The Barandes Basics

In his Introduction to Theoretical Physics course, Jacob Barandes delays dealing with anything vaguely resembling physics and instead starts by defining a "physical system". He ends up developing a definition which is general and illuminating, specifically in the way it characterizes time (and our attempt to parameterize our world through time) as the unifying motif of all of physics.

1 What is a Physical System?

During the summer of 2016, I was fortunate enough to be part of Jacob's Introduction to Theoretical Physics course offered in Harvard's summer school The course was advertised to high school students who had sufficient math background to not be terrified by the prospect of learning calculus in three days.

He began his lectures without talking about physics at all. Instead, he started by talking about systems, configuration spaces, and states. Using these terms he developed the following, rather non-physical, definition of a physical system:

> A physical system is a situation characterized by a **configuration space** (i.e., a set of states which define the system) along with the rules which define how the system can change from one state to another state all within that space.

Jacob's initial examples of configuration spaces and states were similarly non-physical. He considered positions on the real-line, the six sides of a die, and the two sides of a coin to give the students a concrete sense of the diversity of possible states and their associated configuration spaces. I should note that in physics, the word "states" has a typically quantum mechanical connotation but in the class Jacob used it generally to represent anything we take to be definitive of our physical system at a certain time. For example, both the value of an electric field in space or the position of a launched projectile can model how their corresponding systems evolve in time, and we can thus interpret both as defining states of their systems.

After this introduction to configuration spaces, Jacob then introduced the dual concepts dynamics and kinematics. He defined **dynamics** as the rules which determine how states evolve in time, and **kinematics** as the description of a state's evolution. And with these preliminary definitions, Jacob was finally able to introduce a few non-physics examples of systems and their evolution, well before he discussed any apparently foundational physics concepts such as energy or Newton's laws. Two of his examples are reproduced below.

Example System 1

- Configuration Space: $\{0, 1, 2\}$
- Dynamics: $c_{n+1} = (2c_n) \bmod 3$
- Kinematics Example: $2, 1, 2, 1, 2, \ldots$

Example System 2

- Configuration Space: $x \,|\, \forall x \in \mathbb{Z}$
- Dynamics: $x_{n+1} = 2x_n - 3$
- Kinematics Example: $1, -1, -5, -13, -29, \ldots$

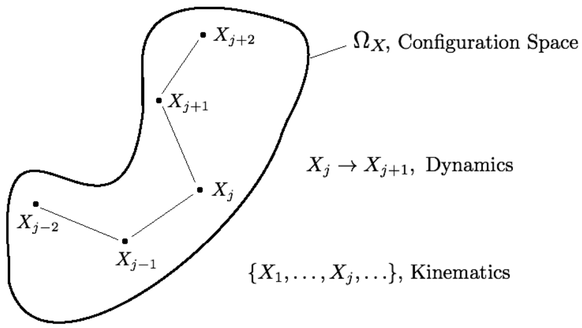

Figure 1: Configuration Space: Schematic of the relationship between configuration spaces, dynamics, and kinematics. The kinematics refers to the sequence of states X_j which move in the configuration space Ω_X, such that $X_j \in \Omega_X$. The rules defining this evolution (namely how X_j goes to X_{j+1}) are the dynamics. Although kinematics can specifically be seen as defining the 'what' of a trajectory, for most practices, trajectory and kinematics are used as interchangeable concepts.

The distinction between dynamics and kinematics can be clarified by introducing a new word: trajectory. A **trajectory** is a sequence of states which represent how a system evolves in time (or according to an index which is analogous to time). By this definition, kinematics refers to the what (as in "what is the trajectory?") of the trajectory, while dynamics refers to the why (as in "why does the trajectory take the form it does?").

Jacob's definition of a physical system is fitting for two reasons: One, we can now consider all areas of physics from a common perspective of states, dynamics, and kinematics of the physical system. That is, we can characterize an area of physics according to what we consider to be the relevant state which defines it, the rules which define transitions between states, and some physically relevant solutions to those rules.

Second, this definition reveals that although different areas of physics are defined by different states, they are all unified by their desire to understand how these states evolve in time. Thus time is the only universal physical variable/parameter and we can therefore compare different areas of physics, not only according to the states and dynamics which define them but also according to how they treat time.

2 Attractors

In class today Jacob gave many wonderful examples of attractors. First he defined a system with an attractor as one in which the dynamics flows to one or collection of states. ?States? were defined as the various possible configurations of a system, and ?dynamics? were defined as rules which take one state to another. Here were his examples.

We need not provide kinematics example, because for all attractor systems all kinematics trajectories are part of the larger space of trajectories which lead to the same final state (or space of final states).

1. **Letters and numbers**

   – State Space: possible english words/phrases (e.g., Mississipppi, The United States of America, onamonapia)

   • Count the number of characters in that phrase; The new phrase is the number of characters written as a word.

   Whatever phrase you begin with you ultimately evolve to a state where the phrase is four. This is because four is the only phrase where the dynamics brings you back to the starting phrase. All other phrases evolve away from this value.

2. **Collatz Conjecture**

   – State Space: All positive integers

   – Dynamics: If the integer n is odd the new integer is $3n + 1$; if the integer is even the new integer is $n/2$.

   The attractors in this system are the set of states 1, 4, 2. There apparently is no formal proof of this.

3. **All roads lead to philosophy**

   – State Space: Wikipedia encyclopedia page

   – Dynamics: On an existing page click on the the first link (besides pronunciation and etymology) which takes you to a new page.

   This is a more of a qualitative example, but Jacob elaborated it excellently. Beginning on an arbitrary page, you eventually reach the page for philosophy about 95% of the time. In the remaining 5% of cases you reach a page without any links, a page which doesn?t exist, or a series of pages which are a self-contained loop.

   In contrast to simpler attractor systems, this system does not remain at the final attractor state ("philosophy") once it is reached. Instead, following the dynamics from the starting point of "philosophy", we proceed through other pages before returning again to "philosophy". Thus "philosophy" and its associated looped pages are much like the attractor states of the Collatz conjecture in that the system evolves towards a cycle of states rather than a single one.

3 Areas of Physics: States, Dynamics, Kinematics

Motivated by Jacob's definition of a physical system, we review the standard areas of physics and outline the various "states" that define them along with the dynamics which define the evolution of those states, and important kinematic solutions of those dynamics. The intention is to provide an abstract but nevertheless unified picture of physics without any discussion of the typically unifying concepts of energy or symmetry.

- Electrodynamics
  - **States:** Particle positions, Electromagnetic fields, Electromagnetic Potentials
  - **Dynamics:**: Lorentz Force Law, Maxwell's Equations, Jefimenko's Equations
  - **Kinematics Examples:** Coulomb's Law, Dipole Radiation, Magnetic Field of of a Solenoid
- Classical Mechanics and Special Relativity
  - **States:** Particle positions, Velocities, Rotation about an Axis
  - **Dynamics:**: Newton's Laws, Newton's Law of Gravitation, Euler-Lagrange Equations, Hamilton's Equations, or the corresponding relativistic versions (if they exist) of these dynamics.
  - **Kinematics Examples:** Elliptical orbit of a planetary object, Projectile Motion, Simple Harmonic Motion, Precession/Nutation of a Top, Rocket motion, Relativistic Harmonic Oscillator
- Quantum Physics
  - **States:** "Vectors" in Hilbert Space expressed in various bases (e.g., position, angle, spin-up/down); For Quantum Mechanics states can be defined by positions, spins, angles, and energy. For Quantum Field Theory states are the eigenkets of the Hamiltonians for various fields.
  - **Dynamics:**: Schrödinger Equation, Measurement Evolution (i.e., wave function collapse)

  - **Kinematics Examples:** Rabi Oscillations, Fermi's Golden Rule, LSZ reduction formula

- Thermodynamics and (Equilibrium) Statistical Mechanics/Physics
  - **States:** Macroscopic Variables (e.g., energy, magnetization, pressure, volume, number of correct element); Microscopic Variables (e.g., position, momentum, spin, permutation)
  - **Dynamics:**: Associated Laws of Classical Physics, or (by virtue of ergodic theorem) dynamics are encoded into ensembles of state space
  - **Kinematics Examples:** None because time is made irrelevant.

- (Non-equilibrium) Statistical Mechanics
  - **States:** Macroscopic Variables (e.g., energy, magnetization, pressure, volume, number of correct element); Microscopic Variables (e.g., position, momentum, spin, permutation)
  - **Dynamics:**: Associated Laws of Classical Physics, or Master-Equation, Fokker-Planck Equation, Stochastic Differential Equation
  - **Kinematics Examples:** Solution to Diffusion Equation, Kinetic Ising Model, Poisson Process, Decay Process, Weiner Process

- General Relativity
  - **States:** Particle Position, Metric
  - **Dynamics:** Geodesic Equation, Einstein's Equations
  - **Kinematics Examples:** Gravitational Lensing, Precession of Mercury orbit, Gravitational Waves from Binary Black Holes, Schwarzchild Solution, AdS space, Kerr Metric

4 Treatment of Time in Various Areas of Physics

The delineation of different areas of physics according to their dynamics, kinematics, and their states places time as the unifying motif across all of physics. We can make this unification more precise and thus hopefully gain a better understanding of the relevance of time in our representation of the physical world by specifically considering how time is treated (or not treated) in various areas of physics.

- **Classical Mechanics (sans Relativity):** Time is a parameter. It is absolute, crystalline, and unchanging.
- **Electrodynamics:** Time is often treated as a parameter akin to its role in classical mechanics. However, since some solutions to Maxwell's equations involve the propagation of waves, time must be incorporated in such a way as to preserve causality (i.e., that the speed of light is the fundamental speed limit to causal signal travel). Moreover, electrodynamics is a Lorentz covariant physical theory and thus treats time relativistically in systems with high speed.
- **Equilibrium Statistical Mechanics and Thermodynamics:** Time is not relevant. Studying the evolution of microscopic variables in time is too complicated so we study their time average values. The ergodic theorem allows us to replace these time averages with ensemble averages, which are probability weighted integrations over configuration space.
- **Non Equilibrium Statistical Mechanics:** Time is a parameter akin to its role in classical mechanics.
- **Relativity:** Time is not merely just a parameter. Time itself is a "dynamical variable" which is a function of proper time. Just as particles can be defined according to their positions, they can also be defined according to their times as such times are compared to the times in an inertial coordinate system.

- **Quantum Mechanics:** Time is a parameter and comes into the Schrödinger Equation to define state evolution. The time of measurement is important for knowing when a wave function collapses.
    - **Quantum Field Theory:** Time appears as a space-time parameter for field operators but is generally unimportant as a characteristic of states since we typically only consider states in the asymptotically far future or past. In string theory, time is promoted to an operator on the same level as the position operator.

5 Conclusion

In the same way General Relativity did not really change Newtonian Gravitation but allowed us to understand how Newtonian Gravitation is corrected at high energy densities, if someone were to somehow develop a more fundamental theory of time it likely would not change any of our existing physical theories which depend on time, but it would certainly change the way we understand and interpret them. Einstein at the beginning of the 20th century, by revealing that time's passage is not absolute and eternal the way previous physical theories had always assumed, made the first major step beyond Newton to supplement our understanding of time. Indeed today, it is only theories that contain relativity that incorporate any non-intuitive definition of time.

Consequently, a reasonable conjecture concerning our understanding of time is that were we to develop an even more fundamental theory of gravity, then our notions of time would be further corrected. However, this best guess is somewhat like the man who searches for his keys in the lamp light because that is where he could best see. Rather it's possible a better theory of time could come from a direction completely orthogonal to our expectations. In any case the only way we could progress in developing such a theory is by asking questions which could move us closer to it.

Questions concerning modern physics and time

- **Quantum Gravity and Time:** How does string theory change/supplement our understanding of time? How generally might a theory of quantum gravity change our understanding of time?
- **Quantum Mechanics and Time:** Quantum Mechanics treats time much like other classical physical theories do (i.e., as a parameter), except for the case of measurement of a quantum state. Does this property of quantum systems lead to any novel questions concerning time?
- **Information and Time:** The second law of thermodynamics asserts that the entropy of the universe increases in time. If physicists developed a more fundamental theory of entropy, perhaps through the second law of thermodynamics, our understanding of time would be corrected as well.
- **Multiverses and the 2nd Law:** Moreover, relativity asserts that the passage of time is relative. Would it be possible to imagine a scenario where non-causally separated multiverses, can observe each other violating the second law of thermodynamics?