Memorandum

To: Roland Nord

From: Naheem Olaniyan (Eng 4/575)

Date: April 3, 2019

Re: Equations (Equations\_ex\_no.docx)

The following headings and text have been taken from Peter Alfeld’s webpage “[Famous Equations and Inequalities](http://www.math.utah.edu/~alfeld/math/equations/equations.html).”[[1]](#footnote-1) Copyedit the document using comments and track changes.

# Mathematics

## Mathematical Constants

This beautiful equation connects three major constants of mathematics, Euler's Number e, the ratio of the circumference of a circle to its diameter, pi, and the square root of -1, i.e., i.

## The definition of Pi

Pi is defined to be the ratio of the circumference c of any circle divided by its diameter, d. Most people when asked will tell you that pi=3.14..., but that's just an accident. You can see [pi to 10,000 digits right here](http://www.math.utah.edu/~alfeld/math/pi.html) if you like, but that expression completely obscures the definition.

## The definition of *e*

One way of defining Euler's number e is by this formula. It can be interpreted as saying that if you collect 100 percent interest annually and compound continuously then you multiply your capital with e every year.

## A differential equation

A more mathematical definition of e is obtained by asking which function f equals its own derivative. The answer to that question is f(x) = e*x*. If that approach is chosen, the statement in the previous equation becomes a Theorem. Again, you can see [*e* to 10,000 digits.](http://www.math.utah.edu/~alfeld/math/e.html)

## The Pythagorean Theorem

If a and b are the lengths of the two short sides of a right triangle and c is its long side, then this formula holds. Conversely, if the formula holds then a triangle whose sides have length a, b and c is a right triangle. This formula is about 2,350 years old and due to Pythagoras of Samos. It is used all over mathematics. The Greek thought of the Theorem not algebraically as it is presented here, but geometrically with the square numbers being represented as squares attached to the edges of the triangle.

## The Fundamental Theorem of Calculus

This formula expresses the fact that differentiation and integration are inverse operations of each other.

## Taylor Series

This formula shows how to express an analytic function in terms of its derivatives.

## Eigenvalue Problems

In this equation, A is a square matrix (often a very large one), x is an unknown vector, and lambda is an unknown real or complex number. Many physical problems lead to equations like this. Usually, the numbers lambda that satisfies the equation are significant to the dynamic behavior of the physical system, i.e., the behavior as time goes on.

## Linear Systems

In this equation, A and x are as before and b is a known vector. The equation also describes many physical systems and the solution x often describes a physical situation either at one point in time or for all time.

## A Variational Principle

Nature likes to minimize things (like energy) and this equation describes one minimization problem. Given a function F, one wants to find a function u=u(x) such that the integral is as small as possible.

## The Mandelbrot Set

The Mandelbrot Set is an extremely complex object that shows a new structure at **all** magnifications. It is the set of complex numbers c for which the iteration indicated nearby remains bounded. Click on the equation to see a page that has lots of pictures and that offers and documents a Java applet you can use to explore the Mandelbrot set yourself.

## Loan Payments

This is arguably the most important equation of the bunch. If you borrow an amount L dollar and pay it back over N months at an annual interest rate of p percent your monthly payment will be m dollars.

## The Triangle Inequality

Let x and y be vectors that form two sides of a triangle whose third side is x+y. The expression ||x|| denotes the length of a vector x. (It's more generally called a norm in mathematics.) The triangle inequality expresses the fact that the sum of the lengths of any two sides of a triangle cannot be less than the length of the third side. It is used ubiquitously throughout mathematics. As an exercise, you may want to prove the

## Reverse Triangle Inequality

You can do it! The main use of the reverse triangle inequality is to provide a challenging exercise to students. The argument is very short and simple, but you have to think of it. I once had a graduate student who said he spent a total of 20 hours finding a four-line proof, but, he said, "it made me feel **really good!**". Mail me your proof if you like. It may spoil your fun, but if you can't resist the temptation [a proof](http://www.math.utah.edu/~alfeld/math/equations/proof.html) is just a click away.

## Cantor's Theorem.

Let S be a set and let |S| denote its cardinality. If S is a finite set then its cardinality is the number of elements in it, and things are not very interesting. But the concept of cardinality makes sense also for infinite sets. That story makes [a fascinating webpage.](http://www.math.utah.edu/~alfeld/math/sets.html) The power set of a set is the set of its subsets. It is easy to see that for finite sets S the cardinality of the power set equals 2*|S|*. Thus, we denote by 2*|S|* the cardinality of the power set even for infinite sets S. Cantor's Theorem states that the cardinality of the power set of a set S always exceeds the cardinality of S itself. That's obvious for finite sets but far from trivial for infinite sets. You are invited to look at the [proof](http://www.math.utah.edu/~alfeld/math/sets/largerproof.html) of this remarkable fact.

# Physics

## Energy, mass, and the speed of light.

Einstein's famous equations say that mass m is equivalent to energy E, and the amount of energy contained in a piece of mass is equal to the mass multiplied with the square of the speed of light, c. Without the fact described by this equation, we wouldn't be around since the energy we obtain from the Sun is generated by converting mass to energy in the process of nuclear fusion.

## Gravity

If you have two objects of mass m *1* and m *2* at a distance d, then these two objects will attract each other with a force F given in this formula. G is the gravitational constant. It equals approximately 6.67\*10*-11*Nm*2*kg*-2*. This formula determines the destiny of our Universe (i.e., whether it will expand forever or whether it will ultimately collapse in a Big Crunch after having originated in the Big Bang).

1. Alfeld, Peter. 15 March 1998. Famous equations and inequalities. University of Utah <<http://www.math.utah.edu/~alfeld/math/equations/equations.html>> 19 March 2014. [↑](#footnote-ref-1)