that are much different than the approximating model p_{t+1}, while small values of θ let the evil agent select large distortions without much cost.

The distortions by the evil agent are what induce robust decision rules on behalf of the policymaker. Since the evil agent aims to reduce the policymaker's welfare, the distortions it selects skew the policymaker's perceived expected continuation value downward. This induces her to select policies that are designed for lower welfare worlds (i.e., worlds where the damage function is more severe). By implicitly changing the policymaker's beliefs about the damage function to be more pessimistic, it induces her to uptake more precautionary action. How strongly she guards against misspecifications is inversely related to the size of θ since it controls the cost of the evil agent selecting a more distorted model q_{t+1}.

The policymaker maximizes her objective while accounting for potential distortions to her approximating model's probability measure, weighted by their Kullback-Leibler divergence. Equation (7) is equivalent to replacing the continuation value in the policymaker's Bellman equation in equation (1) with a risk sensitivity operator $\mathcal{T}^1$ (Hansen and Sargent 2007) that is tractable to use in dynamic programming settings:[21]

$$\mathcal{T}^1\big(\beta_t V_{t+1}(\mathbf{S}_{t+1})\,|\,\theta\big) \;=\; -\theta \log\left(\mathbb{E}_{d_1,\omega_{t+1}}\left[\exp\left(-\frac{\beta_t V_{t+1}(\mathbf{S}_{t+1})}{\theta}\right)\right]\right),$$

where the expectation operator is over d_1 and ω_{t+1}. This results in a new Bellman equation:

$$V_t(\mathbf{S}_t) \;=\; \max_{c_t,\alpha_t}\Big\{U(c_t) + \mathbb{E}_{d_2}\Big[\mathcal{T}^1\big(\beta_t V_{t+1}(\mathbf{S}_{t+1})\,|\,\theta\big)\Big]\Big\}, \tag{8}$$

subject to

$$\mathbf{S}_{t+1} \;=\; f(\mathbf{S}_t, c_t, \alpha_t),$$

[21] The distortions induced by the risk sensitivity operator are equivalent to the policymaker facing state-dependent shocks to the transition distributions, so that the distortions can be temporally linked and persist over time. This is what allows the distortions to represent real misspecifications to the policymaker's model.

where the expectation acting outside the risk sensitivity operator is over d_2.[22] All expectations are taken using the time t information set.

Figure 4 shows three simple cases to illustrate how the risk sensitivity operator distorts the policymaker's expected continuation value. Consider a case where the continuation value, V_{t+1}, has a normal distribution with mean equal to 1, and variance equal to 1, 8, or 24. For simplicity, let the discount factor $\beta_t = 1$ and assume d_2 is known so we can ignore the expectation over d_2 outside the risk sensitivity operator.

When we apply the risk sensitivity operator to V_{t+1}, the exponential term inside the expectation over d_1 and ω_{t+1} has a lognormal distribution, which allows us to obtain a closed-form solution for the expected continuation value: $1 - (\sigma^2/2\theta)$, where σ^2 is the variance.[23] Figure 4 plots these distorted continuation values as a function of the penalty parameter and illustrates two key points. First, a smaller θ allows the evil agent to select transition probability measures that are increasingly distorted: for any given distribution, a smaller θ results in a lower expected value relative to the undistorted expected value. Second, a higher variance baseline continuation value, or equivalently a higher variance baseline transition measure, results in a more distorted expected value for some given θ. A higher variance baseline continuation value means that the policymaker is putting more weight on potential extreme outcomes. This frees up the evil agent to select larger distortions since this additional weight on extreme values in the baseline model will tend to reduce Kullback-Leibler divergence.

Figure 4 shows that as $\theta \rightarrow \infty$, the distorted expected value approaches its undistorted value and we approach a subjective expected utility setting. As θ decreases, the distorted expected continuation value declines. Smaller values of θ make her expected future look worse and induces her to select policies that better guard against misspecifications that would reduce her future welfare. There typically exists some point $\bar{\theta}$, where the problem "breaks down" for $\theta < \bar{\theta}$.[24] I calibrate θ by selecting close to the breakdown point that would induce high levels of concern for model misspecification.[25] For both frameworks I set $\theta = 3.9$. In Section IIIB, I show how varying the concern for model misspecification alters policy.

II. The Policy Effects of Uncertainty and Concern for Model Misspecification

The time t optimal carbon tax is the shadow cost of time t emissions, e_t, and is brought into dollar terms using the optimized time t marginal utility of

[22] Since the risk sensitivity operator is acting directly on the time-invariant distributions of d_1 and ω_{t+1}, learning about d_2 will not result in complete dissipation of the effects of robust control, although changing beliefs about d_2 may affect how the risk sensitivity operator alters the policymaker's decisions.

[23] The general takeaways will hold for other types of transition distributions but the normal distribution delivers a closed form solution.

[24] The problem breaks down because when $\theta < \bar{\theta}$, the exponential term in $T^1(\beta V_{t+1}(\mathbf{S}_{t+1})|\theta)$ is large (e.g., it is much larger than 10^{100}) so that it is equivalent to numerical infinity. The problem is then no longer well-defined.

[25] Hansen and Sargent (2008) demonstrate how to calibrate θ using detection error probabilities in a linear control setting. Athanassoglou and Xepapadeas (2012) formally derive a closed form solution for the worst case misspecification exploiting a linear-quadratic model. Since the DICE model is highly nonlinear I take an approach similar to Gonzalez (2008), where I explore how policy changes under different values of the penalty parameter in Section IIIB.

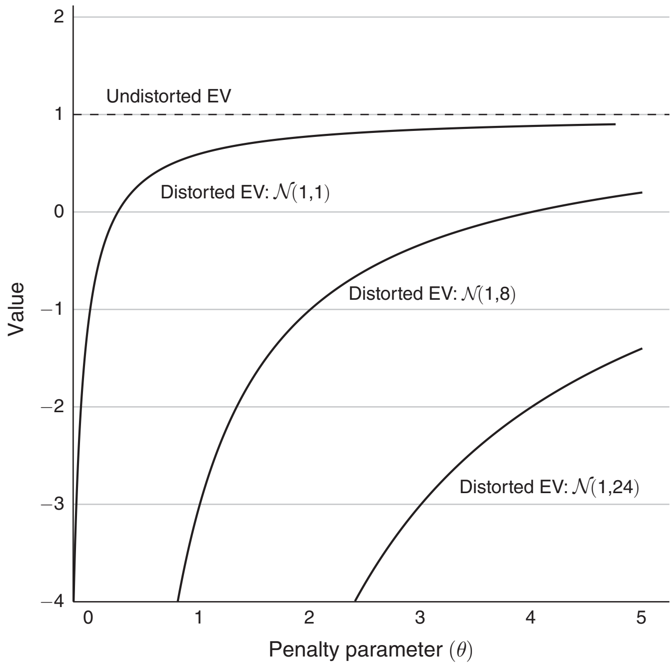

FIGURE 4. THREE EXAMPLES OF THE RISK SENSITIVITY OPERATOR DISTORTING DISTRIBUTIONS

Notes: The plot displays the distorted expected value of three normal distributions as a function of the the penalty parameter. All three distributions have mean 1, but have variances equal to 1 (top), 8 (middle), or 24 (bottom). The undistorted expected value is the dashed line and is independent of the penalty parameter.

consumption. Time t emissions affect the following time $t+1$ states: cumulative emissions (and thus temperature), the fraction of output remaining after damages ($\mathcal{L}_{t+1} = 1/(1 + d_1[T^s_{t+1}]^{d_2}\omega_{t+1})$), and the location and scale parameters.[26] From here on I call $\mathcal{L}_{t+1}$ fractional net output.[27] Without loss of generality, consider the optimal carbon tax for a policymaker using robust control. The shadow cost of emissions is the negative partial derivative of the right-hand side of equation (8) with respect to time t emissions, e_t,

$$Tax_t = \frac{\beta_t}{u'(c_t)}\mathbb{E}_{d_2}\left[\frac{\mathbb{E}_{d_1,\omega_{t+1}}\left[\exp\left(\frac{-\beta_t V_{t+1}}{\theta}\right)\left(\frac{-\partial V_{t+1}}{\partial \mathbf{S}_{t+1}}\frac{\partial \mathbf{S}_{t+1}}{\partial e_t}\right)\right]}{\mathbb{E}_{d_1,\omega_{t+1}}\left[\exp\left(\frac{-\beta_t V_{t+1}}{\theta}\right)\right]}\right],$$

where the expectations are separated because of the risk sensitivity operator T^1. The application of robust control introduces a new term, $\exp((-\beta_t V_{t+1})/\theta)$, which when normalized by its expectation, corresponds to the twisting factor of the worst-case transition distortion induced by the risk sensitivity operator (Hansen and Sargent 2007).

[26] In expectation, the effect of time t emissions on the time $t+1$ location parameter is zero, but we will see that a covariance term arises that may not be zero.

[27] I use fractional net output as a state instead of damages to obtain a bounded domain, $\mathcal{L}_{t+1} \in [0,1]$.

Now I will focus on only the key steps and terms in the derivation in order to convey the intuition for the different carbon tax channels as cleanly as possible. The full carbon tax derivation can be found in the online Appendix. Passing the expectations through and exploiting covariance identities, the carbon tax equation can be rearranged to recover a conventional carbon tax expression (first three lines) and an additively separable adjustment for robust control (fourth line):

$$
(9)\ Tax_t = \frac{\beta_t}{u'(c_t)}\Bigg\{\mathbb{E}_{d_2,d_1,\omega_{t+1}}\left[\frac{-\partial V_{t+1}}{\partial E_{t+1}}\right]\frac{\partial E_{t+1}}{\partial e_t} + \mathbb{E}_{d_2,d_1,\omega_{t+1}}\left[\frac{-\partial V_{t+1}}{\partial \mathcal{L}_{t+1}}\right]E_{d_2,d_1,\omega_{t+1}}\left[\frac{\partial \mathcal{L}_{t+1}}{\partial e_t}\right]
$$

$$
+ \operatorname{cov}_{d_2,d_1,\omega_{t+1}}\left(\frac{-\partial V_{t+1}}{\partial \mathcal{L}_{t+1}}, \frac{\partial \mathcal{L}_{t+1}}{\partial e_t}\right)
$$

$$
+ \operatorname{cov}_{d_2,d_1,\omega_{t+1}}\left(\frac{-\partial V_{t+1}}{\partial \mu^e_{t+1}}, \frac{\partial \mu^e_{t+1}}{\partial e_t}\right) + \mathbb{E}_{d_2,d_1,\omega_{t+1}}\left[\frac{-\partial V_{t+1}}{\partial \Sigma^e_{t+1}}\right]\frac{\partial \Sigma^e_{t+1}}{\partial e_t}
$$

$$
+ \mathbb{E}_{d_2}\left[\operatorname{cov}_{d_1,\omega_{t+1}}\left(\frac{-\partial V_{t+1}}{\partial \mathbf{S}_{t+1}}\frac{\partial \mathbf{S}_{t+1}}{\partial e_t}, \frac{\exp\left(\frac{-\beta_t V_{t+1}}{\theta}\right)}{\mathbb{E}_{d_1,\omega_{t+1}}\left[\exp\left(\frac{-\beta_t V_{t+1}}{\theta}\right)\right]}\right)\right]\Bigg\}.
$$

Next, I introduce two state vectors. The first is the expected time $t+1$ state vector from the perspective of the time t information set:

$$
\upsilon := \left\{k_{t+1}, E_{t+1}, \mathbb{E}_{d_2,d_1,\omega_{t+1}}\left[\mathcal{L}_{t+1}\right], \mu^e_t, \Sigma^e_{t+1}\right\},
$$

which recognizes that $\mathbb{E}_{d_2,d_1,\omega_{t+1}}[\mu^e_{t+1}] = \mu^e_t$ given time t beliefs, and the other states without expectations transition deterministically.[28] The second is the certainty state

$$
ce := \left\{k_{t+1}, E_{t+1}, \mathcal{L}_{t+1}\big|_{d_1=\exp\left(\mu_c+\frac{1}{2}\sigma_c^2\right),\omega_{t+1}=\exp\left(\mu_\omega+\frac{1}{2}\sigma_\omega^2\right)}, \mu^e_t, 0\right\},
$$

which is the time $t+1$ state that would be reached if all random variables were set equal to their means and the prior variance was set to zero.

Last, I decompose the tax into channels that map closely into the consumption-saving literature with a two-step procedure. In the first step of the decomposition, I perform a second-order Taylor expansion of the expectations over the value function partial derivative terms on the first three lines of equation (9)

[28] μ^e_{t+1} has a normal distribution and $\mathcal{L}_{t+1}$ does not have a named distribution.

around the expected state υ.[29] In the second step, I add and subtract two sets of identical terms evaluated at the certainty state *ce* to the Taylor expansion. This results in an expression that I will group into seven different channels.[30]

To economize on space, I omit both the leading $\beta_t/u'(c_t)$ term in equation (9) and the Taylor expansion terms for the value function partial derivatives with respect to the scale parameter Σ^e_{t+1} since they will be the same as the other states. The derivation of the full Taylor expansions are in the online Appendix. For the subsequent analysis, recognize that $\partial E_t/\partial e_t = 1$, and that fractional net output is decreasing in emissions $(\partial \mathcal{L}_{t+1}/\partial e_t < 0)$.

Channel 1 (Certainty Tax):

$$\left.\frac{-\partial V_{t+1}}{\partial E_{t+1}}\right|_{ce} + \left.\frac{-\partial V_{t+1}}{\partial \mathcal{L}_{t+1}}\right|_{ce} \left.\frac{\partial \mathcal{L}_{t+1}}{\partial e_t}\right|_{ce}.$$

The first channel is the *certainty tax*. The certainty tax arises from adding $\big(-\partial V_{t+1}/\partial E_{t+1}\big)|_{ce} + \big(-\partial V_{t+1}/\partial \mathcal{L}_{t+1}\big)|_{ce}\big(\partial \mathcal{L}_{t+1}/\partial e_t\big)|_{ce}$ to the full Taylor expansion around the expected state υ. The certainty tax is the shadow cost of an additional unit of emissions when all uncertain terms are perfectly known and fixed at their time t means. This is the tax that would be set by a policymaker that happened to be at the current time t state $\mathbf{S}_t$ and states transitioned deterministically (Lemoine and Rudik 2017).

Channel 2 (State Uncertainty Adjustment):

$$\left(\left.\frac{-\partial V_{t+1}}{\partial E_{t+1}}\right|_{\zeta} + \left.\frac{-\partial V_{t+1}}{\partial \mathcal{L}_{t+1}}\right|_{\zeta} \mathbb{E}_{d_2,d_1,\omega_{t+1}}\left[\frac{\partial \mathcal{L}_{t+1}}{\partial e_t}\right]\right)$$
$$- \left(\left.\frac{-\partial V_{t+1}}{\partial E_{t+1}}\right|_{ce} + \left.\frac{-\partial V_{t+1}}{\partial \mathcal{L}_{t+1}}\right|_{ce} \left.\frac{\partial \mathcal{L}_{t+1}}{\partial e_t}\right|_{ce}\right).$$

The second channel is the *state uncertainty adjustment*. This channel alters the certainty tax so that it correctly accounts for how uncertainty affects the expected time $t+1$ state. It is the difference in the tax evaluated at the expected state and the tax evaluated at the certainty state; the second term arises from subtracting the certainty tax terms from the full Taylor expansion. The rest of the channels account for how uncertainty affects the expected shadow cost of emissions because of curvature in the value function partial derivatives.

[29] I require the value function to be three times differentiable in a number of the arguments. This is not necessarily guaranteed by having the utility function be three times differentiable and proving that my analytic value function is three times differentiable is not trivial. Here I am using high order polynomial approximations to the value function that are greater than third order so my approximating value function is trivially three times differentiable.

[30] Lemoine and Rudik (2017) similarly demonstrate how to use Taylor expansions of the value function to gain insight into how uncertainty over the climate's sensitivity to emissions drives optimal policy.

Channel 3 (Precautionary Abatement):

$$\frac{1}{2}\frac{-\partial^3 V_{t+1}}{\partial E_{t+1}\mathcal{L}_{t+1}^2}\bigg|_{\zeta} \operatorname{var}_{d_2,d_1,\omega_{t+1}}(\mathcal{L}_{t+1})$$

$$+\frac{1}{2}\frac{-\partial^3 V_{t+1}}{\partial \mathcal{L}_{t+1}^3}\bigg|_{\zeta} \mathbb{E}_{d_2,d_1,\omega_{t+1}}\left[\frac{\partial \mathcal{L}_{t+1}}{\partial e_t}\right]\operatorname{var}_{d_2,d_1,\omega_{t+1}}(\mathcal{L}_{t+1})$$

$$+\frac{-\partial^3 V_{t+1}}{\partial E_{t+1}\partial \mathcal{L}_{t+1}\partial \mu_{t+1}^e}\bigg|_{\zeta} \operatorname{cov}_{d_2,d_1,\omega_{t+1}}(\mathcal{L}_{t+1},\mu_{t+1}^e)$$

$$+\frac{-\partial^3 V_{t+1}}{\partial \mathcal{L}_{t+1}^2\partial \mu_{t+1}^e}\bigg|_{\zeta} \mathbb{E}_{d_2,d_1,\omega_{t+1}}\left[\frac{\partial \mathcal{L}_{t+1}}{\partial e_t}\right]\operatorname{cov}_{d_2,d_1,\omega_{t+1}}(\mathcal{L}_{t+1},\mu_{t+1}^e).$$

The third channel is the *precautionary abatement* motive. The numerical results indicate that this channel tends to increase the carbon tax when not learning about d_2 and decrease the carbon tax when learning about d_2. The third derivative of utility corresponds to how uncertainty about future consumption affects the policymaker's contemporaneous saving decision (Leland 1968, Drèze and Modigliani 1972, Kimball 1990). If uncertainty about future consumption increases contemporaneous saving, the agent is said to exhibit prudence. In a climate-economy setting, abatement is a form of environmental saving: increasing current abatement means that the policymaker forgoes a sure consumption payoff now for increased consumption later due to lower future damages. The second term captures this most clearly. When not learning, the third derivative of her continuation value with respect to fractional net output is positive so that

$$\frac{1}{2}\frac{-\partial^3 V_{t+1}}{\partial \mathcal{L}_{t+1}^3}\bigg|_{\zeta} \mathbb{E}_{d_2,d_1,\omega_{t+1}}\left[\frac{\partial \mathcal{L}_{t+1}}{\partial e_t}\right]\operatorname{var}_{d_2,d_1,\omega_{t+1}}(\mathcal{L}_{t+1}) > 0,$$

and the policymaker increases abatement and her optimal carbon tax. The opposite is true when learning. The precautionary motive also scales in size with the variance of future output.

The first term on the first line captures similar precautionary abatement motives in the face of uncertain output. If this term increases abatement then the agent would be called cross-prudent (Gollier 2010). The policymaker is cross-prudent if she would prefer to have a mean-zero risk attached to fractional net output when cumulative emissions are lower rather than when they are higher.

The second line captures precautionary abatement motives because of cross-prudence, and these channels generally increase the optimal carbon tax. Here, the policymaker abates more because of covariability between fractional net output and her beliefs about the damage exponent. In the numerical results the covariance terms are generally negative since smaller-than-expected $\mathcal{L}_{t+1}$ is the signal that the policymaker would need to receive to revise her expectations about d_2 upward. This covariance matters because when future beliefs about damages are uncertain, then future output and future consumption appear to be even more variable. Whatever information the policymaker receives about output in the future feeds back onto her payoffs through her expectations.

Channel 4 (Signal Smoothing):

$$\frac{1}{2}\frac{-\partial^3 V_{t+1}}{\partial E_{t+1}\partial {\mu_{t+1}^e}^2}\bigg|_{\zeta} \operatorname{var}_{d_2,d_1,\omega_{t+1}}(\mu_{t+1}^e) + \frac{1}{2}\frac{-\partial^3 V_{t+1}}{\partial \mathcal{L}_{t+1}\partial {\mu_{t+1}^e}^2}\bigg|_{\zeta} \mathbb{E}_{d_2,d_1,\omega_{t+1}}\left[\frac{\partial \mathcal{L}_{t+1}}{\partial e_t}\right]\operatorname{var}_{d_2,d_1,\omega_{t+1}}(\mu_{t+1}^e).$$

The fourth channel is the *signal smoothing* motive, which tends to increase the optimal carbon tax in the numerical results. This channel captures one effect of learning on the optimal tax. The second derivative of the value function with respect to the location parameter, $-\partial^2 V_{t+1}/\partial {\mu_{t+1}^e}^2$, captures how well the policymaker can use new information to smooth welfare over possible values of d_2. Since a higher location parameter for the policymaker's beliefs strictly increases her expectation of d_2, and since $-\partial^2 V_{t+1}/\partial {\mu_{t+1}^e}^2 > 0$ in the simulations, the marginal welfare cost of a higher d_2 belief is increasing and convex. If there is not much curvature in her beliefs and $\partial^2 V_{t+1}/\partial {\mu_{t+1}^e}^2$ is small, then she is able to smooth welfare effectively, but the larger $\partial^2 V_{t+1}/\partial {\mu_{t+1}^e}^2$ is in magnitude, the less she can smooth welfare, and the more costly a bad signal of d_2 becomes.

The triple derivatives then indicate how a marginal increase in cumulative emissions or fractional net output affects the policymaker's ability to smooth welfare in response to new information. Indeed, additional emissions decrease her ability to smooth welfare ($-\partial^3 V_{t+1}/\partial E_{t+1}\partial {\mu_{t+1}^e}^2 > 0$), and having a greater fraction of her gross output would increase her ability to smooth welfare ($-\partial^3 V_{t+1}/\partial \mathcal{L}_{t+1}\partial {\mu_{t+1}^e}^2 < 0$). Additional emissions results in greater damages and less output to be able to use towards abatement, while additional fractional net output has the opposite effect. In these cases, additional variability in future beliefs magnifies this effect and increases the optimal tax.

Channel 5 (Output Insurance):

$$\operatorname{cov}_{d_2,d_1,\omega_{t+1}}\left(\frac{-\partial V_{t+1}}{\partial \mathcal{L}_{t+1}}, \frac{\partial \mathcal{L}_{t+1}}{\partial e_t}\right).$$

The fifth channel is the *output insurance* channel. In simulations, this channel decreases the optimal carbon tax over the first 90 years when not learning before eventually increasing the carbon tax. It always increases the optimal carbon tax in the simulations with learning. Output insurance increases the optimal carbon tax if and only if the covariance is positive. The first term in the covariance captures the marginal welfare cost of reducing fractional net output, and the second term in the covariance captures the marginal reduction in fractional net output from emissions. The policymaker cares about the covariance of returns to emissions reductions with marginal utility. This channel increases the optimal carbon tax if emissions reductions are most effective in preserving output when output is most valuable to welfare.

Channel 6 (**Learning Insurance**):

$$\text{cov}_{d_2,d_1,\omega_{t+1}}\left(\frac{-\partial V_{t+1}}{\partial \mu^e_{t+1}}, \frac{\partial \mu^e_{t+1}}{\partial e_t}\right).$$

The sixth channel is the *learning insurance* channel. This channel generally increases the optimal carbon tax in the simulations. Learning insurance motives are similar to output insurance motives in that the policymaker cares about the covariance between the effect of emissions reductions on expectations and marginal welfare. The covariance term is generally positive so this channel reduces the optimal level of emissions.

Channel 7 (**Misspecification Insurance**):

$$\mathbb{E}_{d_2}\left[\text{cov}_{d_1,\omega_{t+1}}\left(\frac{-\partial V_{t+1}}{\partial \mathbf{S}_{t+1}}\frac{\partial \mathbf{S}_{t+1}}{\partial e_t}, \frac{\exp\left(\frac{-\beta_t V_{t+1}}{\theta}\right)}{\mathbb{E}_{d_1,\omega_{t+1}}\left[\exp\left(\frac{-\beta_t V_{t+1}}{\theta}\right)\right]}\right)\right]. \tag{10}$$

The final channel is the *misspecification insurance* channel. The magnitude and direction of the misspecification insurance channel depends on how the marginal welfare cost of emissions covaries with the worst-case distortion to the transition density. The misspecification insurance channel increases the carbon tax if and only if the covariance is positive. This channel increases the optimal carbon tax when not learning and slightly decreases the optimal carbon tax when learning. A decrease in the optimal carbon tax is consistent with previous findings that concerns about model misspecification increase motives for policy experimentation (Cogley et al. 2008).

III. Results

First, I show the mean optimal carbon tax trajectories and the corresponding climate outcomes over the next century for each of the four frameworks. Next, I vary the robust control penalty parameter to examine how the policymaker's concern for model misspecification affects policy. Then, I decompose the carbon taxes into each channel outlined in Section II to determine how uncertainty matters for policy. Finally, I analyze welfare outcomes under the different frameworks. In each simulation run, I randomly sample a vector of annual damage shocks $\{\omega_{2005}, \ldots, \omega_{2105}\}$ from the damage shock distribution. In cases where I take ex ante expectations of the optimal carbon tax, I also randomly sample a $\{d_1, d_2\}$ pair from their year 2005 prior distributions in each simulation.

A. *Optimal Carbon Tax Trajectories*

Figure 5 displays the first century's optimal carbon tax trajectories in $\$/tCO_2$.[31] The top left panel displays the ex ante expected carbon tax trajectories averaged over 50,000 simulations. Each simulation randomly draws a pair of damage function

[31] Some papers report carbon taxes in terms of dollars per ton of carbon. The unit conversion is 11 tons of CO_2 to 3 tons of carbon.

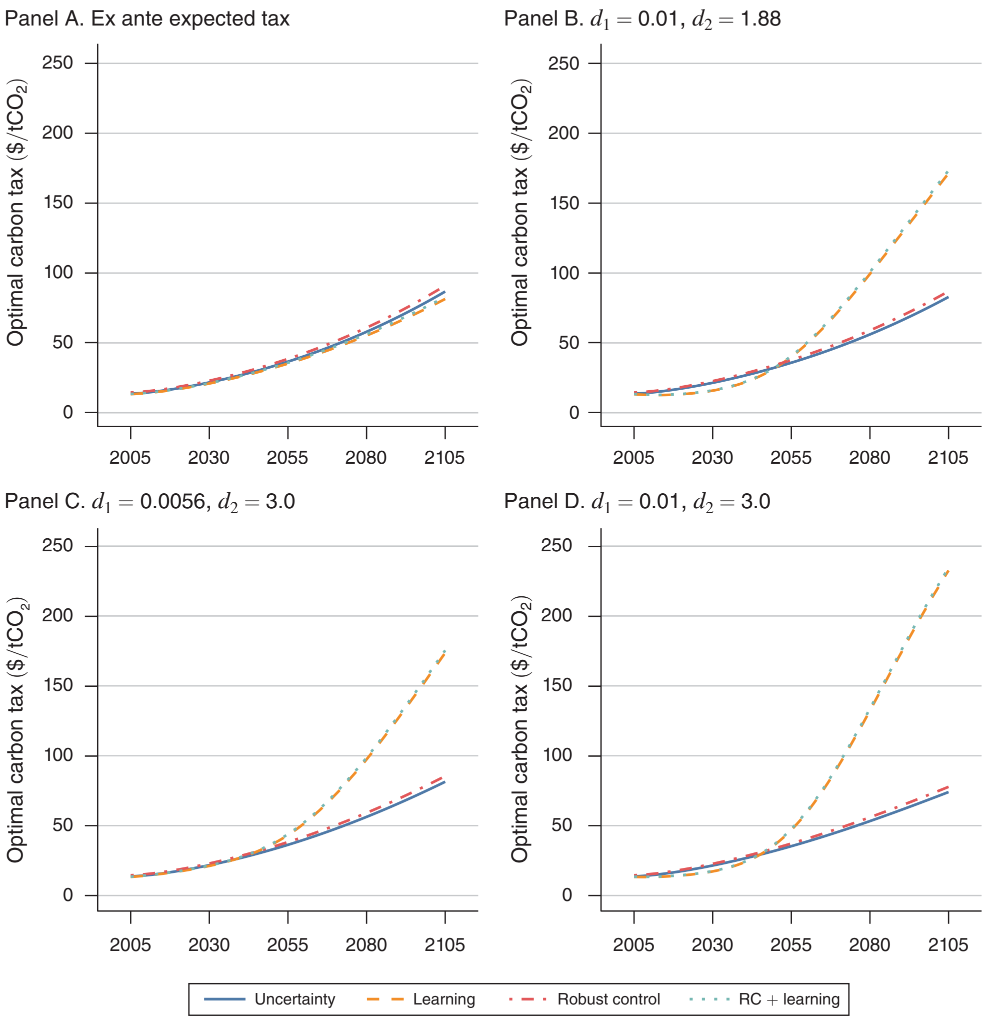

FIGURE 5. THE EX ANTE EXPECTED CARBON TAXES

Notes: The ex ante expected optimal carbon tax over 50,000 simulations for each framework (top left), and the expected optimal carbon tax under specific damage functions over 5,000 simulations for each framework (other three panels). Each of the 50,000 simulations randomly samples a different set of damage parameters and damage shocks $\{d_1, d_2, \omega_{2005}, \ldots, \omega_{2105}\}$. For the other three panels, all simulations use the same damage parameters listed below the panel, and a random sample of damage shocks $\{\omega_{2005}, \ldots, \omega_{2105}\}$.

parameters and a sequence of annual damage shocks $\{d_1, d_2, \omega_{2005}, \ldots, \omega_{2105}\}$ from their distributions defined by the estimated parameters in Table 1. The remaining three panels display results averaged over 5,000 simulations when d_1 and d_2 are fixed to a single value across all 5,000 simulations, but each simulation still randomly draws a sequence of annual damage shocks $\{\omega_{2005}, \ldots, \omega_{2105}\}$. The top right panel sets the true coefficient to 0.01 and the true exponent equal to its expectation, the bottom left panel sets the true coefficient to its expectation and the true exponent to 3.0, and the bottom right panel sets the true coefficient to 0.01 and the true exponent to 3.0.

The top left panel plots the ex ante expected carbon tax trajectories (i.e., the tax trajectories the policymaker expects to set over the first 100 years given her 2005 information set). Under the uncertainty framework, the policymaker begins her optimal tax in 2005 at $13.68/tCO$_2$ and expects to ramp it up to $86.76/tCO$_2$ in 2105. When the policymaker is able to learn d_2 over time, she sets an initial carbon tax 2 percent lower at $13.34/tCO$_2$, which on average rises to $81.44/tCO$_2$ at the end of the century. Applying robust control to guard against potential model misspecification tends to increase the tax when not updating beliefs. In 2005 the robust control tax begins slightly higher than the uncertainty framework at $14.52/tCO$_2$. Over the first century, the robust control tax is expected to increase faster than the uncertainty tax and reaches $90.99/tCO$_2$ at the end of the century. Applying robust control on top of learning increases the initial carbon tax with learning by $0.15/tCO$_2$, and by 2105 the ex ante expected carbon tax with both robust control and learning is $1.41/tCO$_2$ higher than just learning alone.

The top right panel shows the mean carbon tax trajectories when the damage coefficient is about double the policymaker's expectation and the damage exponent is equal to her expectation. Initial carbon taxes start at the same level as in the top left panel since the policymaker is making decisions with the same information set. When facing a higher damage coefficient, the carbon taxes under the uncertainty and robust control frameworks ramp up at a slower rate, reaching $82.89/tCO$_2$ for the uncertainty framework and $86.95/tCO$_2$ for the robust control framework. The large damage coefficient results in more damage and less output to allocate toward abatement so the policymaker sets a lower carbon tax. The greater level of damages acts like a negative income effect. The two frameworks with learning have carbon tax trajectories that initially decrease. Temperature starts below 1°C and the damage function has a monomial form, so when a learning policymaker observes higher damages in the early years she attributes it to a smaller damage exponent and revises her expectation about d_2 downward to approximately 1.6. When temperature crosses the 1°C threshold, she starts revising her beliefs about d_2 upward and begins quickly increasing her carbon tax, which reaches $171.34/tCO$_2$ and $173.37/tCO$_2$ in 2105 under the learning and robust control and learning frameworks.[32]

The bottom left panel shows the mean carbon tax trajectories when the damage coefficient is equal to the policymaker's expectation and the damage exponent is about double her expectation. Similar to the top right panel, the nonlearning frameworks' carbon taxes are lower than ex ante expectations because the policymaker has less output to allocate toward abatement under a more severe damage function. Optimal carbon taxes reach only $81.42/tCO$_2$ and $85.44/tCO$_2$ in 2105 for the uncertainty and robust control frameworks. Over the first few decades, carbon taxes under the learning and robust control and learning frameworks are similar to ex ante expected levels plotted in the top left panel. Damage functions with different exponents yield similar levels of damages at low temperatures so it's difficult to statistically distinguish them. In this setting, the policymaker's mean belief under the learning and RC+L frameworks only increases to about 1.9 by 2035 when

[32] The policymaker expects $d_2 \approx 2.8$ at the end of the first century.

Table 2—The Ex Ante Expected Optimal Carbon Tax, CO_2, and Temperature

	Framework			
	Uncertainty	Learning	RC	RC+L
2055 tax ($/t$CO_2$)	37	35	39	36
	(36, 37)	(25, 46)	(38, 39)	(25, 47)
2105 tax ($/t$CO_2$)	87	81	91	83
	(82, 90)	(30, 138)	(86, 94)	(30, 140)
2105 CO_2 (parts per million)	613	620	611	618
	(612, 613)	(575, 666)	(610, 611)	(573, 665)
2105 temperature (°C)	2.08	2.10	2.07	2.10
	(2.08, 2.08)	(1.95, 2.26)	(2.07, 2.07)	(1.94, 2.25)

Notes: Table 2 displays the ex ante expected optimal carbon tax in 2055 and 2105, CO_2, and temperature in 2105, and the fifth and ninety-fifth percentiles (in parentheses) for each framework over 50,000 simulations. Each of the 50,000 simulations randomly samples a different set of damage parameters and damage shocks $\{d_1, d_2, \omega_{2005}, \ldots, \omega_{2105}\}$.

temperature reaches about 1.1°C. In the second half of the century, the policymaker begins learning more rapidly and quickly increases her carbon tax to \$173.62/tCO_2 and \$175.41/tCO_2 in 2105 for the learning and RC+L frameworks.

The bottom right panel plots mean carbon tax trajectories when the damage coefficient and damage exponent are both higher than the policymaker's ex ante expectation. This is the most severe damage function so the nonlearning frameworks face their smallest output budgets and set carbon taxes of only \$74.04/tCO_2 and \$77.77/tCO_2 in 2105. High damages generate high learning carbon taxes. In 2105 they reach \$232.58/tCO_2 for the learning framework and \$234.47/tCO_2 for the RC+L framework. The learning policymakers attribute the high observed amounts of damages entirely to a large exponent, so that expectations about d_2 overshoot its true value and reach about 3.6 in 2105.

Table 2 displays the mean, fifth and ninety-fifth percentile outcomes for the 2055 carbon tax, the 2105 carbon tax, and 2105 atmospheric CO_2 concentrations and temperature. Learning results in significant variability in realized carbon taxes since the policymaker adapts her policy to the noisy information she receives about each simulation's specific d_2. After 50 years, the learning carbon tax is about \$2/tCO_2 less than the uncertainty tax on average, but in some cases it may be nearly 30 percent lower or 30 percent higher. After 100 years, the learning carbon tax is about 7 percent smaller than the uncertainty carbon tax on average, but it still displays substantial variability depending on the realized values of the damage function parameters and the damage shocks. After 50 years, the mean robust control carbon tax is \$2/tCO_2 higher than the mean uncertainty carbon tax, but after 100 years, the difference grows to \$4/tCO_2.

When learning, the policymaker allows CO_2 to be 7 parts per million (ppm) higher in 2105, which results in greater warming compared to the uncertainty framework. Conversely, the more aggressive carbon tax in the robust control framework tends to keep CO_2 lower than the uncertainty framework by 2 ppm in 2105. There is also significant variability in climatic outcomes when learning. The 90 percent confidence interval for 2105 CO_2 is about 90 ppm wide. This is approximately the same as the real world change in atmospheric CO_2 concentrations from 1960 to 2018.

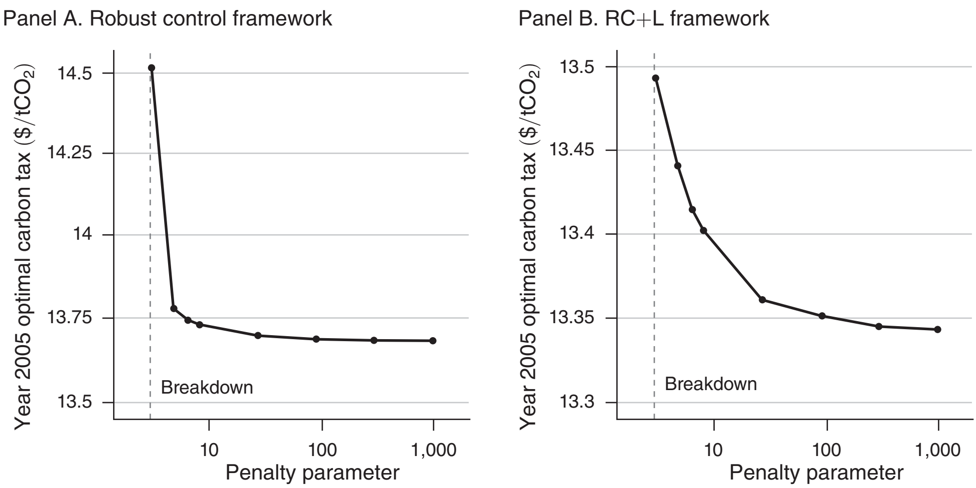

Figure 6. The Year 2005 Optimal Carbon Taxes as a Function of θ for the Robust Control Framework (Left) and the RC+L Framework (Right)

Notes: The dashed line indicates the approximate location of $\bar{\theta}$ where the problem breaks down. Note that each panel has a different *y*-axis scale, and both *x*-axes are on a log10 scale.

B. *The Effect of Concern for Model Misspecification*

Figure 6 shows how changing the level of the policymaker's concern about model misspecification affects the initial optimal carbon tax. Smaller values of the penalty parameter indicate greater misspecification concern. The left panel of Figure 6 displays the optimal carbon tax as a function of the penalty parameter for the robust control framework. When the penalty parameter is sufficiently large, the year 2005 optimal carbon tax is effectively equal to the optimal carbon tax of the uncertainty framework. As the penalty parameter declines and we move to the left on the plot, the optimal carbon tax increases, and on this plot, peaks at $\$14.52/\mathrm{tCO}_2$ when the penalty parameter is 3.9. Decreasing it further results in the problem breaking down so the model no longer solves.

The right panel of Figure 6 displays the initial optimal carbon tax for the robust control and learning framework. Again, a sufficiently large penalty parameter leads the optimal carbon tax to be effectively equal to the learning framework without robust control. Decreasing the penalty parameter to 3.9, right before the problem breaks down, increases the 2005 carbon tax to $\$13.49/\mathrm{tCO}_2$.

C. *Decomposing the Optimal Carbon Tax*

Figure 7 plots the six channels that capture uncertainty's effect on the optimal carbon taxes plotted in the top left panel of Figure 5.[33] Note that all panels have

[33] The total effect of these channels is to increase the optimal carbon tax above the certainty tax by up to 1 percent for the uncertainty framework, 5 percent for the robust control framework, 5 percent for the learning framework, and 7 percent for the RC+L framework.

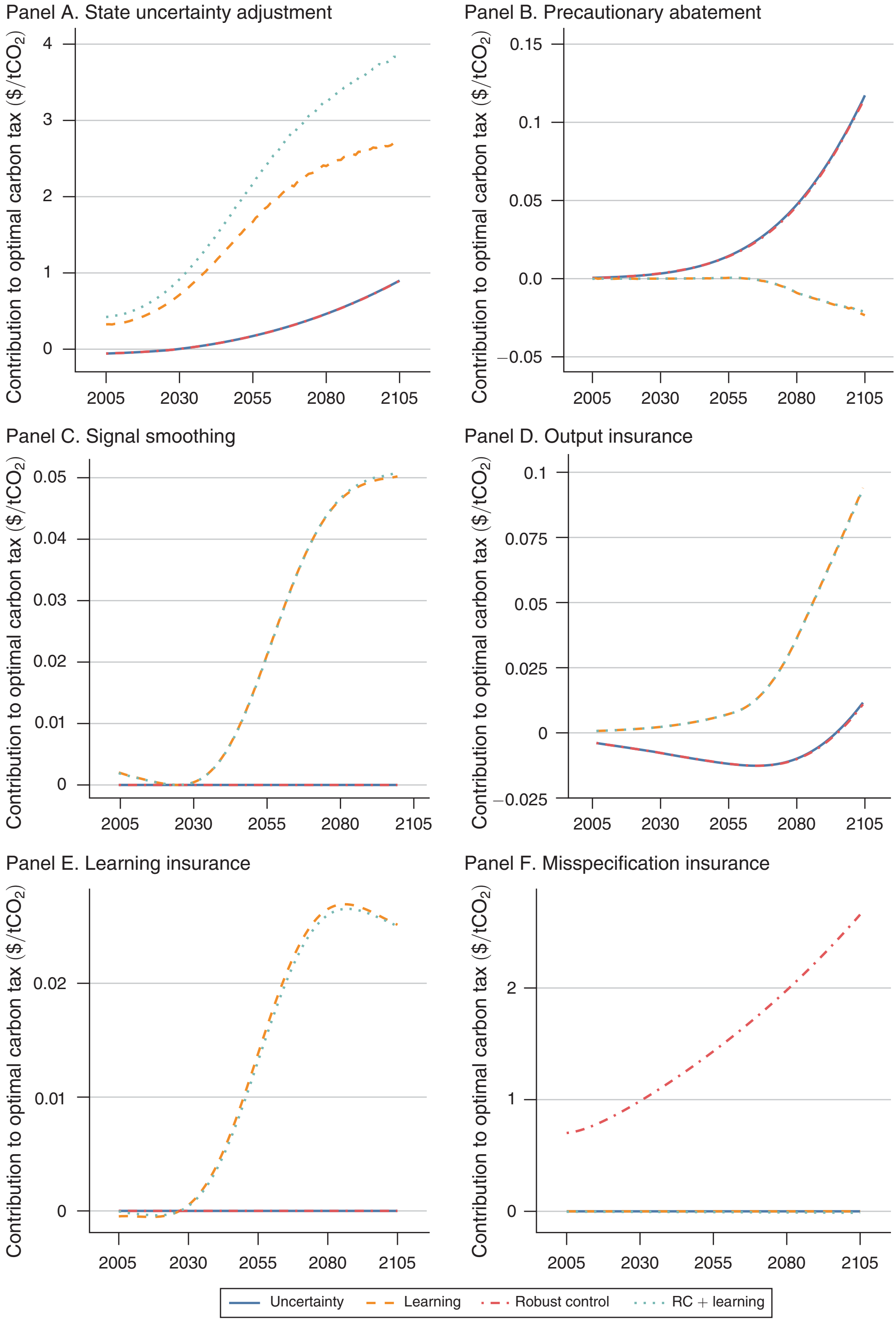

FIGURE 7. THE EX ANTE EXPECTATIONS OF THE SIX CARBON TAX CHANNELS RELATED TO UNCERTAINTY

Notes: The expectations are taken over 50,000 simulations, where each simulation randomly samples a different set of damage parameters and damage shocks $\{d_1, d_2, \omega_{2005}, \ldots, \omega_{2105}\}$. Note that each panel has a different y-axis scale.

different scales to better display differences between the frameworks. The certainty tax is in the online Appendix. Panel A plots the state uncertainty adjustment, the strongest of the channels. For the uncertainty framework and robust control framework, the state uncertainty adjustment begins slightly below zero but becomes positive at 2030 and then grows in magnitude over time to about $\$1/\text{tCO}_2$ by 2105. The change in sign is driven by the curvature of fractional net output in the three random variables. At lower levels of warming, fractional net output is convex in d_1 and ω_{t+1}, and concave in d_2. The convexity in d_1 and ω_{t+1} dominates early on and makes fractional net output larger at the expected state versus the certainty state. This results in a negative state uncertainty adjustment because on the margin, the shadow cost of emissions is smaller if output is higher. As temperature rises, the concavity in d_2 begins to dominate, making the uncertainty adjustment positive. For the two frameworks with learning, the state uncertainty adjustment is always positive, larger in magnitude than without learning, and monotonically increases in size from $\$0.33/\text{tCO}_2$ and $\$0.42/\text{tCO}_2$ in 2005 to $\$2.71/\text{tCO}_2$ and $\$3.86/\text{tCO}_2$ in 2105. Learning makes future beliefs variable and thus future states are more variable. This tends to increase the optimal carbon tax.

Panel B plots the precautionary abatement motive. Initially, precautionary abatement is effectively zero since variability in damages is negligible when temperature is low. Damage variability increases as temperature rises, leading to more precautionary abatement for the nonlearning frameworks because the third derivative of their value functions with respect to fractional net output is positive, and less precautionary abatement for the frameworks with learning because the third derivative of their value functions with respect to fractional net output is negative. In the learning frameworks, increased damage variability is partially offset by the policymaker resolving uncertainty so this channel's magnitude is dampened.

Panel C plots the signal smoothing channel, which is specific to the frameworks with learning. This channel increases the optimal carbon tax because emissions hinder the policymaker's ability to smooth welfare. The signal smoothing channel starts below $\$0.01/\text{tCO}_2$ in 2005 and declines as her future beliefs become less variable. As temperature rises, emissions increasingly reduce the policymaker's ability to smooth welfare to new information. This begins dominating the decrease in belief variability around 2030 and the signal smoothing channel increases thereafter to $\$0.05/\text{tCO}_2$ in 2105.

Panel D displays the effect of output insurance. This channel eventually increases the optimal carbon tax for all frameworks. This channel is near zero at first since surface temperature is low and there is not much variability in the marginal effect of emissions on fractional net output. For the nonlearning frameworks this channel decreases initially because the covariance between the effect of emissions on fractional net output with the marginal welfare cost of less fractional net output is negative, but it eventually becomes positive and reaches $\$0.01/\text{tCO}_2$. For the learning frameworks the size of the channel grows at an increasing rate to approximately $\$0.09/\text{tCO}_2$.

Panel E displays the effect of learning insurance. This channel tends to have the smallest impact out of all six channels. It generally increases the optimal carbon tax because additional emissions tend to increase future mean beliefs about

d_2 $(\partial\mu^e_{t+1}/\partial e_t > 0)$ when welfare is most sensitive to increases in the mean belief about d_2 $(-\partial V_{t+1}/\partial\mu^e_{t+1} > 0)$. This channel is near zero at first since temperature is low and there is not much variability in the marginal effect of emissions on fractional net output. The size of the channel grows at an increasing rate to approximately \$0.03/tCO_2 at 2085 before declining.

Panel F plots the misspecification insurance channel. The channel increases the carbon tax when not learning and decreases the carbon tax when learning. In both cases, this channel begins small as there is less variability early on when temperature is low, but the size of misspecification insurance grows over time for both frameworks using robust control. This is because the marginal benefit of emissions reductions and the distortion to the transition density become larger and increasingly correlated. The size of misspecification insurance reaches over \$2/tCO_2 without learning, and about −\$0.02/tCO_2 when learning. A negative effect of robust control on a learning carbon tax is consistent with findings of increased experimentation motives in other settings (Cogley et al. 2008). A learning policymaker can accelerate the learning process by allowing faster warming. Robust control amplifies this experimentation motive.[34]

D. *Welfare Implications of Learning and Robust Control*

Next I investigate the relative welfare performance of the frameworks. First, I quantify ex ante welfare differences assuming that the policymaker has correctly specified her model so that the true damage function parameters are drawn from the distributions in Section IC. Results for the learning framework gives us the expected benefits of updating the distribution over d_2, while the results for the robust control framework gives us the costs of designing policy to guard against misspecifications when we have actually specified damages correctly.[35]

Second, I quantify ex post welfare when the policymaker's approximating damage model may be misspecified and the true damage function is instead one of three catastrophic damage functions. The first two are the damage functions of Weitzman (2012) and Dietz and Stern (2015), where damages follow close to DICE at low levels of warming, but rise rapidly to 50 percent of output at 6°C and 4°C respectively.[36] The third is a more extreme version of these two damage functions where damages hit 50 percent of output at only 3°C. Damages are subject to the same sequence of damage shocks with the parameters estimated in Section IC. Since I am exploring outcomes when models are misspecified in the second part of the welfare analysis, I must simulate the model and calculate the welfare outcomes over a finite horizon. The frameworks' value functions only yield the true ex ante expected welfare when the model is specified correctly.

[34] The scale parameter is strictly decreasing in temperature so allowing temperature to rise quicker will reduce the variance of beliefs.

[35] Recall that the robust control framework does not distort the actual state transitions, only the policymaker's beliefs about them.

[36] Except for very high levels of warming, the DICE damage function implies less damage than a damage function with parameters given by the distributional means associated with Table 1.

Table 3—Measures of Ex Ante Welfare of the Learning, Robust Control, and Robust Control and Learning Frameworks Relative to the Uncertainty Framework

Framework	Learning	RC	RC + L
2005 lump sum (billions of $)	758	−156	615
2005 per capita ($)	116	−24	94
BGE gain (percent)	0.039	−0.008	0.032

Table 3 displays ex ante welfare results for each of the frameworks relative to the uncertainty framework when the policymaker has correctly specified her damage model. The first line displays the lump sum present value benefit of using one of the frameworks over the uncertainty framework, the second line displays the present value benefit in per capita terms, and the final line shows this in terms of balanced growth equivalent (BGE) consumption gain (Mirrlees and Stern 1972, Lemoine and Traeger 2014, Jensen and Traeger 2016). The BGE is the difference in growth rates between two counterfactual consumption trajectories that grow at a constant rate and also yield the same welfare as the uncertainty framework and the comparison framework.

When damages are specified correctly, learning is worth $758 billion, a one-time payment in 2005 of $116 per person, or a permanent gain in consumption of 0.039 percent. These welfare gains come about because the policymaker uses observations of damages to better match her policy to the actual damage function she faces. When the policymaker uses robust control but the damage function is correctly specified, she incurs losses of $156 billion or equivalently $24 per capita. Robust control induces the policymaker to use too high of a carbon tax, which results in less consumption and lower welfare. Below, I explore outcomes where the damage function is misspecified since that is the motivating factor for using robust control. The RC+L framework yields a present value benefit of $615 billion, 20 percent less than without robust control.

Figure 8 displays the present value ex post welfare gain for each of the frameworks relative to the uncertainty framework for four different time horizons. I compute ex post welfare for each combination of framework, damage function, and time horizon by first performing 10,000 simulations, each with a randomly drawn vector of annual damage shocks. I then average the sum of the present value of flow utilities across the 10,000 simulations. Finally I translate the expected present value from utils into dollars using the initial marginal utility of consumption from the uncertainty framework's optimal trajectory.

The horizontal axis on each plot in Figure 8 shows the four different time horizons over which ex post welfare is evaluated: 50, 100, 150, and 200 years. The vertical axis shows the present value expected welfare gain relative to the uncertainty framework. The uncertainty framework is omitted since it is the baseline framework. An equivalent level of welfare as the uncertainty framework is denoted by the dashed line. Plots above the dashed line indicate gains relative to the uncertainty framework and plots below the dashed line indicate losses. Each plot corresponds to one of the

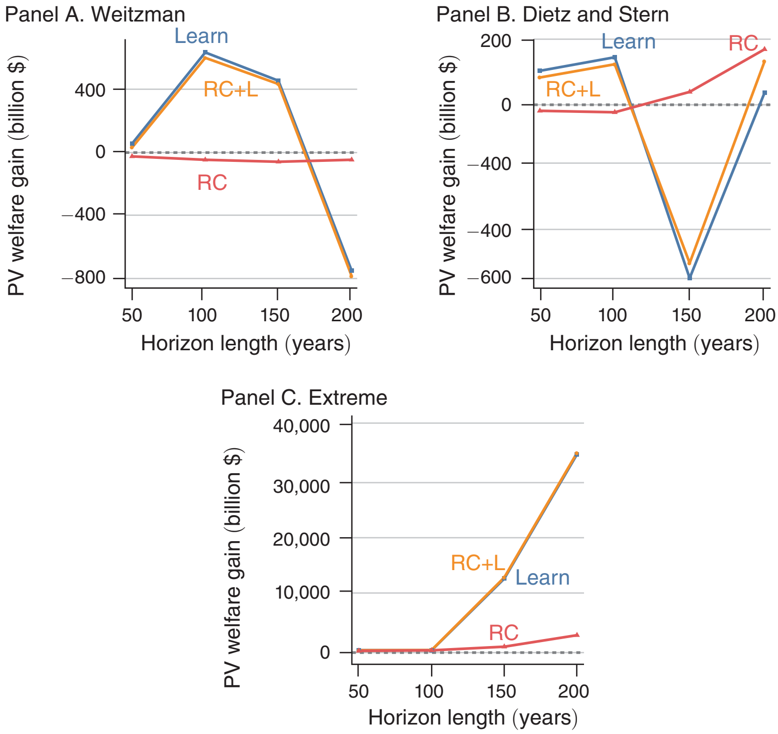

Figure 8. The Present Value Welfare Gain of the Learning, Robust Control, and Robust Control and Learning Frameworks Relative to the Uncertainty Framework for Three Different Catastrophic Damage Functions

Notes: Panel A is for the Weitzman (2012) damage function, where damages reach 50 percent of GDP at 6° C; panel B is for the Dietz and Stern (2015) damage function, where damages reach 50 percent of GDP at 4° C; and panel C is for an extreme damage function, where damages reach 50 percent of GDP at 3° C. Ex post welfare gains are averaged over 10,000 simulations, each with a random sample of damage shocks $\{\omega_{2005}, \ldots, \omega_{2105}\}$.

three catastrophic damage functions. All three catastrophic damage functions have the following form:

$$D\left(T^s_{t+1}, \omega_{t+1}\right) = \left(d_1 \left[T^s_{t+1}\right]^{d_2} + d_3 \left[T^s_{t+1}\right]^{d_4}\right) \omega_{t+1}.$$

The damage functions have the following common parameter values: $d_1 = 0.00284$, $d_2 = 2$, $d_4 = 6.754$. For the Weitzman damage function $d_3 = 5.07 \times 10^{-6}$, for the Dietz and Stern damage function $d_3 = 8.19 \times 10^{-5}$, and for the extreme damage function $d_3 = 5.85 \times 10^{-4}$. Here, ω_{t+1} is the damage shock drawn from the same distribution as before.

Panel A displays outcomes when the policymaker's damage model is misspecified and the true damage function is the one developed in Weitzman (2012). Learning (with or without robust control) delivers ex post welfare gains over time horizons of 50–150 years. Although the policymaker has misspecified her model by omitting the extra catastrophic term, and she mislearns by using the information from new damage observations to update d_2 instead of changing the structure of her damage function, learning a misspecified damage function improves ex post welfare for a set of time horizons. Until temperature rises substantially more than the initial level, the Weitzman damage function results in relatively low damages. The policymaker believes this is because she faces a low true value of d_2 and revises her expectation down to about 1.28 in 2105 when temperature is only 2.22°C. Despite the damage function misspecification, this helps her push policy in the right direction for the near-term low damage world and she only sets a carbon tax of $\$32/tCO_2$ in 2105. Welfare gains are over \$400 billion for 100 and 150 year time horizons. Over time, this low carbon tax results in accelerated warming and eventually, and unexpectedly to the policymaker because of her misspecification, massive damages. Damages are nearly four times the levels of the uncertainty framework in the last 50 years of the simulation, and consumption under the learning framework is lower than under the uncertainty framework by tens of trillions of dollars per year. The accumulation of damage more than offsets the early welfare gains and generates total losses of over \$750 billion in present value terms by 2205.

The robust control framework results in ex post welfare losses of up to \$60 billion even out to 200 years in the future. This is because when a nonlearning policymaker is optimizing carbon taxes under her misspecified damage function, temperature never crosses 3°C, so the catastrophic part of the Weitzman (2012) damage function does not kick in. Over time horizons up to 200 years, the robust control policymaker implements too stringent of a carbon tax relative to the uncertainty framework.

Panel B shows welfare outcomes when the true damage function is the one developed in Dietz and Stern (2015). Here learning results gains of about \$100 billion relative to the uncertainty framework over the first 100 years, large losses at 150 years, and small gains at 200 years. Similar to panel A, the policymaker believes early on that she is not in a high damage world because temperature does not reach the point where the catastrophic part of the damage function kicks in. Near the end of the first century, she believes d_2 to be as low as 1.72. Correspondingly, she sets a moderate carbon tax of $\$43/tCO_2$ in 2075 as opposed to a tax of $\$54/tCO_2$ under the uncertainty framework, and she reallocates output toward consumption, which increases present value ex post welfare at a 100 year horizon. In the second century, temperature crosses 2°C and gets closer to the catastrophic part of the damage function. The policymaker begins revising her beliefs upward and by 2155 she expects d_2 to be 2.30. This rapid shift in her beliefs leads to a reallocation of output from consumption toward abatement. The learning carbon tax triples over a 50-year time span, from $\$79/tCO_2$ in 2105 to $\$255/tCO_2$ in 2155, while under the uncertainty framework the carbon tax only about doubles over this timespan, from $\$87/tCO_2$ to $\$159/tCO_2$. This aggressive climate action results in short-term welfare losses between 2105 and 2155, but it improves ex post welfare relative to the uncertainty framework by 2205 through reduced warming and damages. Adding robust control on top of learning increases welfare by about \$140 billion in 2205 relative to learning alone because of more aggressive climate action.

When using robust control but not learning, positive net benefits can be achieved relative to the uncertainty framework after 150 years. Expected ex post welfare gains are \$140 billion at 200 years in the future. Over the first century, temperature is below 2°C and the Dietz and Stern (2015) damage function closely follows the actual DICE damage function. Over this timeframe guarding against misspecification results in a carbon tax that is too high for a noncatastrophic damage function and slight welfare losses. After 150 years, temperature crosses 2.5°C for both the robust control and uncertainty frameworks, and damages start growing quicker as temperature begins reaching catastrophic levels. By the end of the second century, the avoided damages from the robust control framework's more aggressive carbon tax preserves an additional half a percentage point of output per year.

Panel C shows welfare outcomes under the extreme damage function, where damages reach 50 percent of output at only 3°C. Here learning delivers large ex post welfare gains after the first century. The catastrophic part of the damage function kicks in rapidly and the learning policymaker quickly adapts her carbon tax in the right direction despite her misspecified damage function. At the end of the first century, the learning and RC+L policymakers set carbon taxes over twice as high as under the uncertainty framework and have nearly achieved full abatement of carbon emissions. By the end of the second century the learning and RC+L carbon taxes are eight times as large. Learning achieves welfare gains of up to $35 trillion dollars after 200 years. Using robust control alone also begins delivering positive welfare gains at around 100 years. At this point, the robust control tax is only about 5 percent larger than the uncertainty tax. After 200 years, robust control welfare gains are $3 trillion and the robust control tax is now 15 percent larger. Learning welfare gains are an order of magnitude larger than robust control welfare gains.

IV. Conclusion

Most economists believe that damage functions in IAMs are misspecified. Yet, analyses of optimal climate policy have not investigated the effects of endogenously improving the damage function over time, or for adapting policy to take account of the widely noted concerns about damage function misspecification. I fill this gap in the literature by comparing the performance of four policy frameworks with different degrees of damage assumptions. The uncertainty framework accounts for uncertainty over all parameters of the damage function. The learning framework acknowledges that economists do learn over time and allows the policymaker to endogenously update her distribution over the uncertain temperature elasticity of damages. The robust control framework borrows techniques from the macroeconomic literature to incorporate concerns that the damage function is misspecified in unknown ways. The robust control policymaker constructs policies that guard against model misspecification. Last, the robust control and learning framework combines concern for model misspecification with updating of beliefs over the temperature elasticity of damages. For each of the frameworks, I demonstrate how misspecification concerns, and uncertainty about damages and future beliefs feed into the optimal carbon tax.

I find that uncertainty over the damage parameters generally increases the optimal carbon tax through the state uncertainty adjustment. A decomposition of the carbon tax into six uncertainty-related channels reveals that there are conflicting effects of uncertainty on the optimal carbon tax due to precaution and insurance motives. Depending on whether the policymaker learns about the damage function, motives to insure against model misspecification may increase or decrease the carbon tax, but the total effect of robust control is to increase the carbon tax.

Learning about the temperature elasticity of damages can deliver ex ante welfare gains worth hundreds of billions of dollars, indicating that current research aimed at updating damage functions has the potential to be extremely valuable. Ex ante welfare losses from using robust control when the model has been correctly specified are $156 billion. In cases where the model's damage function is misspecified and the true damage function instead allows for catastrophe at temperatures of 3° C or higher, robust control and (incorrect) learning can both achieve higher welfare than accounting for parametric damage uncertainty alone, but with several caveats. Learning and robust control both perform well when the damage function has been highly misspecified, such as in worst-case scenarios where massive damages occur at temperatures we are expected to reach well before the end of the century along a business-as-usual trajectory. However if damages are expected to be low and noncatastrophic until 4° C or higher, this may not be true. Learning about a misspecified damage function can backfire and reduce welfare by erroneously ruling out that there is catastrophe further down the line.

This paper's results make several contributions to the broader discussions about the specification of damage functions and the updating of damage functions.[37] The first is to raise the question about whether a focus on protecting against Weitzman (2012) and Dietz and Stern (2015) style catastrophe is of first-order importance since the effects of a common method to guard against misspecification are small relative to that of learning. One omitted factor in my analysis is that uncertainty about climate sensitivity should make these catastrophes more relevant; however, I leave adding climate sensitivity uncertainty into this framework to future work.[38] The second contribution is to emphasize the high stakes of current research on estimating damages. The welfare effects of updating beliefs are large, and their magnitude and sign depend on the form of the true damage function. The damage functional form onto which updated estimates are placed will have a major influence on the real world effectiveness of IAM policy prescriptions.

The sensitivity of welfare to damage functional forms suggests that damage functions must simultaneously be more flexible and also better grounded in science in order to improve the utility of IAMs for policymaking. Continually updating quadratic or even general monomial damage functions will not adequately capture nonlinearities. However, in order to estimate future damages under temperatures never before seen in human history, we cannot simply rely on flexible polynomial forms and statistical updating. Future temperatures will be far outside our historical sample and extrapolating without any structure on the damage function will give us poor estimates of future damages. More scientific guidance is needed to tell modelers what damage observations today might mean for the structure of the real world damage function at never-before-seen temperatures. The dynamic stochastic IAM

[37] Beyond the greater damage function debate, this paper also points out the relevance of intergenerational equity. The costs of a more robust climate policy will be borne by current generations but the benefits will not be felt until potentially hundreds of years in the future.

[38] Future work may also focus on learning about both parameters of the damage function or modeling learning in a way that is more agnostic about the damage functional form.

literature has begun making inroads in using scientific knowledge to better incorporate tipping points into models (e.g., Lemoine and Traeger 2014, 2016b; Cai and Lontzek 2019), but there is much work left to be done to address other areas of the climate-economy system.

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