Mobolaji Williams (mwilliams@physics.harvard.edu➦) First Version: August 19, 2016

Non-Traditional Physics Problems

In standard physics coursework, students often encounter classical problem solving assignments where they are tasked with working through some analytic or computational procedure to answer a question posed by the instructor. Such problems build the typically termed "problem solving skills" but they comprise only a small part of the spectrum of problems physics courses can use. The hope for these notes is that they can serve as a collection of examples of non-traditional types of physics problems, problems which not only build problem solving skills but also build the softer and less-often practiced skills needed to structure knowledge in physics.

The Problem Types and Examples

- **Equation Diagramming:** This idea comes from *A student's guide to Maxwell's Equations* [1]. You take an example of an equation or a part of a derivation in physics and you describe in detail the meaning of the symbol used therein. In a more advanced treatment, you can simply describe the equation to the student and then ask the student to write the equation and write out a description of each discrete symbol used in the equation.

  > **Example Problem:** Diagram the most general (i.e. basis independent form) of the Schrödinger equation.

  Solution:

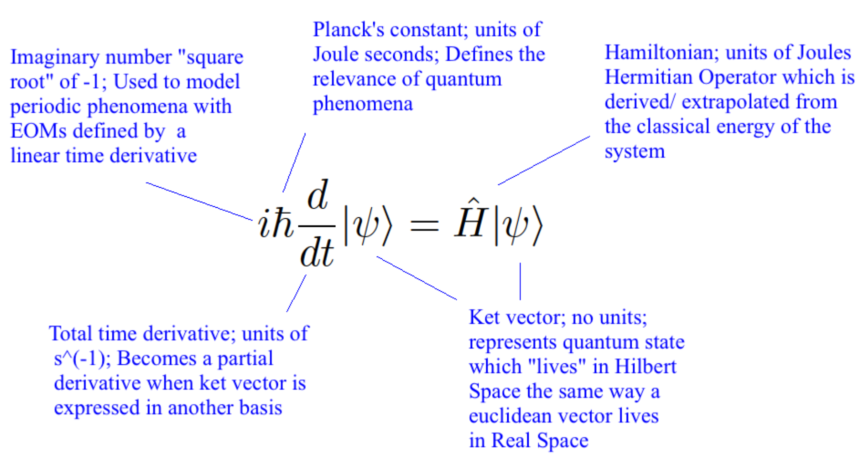

Figure 1: Diagram for Schrödinger's Equation

- **What is the Question:** This is a flip of the standard problem solving question. Instead of giving students a physical situation and a question they must answer about the situation, you give the students the situation and ask them to generate questions about it. As a second part, these student-generated questions can then be answered by the students.

The goal is to have students explore physical systems through questions they ask themselves. Whenever students attempt to use the knowledge they gained outside their courses they will often have to begin with such questions.

This flipped problem, will generally be easier to implement for conceptually transparent systems like those in classical mechanics. In my mind, it would be difficult to generate such questions for quantum field theory or quantum mechanics.

> **Example Problem:** A uniform sphere of dielectric constant ε is spinning with an angular velocity ω about the z axis. A uniform electric field is pointing in a direction perpendicular to the angular velocity direction. What are some questions you can ask (and possibly answer) about this physical situation?

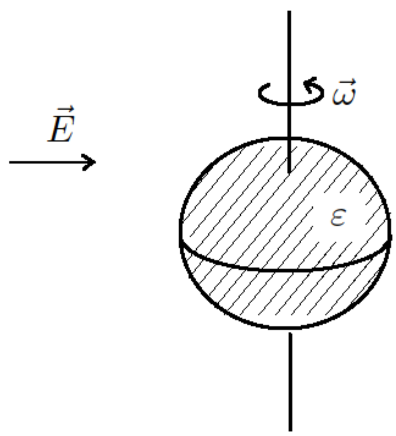

Figure 2: Spinning Dielectric

Solution: You can ask (for example)

- What is the time-dependent electric field generated by the dielectric?
- What happens when we make ω very large? Does the dielectric become impermeable to electric fields?
- What if we include a non-zero magnetic field?
- What if we have an electromagnetic wave of frequency much faster than the angular frequency of the dielectric?

- **Suggest Another Problem:** If the given problem in an assignment is too difficult, or just can't be solved with the student's current knowledge then the student can propose an alternative problem. This proposal will allow the student to practice navigating the knowledge space around a question and to practice generating auxiliary questions, which although not automatically leading to the problem solution, still create a better understanding of it.

  Generally this could only work for problems which are flexibly structured.

  Example Types:

1. Given a Newtonian dynamics problem, a student can ask to solve it with Lagrangian dynamics.
2. Given a difficult integral, a student could suggest a simpler integral which approximates the result or one related (through an inequality for example) to the first.

Example Problem: A chain of length ℓ and mass density σ is held such that it hangs vertically above a scale. It is then released. What is the reading on the scale as a function of the height of the top of the chain?

Examples of New/Different Question:

- What if there were a series of N disconnected falling particles instead of a continuous chain? What would be the reading on the scale as a function of the number of particles on the scale (Assuming they each collided inelastically with the scale)?
- What if there was one big ball? What would be the reading on the scale as a function of time?
- What if the chain had a non-uniform mass density?
- What if the chain was negligibly massless except for both ends which were massive?

- **Reverse Instruction Manual Problems:** Give the students an ill-defined problem and ask them to devise (and possibly implement) the series of steps needed to solve the problem.

  Such problems would have to be introduced later in the course after students have had some time to work through instructor-provided ”instruction manual problems. For such problems students should be asked to pay attention to the structure of those problems in anticipation of when they will create that structure themselves.

  These problems will give students practice in generating problem frameworks and will teach them to be cognizant of how the frameworks they develop constrain how they answer a question. These problems will also teach students how to contend with ambiguity.

  As an archetype, the problems in *Thinking Like a Physicist* by Thompson [**?**] are general and ill-defined enough that they can act as a model for ”reverse instruction manual” problems.

Example Problem: Determine the intermediary steps/questions you have to work through to answer the following question:

> *Thinking Like a Physicist* Problem 128: A man jumps as high as possible on a small planet. Estimate how small it would have to be for him to be able to jump off altogether? (The radius of the earth is 6380 km).

Solution: To answer this question you would have to:

1. Determine how high a man (typically) can jump on earth.
2. Determine with how much energy or with what initial speed a man jump's (typically) on earth.
3. Determine a relationship between escape velocity and the mass and radius of a planet.
4. Investigate if this relationship is the same if the man's mass and the planet's mass are of the same order (i.e., if the man jumping off the planet, gives the planet a nonzero velocity in the opposite direction.)
5. Find some constraint between mass and radius (one that allows you to write mass in terms of radius)

Note: It's possible to consider problems of this type in multiple directions. You could ask students to generate (no matter how vaguely defined) outlined steps to answer the question, or you could ask students to generate an explicit and as well articulated as possible problem (the kind an instructor would create).

- **Find the error (if there is one):** Students are given a conceptual or a mathematical argument for a problem and are then tasked with finding the error in the argument, explaining why it is an error, and then writing a correct argument. For conceptual problems of this type, students are allowed to practice applying and checking their qualitative understanding of a subject. For analytical problems of this type, students are practicing critically evaluating the assumptions and approximations of a stated solution. This latter practice is especially important (just because a mathematical explanation exists does not mean it is correct), and in order to understand a correct explanation students need should also understand why alternative explanations are incorrect.

Analytical Examples:

1. The force on an object of mass m is $F = me^{-bt}$. Find $x(t)$ given that the initial speed and position are zero.

   Solution: By Newton's 2nd law, we have

   $$F = me^{-bt} = m\frac{dx}{dt}. \tag{1}$$

   Solving this equation gives us

   $$\frac{dx}{dt} = e^{-bt} \quad \longrightarrow \quad x(t) = x_0 e^{-bt}. \tag{2}$$

   Given $x_0 = 0$, we thus find $x(t) = 0$.

2. A ball is thrown from the edge of a cliff of height h. At what angle should it be thrown so that it travels a maximum horizontal distance? Assume the ground below the cliff is level.

   Solution: We assume the ball is thrown with speed v_0 and angle θ. We begin with the kinematic equations

   $$x(t) = v_0 \cos\theta t \tag{3}$$
   $$y(t) = v_0 \sin\theta t - \frac{1}{2}gt^2. \tag{4}$$

   Solving for $y(t_f) = 0$ gives us $t_f = 2v_0 \sin\theta/g$. Plugging this into the $x(t)$ equation gives us

   $$x(t_f) = 2v_0^2 \sin\theta\cos\theta/g = v_0^2\cos 2\theta/g. \tag{5}$$

   Differentiating $x(t_f)$ with respect to θ and setting the result to zero we find

   $$\frac{d}{d\theta}x(t_f) = 0 = -2v_0^2\sin(2\theta)/g. \tag{6}$$

   Thus $2\theta = 0, \pi$, and the distance is maximized for $\theta = 0$ or $\theta = \pi/2$.

Conceptual Examples:

1. (From Physics - S1a:) Alan and Betty are studying physics together. Determine if Alan is correct.

   Alan : I think Newton's 3rd law is wrong.

   Betty: . . . um . . . why?

   Alan: Well, think about someone walking. I know there's an equal and opposite force between the ground and me. That makes the net force zero, so I shouldn't go anywhere but I clearly am able to walk places, so the law must be wrong, right?

2. (From Morin's Introduction to Classical Mechanics) A train of length L (measured when it is at rest) travels past you at speed v. A person on the train stands at the front, next to a clock that reads zero. At this moment in time (as measured by you), a clock ta the back of the train reads Lv/c^2. Evaluate whether the following statement can be true or not true. Explain either case you decide on.

   > ”The person at the front of the train can leave the front right after the clock there reads zero, and then turn to the back and get there right before the clock there reads Lv/c^2. You (on the ground) will therefore see the person simultaneously at both the front and the back of the train when the clocks there read zero and Lv/c^2, respectively.”

- **Find the logical/assumptive gap:** The student is given an analytical derivation of a result in physics and wherever there is a gap (i.e., some missing step or assumption) in the derivation, the student must identify the gap, generate a question which isolates it, and answer the question.

  This type of problem supplies practice in finding holes in analytical derivation and using these holes to create new questions and ways to extend the derivation. Also serves as practice in formulating well defined questions.

Example Problem: The following calculation derives the initial angle dependent angular frequency of a pendulum. Identify the missing aspects and formulate these missing aspects as questions which if answered make the calculation more complete:

A pendulum is swinging with total energy $E_0 = mg\ell(1 - \cos\theta_0)$. Thus we know the angular velocity of the pendulum is

$$\frac{d\theta}{dt} = \sqrt{\frac{g}{\ell}}\sqrt{2(\cos\theta - \cos\theta_0)}. \tag{7}$$

We then find the angular frequency as a function of initial angle is

$$\omega(\theta_0) = \frac{\omega_0}{2\pi\Omega(\theta_0)\sqrt{2}} \tag{8}$$

where $\omega_0 = \sqrt{g/\ell}$ and

$$\Omega(\theta_0) = \int_0^{\theta_0} d\theta \, \frac{1}{\sqrt{\cos\theta - \cos\theta_0}}. \tag{9}$$

Solution: The derivation is missing steps between each successive result. To move from the statement of the total energy of the system toward Eq.(7), we need to ask (and answer):
– What is the total energy in terms of angular velocity?
– What does conservation of energy require for this system?
– What is the angular velocity in terms of initial angle and current angle?

To move from Eq.(7) to Eq.(8) and Eq.(9), we need to ask (and answer):
– What is the period of the pendulum motion?
– How is period related to angular frequency?
– What is the quantity of units $\sec^{-1}$ we can define as the angle-independent angular frequency ω_0.

- **Write an Essay/Paragraph:** The communicative aspects of physics are criminally absent in most physics courses. There is great educational benefit in having students explain exclusively through words what their thought processes are. Such explanations can clarify student thinking by hopefully making them more cognizant of the logical errors (at least at the level of argument) they are making.

Example Problem: Explain (primarily through words) how one can obtain the general solution to the simple harmonic oscillator (SHO) equation of motion.

Solution: The analytically simplest way to obtain the general solution to the SHO equation of motion is to note that the phenomena or systems the differential equation describes all exhibit prolonged oscillatory behavior. The simplest function which exhibit that behavior are sine and cosine functions. The periods of these functions are 2π if their arguments just consist of the independent variable, and are $2\pi/A$ if the arguments are some parameter A times the independent variable. Because the differential equation is a linear equation, to find the general solution we add the possible solutions as linear combinations. Doing so gives us the general solution as a linear combination of sine and cosine functions.

- **Concept Map:** In order for knowledge to be effectively retained and employed it should be well organized. For students, this means they should be cognizant of how physical results/predictions are related to the principles and techniques through which they are obtained. A visual is useful in helping students understand the structure of a subject, specifically how various ideas relate to each other.

  In incorporating concept maps into course work, you can begin giving students a list of terms and asking them to connect them as the logical development of the subject entails. Later on you can just give them the starting and ending points of a derivation and have them fill in the body of the concept map (searching for relevant terms and determining how they're connected) themselves.

Example Problem: Using the following concepts/results, create (**and explain**) a concept map which shows how Maxwell's equations lead to the Abraham Lorentz formula for the radiative reaction force:

— Maxwell's Equations

— Theory of Green's Functions
— Poynting Vector
— Liénard-Wierchat potential for Φ and $\vec{A}$
— General retarded solution for Φ and $\vec{A}$ of local charge and current densities
— Inhomogeneous wave equations for Φ and $\vec{A}$
— Current and Charge densities of point charge
— Liénard Generalization of Larmour Formula
— Electric field of a moving point charge;
— Abraham-Lorentz Radiation Reaction Force
— Definition of Electric and Magnetic fields in terms of potentials
— Lamour Formula

Solution: In moving from Maxwell's Equations to the Abraham-Lorentz Radiation Reaction Force on a moving charged particle, one must employ the inhomogeneuous wave equations for the potentials. These equations yield retarded solutions for Φ and $\vec{A}$ given charge and current distributions. Once we consider the charge and current densities of a moving point particle, we obtain the Lienard-Wierchert potentials (the Φ and the $\vec{A}$) of classical electromagnetism. Using the gauge invariant definition of the electric field, we can then obtain the electric field (and the magnetic field) of a moving point particle. Applying the Poynting Vector to this result gives us the Larmour formula for the power radiated from an accelerating charged particle. This power can be interpreted as a force acting on the particle, which thus gives us the Abraham-Lorentz Radiation Reaction Force.

References

[1] D. Fleisch, *A Student's guide to Maxwell's equations*. Cambridge University Press, 2008.

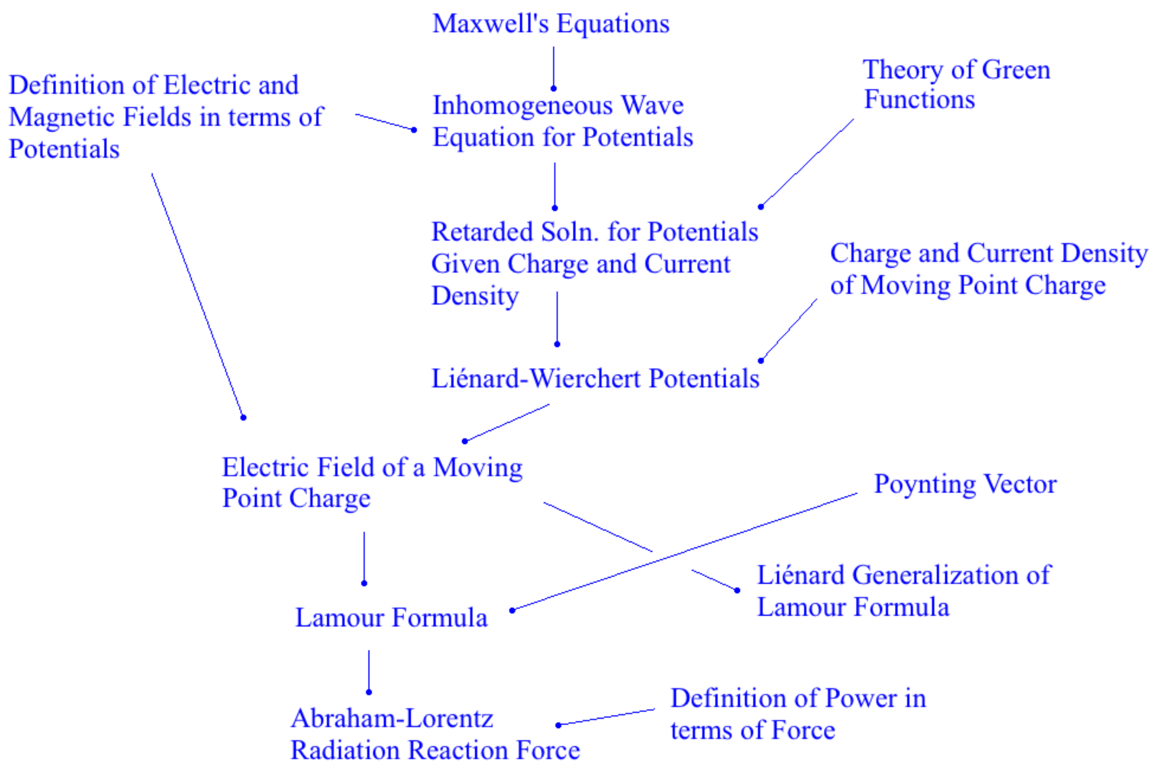

Figure 3: Concept Map for moving from Maxwell's Equations to the Abraham-Lorentz Radiation Reaction Force. The filled circles at the end of lines stand in for arrow heads and mean that the node closest to the filled circle is derived from the node at the opposite end of the line.