DEAR JOKE,

BECAUSE I AM POOR AN UNORIGINAL, I AM GIVING YOU THE MUG I BOUGHT FROM THE CERN GIFT SHOP AS YOUR 26TH BIRTHDAY PRESENT. I RECALL YOU ~~OF~~ OPPOSING THIS GIFT BECAUSE YOU WERE AFRAID PEOPLE MIGHT ASK YOU ABOUT THE EQUATIONS WRITTEN ON THE SIDE. ALAS! THERE IS HOPE! NOT ONLY WILL I GIVE YOU THE MUG, BUT I WILL ALSO USE MY RUDIMENTARY AND UNDERUTILIZED KNOWLEDGE OF PARTICLE PHYSICS TO GIVE YOU A RUNDOWN OF THE BASIC CONCEPTS YOU NEED TO UNDERSTAND THIS EQUATION.

THE EQUATION IS AS FOLLOWS

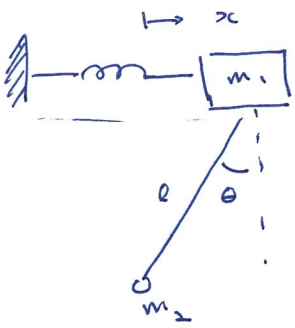
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (i \bar{\Psi} \not{D} \Psi + h.c.) + (\Psi_i y_{ij} \Psi_j + h.c) + |D_\mu \Phi|^2 - V(\Phi)$$

(I ADDED THE PARENTHESES FOR CLARITY)

SO MANY SYMBOLS! HOW STRANGE AND EXOTIC THEY ARE! DO NOT WORRY. WE'RE NOT GOING TO TALK TOO MUCH ABOUT THE MATHEMATICS OF THESE SYMBOLS. INSTEAD WE WILL FOCUS ON THE CONCEPTS THEY REPRESENT.

IN GENERAL, HOWEVER, THE ENTIRE EQUATION DEFINES HOW ELEMENTARY PARTICLES (E.G., ELECTRONS, QUARKS, NEUTRINOS) INTERACT THROUGH THE FUNDAMENTAL FORCES OF NATURE: ELECTROMAGNETISM, THE WEAK FORCE, THE STRONG FORCE, & THE HIGGS FORCE.

ABOVE WE HAVE AN EQUATION WHICH STATES $\mathcal{L}$ IS EQUAL TO SOME COMPLICATED SUM OF TERMS. THIS $\mathcal{L}$ STANDS FOR LAGRANGIAN. IN CLASSICAL PHYSICS, THE LAGRANGIAN IS TYPICALLY USED TO DETERMINE HOW DYNAMICAL SYSTEMS EVOLVE IN TIME. IT IS SEEN AS AN ALTERNATIVE TO NEWTON'S 2ND LAW. IN ORDER TO FIND OUT HOW THE POSITIONS, ANGLES, ETC. IN YOUR SYSTEM CHANGE OVER TIME YOU CAN EITHER WRITE DOWN ALL THE FORCES IN THE SYSTEM AND APPLY NEWTON'S 2ND LAW OR YOU CAN WRITE DOWN A LAGRANGIAN.



"Complex Mechanical System"

$\Rightarrow$ What are $\theta(t)$ & $x(t)$?

(Solve Newton's 2ND Law)

$$\Sigma F_1 = m_1 \ddot{x}$$

$$\Sigma F_2 = m_2 \ell \ddot{\theta}$$

(Compute Lagrangian)

$$L = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \ell^2 \dot{\theta}^2 - \dots$$

IN QUANTUM PHYSICS, LAGRANGIANS ARE ALSO USED TO COMPUTE THE DYNAMICS OF SYSTEMS, BUT THEY DO NOT REALLY PROVIDE THE SAME INFORMATION AS IN THE TYPICAL QUANTUM MECHANICAL APPROACH. PROBLEMS IN QUANTUM MECHANICS ARE TYPICALLY SOLVED BY WRITING OUT AN ENERGY OPERATOR FOR THE SYSTEM AND THEN SOLVING THE SCHRÖDINGER EQUATION. AT THE END OF THIS PROCESS, YOU ARE LEFT WITH INFORMATION ABOUT THE ENERGY SPECTRUM OF THE SYSTEM, THE PROBABILITY OF TRANSITIONS, AND THE PROBABILITY TO BE IN VARIOUS STATES

SCHRÖDINGER EQUATION

$$i\hbar \frac{d}{dt} |\Psi\rangle = \hat{H} |\Psi\rangle$$

ENERGY OPERATOR

TELLS US $\Rightarrow$

ENERGY SPECTRUM

E_n ——
.
.
.
E_2 ——
E_1 ——

PROBABILITY TO TRANSITION BETWEEN STATES

$|m; t\rangle \longrightarrow |k; t'\rangle$

$P_{m \to k}(t', t)$

PROBABILITY TO BE IN VARIOUS STATES

$|1\rangle \quad |2\rangle \quad \dots \quad |n\rangle$

$P_1 \quad P_2 \quad P_n$

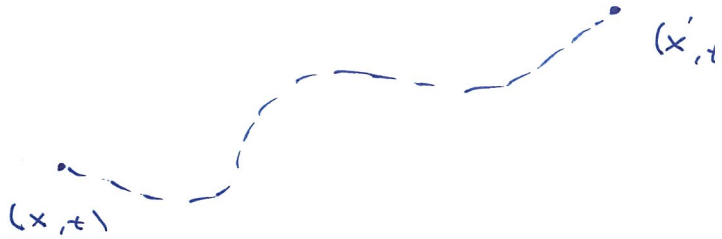
LAGRANGIANS APPEAR IN QUANTUM PHYSICS IN WHAT IS KNOWN AS THE PATH INTEGRAL. THE PATH INTEGRAL COMPUTES THE PROBABILITY FOR A QUANTUM MECHANICAL SYSTEM WHICH IS INITIALLY AT ONE POSITION x AT TIME t TO TRANSITION TO ANOTHER POSITION x' AT TIME t'. WHEN WE STUDY THE QUANTUM THEORY OF FIELDS (AS OPPOSED TO QUANTUM MECHANICS WHICH IS THE QUANTUM THEORY OF PARTICLES) THE PATH INTEGRAL GENERALLY CONSTRAINS HOW QUANTUM FIELDS INTERACT

PATH INTEGRAL IN QUANTUM MECHANICS

$$\langle X';t' \mid X;t \rangle = \int_{X(t)}^{X'(t')} \mathcal{D}x \exp\left[\frac{i}{\hbar}\int_t^{t'} dt_1\, L\right]$$

LAGRANGIAN

$\langle X';t' \mid X;t \rangle$ = Probability* for particle to move from X at time t to X' at time t'



* Probability Amplitude, to be specific

ONE OF THE GREAT HISTORICAL IRONIES OF MODERN PHYSICS IS THAT AS AN UNDERGRADUATE RICHARD FEYNMAN, WHO IS PRIMARILY RESPONSIBLE FOR THE INCORPORATION OF LAGRANGIANS INTO QUANTUM PHYSICS, SPURNED THEIR USE IN CLASSICAL PHYSICS BECAUSE, HE CLAIMED, THEY OBSCURED A DEEPER UNDERSTANDING OF THE RELEVANT SYSTEM. FEYNMAN STARTED (AND ESSENTIALLY COMPLETED) HIS WORK ON PATH INTEGRALS WHEN HE WAS A GRADUATE STUDENT AND HIS VERY READABLE (AND NOW WIDELY ACCESSIBLE) PH.D THESIS OUTLINES HIS EARLY DISCOVERIES.

SO THAT'S $\mathcal{L}$. IN THE EQUATION ON THE MUG "$\mathcal{L}$ =" ESSENTIALLY MEANS THAT EVERYTHING ON THE RIGHT HAND SIDE OF THE EQUALITY DEFINES THE DYNAMICS OF OUR SYSTEM, THAT IS, DEFINES HOW ELEMENTARY PARTICLES INTERACT.

NOW A GENERAL COMMENT ABOUT THE PROPERTIES OF TERMS IN THE EQUATION ON THE MUG. THE TERMS $F_{\mu\nu}$, ψ, and ϕ ARE ALL FIELDS ($F_{\mu\nu}$ IS ACTUALLY THE DERIVATIVE OF A FIELD BUT THAT DOESN'T CHANGE ITS PROPERTIES). MUCH IN THE SAME WAY WE SPEAK OF ELECTRIC AND MAGNETIC FIELDS, THESE FIELDS TAKE ON PRECISE VALUES AT EACH POINT IN SPACE AND TIME. HOWEVER IN QUANTUM FIELD THEORY, THESE FIELDS ALSO REPRESENT ~~PARTICLE~~ PARTICLES. SO IN THE SUBSEQUENT DISCUSSION WHENEVER YOU SEE FIELD TERMS THINK PARTICLE. (THE RELATIONSHIP BETWEEN QUANTUM FIELDS AND PARTICLES IS ACTUALLY SUBTLE BUT WE CAN IGNORE THIS SUBTLETY FOR NOW)

FIELDS	PARTICLES
$F_{\mu\nu}$	photons, gluons, W & Z bosons
ψ	electrons, neutrinos, quarks
ϕ	Higgs boson

AS I PREVIOUSLY MENTIONED, THE EQUATION ON THE MUG DEFINES HOW ELEMENTARY PARTICLES INTERACT. BY ELEMENTARY WE MEAN INDIVISIBLE. FOR EXAMPLE ATOMS AND PROTONS ARE NOT ELEMENTARY BECAUSE THEY ARE COMPOSED OF OTHER PARTICLES. ALSO, THE EQUATION DOES NOT DEFINE HOW THE GRAVITON, THE ~~HYPOTHETICAL~~ PARTICLE RESPONSIBLE FOR GRAVITATION, INTERACTS. ANYWAY, THE EQUATION AND THE SET OF CONCEPTS/DEFINITIONS ASSOCIATED WITH IT ARE TERMED "THE STANDARD MODEL" AS IN "THE STANDARD (ACCEPTED) MODEL (OF ELEMENTARY PARTICLE INTERACTIONS)"

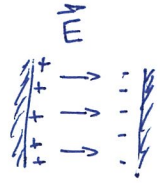
EQUATION ON MUG $\Longleftrightarrow$ THE STANDARD MODEL

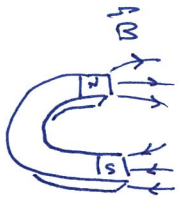
THE $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ TERM IS CALLED THE GAUGE FIELD STRENGTH. IT DEFINES THE PROPERTIES OF GAUGE FORCES WHICH THEMSELVES ALLOW ELECTRONS, NEUTRINOS, AND QUARKS TO INTERACT. THE TERM "GAUGE" REFERS TO THE UNIQUE SYMMETRY PROPERTIES OF THESE INTERACTIONS.

AS FAR AS WE KNOW, THERE ARE THREE GAUGE FORCES IN THE PHYSICAL UNIVERSE. THERE IS THE ELECTROMAGNETIC FORCE WHICH GIVES RISE TO ELECTRIC AND MAGNETIC PHENOMENA; THE STRONG FORCE ALSO KNOWN AS THE NUCLEAR FORCE WHICH BINDS THE NUCLEI OF ATOMS; AND THE WEAK INTERACTION WHICH IS THE FORCE RESPONSIBLE FOR RADIOACTIVE DECAY. THE $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ TERM REPRESENTS ALL THREE OF THESE FORCES.

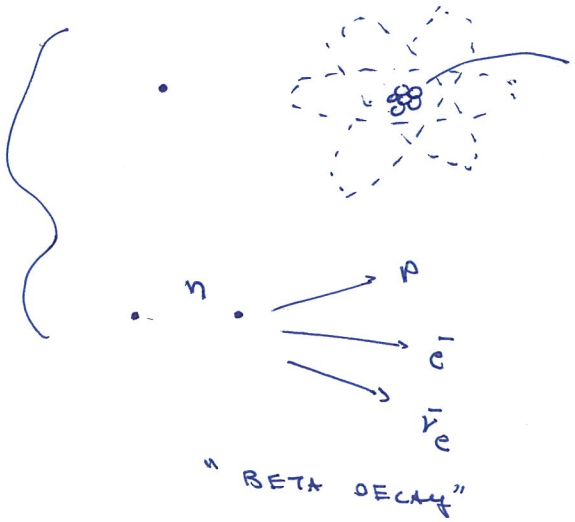
GAUGE FIELD STRENGTH TERM

$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ REPRESENTS





(ELECTRICITY & MAGNETISM)



(STRONG INTERACTION)

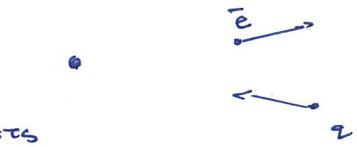
(WEAK INTERACTION)

"BETA DECAY"

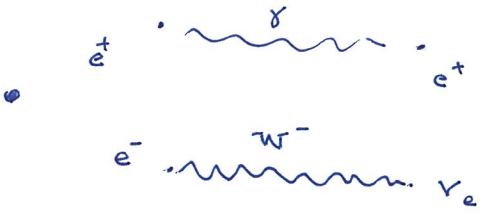
THE $i\bar{\psi}\not{D}\psi$ TERM IS CALLED THE "FERMION KINETIC TERM". "FERMION" BECAUSE IT ONLY MODELS PARTICLES WHOSE SPIN ANGULAR MOMENTUM IS $\hbar/2$, AND "KINETIC" BECAUSE IT DEFINES HOW THESE PARTICLES PROPAGATE THROUGH SPACETIME. THIS TERM ALSO CONTAINS A HIDDEN PART WHICH CONSTRAINS HOW THESE FERMIONS INTERACT THROUGH THE ELECTROMAGNETIC, STRONG, & WEAK INTERACTIONS

FERMION KINETIC TERM

$i\bar{\psi}\not{D}\psi$ REPRESENTS



(FERMIONS MOVING THROUGH SPACETIME)

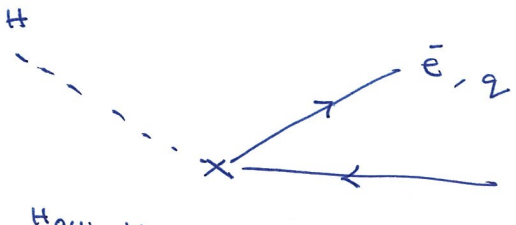


(FERMIONS INTERACTING THROUGH GAUGE FORCES

THE $\psi_i y_{ij} \psi_j \phi$ TERM IS CALLED THE YUKAWA INTERACTION. IT DEFINES HOW THE HIGGS BOSON INTERACTS WITH THE FERMIONS OF THE STANDARD MODEL. THROUGH THIS INTERACTION THE ELECTRONS AND QUARKS (AND THEIR ANTI-PARTICLES, WHICH WE WON'T TALK ABOUT) OBTAIN A MASS.

YUKAWA INTERACTION

$\psi_i y_{ij} \psi_j \phi$ REPRESENTS



How higgs interacts with electrons and quarks (and their antiparticles)

AND

gives electrons and quarks (and their antiparticles) their mass

THE $|D_\mu \phi|^2$ TERM IS THE HIGGS KINETIC TERM. IT DEFINES HOW THE HIGGS PROPAGATES THROUGH SPACETIME AND HOW THE HIGGS INTERACTS THROUGH ELECTROMAGNETISM AND THE WEAK INTERACTION.

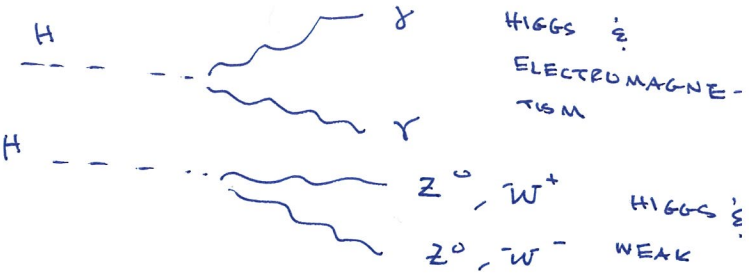
FEBRUARY 13, 2016

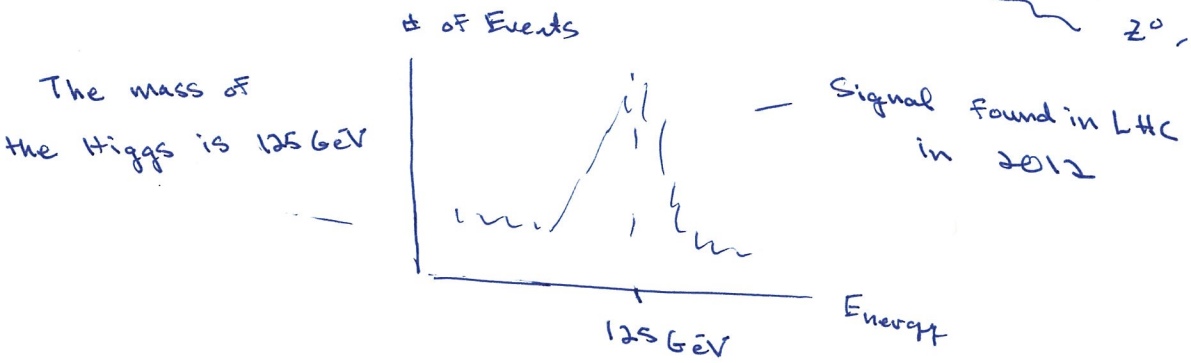
6/8

THEORETICALLY THE HIGGS BOSON PROVIDES ALL THE MASSIVE PARTICLES OF THE STANDARD MODEL WITH THEIR MASS. THE HIGGS BOSON IS THE LAST AND MOST RECENTLY DISCOVERED PARTICLE OF THE STANDARD MODEL. IT WAS FOUND AT THE LHC IN GENEVA, SWITZERLAND IN 2012

HIGGS BOSON KINETIC TERM

$|D_\mu \phi|^2$





THE $V(\phi)$ TERM IS CALLED THE BOSON POTENTIAL. IN A ~~WAY~~ IT IS THE REAL SOURCE OF THE MASS OF THE MASSIVE PARTICLES IN THE STANDARD MODEL. THE EXACT FORM OF THIS POTENTIAL LEADS TO A PHENOMENON CALLED SPONTANEOUS SYMMETRY BREAKING (SSB). SSB IS JUST A FANCY ~~F~~ NAME FOR WHEN THE PHYSICAL PROPERTIES OF A SYSTEM DO NOT SHOW THE ORIGINAL SYMMETRIES IN THE STARTING MATHEMATICAL DESCRIPTION OF THE SYSTEM. IN THE STANDARD ~~MODEL~~ MODEL, SSB IS RESPONSIBLE FOR MAKING THE WEAK INTERACTION HAVE A SHORTER RANGE OF INTERACTION THAN ELECTROMAGNETISM.

BOSON POTENTIAL

$V(\phi) \Rightarrow$ INDUCES SPONTANEOUS SYMMETRY BREAKING $\Rightarrow$

ELECTROMAGNETIC INTERACTION HAS INFINITE RANGE

WHILE

WEAK INTERACTION HAS A FINITE RANGE

AND

ELECTRONS AND QUARKS OBTAIN A MASS

SO THAT'S BASICALLY IT. ALL THE TERMS ON THE MUG AND WHAT THEY MEAN OF COURSE THERE'S MORE TO EACH ONE BUT THAT IS THE BASIC IDEA.

SOME THINGS I DIDN'T TALK ABOUT

FEBRUARY 13, 2016

HERMITIAN CONJUGATE

THE "h.c." TERM YOU SEE NEXT TO THE $i\bar{\psi}\partial\!\!\!/\psi$ AND $\psi_i \gamma^\mu_{ij} \psi_j A_\mu$ TERMS STANDS FOR "HERMITIAN CONJUGATE". THE HERMITIAN CONJUGATE IS FOR MATRICES AND OPERATORS WHAT COMPLEX CONJUGATE IS FOR NUMBERS. FOR EXAMPLE, A MATRIX ADDED TO ITS HERMITIAN CONJUGATE YIELDS A NEW MATRIX WITH REAL EIGENVALUES.

$$A + \text{h.c.} \equiv A + A^\dagger = M \longrightarrow M\vec{v} = \lambda\vec{v}, \quad \lambda \in \mathbb{R}$$

$M = M^\dagger$ SO WE SAY M IS "HERMITIAN"

THE EIGENVALUES OF M ARE REAL NUMBERS AND CAN REPRESENT PHYSICAL QUANTITIES

IN PHYSICS IT IS IMPORTANT FOR MATRICES TO HAVE REAL (INSTEAD OF COMPLEX) EIGENVALUES BECAUSE ONLY REAL NUMBERS CAN REPRESENT PHYSICAL QUANTITIES. THE LAGRANGIAN IS CONSIDERED A PHYSICAL QUANTITY AND SO TERMS IN $\mathcal{L}$, WHICH ARE NOT ALREADY HERMITIAN, ARE MADE HERMITIAN BY ADDING THEM TO THEIR HERMITIAN CONJUGATE.

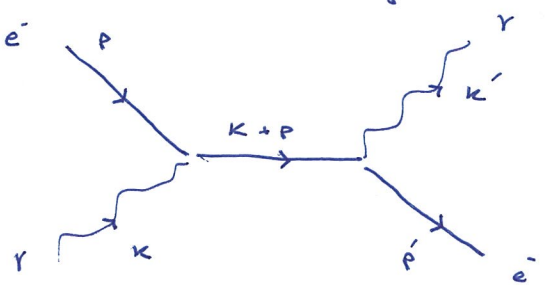
THE MUG, HOWEVER, HAS A MISTAKE. THE FIRST "+ h.c." NEXT TO $i\bar{\psi}\partial\!\!\!/\psi$ IS NOT NECESSARY BECAUSE $i\bar{\psi}\partial\!\!\!/\psi$ IS ALREADY HERMITIAN. THE SECOND "+ h.c." IS CORRECT.

FEYNMAN DIAGRAMS

SINCE I WAS FOCUSING ON THE MUG, I NEVER MENTIONED ONE OF THE MOST FAMOUS CALCULATION TOOLS IN PARTICLE PHYSICS. FEYNMAN DIAGRAMS! NAMED AFTER THEIR PROGENITOR RICHARD FEYNMAN, ~~FEET~~ FEYNMAN DIAGRAMS ARE SIMPLE WAYS OF REPRESENTING PARTICLE INTERACTION PROCESSES. THEY ALSO SERVE AS MNEMONICS FOR WRITING DOWN MATHEMATICAL EXPRESSIONS FOR THE PROBABILITY OF OCCURRENCE OF THE PROCESSES THEY REPRESENT. THESE PROBABILITIES CAN IN TURN BE CONVERTED INTO PHYSICAL QUANTITIES MEASURED AT PLACES LIKE CERN.

Ex. COMPTON SCATTERING

$e^- + \gamma \to e^- + \gamma$



$=$ SCATTERING AMPLITUDE $\langle |T^2| \rangle$

↓ USED TO COMPUTE

$\frac{d\sigma}{d\Omega}$ "DIFFERENTIAL CROSS SECTION"

– CAN BE MEASURED AT PARTICLE ACCELERATORS

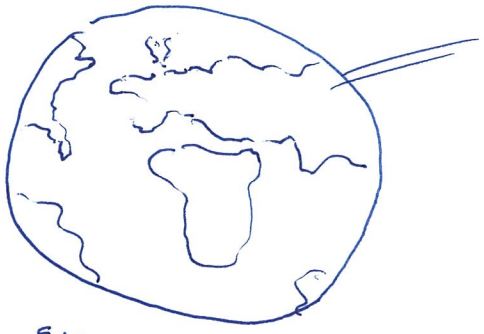
FEBRUARY 13, 2016

MOST OF THE MASS OF EARTH

THIS IS REALLY TANGENTIAL BUT I MENTION IT BECAUSE IT'S INTERESTING. ALTHOUGH THE HIGGS BOSON SUPPLIES THE PARTICLES OF THE STANDARD MODEL WITH MASS, AND THESE PARTICLES MAKE UP THE ATOMS WHICH MAKE UP THE WORLD, MOST OF THE MASS OF EARTH COMES, NOT FROM THE HIGGS, BUT FROM THE GAUGE FIELD OF THE STRONG INTERACTION.

HERE'S WHY. THE MASS OF MATTER COMES PRIMARILY FROM THE MASS OF PROTONS AND NEUTRONS BECAUSE PROTONS/NEUTRONS ARE MUCH HEAVIER THAN ELECTRONS. BUT PROTONS/NEUTRONS ARE COMPOSED OF THREE INTERACTING QUARKS, QUARKS WHOSE MASSES ARE MUCH MUCH SMALLER THAN THE MASS OF AN ELECTRON. CONTRADICTION?

THE RESOLUTION COMES QUALITATIVELY FROM THE EINSTEIN MASS-ENERGY EQUIVALENCE. ALTHOUGH EACH PROTON & NEUTRON IS MADE UP OF THREE QUARKS WITH VERY SMALL MASSES, THE ENERGY WHICH BINDS THESE QUARKS TOGETHER IS COMPARATIVELY HUGE. THIS LARGE ENERGY COMES FROM THE STRONG FORCE DESCRIBED BY $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. AND BY $M = E/c^2$ THIS ENERGY COMPRISES MOST OF THE MASS OF PROTONS & NEUTRONS AND HENCE MOST OF THE MASS OF EARTH.



EARTH

MOST OF THE MASS COMES FROM $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ FOR STRONG INTERACTION

NOT FROM THE HIGGS BOSON