## Time and Dynamics

These notes are part of a series concerning "Motifs in Physics" in which we highlight recurrent concepts, techniques, and ways of understanding in physics. This week we discuss the role of time in physics.

## States, Dynamics, and Kinematics

With the introduction of the Schrödinger equation we have at last put our study of quantum mechanics in contact with one of the unifying motifs in physics: time. Until now we have dealt almost exclusively with time-independent states and phenomena, but in the coming weeks we will supplement this study by considering how these states and their associated probabilistic predictions evolve in time.

More generally throughout all areas of physics, our study of a physical system is principally concerned with how said system changes (i.e., evolves) in time. In understanding and describing the aspects of this time evolution, there are three related concepts:

A physical system is characterized by a **configuration space** (i.e., a set of states which define the system) along with the rules which define how states of the system change in time. These rules are referred to as dynamics which the description of a state's evolution is called **kinematics**.

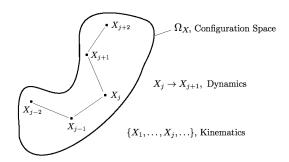


Figure 1: Configuration Space: Schematic of the relationship between configuration spaces, dynamics, and kinematics. The kinematics refers to the sequence of states  $X_j$  which move in the configuration space  $\Omega_X$ . The rules defining this evolution (namely how  $X_j$  goes to  $X_{j+1}$ ) are the dynamics.

## **States in Physics**

In quantum physics, the states are vectors in Hilbert Space, the dynamics are defined by the Schrödinger equation and the property of wave function collapse (both of which are taken as postulates), and we use these dynamics to compute how states and probabilities evolve in time.

Table 1: States, Dynamics, and Kinematics in Quantum Physics				
Physical Theory	States in Configuration Space	Dynamics	Kinematics Examples	
Quantum Physics	Vectors in Hilbert Space	Schrödinger Equation; Wave function Collapse	Rabi Oscillations, Fermi's Golden Rule,	

This interplay between states, kinematics, and dynamics (all of which are unified by the fact that they concern physical systems and time) occurs throughout physics. Indeed we can outline all foundational subjects according to how they engage with this trichotomy.

Table 2: States, Dynamics, and Kinematics Throughout Physics				
Physical Theory	States in Configuration Space	Dynamics	Kinematics Examples	
Classical Mechanics	Center of mass positions and velocities	Newton's Laws, Postulates of Special Relativity, Euler-Lagrange Equations	Conic section orbits, Simple Harmonic Oscillator, Rocket Motion	
Electrodynamics	Particle positions, Electromagnetic fields and potentials	Maxwell's Equations; Lorentz Force Law; Jefimenko's Equations	Coulomb's Law, Dipole Radiation, Liénard–Wiechert potential	
Equilibrium Statistical Physics	Macroscopic variables (e.g., energy, pressure); Microscopic variables (e.g., position, momentum)	Dynamics are encoded into ensembles of state space through ergodic theorem	(None because time is made irrelevant)	
General Relativity	Particle Position, Metric	Geodesic Equation, Einstein's Equations	Gravitational Lensing, AdS Space, Precession of Mercury Orbit, Gravitational Waves	
Non-Equilibrium Statistical Physics	(Same as in Equilibrium Statistical Physics)	Associated Laws of Classical Physics, Master-Equation, Stochastic Differential Equations	Solution to Diffusion Equation, Kinetic Ising Model, Poisson Process, Weiner Process	

## Time's Ubiquity

Given the ubiquity of time it is perhaps not so surprising that time exists as an epistemic construct beyond which most physical theories cannot progress. That is, although many physical theories are defined by quantities (like position, charge density, or field) which obtain a more precise definition by some other physical theory at a shorter length scale, no currently accepted physical theory serves as a foundational *explanation* of time<sup>1</sup> which can change the way time is understood in other theories.

<sup>&</sup>lt;sup>1</sup>The natural rebuttal is to cite Special Relativity. However, one could argue special relativity did not so much as explain time as correct time's role from a parameter which is constant and crystalline to something which is itself dynamical.