Differential Equations Student Notebook

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Chapter 1

Preface

Welcome to my *Differential Equations Student Notebook*. I created this "borderline textbook" notebook as a way for me and others to study and/or learn Differential Equations from. You may be asking: "What qualifications do you have to be making this *Differential Equations Notebook*?" and my response to that is that I am not. **I AM NOT QUALIFIED**. I am just a student who thought that making a textbook would assist me in learning about DEs. However, I have taken Calculus I, II, III, Discrete Mathematics, Linear Algerbra, and any other foundation-building mathematical class that University of Houston - Clear Lake provides (if that makes you trust me more).

It would be greatly appreciated if you simply took this as a study resource for any current, soon-to-be-current, already-took-this-and-just-forgot student. Therefore, without further ado, lets begin our dive into DIFFERENTIAL EQUATIONS.

Chapter 2

The Very Beginning

2.1 Requirements

As it is in every area of mathematics, there is a certain level of knowledge that is to be expected to be known by students. Requirements for *Differential Equations* is prior experience in Calculus I and II.

Of course, it is **expected that you are a BORDERLINE SAVANT in Calculus**. If you are not absolutely comfortable with Calculus, I reccommend that you REALLY go over it.

2.2 Types of Differential Equations

There are two types of Differential Equations: Ordinary Differential Equations (ODEs) and Partial Differential Equations (PDEs). The two differences between a **ODE** and a **PDE** is that an ODE is (as the name suggests) an equation that contains ordinary differentials. An example of an ODE is as follows:

$$\frac{dy}{dx} = 4y - 2x \tag{2.1}$$

An example of a PDE is as follows:

$$\frac{\partial y}{\partial x} = 17x^2 - 3y * \sin y \tag{2.2}$$

Did you spot the difference? The differentiating factor between example 2.1 and 2.2 is $\frac{dy}{dx}$ and $\frac{\partial y}{\partial x}$. *One contains partials while the other does not.*

Definition 2.2.1: Differential Equation

An equation that contains the derivatives of one or more unknown functions or variables ($f(x), y$, etc.), with respect to one or more independent variables is a **DIFFERENTIAL EQUATION**.

2.3 Classification By Order

The ordering of a differential equation is set by these three tenets:

- Type
- Order
- Linearity

2.3.1 Type

The type of DQ is nothing more that it being an ODE or a PDE (as was explained above). Of course there are differences in notation, such as the **Leibniz notation**, **prime notation**, and **dot notation**, with their respective notations being written as such: $\frac{dy}{dx}$, y', and $\dot{y}$

2.3.2 Order

The **order of a differential equation** is simply the the order of the highest derivative in an equation. An example of a 2nd order differential equation is:

$$\frac{d^2y}{dx^2} + 21(\frac{dy}{dx}) + 2y = 0 \tag{2.3}$$

In equation (2.3) we can tell that it is a 2nd order equation due to the fact that the number 2 appears in $\frac{d^2y}{dx^2}$. As such, it can be said that this equation is a 2nd order equation. Additionally, it can be said that equation (2.3) can be written in the **differential form** (possibly).

Definition 2.3.1: Differential Form

The **differential form** of an equation is an equation that is written as $M(x, y)dx + N(x, y)dy = 0$. In other words, differential form is when the fraction is turned into not being a fraction.

In addition to there being a differential form of a differential equation, there also exists the **normal form** of a differential equation.

Definition 2.3.2: Normal Form

The **normal form** of a differential equation is an equation that contains the highest-order differential on one side of the equation, such as

$$\frac{d^2y}{dx^2} = f(x, y, y') \tag{2.4}$$

Of course, it is expected that you can aleady manipulate any given equation to normal or differential form. If you are not able to do so, I am sorry.

2.3.3 Linearity

Any arbitrary ordinary differential equation is **linear** if F is linear in $y, y', y'', ..., y^{(n)}$. Therefore, any nth-order ODE is linear when it can be represented as:

$$a_n(x)\frac{d^ny}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x) \tag{2.5}$$

Two important properties of ODEs are:

- The dependent variable and all of its derivatives are linear.
- The coefficients of the dependent variables $y, y', y'', ..., y^{(n)}$ contain at most an independent variable.

An example of a linear ODE is:

$$4x^2\frac{d^2y}{dx^2} + 21x\frac{dy}{dx} - 7y = 0 \tag{2.6}$$

An example of a nonlinear ODE is:

$$3yx\frac{d^2y}{dx^2} - 15x\frac{dy}{dx} + 12xy = 0 \tag{2.7}$$

THE REASON: Equation (**??**) has no dependent variables in its coefficients, while (**??**) does.