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Source: *American Economic Journal: Economic Policy*, May 2020, Vol. 12, No. 2 (May 2020), pp. 228-257

Published by: American Economic Association

Stable URL: <https://www.jstor.org/stable/10.2307/26921731>

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## Does School Desegregation Promote Diverse Interactions? An Equilibrium Model of Segregation within Schools<sup>†</sup>

By ANGELO MELE\*

*This paper studies racial segregation in schools using data on student friendships from Add Health. I estimate an equilibrium model of friendship formation, with preferences allowing both homophily and heterophily in direct and indirect ties. I find that homophily goes beyond direct links: students also prefer racially homogeneous indirect friends, while there is heterophily in income. I simulate policies reallocating students across schools. Race-based policies have non-linear effects on within-school segregation and other network features such as clustering and centrality. Policies increasing diversity through reallocations based on income have less impact on racial segregation. (JEL H75, I21, I28, J15)*

Social networks are important determinants of individuals' socioeconomic performance. An increasing amount of evidence shows that the number and composition of social ties affect employment prospects, school performance, risky behavior, adoption of new technologies, diffusion of information, and health outcomes (Topa 2001; Laschever 2005; Cooley 2010; De Giorgi, Pellizzari, and Redaelli 2010; Nakajima 2007; Bandiera and Rasul 2006; Conley and Udry 2010; Golub and Jackson 2011; Acemoglu et al. 2011; Calvó-Armengol, Patacchini, and Zenou 2009). An empirical challenge in estimating such network effects is the endogeneity of social relationships: individuals choose their peers and friends according to their socioeconomic characteristics and relationships. As a consequence, in socially generated networks, the agents are likely to interact with similar individuals

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<sup>†</sup>Go to <https://doi.org/10.1257/pol.20170604> to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

(homophily), segregating along socioeconomic attributes (Currarini, Jackson, and Pin 2009; Boucher 2015; De Martí and Zenou 2009).

Empirical estimates of peer effects suggest that segregation may have an effect on school outcomes of minorities (Echenique and Fryer 2007; Cutler and Glaeser 1997; Ananat 2011; Angrist and Lang 2004; Echenique, Fryer, and Kaufman 2006). Desegregation programs are an attempt to reengineer this process by reallocating minority students to predominantly white/Caucasian schools. The underlying premise of desegregation programs is that by exposing students of different racial backgrounds to each other, they will necessarily interact more. In practice, these programs face a Lucas critique: if desegregation plans are designed without taking into account the equilibrium choices of peers and their effects on outcomes, they may backfire (Carrell, Sacerdote, and West 2013; Badev 2013). In an experimental study that reassigned students to peer groups to maximize educational outcomes based on peer effects estimates, Carrell, Sacerdote, and West (2013) show that after being assigned to another group, students modify the way they interact within the newly assigned group, decreasing the expected effect of the intervention. Recent structural models suggest that ignoring the equilibrium network formation leads to underestimating peer effects (Badev 2013, Hsieh and Lee 2016).

This paper contributes to this debate by estimating an equilibrium model of network formation in schools and measuring the extent to which reallocating students among schools leads to changes in segregation patterns and interactions, *even in the absence of peer effects*. While, from a policy perspective, it is not clear whether we should focus on segregation across schools or within schools in order to improve outcomes, my simulations show that there is a relationship between the demographic composition of a school and the way its students interact in groups.<sup>1</sup> These simulations can thus be interpreted as measuring a first-order effect of such reallocation policies. In the model, students are heterogeneous on several (observed) dimensions, and they decide whether to form links with other students based on their preferences. These preferences are flexible and allow for both homophily (a bias for similar people) and heterophily (a bias for different people) on different characteristics (Mele 2017a; Boucher 2015; dePaula, Richards-Shubik, and Tamer 2018). For example, there may be homophily by race and heterophily by age. Preferences also depend on link externalities, such as having common friends or reciprocated links.

The link formation is sequential, and we focus on stationary equilibria. The reason for this is that data usually come from a single snapshot of the network, and therefore the dynamics of the model are harder to identify and estimate. The stationary equilibrium is characterized by the probability of observing a particular network in the long run. This distribution has peaks in the networks that are Nash equilibria. Assuming that the network we observe in the data is drawn from the stationary equilibrium of the model, this distribution can be interpreted as the likelihood of observing a network in the long run.

<sup>1</sup> While most of the literature focuses on the effect of segregation across schools (Clotfelter 2004, Lutz 2011, Angrist and Lang 2004), there are recent works that analyze the effect of segregation within schools (Echenique and Fryer 2007; Echenique, Fryer, and Kaufman 2006; Badev 2013; Mele 2017b) on outcomes.

Compared to a standard logit model of link prediction, the model used in this paper incorporates externalities and equilibrium effects that change the incentives of a player to form or delete links. Indeed, if the equilibrium effects are absent, the only relevant determinants of a link are the demographics of the players. An individual will form a link to another student if the direct utility derived from the demographic characteristics is sufficiently high (net of idiosyncratic shocks). However, this mechanism does not take into account that when a student forms a new link, she is also creating an additional indirect link for someone else. While this indirect connection may not bear the full extent of the benefits of a direct link, it may increase or decrease the payoffs. This equilibrium mechanism generates additional incentives (positive or negative) to form links and may as well depend on the demographics and compositions of indirect ties. We focus on the equilibrium of such a process of link formation, that is, networks that are stable in a well-defined sense (game theoretical equilibrium).

Using this theoretical framework, I estimate preferences for link formation in school friendship networks, using data from the *National Longitudinal Study of Adolescent Health* (Add Health). This unique dataset contains detailed information on the friendship networks of students enrolled in a representative sample of US schools. The final sample includes 16 high schools from the *saturated sample* with a total of 3,604 students and more than 3.5 million pairs.<sup>2</sup> I find that race, gender, parental income, attractiveness, and grade level are important determinants of network formation in schools. There is overwhelming evidence of homophily: students tend to interact and form social ties with similar people, controlling for the structure of their network. Furthermore, I find that homophily effects extend well beyond direct links: for example, students also prefer a homogeneous racial composition of friends of friends. I also find evidence of heterophily in parental-income levels, suggesting that students tend to sort by socioeconomic status. Interestingly, Hispanics prefer a diverse group of friends, as well as a diverse group of indirect friends. Therefore, the creation of a new link with a student of the same race will change the incentive for other students to form links within the same racial group. If these incentives are positive (homophily), this effect will lead to increased segregation levels and the clustering of the network. Conversely, if there is heterophily, this leads to a decrease in segregation and clustering.

I use the estimated model to predict how a change in the composition of the student population affects the structure of the network.<sup>3</sup> Because the structure of the network is correlated with individual and aggregate outcomes, this effect is of first-order importance (Carrell, Sacerdote, and West 2013; Echenique and Fryer 2007; Badev 2013; Calvó-Armengol, Patacchini, and Zenou 2009). First, I simulate a desegregation program by swapping students in two homogeneous schools that are (almost)

<sup>2</sup>I use only the schools from the *saturated sample*. The sampling scheme of Add Health involved in-school interviews for all 90,118 students. A subsample of 20,745 students was also interviewed at home to collect detailed individual information. The saturated sample contains schools for which both interviews were administered to *each* student enrolled. Therefore this sample does not contain any missing information about individual controls. This is not the case for most schools in Add Health.

<sup>3</sup>Alternatively, the model could be used as a guide for the design of randomized experiments that modify students' assignments.

completely white and black, respectively. Interestingly, the results depend on which measure of network segregation we use. The most intuitive measure of segregation, the Freeman Segregation Index (FSI), measures the difference between the fraction of mixed-group friendships and their expected value under no homophily. The conclusion of this experiment is that trying to make schools equally diverse by equalizing their racial share may not decrease segregation within schools, according to FSI. Indeed, segregation is lower in the presence of a relatively small minority that is not able to self-segregate. While the pattern of segregation as a function of the fraction of blacks in the school is similar, there are differences in the magnitude across schools due to the difference in how whites and blacks respond to changes in the fraction of their own groups in the school. Income segregation slightly decreases in the (originally) white school, while it increases in the (originally) black school. These conclusions change if we measure segregation using the Spectral Segregation Index (SSI) of Echenique and Fryer (2007). SSI measures segregation of a group in a way similar to how the page-rank algorithm computes scores for webpages. That is to say, the higher the segregation of a student, the higher the segregation of the students she is linked to. We use the mean SSI to measure segregation in the school, and our results show that an increase in the fraction of blacks in the school will always increase segregation. Therefore, a policy that makes schools equally diverse would be a good compromise in terms of racial and income segregation.

Second, in another policy experiment, I take some students whose parent incomes are above median in a homogeneously white school and swap them with students with incomes below median in a homogeneously black school. This policy increases income segregation across schools while also increasing racial diversity within schools.<sup>4</sup> There is almost no effect on the level of racial segregation within schools according to the FSI. However, the effect on income segregation is different across the two schools. The school that is homogeneously white becomes poorer and sees an increase in income segregation. The other school becomes richer and sees almost no change in income segregation. According to the SSI, racial and income segregation will increase in the white school and decrease in the black school. Finally, while in the black school the most popular students are black, in the white school this policy increases the probability that the most central (and popular) students will be black.

In summary, I show that even absent peer effects or other outcome-related motives for link formation, a reassignment of students will result in a new equilibrium set of links. The presence of link externalities in the payoffs may amplify this mechanism, resulting in more or less segregation by socioeconomic characteristics, a point of practical importance when designing desegregation programs.

This paper contributes to the empirical network literature by estimating an equilibrium model of homophily and segregation (Jackson 2008; dePaula 2017, Graham 2017, Chandrasekhar 2016),<sup>5</sup> whose results are consistent with recent evidence on homophily in networks (Currarini, Jackson, and Pin 2009; Boucher 2015; Boucher

<sup>4</sup>This simulation is motivated by some evidence that the disparity in average school poverty rates between white and black students' schools is a strong predictor of racial school achievement gaps (Reardon 2016).

<sup>5</sup>For additional structural models of network formation see Menzel (2016); Sheng (2012); dePaula, Richards-Shubik, and Tamer (2018); Leung (2015b); Leung (2015a).

and Mourifie forthcoming; Mele 2017b). I also find homophily for indirect connections. Models of network formation that exclude link externalities in the payoffs are not able to capture this feature (see, for example, Graham 2017, Dzemski 2017), attributing all the homophily to direct links or unobserved characteristics.

The rich dataset used in my estimation partially solves the identification issues highlighted in Mele (2017a). Since I estimate the model using multiple independent school networks, the identification of the structural parameters relies on the variation of network features both across schools and within schools (Lehman 1983, Wainwright and Jordan 2008).<sup>6</sup>

The paper also contributes to the literature on racial segregation in schools. This body of work focused on the effects of residential and school segregation on minority socioeconomic outcomes (Cutler and Glaeser 1997, Echenique and Fryer 2007, La Ferrara and Mele 2007, Ananat 2011). Other work has considered the effect of school segregation on educational attainment (Angrist and Lang 2004). Most studies analyze segregation among the schools in a district, but few authors have used more detailed data at the school level to understand the patterns of racial segregation within schools (Echenique and Fryer 2007; Echenique, Fryer, and Kaufman 2006; Mele 2017b; Boucher 2015; Badev 2013). While it is not clear whether within- or between-school segregation is more important in determining outcomes, my approach in this paper provides a structural interpretation of the segregation levels within schools and takes between-school segregation as given. In the policy experiments, I modify between-school segregation by simulating reallocations of the students, focusing on the equilibrium implications within schools.

This model also provides insights about possible mechanisms behind the moderate increase in racial segregation after the end of court-ordered desegregation programs (Lutz 2011) and the modest effects of desegregation program on educational outcomes (Angrist and Lang 2004).

The rest of the paper is organized as follows: Section I briefly describes the theoretical model and provides the intuition about the equilibrium effects. Section II develops the estimation strategy and provides an overview of the data and identification. Section III reports the posterior estimates and the policy experiments. Some of the computational details are provided in the online Appendix.

## I. A Model of Network Formation

In this section, I briefly present the setup of Mele's (2017a) model and the equilibrium likelihood. The proofs and more theoretical results are all contained in that paper. Time is discrete, and there are  $n$  players in the network. Each player is characterized by a vector of observable covariates,  $X_i$ , that may contain information about race, gender, wealth, age, location, etc. The matrix  $X = \{X_1, X_2, \dots, X_n\}$  contains these vectors for all the players, stacked by column.

The social network of friendship nominations is represented by an  $n \times n$  adjacency matrix  $g$ , with entries  $g_{ij} = 1$  if individual  $i$  nominates individual  $j$  as a friend,

<sup>6</sup>Similar identification with multiple networks is exploited in Nakajima (2007), Badev (2013), Sheng (2012).



and  $g_{ij} = 0$  otherwise. There are no self-loops,  $g_{ii} = 0$ , for any  $i$ , and the network is *directed*: the existence of a link from  $i$  to  $j$  does not imply the existence of the link from  $j$  to  $i$ . This modeling choice reflects the structure of the Add Health data, where friendship nominations are not necessarily mutual. Some authors refer to this data as *perceived networks*.<sup>7</sup> Let the *realization* of the network at time  $t$  be denoted as  $g^t$  and the *realization* of the link between  $i$  and  $j$  at time  $t$  be  $g_{ij}^t$ . The network including all the current links but  $g_{ij}^t$ , i.e.,  $g^t \setminus g_{ij}^t$ , is denoted as  $g_{-ij}^t$ ; while  $g_{-i}^t$  denotes the network matrix excluding the  $i$ th row (i.e., all the links of player  $i$ ).

The network formation process follows a stochastic best-response dynamic as in Blume (1993). At the beginning of each period, a player  $i$  is randomly selected from the population, and he meets individual  $j$  according to a meeting probability  $\rho(ij|g^{t-1}, X)$ . Notice that  $\rho$  may depend on the previous period network and the observable characteristics. For example, people that have many friends in common may meet with higher probability than people without common friends. Or students with similar demographics may have a higher probability of interaction than students with different backgrounds do. An implicit assumption of the model is that the player can observe the entire network and the covariates of all the agents before making their choice about linking.

Upon meeting agent  $j$ , player  $i$  decides whether to update his link  $g_{ij}$ . The preferences of  $i$  are defined over networks and covariates. The utility of player  $i$  from network  $g$  and covariates  $X$  is given by

$$(1) \quad U_i(g, X) = \underbrace{\sum_{j=1}^n g_{ij} u_{ij}}_{\text{direct friends}} + \underbrace{\sum_{j=1}^n g_{ij} g_{ji} m_{ij}}_{\text{mutual friends}} + \underbrace{\sum_{j=1}^n g_{ij} \sum_{\substack{k=1 \\ k \neq i, j}}^n g_{jk} v_{ik}}_{\text{friends of friends}} + \underbrace{\sum_{j=1}^n g_{ij} \sum_{\substack{k=1 \\ k \neq i, j}}^n g_{ki} w_{kj}}_{\text{popularity}},$$

where  $u_{ij} \equiv u(X_i, X_j)$ ,  $m_{ij} \equiv m(X_i, X_j)$ ,  $v_{ij} \equiv v(X_i, X_j)$ , and  $w_{ij} \equiv w(X_i, X_j)$  are (bounded) real-valued functions of the attributes. The utility of the network is the sum of the net benefits received from each link. The total benefit from an *additional link* has four components.

When a player creates a link to another individual, he receives a *direct* net benefit  $u_{ij}$ . The direct utility includes both costs and benefits, and it may possibly be negative. When only homophily enters payoffs of direct links, the net utility  $u_{ij}$  is positive if  $i$  and  $j$  belong to the same group, while it is negative when they are of different types. In many models, this component is parameterized as  $u_{ij} = b_{ij} - c_{ij}$ , where  $b_{ij}$  indicates the (gross) benefit and  $c_{ij}$  the cost of forming the additional link  $g_{ij}$ . I use the notation  $u_{ij}$  since it does not require assumptions on the cost function.

The players receive additional utility  $m_{ij}$  if the link is mutual; friendship is valued differently if the other agent reciprocates. An agent may perceive another individual as a friend, but that person may not perceive the relationship in the same way.

The players value the composition of friends of friends. When  $i$  is deciding whether to befriend  $j$ , she observes  $j$ 's friends and their socioeconomic characteristics. Each of  $j$ 's friends provides additional utility  $v(X_i, X_k)$  to  $i$ . In this model, an

<sup>7</sup> See Wasserman and Faust (1994) for references.

agent who has the opportunity to form an additional link values a white student with three Hispanic friends as a different good than a white student with two white friends and one African American friend.<sup>8</sup> In other words, individuals value both *exogenous* heterogeneity and *endogenous* heterogeneity: the former is determined by the socioeconomic characteristics of the agents, while the latter arises endogenously with the process of network formation. I assume that only friends of friends are valuable and that they are perfect substitutes: individuals do not receive utility from two-links-away friends.

The fourth component corresponds to a *popularity effect*. When an agent forms a link, he/she automatically creates an indirect link for other agents that are connected to him/her, thus generating externalities. This makes him/her more or less appealing to her friends, thus impacting his/her “popularity.”

Conditional on the meeting  $m^t = ij$ , player  $i$  updates the link  $g_{ij}$  to maximize his current utility, taking the existing network  $g_{-ij}^t$  as given. I assume that the agents do not take into account the effect of their linking strategy on the future evolution of the network. The players have *complete information*, since they can observe the entire network and the individual attributes of all agents. Before updating his link to  $j$ , individual  $i$  receives an idiosyncratic shock  $\varepsilon \sim F(\varepsilon)$  to his preferences that the econometrician cannot observe. This shock models unobservables that could influence the utility of an additional link, e.g., mood, gossips, fights, etc. Player  $i$  links agent  $j$  at time  $t$  if and only if it is a best response to the current network configuration, i.e.,  $g_{ij}^t = 1$  if and only if

$$(2) \quad U_i(g_{ij}^t = 1, g_{-ij}^{t-1}, X) + \varepsilon_{1t} \geq U_i(g_{ij}^t = 0, g_{-ij}^{t-1}, X) + \varepsilon_{0t}.$$

I assume that when the equality holds, the agent plays the status quo.<sup>9</sup>

The model satisfies the following assumptions.

**ASSUMPTION 1 (Preferences):** *The payoffs are such that  $m(X_i, X_j) = m(X_j, X_i)$  and  $w(X_k, X_j) = v(X_k, X_j)$  for all players  $i, j, k$ .*

**ASSUMPTION 2 (Meetings):** *Any meeting is possible, i.e.,  $\rho(ij|g^{t-1}, X) = \rho(ij|g_{-ij}^t, X) > 0$  for any pair of players  $i, j$ .*

**ASSUMPTION 3 (Shocks):** *Before deciding whether to update a link, players receive a stochastic shock that follows a Type I extreme value distribution, i.i.d. among links and across time.*

The first assumption about symmetry of  $m_{ij}$  is needed for identification: two individuals with the same exogenous characteristics  $X_i = X_j$  (say two males, white, enrolled in eleventh grade) who form a mutual link receive the same  $u_{ij}$  and  $m_{ij}$ , but

<sup>8</sup>A similar assumption is used in De Martí and Zenou (2009), where the agents' cost of linking depends on the racial composition of friends of friends. Their model is an extension of the connection model of Jackson and Wolinsky (1996), and the links are formed with mutual consent. The corresponding network is undirected.

<sup>9</sup>This assumption does not affect the main result and is relevant only when the distribution of the preference shocks is discrete.



they may have different utilities from that additional link because of the composition of their friends of friends and their popularity. Therefore, this part of the assumption helps in identifying the utility from indirect links and popularity.

When  $i$  forms a link to  $j$ ,  $i$  creates an externality for all  $k$ s who have linked her: any such  $k$  now has an additional indirect friend, i.e.,  $j$ , who agent  $k$  values by an amount  $v(X_k, X_j)$ . When  $w(X_k, X_j) = v(X_k, X_j)$ , an individual  $i$  values his popularity effect as much as  $k$  values the indirect link to  $j$ , i.e.,  $i$  internalizes the externality she creates.<sup>10</sup>

The second assumption on the meeting process guarantees that any pair of agents can meet. The main implication is that any equilibrium network can be reached with positive probability. For example, a discrete uniform distribution satisfies this assumption.

Finally the third assumption allows the Markov chain to escape from the nash networks, eliminating absorbing states and making the model ergodic.

As a consequence of these assumptions, the network formation process is a potential game, where all the incentives of the players can be summarized by an aggregate function of the network  $Q$ :

$$(3) \quad Q(g, X) = \sum_{i=1}^n \sum_{j=1}^n g_{ij} u_{ij} + \sum_{i=1}^n \sum_{j>i}^n g_{ij} g_{ji} m_{ij} + \sum_{i=1}^n \sum_{j=1}^n \sum_{\substack{k=1 \\ j \neq i, k \neq i, j}}^n g_{ij} g_{jk} v_{ik}.$$

The potential is such that for any player  $i$  and any link  $g_{ij}$ , we have

$$Q(g_{ij}, g_{-ij}, X) - Q(1 - g_{ij}, g_{-ij}, X) = U_i(g_{ij}, g_{-ij}, X) - U_i(1 - g_{ij}, g_{-ij}, X).$$

Consider two networks,  $g = (g_{ij}, g_{-ij})$  and  $g' = (1 - g_{ij}, g_{-ij})$ , that differ only with respect to one link,  $g_{ij}$ , chosen by individual  $i$ : the difference in utility that agent  $i$  receives from the two networks,  $U_i(g, X) - U_i(g', X)$ , is exactly equal to the difference of the potential function evaluated at the two networks,  $Q(g, X) - Q(g', X)$ . That is, the potential is an aggregate function that summarizes both the state of the network and the deterministic incentives of the players in each state.

The model generates a Markov Chain of networks that converges to a unique stationary distribution  $\pi$ ,

$$(4) \quad \pi(g, X) = \frac{\exp[Q(g, X)]}{\sum_{\omega \in \mathcal{G}} \exp[Q(\omega, X)]},$$

where  $\mathcal{G}$  is the set of all networks with  $n$  nodes. In the long run, the system spends more time in network states that have high potential. It can be shown that these networks correspond to Nash equilibria of a model without any shock to the

<sup>10</sup>This restriction of the preferences guarantees the model's coherency in the sense of Tamer (2003). In simple terms, this part of the assumption guarantees that the system of conditional linking probabilities implied by the model generates a proper joint distribution of the network matrix. Similar restrictions are also encountered in spatial econometrics models (Besag 1974) and in the literature on qualitative response models (Heckman 1978, Amemiya 1981).

preferences (Mele 2017a, Jackson and Watts 2001, Monderer and Shapley 1996). The function

$$(5) \quad c(\mathcal{G}, X, \theta) = \sum_{\omega \in \mathcal{G}} \exp[Q(\omega, X, \theta)]$$

is a normalizing constant that guarantees that (4) is a proper probability. This is the sum of exponential potential functions over the set  $\mathcal{G}$  of all networks with  $n$  nodes.

### A. The Role of Indirect Payoffs in Equilibrium

To develop the intuition about the role of indirect payoffs in the determination of the equilibrium, let's consider a simplified version of this model with only two types of students (e.g., boys and girls, whites and nonwhites, etc.). The type is a binary variable  $x_i \in \{0, 1\}$ ; let the indicator function  $\mathbf{1}_{ij} = 1$  if  $x_i = x_j$  and  $\mathbf{1}_{ij} = 0$  otherwise. Let the direct payoff be  $u_{ij} = \alpha + \beta \mathbf{1}_{ij}$ , the mutual utility  $m_{ij} = 0$  and the indirect utility  $v_{ij} = \gamma \mathbf{1}_{ij}$  for any  $i, j = 1, \dots, n$ . I assume  $\alpha < 0$  for the rest of this section, and interpret  $\alpha$  as the cost of forming a link; so for every link  $g_{ij}$ , student  $i$  pays a cost  $\alpha$ ; if  $j$ 's type is the same as  $i$ , that is  $x_i = x_j$ , student  $i$  receives an additional direct payoff  $\beta$ ; finally, student  $i$  receives a payoff  $\gamma$  for each indirect friend of  $j$  whose type is the same as  $i$ 's. Therefore the utility of player  $i$  is

$$(6) \quad U_i(g, X) = \alpha \sum_{j=1}^n g_{ij} + \beta \sum_{j=1}^n g_{ij} \mathbf{1}_{ij} + \gamma \sum_{j=1}^n g_{ij} \sum_{\substack{k=1 \\ k \neq i, j}}^n (g_{jk} \mathbf{1}_{ik} + g_{ki} \mathbf{1}_{kj}).$$

There is same-type bias (homophily) in preferences when  $\beta > 0$  and indirect same-type bias when  $\gamma > 0$ . If  $\beta < 0$ , we have bias for other types, and  $\gamma < 0$  implies indirect bias for other types. Depending on the magnitude of  $\beta$  and  $\gamma$ , we can observe homophily or heterophily in aggregate. The role of  $\alpha$  is to drive the density of the network. If  $\alpha < 0$  and large enough, we will have relatively sparse networks, while  $\alpha > 0$  will necessarily generate dense networks. More generally, the density of the network will depend on the relative magnitude of  $\alpha$ ,  $\beta$ , and  $\gamma$ .

**No Indirect Effects:** To understand the equilibrium feedbacks implied by the model, let's first shut down all payoffs from indirect links, setting  $\gamma = 0$ . Conditional on  $i$  meeting  $j$ , a link is formed with probability  $p_{ij}^s = \propto \exp(\alpha + \beta)$  if  $x_i = x_j$ ; and  $p_{ij}^d \propto \exp(\alpha)$  if  $x_i \neq x_j$ . Therefore, we expect the model to converge to a network with  $n(n-1)p_{ij}^s$  links among people of the same type and  $n(n-1)p_{ij}^d$  links among people of different types. Because there are no indirect payoffs, the equilibrium network will be welfare-maximizing. The level of homophily depends on the parameter  $\beta$ : if  $\beta > 0$  and large, we will observe most links between people of same type; conversely, if  $\beta < 0$ , most links will be between people of different type.

**Positive Indirect Payoffs:** If  $\gamma > 0$  and  $\beta > 0$ , the homophily effect will be amplified by the indirect payoff. Indeed, students will tend to form links to same type people with a homogeneous set of friends of the same type. Compared with the case  $\gamma = 0$  above, this case will lead to a network with more same-type links and more clustering, with the creation of very dense communities of the same type. Indeed, when a player

forms a link, the direct effect of homophily  $\beta > 0$  will give incentive to form a link with a person of the same type; the indirect effect is that an increase in the incentive to form links to same type individuals if they have an homogeneous set of friends. If  $\beta < 0$  and  $\gamma > 0$  is large enough, we could still observe homophily in the aggregate because the effect of indirect payoffs may offset the direct heterophily. If  $\gamma$  is large enough, it may be profitable to form links to people of different type, as long as their set of friends is homogeneous. This shows that the model has the ability to match many possible outcomes in terms of network configurations.

**Negative Indirect Payoffs:** If  $\gamma < 0$ , there is a form of competition for the indirect friends, and having friends of friends of the same type will decrease utility. In such a setting, we can still observe homophily in aggregate as long as  $\beta > 0$  and large enough. If  $\beta < 0$ , we will have a network displaying aggregate heterophily.

Finally, while the model is able to match complex network configurations in the data, the sign and magnitude of the payoffs is ultimately an empirical question.

## II. Estimation Strategy

The estimation recovers the payoffs of the players that depend on parameters  $\theta = (\theta_u, \theta_m, \theta_v)$ :

$$u_{ij}(\theta_u) = u(X_i, X_j, \theta_u); \quad m_{ij}(\theta_m) = m(X_i, X_j, \theta_m); \quad v_{ij}(\theta_v) = v(X_i, X_j, \theta_v).$$

Assuming that the observed network is a draw from the stationary distribution of the theoretical model, we can use the distribution in (4) as likelihood of the network data. However, we cannot perform exact maximum likelihood because the distribution  $\pi(g, X; \theta)$  is proportional to the normalizing constant  $c(\mathcal{G}, X, \theta)$ , whose exact evaluation is infeasible, even in small networks. To be concrete, consider a network with  $n = 10$  agents. To compute the constant at the current parameter  $\theta$ , we would need to evaluate the potential function for all  $2^{90} \cong 10^{27}$  possible networks with 10 agents and compute their sum. Assuming that a state-of-the-art supercomputer can evaluate  $10^{12}$  potential functions in one second, it would take almost 40 million years to compute the constant at a single parameter vector.<sup>11</sup> Therefore, direct evaluation of the likelihood is impossible. It is easy to show that the first- and second-order conditions for the maximum likelihood problem also depend on the normalizing constant. The same problem arises if we use a Bayesian approach and standard Markov Chain Monte Carlo samplers to estimate the posterior

$$(7) \quad p(\theta | g, X) = \frac{\pi(g, X, \theta) p(\theta)}{\int_{\Theta} \pi(g, X, \theta) p(\theta) d\theta},$$

because equation (7) contains the normalizing constant in the likelihood.

<sup>11</sup> A network with  $n$  players has  $2^{n(n-1)}$  possible network configurations. The schools used in the empirical section have between 20 and 159 enrolled students. This translates into a minimum of  $2^{380}$  and a maximum of  $2^{25,122}$  possible network configurations.

### A. Estimation Algorithm

To solve this estimation problem, I use the *exchange algorithm*, first developed by Murray, Ghahramani and MacKay (2006) for distribution with intractable normalizing constants and adapted to network models by Caimo and Friel (2011) and Mele (2017a). This algorithm uses a double Metropolis-Hastings step to avoid the computation of the normalizing constant  $c(\mathcal{G}, X, \theta)$  in the likelihood.<sup>12</sup>

While several authors have proposed similar algorithms in the related literature on Exponential Random Graphs Models (ERGM),<sup>13</sup> the models estimated with this methodology typically have very few parameters and use data from very small networks. To the best of my knowledge, this is the first attempt to estimate a high-dimensional model using data from multiple networks. In this section, I describe the algorithm for a single network, while in the online Appendix, I provide the extension for multiple independent networks.<sup>14</sup>

The idea of the algorithm is to sample from an augmented distribution using an auxiliary variable. At each iteration, the algorithm proposes a new parameter vector  $\theta'$  drawn from a suitable proposal distribution  $q_\theta(\theta'|\theta)$ ; in the second step, it samples a network  $g'$  (the auxiliary variable) from the likelihood  $\pi(g', X, \theta')$ ; finally, the proposed parameter is accepted with a probability  $\alpha_{ex}(\theta, \theta')$ , such that the Markov chain of parameters generated by these updates has the posterior (7) as unique invariant distribution.

#### ALGORITHM

Fix the number of simulations  $R$ . At each iteration  $t$ , with current parameter  $\theta_t = \theta$ , network data  $g$ , and control variables  $X$ :

- (i) Propose a new parameter  $\theta' \sim q_\theta(\cdot|\theta)$ .
- (ii) Simulate  $R$  networks from the stationary distribution of the model and collect the last simulated network  $g' \sim \mathcal{P}_{\theta'}^{(R)}(g'|g)$  using the following steps (a) and (b) at each iteration:
  - (a) At iteration  $r$ , with current network  $g_r$  and proposed parameter  $\theta'$ , start the simulations at network  $g$  and propose a network  $g^*$  from a proposal distribution  $g^* \sim q_g(g^*|g_r)$ .

<sup>12</sup>This improvement comes with a cost: the algorithm may produce MCMC chains that have very poor mixing properties (Caimo and Friel 2011) and high autocorrelation. I partially correct for this problem by carefully calibrating the proposal distribution. In this paper I use a random walk proposal. Alternatively one could update the parameters in blocks or use recent random block techniques as in Chib and Ramamurthy (2010) to improve convergence and mixing.

<sup>13</sup>Caimo and Friel (2011) use the exchange algorithm to estimate ERGM. They improve the mixing of the sampler using the snooker algorithm. Koskinen (2008) proposes the Linked Importance Sampler Auxiliary variable (LISA) algorithm, which uses importance sampling to provide an estimate of the acceptance probability. Another variation of the algorithm is used in Liang (2010).

<sup>14</sup>When the data consist of several independent school networks, I use a parallel version of the algorithm that stores each network in a different processor. Each processor runs the simulations independently and the final results are summarized in the master processor, that updates the parameters for next iteration. Details in the online Appendix.

- (b) Update the network:  $g_{r+1} = g^*$  with probability  $\alpha_{mh}(g_r, g^*)$  and  $g_{r+1} = g_r$  with probability  $1 - \alpha_{mh}(g_r, g^*)$ , where

$$(8) \quad \alpha_{mh}(g_r, g^*) = \min \left\{ 1, \exp[Q(g^*, X, \theta) - Q(g_r, X, \theta)] \frac{q_g(g_r | g^*)}{q_g(g^* | g_r)} \right\}.$$

- (iii) Update the parameter vector:  $\theta_{t+1} = \theta'$  with probability  $\alpha_{ex}(\theta, \theta', g', g)$  and  $\theta_{t+1} = \theta$  with probability  $1 - \alpha_{ex}(\theta, \theta', g', g)$ , where

$$(9) \quad \alpha_{ex}(\theta, \theta', g', g) = \min \left\{ 1, \frac{\exp[Q(g', X, \theta)] p(\theta') q_\theta(\theta | \theta') \exp[Q(g, X, \theta')]}{\exp[Q(g, X, \theta)] p(\theta) q_\theta(\theta' | \theta) \exp[Q(g', X, \theta')]} \right\}.$$

The appeal of this algorithm is that all quantities in the acceptance ratio (9) can be evaluated; there are no integrals or normalizing constants to compute. The sampler is likely to accept proposals that move toward high-density regions of the posterior distribution, and it is likely to reject proposals that move toward low-density regions of the posterior. Therefore, it produces samples of parameters that closely resemble draws from the posterior distribution (7). A formal statement about convergence to the posterior distribution (7) is in Theorem 6 of Mele (2017a).<sup>15</sup> In the empirical application, we have multiple school networks and therefore step (ii) is parallelized, with each school being simulated in a different processor. This speeds up computations and allows inference in complex models.

### B. Likelihood of Multiple Independent Networks

In the empirical analysis, I specify utility functions linear in parameters. Let  $\theta_u = (\theta_{u1}, \theta_{u2}, \dots, \theta_{uP})'$ ,  $\theta_m = (\theta_{m1}, \theta_{m2}, \dots, \theta_{mL})'$ , and  $\theta_v = (\theta_{v1}, \theta_{v2}, \dots, \theta_{vS})'$  be the utility parameters for  $u$ ,  $m$ , and  $v$ , respectively. Let functions  $h: \mathbb{R}^A \times \mathbb{R}^A \rightarrow \mathbb{R}$  be defined as

$$u_{ij}(\theta_u) = \sum_{p=1}^P \theta_{up} h_{up}(X_i, X_j); \quad m_{ij}(\theta_m) = \sum_{l=1}^L \theta_{ml} h_{ml}(X_i, X_j);$$

$$v_{ij}(\theta_v) = \sum_{s=1}^S \theta_{vs} h_{vs}(X_i, X_j).$$

The functions  $h$  are quite general. In particular, they can include interactions of multiple control variables, like gender and race or interactions of individual and network-level attributes, such as race interacted with share of a racial group in the network. In principle, the functions used for direct, mutual, and indirect utility may

<sup>15</sup> Practical implementation of the algorithm is discussed in Mele (2017a).

be different. Because of the linearity of  $u_{ij}$ ,  $m_{ij}$ , and  $v_{ij}$ , the likelihood is an exponential family distribution (Lehman 1983)

$$(10) \quad \pi(g, X) = \frac{\exp[\boldsymbol{\theta}'\mathbf{t}(g, X)]}{\sum_{\omega \in \mathcal{G}} \exp[\boldsymbol{\theta}'\mathbf{t}(\omega, X)]},$$

where  $\boldsymbol{\theta} = (\theta_u, \theta_m, \theta_v)'$  is a vector of parameters and  $\mathbf{t}(g, X) = (t_1(g, X), \dots, t_K(g, X))$  is a vector of *sufficient statistics* for the network formation model, such as number of links, the number of links among black students, the number of reciprocated links, etc.<sup>16</sup>

The estimation is performed with a sample of school network data. For each school we observe the friendship nominations  $g_c$  and student's control variables  $X_c$ , for  $c = 1, \dots, C$ . The nature of the sample and the survey we use (see section about Add Health data) makes the school networks independent. Therefore, the log-likelihood of our data is the sum of the log-likelihoods of each school:

$$(11) \quad \ell(g, X) = \sum_{c=1}^C \boldsymbol{\theta}'\mathbf{t}(g_c, X_c) - \kappa(\mathcal{G}, X, \boldsymbol{\theta})$$

where the log-normalizing constant  $\kappa(\mathcal{G}, X, \boldsymbol{\theta})$  is

$$(12) \quad \kappa(\mathcal{G}, X, \boldsymbol{\theta}) = \sum_{c=1}^C \log \left( \sum_{\omega_c \in \mathcal{G}_c} \exp[\boldsymbol{\theta}'\mathbf{t}(\omega_c, X_c)] \right).$$

This is the log-likelihood of independent observations from an exponential family (Lehman 1983, Wainwright and Jordan 2008, Snijders 2002, Geyer and Thompson 1992).

### C. Identification

The model generates a dynamic sequence of networks, converging to stationarity. In principle, if the researcher has access to data where the same network is observed over time, we can estimate and identify the transition probability of the Markov Chain, that is the meeting probability and the conditional linking probability implied by the payoffs and extreme value distributed and independent shocks.

However, the data consists of only one observation at a single point in time. With such data, we cannot identify the transition probability of the model. The crucial assumptions to get identification of the stationary distribution are that shocks are independent extreme value and the meeting process does not depend on the existence of a link between two players. These assumptions make the stationary distribution independent of the specific meeting process, while allowing us to write it in closed-form as an exponential family distribution.

<sup>16</sup>This likelihood defines an exponential random graph model (ERGM), a statistical model of networks used in many applications (Snijders 2002, Frank and Strauss 1986, Moody 2001, Boucher and Mourifié 2017, dePaula 2017, Chandrasekhar 2016). My theoretical model can be interpreted as providing the microfoundations for exponential random graphs. In this sense, we can interpret the ERGM as the stationary equilibrium of a strategic game of network formation where myopic agents follow stochastic best-response dynamics and utilities are linear functions of the parameters.



This allows me to identify the preferences of individuals from a network drawn from the stationary distribution. My assumptions make sure that the conditional link probability of each player is consistent with the likelihood of the whole network (Tamer 2003). This is achieved by the assumptions on the payoff functions that guarantee the existence of a potential.

Since the likelihood of the model belongs to the exponential family (Lehman 1983), when we observe a sample of independent networks (at a single point in time), the parameters are identified through the variation of the sufficient statistics across networks. If the sufficient statistics are not linearly dependent, then the exponential family is minimal and the likelihood is strictly concave; therefore the mode is unique (Geyer and Thompson 1992, Lehman 1983, Wainwright and Jordan 2008). Our priors are relatively flat, so most of the information about the posterior is given by the likelihood.

As an aside, notice that identification may still fail when the observed sufficient statistics are “extreme.” As a simple example, consider the case in which the network does not have any reciprocated links or does not have any links between Hispanics. In that case, it will be impossible to estimate the effect of mutual links and the homophily of Hispanic students.

While in principle it is possible to add unobserved heterogeneity in the model, it is not clear that we can identify such effects using only one network observation (dePaula 2017, Graham 2017, Chandrasekhar 2016). The literature on latent position network models suggests that in some special case this is possible (Airoldi et al. 2008); however, such models consider links that are conditionally independent, while in our model, the links are conditionally dependent.

#### D. The Add Health Data

The *National Longitudinal Study of Adolescent Health* (Add Health) is a dataset containing information on a nationally representative sample of US schools. The survey started in 1994, when the 90,118 participants were entering grades 7–12, and the project collected data in four successive waves.<sup>17</sup> Each student responded to an *in-school* questionnaire, and a subsample of 20,745 students was given an *in-home* interview to collect more detailed information about behaviors, characteristics, and health status. In this paper, I use only data from the *saturated sample* of Wave I, containing information on 16 schools. Each student in this sample completed both the in-school and in-home questionnaires, and the researchers made a significant effort to avoid any missing information on the students.<sup>18</sup>

Most schools in the saturated sample are relatively small and enroll about 100 students, and the largest schools enroll 1,664, 811, and 159 students. The final sample includes 3,604 students in 16 different schools. I also provide estimates excluding the two largest schools, for which the sample is 1,129 students.

<sup>17</sup> More details about the sampling design and the representativeness are contained in Moody (2001) and the Add Health website <http://www.cpc.unc.edu/projects/addhealth/projects/addhealth>.

<sup>18</sup> While this sample contains no missing covariate information for the students, there are several missing values for the parental variables.

The *in-school* questionnaire collects the social network of each participant. A subsample of 20,745 students was also given an *in-home* questionnaire that collected most of the sensible data in addition to the network data. Each student was given a school roster and was asked to identify up to five male and five female friends.<sup>19</sup> I use the friendship nominations as proxy for the social network in a school. The resulting network is *directed*: Paul may nominate Jim, but this does not necessarily imply that Jim nominates Paul.<sup>20</sup> The model developed in this paper takes this feature of the data into account. In Figure 1, we report three school networks from the saturated sample where each dot is a student, its color represents the racial group, and an arrow is a friendship nomination.

In the empirical specification, I control for racial group, grade, and gender. A student with a missing value in any of these variables is dropped from the sample. Each student that declares themselves to be of Hispanic origin is considered Hispanic. The remaining non-Hispanic students are assigned to the racial group they declared. Therefore, the racial categories are: white, black, Asian, Hispanic, and other race. Other race contains Native Americans. I include controls for parental income using a question from the parent questionnaire.<sup>21</sup>

There may be some unobservable variables that affect network formation. For example, some students may be “cool” and receive more friendship links than others. Unobserved heterogeneity can be included in the model at the cost of significant additional computational burden in the forms of random effects for each node.<sup>22</sup> To partially control for such unobserved determinants of link formation, I use information from the interviewer remarks about the *physical attractiveness* and *personality* of the student interviewed. I define a dummy variable “beauty,” which is equal to 1 if the interviewer reported that the student was very attractive. Analogously, the dummy “personality” is equal to 1 if the interviewer reported that the personality of the student was very attractive. Finally, I include school dummies to control for different schools characteristics.

Descriptive statistics are in Tables 1 and 2.

There is a certain amount of variation in the number of links: some schools are more social and form many links per capita, while other schools have very few friendship nominations. The ratio of boys to girls is quite balanced in almost all schools, except school 369, where female students are a larger majority.

Panel A summarizes the racial composition. Many schools are almost racially homogeneous. Schools 1, 28, 77, 126, and 175 are more diverse as reflected in the Racial Fragmentation index (Alesina and La Ferrara 2000; Alesina, Baqir, and Easterly

<sup>19</sup>In the in-home interviews there are 20,745 students. Of these, about 5 percent nominated 5 male friends and 5 percent nominated 5 female friends. Only about 3 percent of the 20,745 students nominated both 5 male and 5 female friends (Moody 2001).

<sup>20</sup>Some authors do not consider this feature of the data and they recode the friendships as mutual: if one student nominates another, the opposite nomination is also assumed.

<sup>21</sup>There are several cases in which the family income is missing. For those observations, I imputed values drawn from the unconditional income distribution of the community. An alternative, albeit a computationally very costly one, is to introduce an additional step in the simulation in which the imputation of missing incomes is done at each iteration. Given the computational burden of my estimation exercise, I did not pursue this alternative here.

<sup>22</sup>See Mele (2017a) for a discussion. Badev (2013) also discusses the introduction of random effects. Mele (2017b) includes unobserved heterogeneity in the form of unobserved communities. The computational burden of the exchange algorithm for such a model is impractical for estimation of networks with more than 200 nodes.

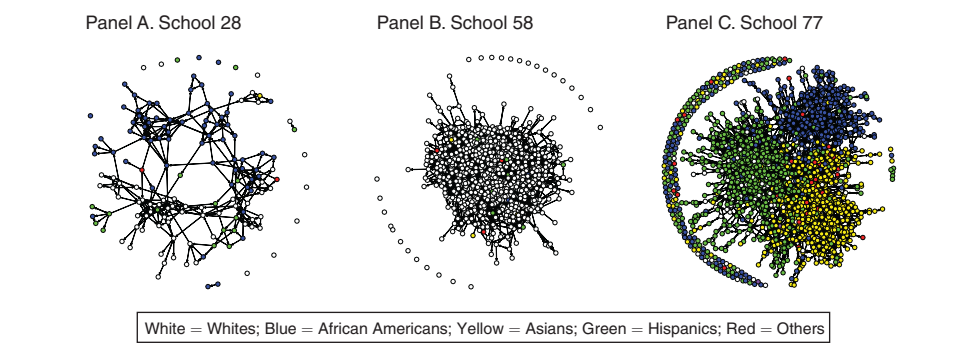


FIGURE 1. THREE SCHOOL FRIENDSHIP NETWORKS FROM ADD HEALTH, SATURATED SAMPLE

Notes: The graphs represent the friendship network of a school extracted from AddHealth. Each dot represents a student; each arrow is a friend nomination. The colors represent racial groups.

TABLE 1—DESCRIPTIVE STATISTICS FOR THE SCHOOLS IN THE SATURATED SAMPLE

School	1	2	3	7	8	28	58	77
Students	44	60	117	159	110	150	811	1,664
Links	12	120	125	344	239	355	3,290	3,604
Females	0.5	0.517	0.419	0.44	0.5	0.587	0.473	0.483
Clustering	0.000	0.421	0.154	0.222	0.282	0.291	0.197	0.193
Density	0.006	0.034	0.009	0.014	0.020	0.016	0.005	0.001
<i>Panel A. Racial composition</i>								
Whites	0.5	0.95	0.983	0.981	0.973	0.42	0.978	0.055
Blacks	0.136	0	0	0.006	0.018	0.453	0.002	0.233
Asians	0	0	0	0	0.009	0.007	0.005	0.299
Hispanics	0.364	0.05	0.017	0.006	0	0.107	0.011	0.392
Others	0	0	0	0	0	0.013	0.004	0.02
Racial fragmentation	0.599	0.095	0.034	0.037	0.053	0.606	0.044	0.699
<i>Panel B. Grade composition</i>								
7th grade	0.159	0.2	0.128	0.145	0.227	0.173	0.002	0.001
8th grade	0.159	0.217	0.154	0.157	0.2	0.173	0.004	0.003
9th grade	0.114	0.2	0.12	0.214	0.136	0.2	0.289	0.004
10th grade	0.273	0.133	0.205	0.157	0.182	0.167	0.277	0.346
11th grade	0.136	0.167	0.179	0.164	0.118	0.14	0.223	0.345
12th grade	0.159	0.083	0.214	0.164	0.136	0.147	0.205	0.301
<i>Panel C. Segregation</i>								
FSI gender	0.348	0.035	0.095	0.263	0.100	0.206	0.142	0.228
FSI race	0.000	0.689	0.180	0.553	0.000	0.671	0.014	0.690
FSI income 90	0.596	0.332	0.000	0.189	0.000	0.118	0.024	0.000
FSI income 50	0.023	0.000	0.027	0.133	0.000	0.013	0.077	0.082
SSI gender	0.305	0.541	0.493	0.697	0.586	0.659	0.798	0.727
SSI race	0.146	0.862	0.791	0.894	0.865	0.754	0.927	0.748
SSI income 90	0.409	0.767	0.641	0.783	0.735	0.803	0.820	0.767
SSI income 50	0.214	0.469	0.444	0.601	0.501	0.547	0.726	0.620

(continued)

1999), an index that measures the probability that two randomly chosen students in the school belong to different racial groups.<sup>23</sup> An index of 0 indicates that there is

<sup>23</sup> If there are  $K$  racial groups and the share of each race is  $s_k$ , the index is  $FRAG = 1 - \sum_{k=1}^K (s_k)^2$ .

TABLE 2—DESCRIPTIVE STATISTICS FOR THE SCHOOLS IN THE SATURATED SAMPLE (continued from Table 1)

School	81	88	106	115	126	175	194	369
Students	98	90	81	20	53	52	43	52
Links	163	308	162	44	123	171	42	48
Females	0.531	0.522	0.531	0.55	0.491	0.538	0.512	0.654
Clustering	0.362	0.202	0.393	0.392	0.284	0.064	0.056	
Density	0.017	0.038	0.024	0.116	0.045	0.064	0.023	0.018
<i>Panel A. Racial composition</i>								
Whites	0.98	0.989	0	1	0.472	0.769	0.977	0.942
Blacks	0	0	0.963	0	0.151	0.019	0	0
Asians	0.01	0	0	0	0.038	0.038	0	0
Hispanics	0.01	0	0.025	0	0.302	0.154	0.023	0.058
Others	0	0.011	0	0	0.038	0.019	0	0
Racial fragmentation	0.04	0.022	0.072	0	0.661	0.382	0.045	0.109
<i>Panel B. Grade composition</i>								
7th grade	0.112	0.144	0.506	0.4	0.491	0.462	0.488	0.538
8th grade	0.153	0.178	0.481	0.6	0.472	0.538	0.488	0.462
9th grade	0.153	0.122	0.012	0	0.038	0	0	0
10th grade	0.214	0.167	0	0	0	0	0	0
11th grade	0.265	0.211	0	0	0	0	0.023	0
12th grade	0.102	0.178	0	0	0	0	0	0
<i>Panel C. Segregation</i>								
FSI gender	0.196	0.107	0.186	0.123	0.095	0.050	0.186	0.000
FSI race	0.819	0.816	0.000		0.403	0.000	0.000	0.560
FSI income 90	0.000	0.077	0.000	0.000	0.000	0.272	0.000	0.384
FSI income 50	0.064	0.116	0.012	0.000	0.269	0.069	0.131	0.000
SSI gender	0.614	0.618	0.601	0.658	0.561	0.696	0.488	0.461
SSI race	0.805	0.921	0.761		0.550	0.632	0.786	0.817
SSI income 90	0.684	0.836	0.705	0.778	0.726	0.825	0.709	0.681
SSI income 50	0.439	0.563	0.460	0.419	0.535	0.611	0.484	0.421

Source: Author’s calculations based on Add Health Wave I data.

only one racial group and the population is perfectly homogeneous. Higher values of the index represent increasing levels of racial heterogeneity. Panel B summarizes the grade composition. Most schools offer all grades from seventh to twelfth, with homogeneous population across grades. Several schools only have lower grades.

Panel C analyzes the racial, gender, and income segregation of each school. Income segregation is measured in two ways: the segregation of the students in the ninetieth percentile from the rest and the segregation of students above median income from the rest. The measures of segregation are the Freeman Segregation Index (FSI) (Freeman 1978) and the Spectral Segregation Index (SSI) (Echenique and Fryer 2007). The FSI varies from 0 to 1 and measures the difference between the expected and actual number of links among individuals of different groups. An index of 0 means that the actual network closely resembles one in which links are formed at random. Higher values indicate more segregation. The index varies between 0 and 1, where the maximum corresponds to a network in which there are no cross-group links. The SSI computes segregation using spectral methods. The intuition is as follows. First-order segregation of a student is the share of a student’s social interactions with students in his/her own group; second order segregation is the average over the student’s own group of first order segregation;  $n$ th order segregation is the average of the students’s own group of  $(n - 1)$ th segregation. The student’s SSI is the limit for  $n$  going to infinity of this

sequence. We take the average student's SSI in a school as measure of segregation in the school. SSI also varies from 0 to 1 as segregation increases.

The schools are quite segregated based on race as long as there is some diversity in the school. Income segregation and gender segregation seem to be lower in these schools. We notice that the SSI measures high segregation in schools that are almost homogeneous by race because the largest connected component of the network contains only individuals of the same group. However, FSI measures very low segregation when the school is homogeneous. We use both measures to understand the effects of our counterfactual simulations in the next section.

### III. Empirical Results

#### A. Posterior Estimates

In Table 3, we report the estimated posterior means for six models. More detailed tables, including posterior mean, median, standard deviation, and 95 percent credible intervals, are in the online Appendix. All the estimated posterior means have small credible intervals; therefore, their sign and magnitude are estimated very precisely.<sup>24</sup>

The models in columns 1–4 are estimated with the smaller 14 schools in the saturated sample. This sample includes a total of 1,129 students and 112,751 pairs of potential links. The latter should be thought of as the number of observations within our model, since we are modeling the links' probabilities and decisions. Columns 5 and 6 are estimated with the full saturated sample, including 16 schools. This includes the two largest schools with 811 and 1,664 students, increasing our student sample to 3,604 and the number of dyads to 3,536,893. In models 1, 3, and 5, preferences are modeled as functions of the directed utility only, thereby omitting the externalities payoffs; in models 2, 4, and 6, we include all the payoffs and estimate the preferences for reciprocity and indirect links. All the models control for other school characteristics with a set of school dummies.

Each estimate measures the marginal effect of the variable on the payoff: for example, the parameter associated with the direct utility of *WHITE-WHITE* measures the marginal utility of a white student when forming a link to another white student, other things being equal.

Model (1) does not include any externality, thus making it a standard (Bayesian) logistic regression estimate. Model (2) has the same specification as (1), but includes mutual and indirect utility in the specification. The signs of the coefficients for the direct utility seem to agree between the two specifications, except in three cases; once we control for mutual and indirect utility, homophily by gender (*SAME GENDER*), physical attractiveness of the alter (*ATTRACTIVE j (Phys)*), and the effect of white share in the school (*SHARES WHITE*) change sign. We notice that the signs in model 2 are consistent with the remaining specifications (3)–(6).

The coefficient for racial homophily is positive in both specifications (*WHITE-WHITE*, *BLACK-BLACK*, *HISP-HISP*). In model (2), the payoff from

<sup>24</sup> All the replication codes are available at <https://github.com/meleangelo/segnet>.

TABLE 3—POSTERIOR MEAN OF ESTIMATED MODELS

	(1)	(2)	(3)	(4)	(5)	(6)
<i>Panel A. Direct utility (<math>u_{ij}</math>)</i>						
CONSTANT	−6.9201	−5.5381	−6.6500	−5.9132	−7.2182	−5.8070
MALE $i$			−0.1517	0.0463	−0.2718	0.2350
WHITE $i$			−0.1710	0.0044 <sup>a</sup>	0.0440 <sup>a</sup>	0.3023
BLACK $i$			1.0451	1.1310	0.7074	1.1801
HISP $i$			2.0990	2.2806	1.4590	2.0295
INCOME $i$ (logs)			−2.0543	−1.6492	−1.8738	−1.4645
SAME GENDER	−0.4545	0.1850	0.2067	0.4851	0.3154	0.7644
SAME GRADE	2.3124	2.2384	2.3817	2.0113	2.5185	2.1800
WHITE-WHITE	0.3504	0.5414	1.0138	0.5720	0.9959	0.2739
BLACK-BLACK	0.1443	0.3660	1.6491	1.1445	1.5347	0.9405
HISP-HISP	1.8597	1.6794	0.3186	−0.2269	0.7130	−0.1394
ATTRACTIVE $i$ (Phys)	0.2757	0.3068	−2.3568	−2.2413	−1.9291	−1.9430
ATTRACTIVE $j$ (Phys)	−0.0410	0.2322	2.5166	1.5861	2.7615	1.2609
ATTRACTIVE $i$ (Pers)	−0.4402	0.0063 <sup>a</sup>	−0.4964	−0.1570	−0.8646	−0.1631
ATTRACTIVE $j$ (Pers)	1.0672	0.8678	−1.0932	−0.7390	−0.6361	−0.3939
INCOME $i$ − INCOME $j$ (logs)	0.1793	0.1462	0.8883	0.9012	0.9938	0.7403
INCOME $i$ + INCOME $j$ (logs)	−0.0882	−0.0806	1.0947	0.9244	0.8977	0.6892
SHARE WHITES	0.9070	−0.4814	−1.7088	−1.4420	−1.5748	−1.6126
SHARE BLACKS	3.2238	3.0985	1.3416	1.8309	0.7645	1.9618
SHARE HISP	2.524	2.444	0.8397	0.7798	1.0078	0.7731
WHITE-WHITE × SHARE WHITES	1.3962	1.0094	4.3915	2.7840	4.7269	2.3272
BLACK-BLACK × SHARE BLACKS	0.4664	0.1478	0.2528	0.4028	0.1172	0.2516
HISP-HISP × SHARE HISP	−1.5643	−1.4255	−1.6908	−1.3630	−1.3872	−1.1400
<i>Panel B. Mutual utility (<math>m_{ij}</math>)</i>						
CONSTANT		1.1853		6.1668		5.3139
SAME GENDER		1.1652		1.0716		1.1539
SAME GRADE		−1.6882		−3.0514		−3.0575
WHITE-WHITE		0.0073 <sup>a</sup>		−0.6017		−0.4960
BLACK-BLACK		0.7468		1.1177		0.7067
HISP-HISP		0.7779		−1.4659		−1.4639
<i>Panel C. Indirect utility and popularity (<math>v_{ij}</math>)</i>						
CONSTANT		−0.2891		−0.4705		−0.4308
SAME GENDER		0.1721		−0.4074		−0.3987
SAME GRADE		−0.3145		0.1136		0.3266
WHITE-WHITE		0.2239		0.1856		0.2978
BLACK-BLACK		−0.1364		0.1372		0.1202
HISP-HISP		0.4328		−0.5067		−0.2859
SCHOOL DUMMIES	YES	YES	YES	YES	YES	YES
<i>Panel D. Sample size</i>						
Number of schools	14	14	14	14	16	16
Number of students	1,129	1,129	1,129	1,129	3,604	3,604
Number of pairs/dyads	112,751	112,751	112,751	112,751	3,536,893	3,536,893

Notes: Models 1–4: posterior sample of 100,000 parameter and 5,000 network simulations per parameter. Models 5–6: posterior sample of 20,000 parameter and 10,000 network simulations per parameter.  
<sup>a</sup>Credible 95 percent interval contains both positive and negative values.

mutual links displays racial homophily as well, while for indirect links there is mixed evidence, as there is a negative coefficient for *BLACK-BLACK* payoff. Models 3–6 are estimated according to our favorite specification. In these models, we control for some demographics of the student nominating friends: gender, race, and income of the parents (in logs).



A comparison of models 3 and 5, as well as 4 and 6, shows that the sample with the 14 small schools and the larger sample including all 16 schools of the saturated sample generate qualitatively similar estimates, while the magnitudes of the parameters are different. Because model 6 is estimated with more than 3.5 million observations (the number of possible student pairs) we are more confident about these estimates and discuss these in detail.

In model 6, the estimated preferences show racial homophily for direct and indirect links except for Hispanic students. For mutual links, only blacks seem to show homophily. Hispanics have the highest propensity to form new links, but, interestingly, prefer links outside their own racial group, especially when the share of Hispanic students in the school is relatively large. The mutual links constant is positive, showing that reciprocity provides positive baseline utility, but the constant for the indirect friends payoff is negative, suggesting congestion or competition for indirect friends. High income reduces the willingness to form links, while males seems to be more active than females in friendship formation.

An increase in the share of blacks and Hispanic students will increase willingness to link, while the opposite happens with the share of Caucasian students. White and black students' homophily is higher in schools with a higher share of whites and blacks. This is important, as different groups will possibly have different responses to desegregation efforts: some groups may engage more in interracial friendships, while some group may segregate even more.

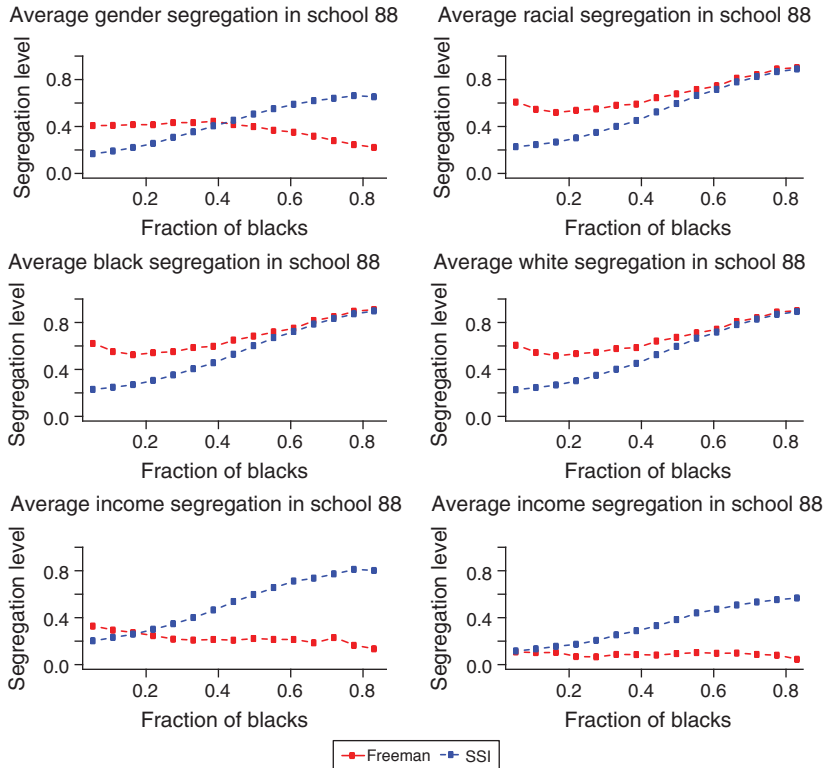
Physically attractive students do not initiate friendships often, but receive friendship nominations with higher frequency. Students with attractive personalities form and receive fewer nominations. Income differences increase the likelihood of friendship, suggesting that students tend to mix based on income levels; higher income levels of the pair involved in the link increase the number of friendships formed. The total number of friends is higher in schools with a higher fraction of minorities.

### B. Policy Experiments

In this section, the estimated model is used to estimate the effect of counterfactual policies on several network characteristics. I simulate 1,000 equilibrium networks for each policy change, using the posterior mean estimated in Table 3, column 6. For each network, I compute measures of segregation, clustering, and centrality.<sup>25</sup>

*Reallocation of Students Based on Race.*—Schools 88 and 106 are very racially homogeneous, being almost completely white and black, respectively. In these simulations, a total of  $x$  white students is moved from school 88 to school 106; conversely, the same number  $x$  of black students is moved from school 106 to school 88. This policy changes the racial heterogeneity levels within schools by directly targeting students based on their race. Our model predicts that, in equilibrium, the students will best respond to the new environment by modifying their linking strategy.

<sup>25</sup> The complete simulation codes are available in Github.



(continued)

FIGURE 2. AVERAGE SEGREGATION IN SCHOOL 88 AND 106

In Figure 2, we show the effect of this policy on the average segregation in the schools. We have computed segregation based on race, gender, and income. In all graphs, the red line represents the Freeman Segregation Index (FSI) of Freeman (1978), while the blue is the Spectral Segregation Index (SSI) of Echenique and Fryer (2007).

Let's first consider the FSI (in red in Figure 2). In school 88, as the policy increases diversity by increasing the fraction of blacks, racial segregation first decreases and then starts increasing again. The intuition for this outcome is that when the fraction of minority students in the school is small enough, they need to find friends in another racial group. However, when the size of the minority in the school is large enough, the homophily effect prevails and minorities will tend to self-segregate.

The results on gender segregation suggest that the segregation would increase slightly as the fraction of black students increases and then decreases.

These results, based on the Freeman Segregation Index, suggest that a policy promoting perfect racial integration among schools would not necessarily increase interactions among students of different racial backgrounds.

On the other hand, the results using the Spectral Segregation Index (SSI, in blue in Figure 2), seem to point to a slightly different conclusion.

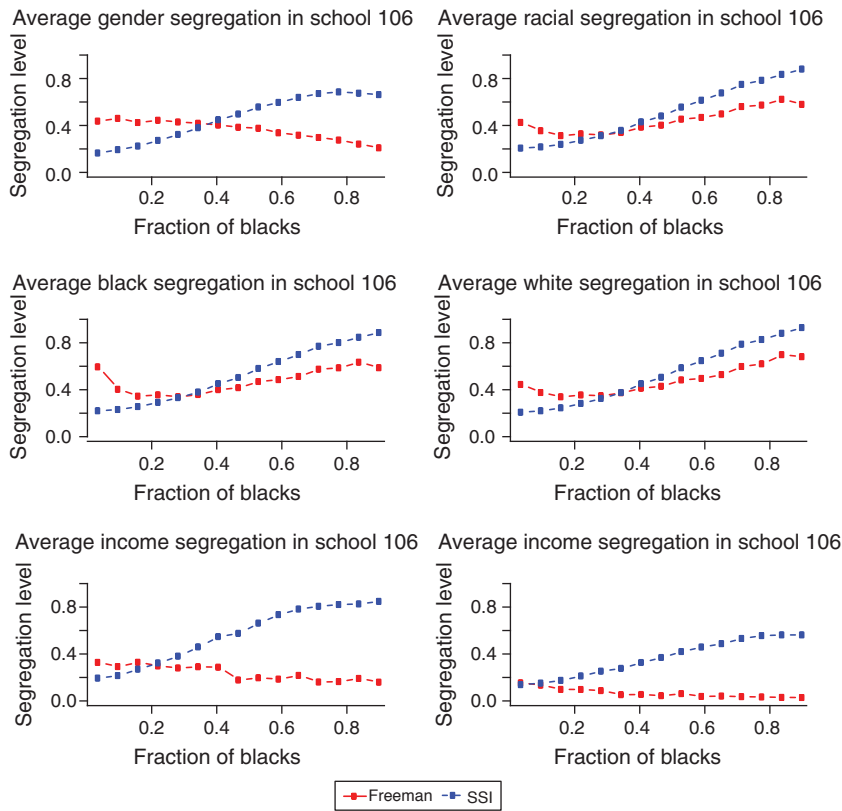


FIGURE 2. AVERAGE SEGREGATION IN SCHOOL 88 AND 106 (continued)

Note: Each policy simulation simulates the network using 1,000 draws from the posterior distribution estimated in model 6 of Table 3.

The result for the SSI should be thought of as what happens to the average student in the school. This index says that, at least in terms of the SSI, the average student will be exposed to higher gender, race, and income segregation as the fraction of blacks in the school increases. According to this measure of segregation, a perfectly integrated school would be a good compromise.

This policy has an effect on several structural characteristics of the network. Figure 3 (left panels) shows the density and transitivity of the network as the diversity of the schools changes. An increase in the fraction of blacks will increase both density and clustering in both schools.

Figure 4 shows race and gender of the most central students as measured by three indicators: (i) indegree centrality measures popularity, (ii) outdegree centrality measures social activity, and (iii) eigenvector centrality measures the most crucial node. These indicators are usually correlated with performance (Calvó-Armengol, Patacchini, and Zenou 2009). The graphs on the top compare schools based on indegree centrality, in the middle, we have outdegree centrality, and at the bottom, the eigenvector centrality.

In many cases, the most popular person in both schools is a girl.

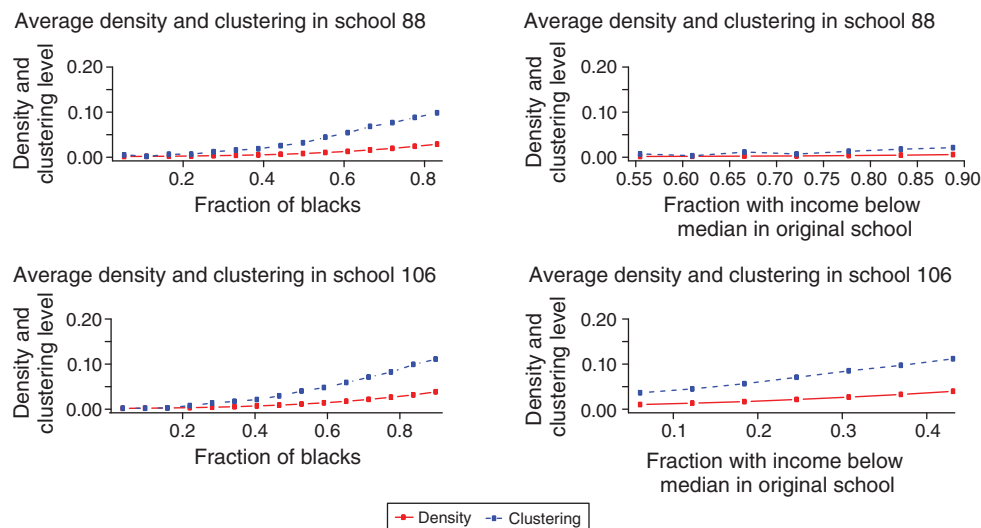


FIGURE 3. DENSITY AND CLUSTERING

Note: Each policy simulation simulates the network using 1,000 draws from the posterior distribution estimated in model 6 of Table 3.

The probability that the most central student is black increases with the fraction of blacks in the school. This pattern seems to be similar in both schools and across the centrality indicators.

According to our most intuitive measure of segregation, the Freeman Segregation Index (FSI), the conclusion of this experiment is that trying to make schools equally diverse by equalizing their racial share may not decrease segregation necessarily. Indeed, it seems better to have a relatively small minority that will not be able to self-segregate. While the patterns of segregation as we increase the fraction of blacks in the school look the same, there are differences in the magnitude of segregation across schools due to the difference in which whites and blacks respond to changes in the fraction of their own groups in the school. Income segregation slightly decreases with the reallocation of blacks to school 88 while it increases for school 106. According to the Spectral Segregation Index (SSI), an increase in the fraction of blacks will always increase segregation. This means that a policy that makes schools equally diverse will be a good compromise in terms of racial and income segregation.

The policy will increase density and clustering in school 88, while decreasing both in school 106. If schools are equally racially diverse, black students will be, on average, more popular and central. This is also true if whites are a small minority.

*Reallocation of Students Based on Income.*—The second policy experiment consists of swapping students based on their parental income. For both schools 88 and 106, we compute the median income of the parents, and we swap students of school 88 with incomes above median with students of school 106 with incomes below median.

This procedure increases segregation by income across the schools, making them more homogeneous by income. School 88 becomes populated by poorer students,

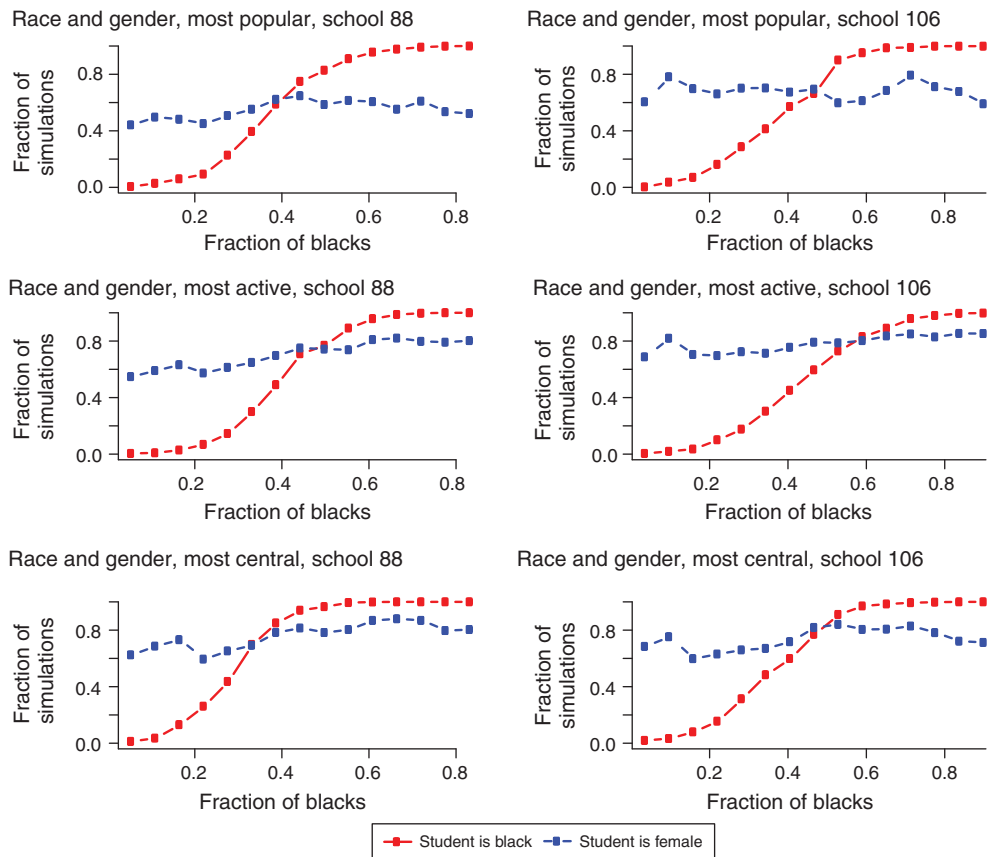


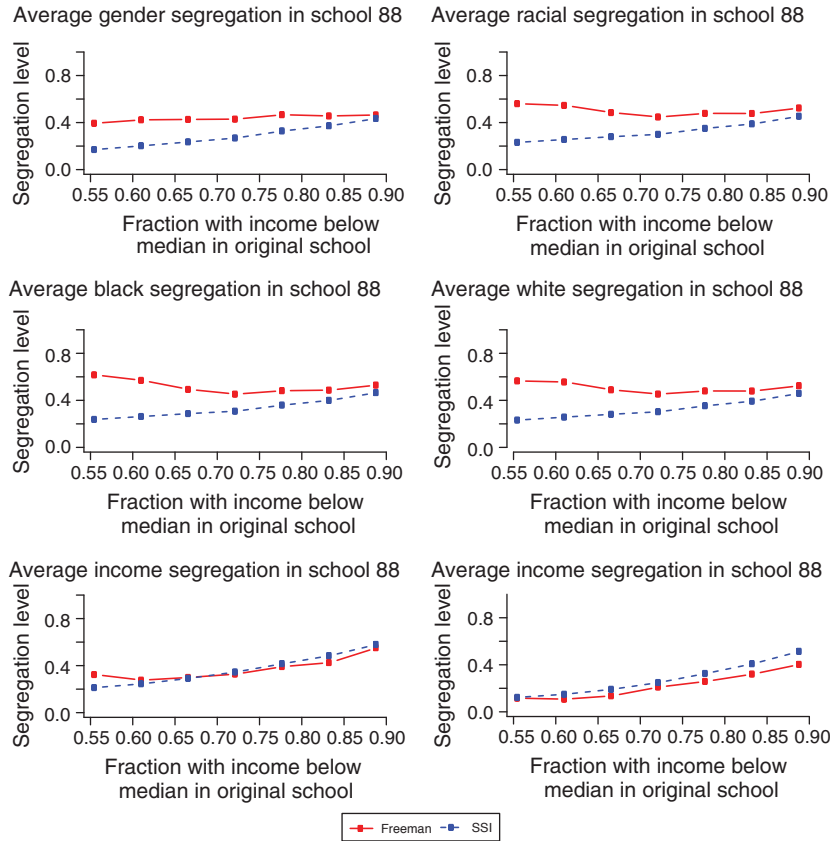
FIGURE 4. GENDER AND RACE OF THE MOST CENTRAL

Note: Each policy simulation simulates the network using 1,000 draws from the posterior distribution estimated in model 6 of Table 3.

while school 106 becomes richer. On the other hand, this policy increases racial diversity within schools, because we are swapping white/Caucasian students from school 88 with black students from school 106, thus making schools more heterogeneous by race.

In Figure 5, we show the effect of this policy on the average segregation in schools 88 and 106. To interpret the results, notice that for school 88, moving left to right on the x-axis makes the school *more homogeneous by income (and poorer) and more diverse by race*. Conversely, for school 106, moving left to right on the x-axis makes the school *more heterogeneous by income and less diverse by race*.

According to the FSI, for school 88 there is a moderate effect on racial segregation, which follows the similar U-shaped pattern we have seen in the previous race-based policy. Income segregation increases with income homogeneity and racial heterogeneity. This reflects the fact that preferences estimated in model 6 of Table 3 show heterophily in income levels. On the other hand, in school 106, there is no effect on income segregation, and gender segregation decreases with income homogeneity and racial diversity.



(continued)

FIGURE 5. AVERAGE SEGREGATION IN SCHOOL 88 AND 106

The Spectral Segregation Index mirrors the patterns of the policy with reallocation based on race.

In Figure 3 (right panels), we show the density and transitivity of the network. While school 88 does not show any substantial change, school 106’s density and clustering increase with income homogeneity and racial diversity. In this school, there is no change in which racial group is most central as a consequence of the policy (see Figure 6, right panel). On the other hand, for school 88, the increase in income homogeneity also increases the popularity of black students.

In summary, this policy increases segregation by income across the schools, making income distribution more homogeneous within schools and simultaneously increasing racial diversity. This has asymmetric effects on the two schools’ racial segregation as measured by the FSI.

IV. Conclusions

This paper analyzed racial segregation in schools, guided by an equilibrium model of network formation. The model generates segregation as an equilibrium outcome and allows me to estimate the preferences of students, which include direct



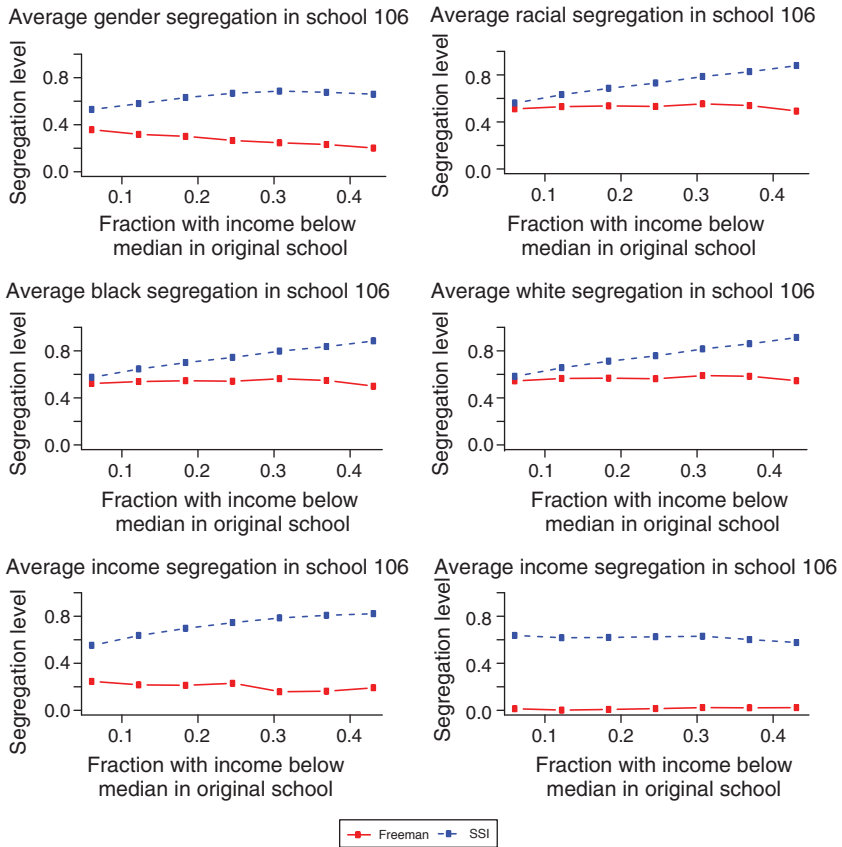


FIGURE 5. AVERAGE SEGREGATION IN SCHOOL 88 AND 106 (continued)

Note: Each policy simulation simulates the network using 1,000 draws from the posterior distribution estimated in model 6 of Table 3.

effects of observables and indirect payoffs for reciprocated relationships and indirect friends. I find homophily by race, both in direct and indirect links. My specification allows homophily to vary within the fraction of the racial groups in the school: an increase in the fraction of white students decreases the propensity of white students to form links within the same racial group; the opposite holds true for Hispanics or blacks. These differences are important in understanding the effect of policies that modify the relative shares of groups in the school. There is evidence of homophily beyond direct links, thus supporting a model that includes indirect effects of linking in the preferences.

To illustrate the implications of such estimated preferences, I simulate two “desegregation” policies using two racially homogeneous schools from my sample in which student populations are almost completely white and black, respectively. The model simulations provide predictions about the expected levels of segregation and network features implied by swapping students among schools.

The first policy simulation swaps black students from one school with white students from the other school, thus modifying the racial balance within the schools.

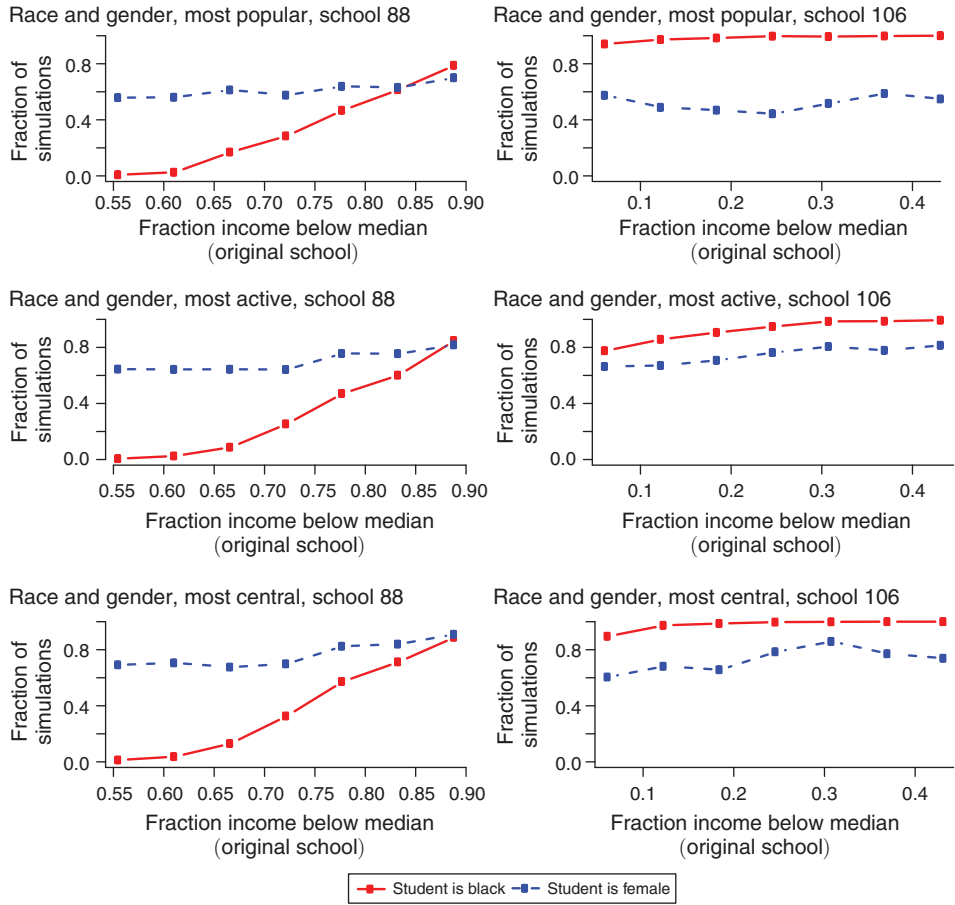


FIGURE 6. GENDER AND RACE OF THE MOST CENTRAL

Note: Each policy simulation simulates the network using 1,000 draws from the posterior distribution estimated in model 6 of Table 3.

Interestingly, the outcomes of these policies vary by the way we measure segregation. According to the Freeman Segregation Index, integrating the two schools may potentially increase segregation. However, according to the Spectral Segregation Index, perfect integration across the schools leads to the best compromise in terms of racial segregation.

The second simulation decreases income heterogeneity within schools by swapping kids with parental income above median in a school with students whose parental income is below median in the other school. This simultaneously increases racial diversity within schools. According to the FSI, there is not much effect on the racial segregation of either school. However, income segregation increases in the school that becomes poorer. In the latter school, the popularity of black students increases with income homogeneity (and racial diversity).

These results suggest that desegregation policies do not necessarily lead to more interactions among students of different ethnic or socioeconomic backgrounds.

## REFERENCES

- Acemoglu, Daron, Munther A. Dahleh, Ilan Lobel, and Asuman Ozdaglar.** 2011. "Bayesian Learning in Social Networks." *Review of Economic Studies* 78 (4): 1201–36.
- Airolidi, Edoardo M., David M. Blei, Stephen E. Fienberg, and Eric P. Xing.** 2008. "Mixed Membership Stochastic Blockmodels." *Journal of Machine Learning* 9: 1981–2014.
- Alesina, Alberto, Reza Baqir, and William Easterly.** 1999. "Public Goods and Ethnic Divisions." *Quarterly Journal of Economics* 114 (4): 1243–84.
- Alesina, Alberto, and Eliana La Ferrara.** 2000. "Participation in Heterogeneous Communities." *Quarterly Journal of Economics* 115 (3): 847–904.
- Amemiya, Takeshi.** 1981. "Qualitative Response Models: A Survey." *Journal of Economic Literature* 19 (4): 1483–1536.
- Ananat, Elizabeth Oltmans.** 2011. "The Wrong Side(s) of the Tracks: The Causal Effects of Racial Segregation on Urban Poverty and Inequality." *American Economic Journal: Applied Economics* 3 (2): 34–66.
- Angrist, Joshua D., and Kevin Lang.** 2004. "Does School Integration Generate Peer Effects? Evidence from Boston's Metco Program." *American Economic Review* 94 (5): 1613–34.
- Badev, Anton.** 2013. "Discrete Games in Endogenous Networks: Theory and Policy." PSC Working Paper Series 2-1-2013.
- Bandiera, Oriana, and Imran Rasul.** 2006. "Social Networks and Technology Adoption in Northern Mozambique." *Economic Journal* 116 (514): 869–902.
- Besag, Julian.** 1974. "Spatial Interaction and the Statistical Analysis of Lattice Systems." *Journal of the Royal Statistical Society Series B (Methodological)* 36 (2): 192–236.
- Blume, Lawrence E.** 1993. "The Statistical Mechanics of Strategic Interaction." *Games and Economic Behavior* 5 (3): 387–424.
- Boucher, Vincent.** 2015. "Structural Homophily." *International Economic Review* 56 (1): 235–64.
- Boucher, Vincent, and Ismael Mourifié.** 2017. "My Friend Far Far Away: A Random Field Approach to Exponential Random Graph Models." *Econometrics Journal* 20 (3): S14–46.
- Caimo, Alberto, and Nial Friel.** 2011. "Bayesian Inference for Exponential Random Graph Models." *Social Networks* 33 (1): 41–55.
- Calvo-Armengol, Antoni, Eleonora Patacchini, and Yves Zenou.** 2009. "Peer Effects and Social Networks in Education." *Review of Economic Studies* 76 (4): 1239–67.
- Carrell, Scott E., Bruce I. Sacerdote, and James E. West.** 2013. "From Natural Variation to Optimal Policy? The Importance of Endogenous Peer Group Formation." *Econometrica* 81 (3): 855–82.
- Chandrasekhar, Arun.** 2016. "Oxford Handbook on the Economics of Networks." In *Econometrics of Network Formation*, edited by Yann Bramoullé, Andrew Galeotti, and Brian Rogers. Oxford: Oxford University Press.
- Chib, Siddhartha, and Srikanth Ramamurthy.** 2010. "Tailored Random Block MCMC Methods with Application to DSGE Models." *Journal of Econometrics* 155 (1): 19–38.
- Clofelter, Charles T.** 2004. *After 'Brown': The Rise and Retreat of School Desegregation*. Princeton: Princeton University Press.
- Conley, Timothy G., and Christopher R. Udry.** 2010. "Learning about a New Technology: Pineapple in Ghana." *American Economic Review* 100 (1): 35–69.
- Cooley, Jane.** 2010. "Desegregation and the Achievement Gap: Do Diverse Peers Help?" <https://www.ssc.wisc.edu/~jcooley/CooleyDeseg.pdf>.
- Curranini, Sergio, Matthew O. Jackson, and Paolo Pin.** 2009. "An Economic Model of Friendship: Homophily, Minorities, and Segregation." *Econometrica* 77 (4): 1003–45.
- Cutler, David M., and Edward L. Glaeser.** 1997. "Are Ghettos Good or Bad?" *Quarterly Journal of Economics* 112 (3): 827–72.
- De Giorgi, Giacomo, Michele Pellizzari, and Silvia Redaelli.** 2010. "Identification of Social Interactions through Partially Overlapping Peer Groups." *American Economic Journal: Applied Economics* 2 (2): 241–75.
- De Martí, Joan, and Yves Zenou.** 2009. "Ethnic Identity and Social Distance in Friendship Formation." CEPR Discussion Paper 7566.
- DePaula, Áureo.** 2017. "Econometrics of Network Models." In B. Honore, A. Pakes, M. Piazzesi, and L. Samuelson, (eds), *Advances in Economics and Econometrics: Eleventh World Congress (Econometric Society Monographs)*, 268–323. Cambridge: Cambridge University Press.
- DePaula, Áureo, Seth Richards-Shubik, and Elie Tamer.** 2018. "Identifying Preferences in Networks with Bounded Degree." *Econometrica* 86 (1): 263–88.

- Dzanski, Andreas.** 2017. "An Empirical Model of Dyadic Link Formation in a Network with Unobserved Heterogeneity." University of Gothenburg Working Paper in Economics 698.
- Echenique, Federico, and Roland G. Fryer, Jr.** 2007. "A Measure of Segregation Based on Social Interactions." *Quarterly Journal of Economics* 122 (2): 441–85.
- Echenique, Federico, Roland G. Fryer, Jr., and Alex Kaufman.** 2006. "Is School Segregation Good or Bad?" *American Economic Review* 96 (2): 265–69.
- Frank, Ove, and David Strauss.** 1986. "Markov Graphs." *Journal of the American Statistical Association* 81 (395): 832–42.
- Freeman, Linton C.** 1978. "Segregation in Social Networks." *Sociological Methods and Research* 6 (4): 411–29.
- Gelman, A., G.O. Roberts, and W.R. Gilks.** 1996. "Efficient Metropolis Jumping Rules." *Bayesian Statistics* 5: 599–607.
- Geyer, Charles J., and Elizabeth A. Thompson.** 1992. "Constrained Monte Carlo Maximum Likelihood for Dependent Data." *Journal of the Royal Statistical Society, Series B (Methodological)* 54 (3): 657–99.
- Golub, Benjamin, and Matthew Jackson.** 2011. "Network Structure and the Speed of Learning: Measuring Homophily Based on its Consequences." *Annals of Economics and Statistics*.
- Graham, Bryan S.** 2017. "An Empirical Model of Network Formation with Degree Heterogeneity." *Econometrica* 85 (4): 1033–63.
- Heckman, James J.** 1978. "Dummy Endogenous Variables in a Simultaneous Equation System." *Econometrica* 46 (4): 931–59.
- Hsieh, Chih-Sheng, and Lung-Fei Lee.** 2016. "A Social Interactions Model with Endogenous Friendship Formation and Selecting." *Journal of Applied Economics* 31: 301–19.
- Jackson, Matthew O.** 2008. *Social and Economics Networks*. Princeton: Princeton University Press. <https://web.stanford.edu/~jacksonm/netbook.pdf>.
- Jackson, Matthew O., and Alison Watts.** 2001. "The Existence of Pairwise Stable Networks." *Seoul Journal of Economics* 14 (3): 299–321.
- Jackson, Matthew O., and Asher Wolinsky.** 1996. "A Strategic Model of Social and Economic Networks." *Journal of Economic Theory* 71 (1): 44–74.
- Koskinen, Johan H.** 2008. *The Linked Importance Sampler Auxiliary Variable Metropolis Hastings Algorithm for Distributions with Intractable Normalising Constants*. University of Melbourne: MelNet Social Networks Laboratory Technical Report 08-01.
- La Ferrara, Eliana, and Angelo Mele.** 2007. "Racial Segregation and Public School Expenditure." <https://pdfs.semanticscholar.org/9bfe/3d497cbe0d7249dfe1f749fa23cbb21e182a.pdf>.
- Laschever, Ron.** 2005. "The Doughboys Network: Social Interactions and Labor Market Outcomes of World War I Veterans." <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.407.9930&rep=rep1&type=pdf>.
- Lehman, E.L.** 1983. *Theory of Point Estimation*. Wiley and Sons.
- Leung, Michael P.** 2015a. "A Random-Field Approach to Inference in Large Models of Network Formation." <https://mpleung.github.io/files/rfnet.pdf>.
- Leung, Michael P.** 2015b. "Two-Step Estimation of Network-Formation Models with Incomplete Information." *Journal of Econometrics* 188 (1): 182–95.
- Liang, Faming.** 2010. "A Double Metropolis-Hastings Sampler for Spatial Models with Intractable Normalizing Constants." *Journal of Statistical Computing and Simulation* 80 (9): 1007–22.
- Lutz, Byron.** 2011. "The End of Court-Ordered Desegregation." *American Economic Journal: Economic Policy* 3 (2): 130–68.
- Mele, Angelo.** 2017a. "A Structural Model of Dense Network Formation." *Econometrica* 85 (3): 825–50.
- Mele, Angelo.** 2017b. "A Structural Model of Homophily and Clustering in Social Networks." [http://www.meleangelo.com/homophily\\_sept2017.pdf](http://www.meleangelo.com/homophily_sept2017.pdf).
- Menzel, Konrad.** 2016. "Strategic Network Formation with Many Agents." <http://sticerd.lse.ac.uk/seminarpapers/em17032016.pdf>.
- Monderer, Dov, and Lloyd S. Shapley.** 1996. "Potential Games." *Games and Economic Behavior* 14 (1): 124–43.
- Moody, James.** 2001. "Race, School Integration, and Friendship Segregation in America." *American Journal of Sociology* 107 (3): 679–716.
- Murray, Iain, Zoubin Ghahramani, and David J.C. MacKay.** 2006. "MCMC for Doubly-Intractable Distributions." <https://dslpitt.org/uai/papers/06/p359-murray.pdf>.
- Nakajima, Ryo.** 2007. "Measuring Peer Effects on Youth Smoking Behavior." *Review of Economic Studies* 74 (3): 897–935.

- Reardon, Sean F.** 2016. "School Segregation and Racial Achievement Gaps." *Russell Sage Foundation Journal of the Social Sciences* 2 (5): 34–57.
- Robert, Christian P., and George Casella.** 2004. *Monte Carlo Statistical Methods*. Secaucus, NJ: Springer.
- Sheng, Shuyang.** 2012. "Identification and Estimation of Network Formation Games." <http://www.its.caltech.edu/~mshum/reading/Sheng2012.pdf>.
- Snijders, Tom A.B.** 2002. "Markov Chain Monte Carlo Estimation of Exponential Random Graph Models." *Journal of Social Structure* 3 (2).
- Tamer, Elie.** 2003. "Incomplete Simultaneous Discrete Response Model with Multiple Equilibria." *Review of Economic Studies* 70 (1): 147–65.
- Topa, Giorgio.** 2001. "Social Interactions, Local Spillovers and Unemployment." *Review of Economic Studies* 68 (2): 261–95.
- Wainwright, Martin J., and Michael I. Jordan.** 2008. "Graphical Models, Exponential Families, and Variational Inference." *Foundations and Trends in Machine Learning* 1 (1–2): 1–305.
- Wasserman, Stanley, and Katherine Faust.** 1994. *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.