

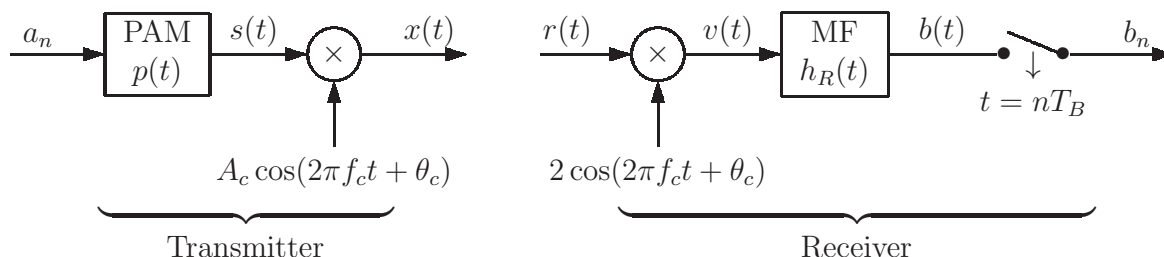
Lab 9: M -ary Amplitude and Frequency Shift Keying, Signal Space

1 Introduction

Amplitude modulation (AM) can easily be used for the transmission of digital data if the (analog) message signal $m(t)$ is replaced by a (digitally) pulse amplitude modulated (PAM) signal $s(t)$. The simplest method to transmit binary data is on-off keying (OOK), whereby the transmitter is off for sending 0's, and a carrier at frequency f_c is transmitted for sending 1's (or vice versa). If the carrier oscillator at the transmitter is just turned on and off, then the phase for each 1 is random and a non-coherent receiver must be used. If phase coherence is maintained at the transmitter, then either coherent or non-coherent reception is possible. Instead of multiplying the carrier oscillator by 0 or 1 for binary data, it can be multiplied by -1 or $+1$, thereby producing phase changes by $\pm 180^\circ$ at the carrier frequency f_c . This method is called binary phase shift keying (BPSK) and requires a coherent receiver. Instead of using a single carrier frequency f_c for transmitting 0's and 1's, two distinct frequencies, e.g., f_{c0} and f_{c1} can be used to transmit 0's and 1's, respectively. The resulting signal is a binary frequency shift keying (BFSK) signal. It can be easily demodulated with a non-coherent receiver. If phase coherence is maintained at the transmitter, it can also be demodulated (with a smaller probability of error) using a coherent receiver. Both, amplitude shift keying (ASK) and frequency shift keying (FSK) can be extended in a straightforward manner from the binary to the M -ary case by using M amplitudes or M carrier frequencies.

1.1 Amplitude Shift Keying

The name "Amplitude Shift Keying" (ASK) refers to a digital bandpass modulation method, whereby the amplitude of a carrier frequency f_c takes on different discrete values (at the sampling time instants), depending on the transmitted DT sequence a_n . The blockdiagram of a coherent ASK communication system looks as follows.



At the transmitter a PAM signal

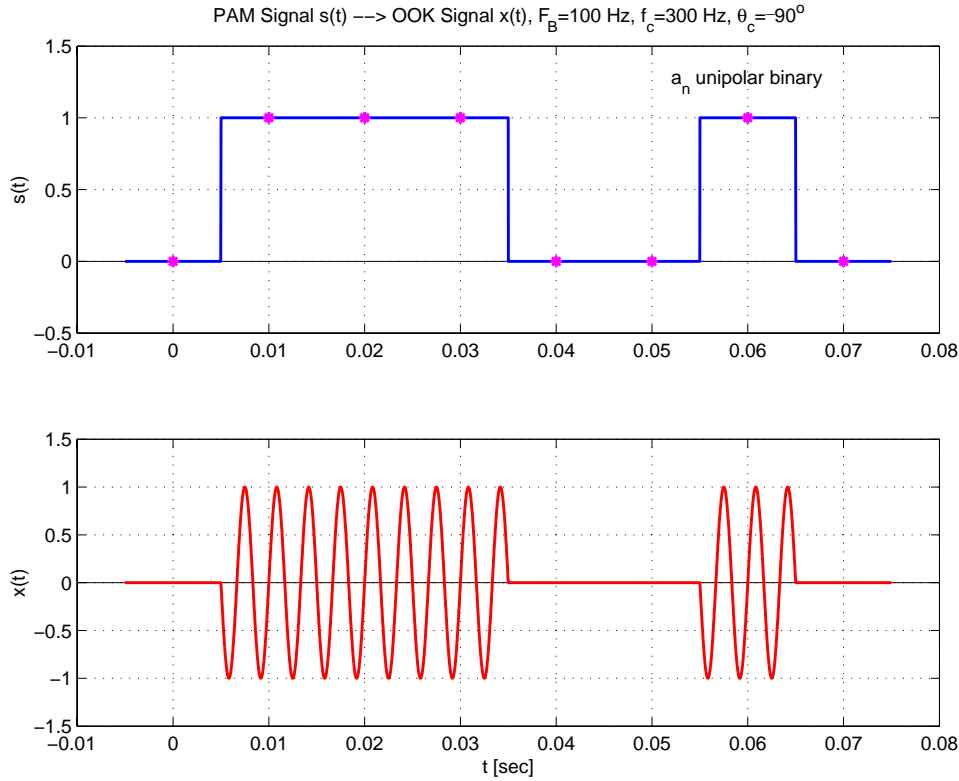
$$s(t) = \sum_{n=-\infty}^{\infty} a_n p(t - nT_B),$$

with baud rate $F_B = 1/T_B$ and pulse $p(t)$ is generated from the (digital) DT sequence a_n . The CT signal $s(t)$ then modulates a carrier with frequency f_c and phase θ_c , so that the transmitted signal is

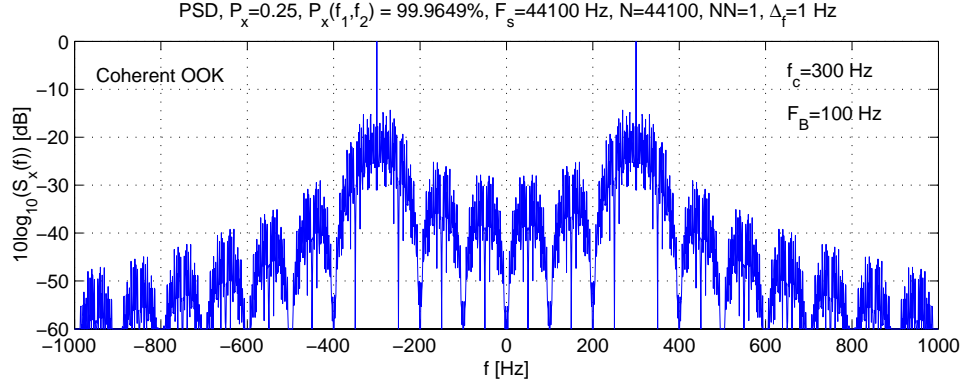
$$x(t) = A_c \sum_{n=-\infty}^{\infty} a_n p(t - nT_B) \cos(2\pi f_c t + \theta_c).$$

At the receiver the signal $r(t)$ is demodulated using a local oscillator that replicates the carrier signal $\cos(2\pi f_c t + \theta_c)$ with exact frequency and phase synchronization. After that, a regular baseband PAM receiver with matched filter (MF), matched to $p(t) * h_{CL}(t)$, where $h_{CL}(t)$ is the lowpass-equivalent channel response (i.e., $h_C(t)$ shifted down in frequency by f_c), is used.

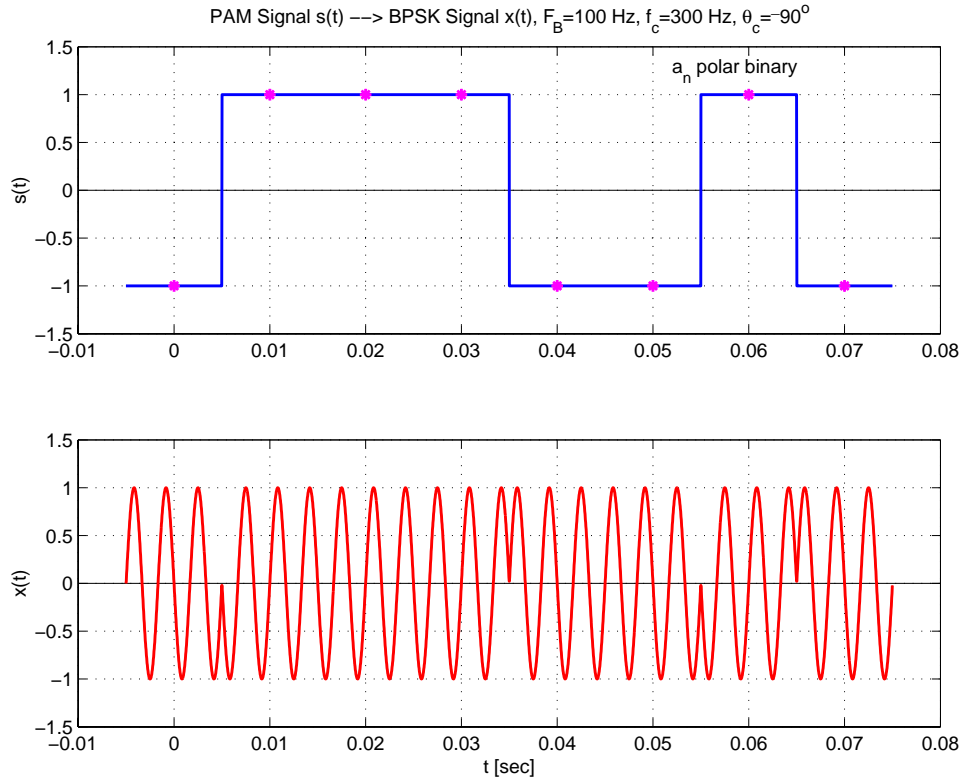
The simplest case of ASK is **on-off keying (OOK)**, which is obtained when a_n is a unipolar binary sequence (i.e., $a_n \in \{0, 1\}$). The following two graphs show $s(t)$ and $x(t)$ for $\{a_n\} = \{0, 1, 1, 1, 0, 0, 1, 0\}$, $F_B = 100$ Hz, rectangular $p(t)$, $f_c = 300$ Hz, and $\theta_c = -90^\circ$.



The PSD of a coherent OOK signal (i.e., an OOK signal for which θ_c does not change over time) with rectangular $p(t)$, $f_c = 300$ Hz, and $F_B = 100$ Hz is shown in the next graph.



If a_n is a polar binary sequence, e.g., $\{a_n\} = \{-1, +1, +1, +1, -1, -1, +1, -1\}$, then $s(t)$ and $x(t)$ look as follows.



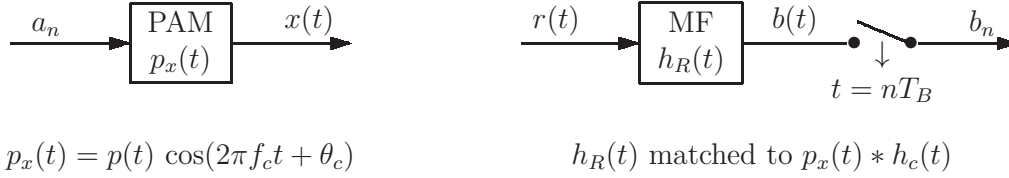
Assuming a rectangular $p(t)$, the carrier is multiplied by either $+1$ or -1 , depending on the data to be transmitted. This leads to phase changes by 180° and because of it this version of ASK is usually called **binary phase shift keying (BPSK)**. Note that for BPSK it is crucial that the transmitter and receiver are phase-synchronized, i.e., BPSK requires a coherent receiver.

Rather than generating a baseband PAM signal first and then using amplitude modulation to make it a bandpass signal at f_c , it is possible to directly generate a bandpass PAM signal

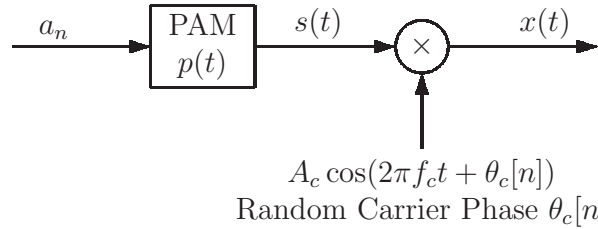
using a pulse $p_x(t)$ that is already shifted to f_c in the frequency domain. Mathematically, this can be expressed as

$$x(t) = \sum_{n=-\infty}^{\infty} a_n p_x(t - nT_B), \quad \text{where} \quad p_x(t) = p(t) \cos(2\pi f_c t + \theta_c).$$

At the receiver one can then use a MF that is directly matched to $p_x(t) * h_c(t)$ where $h_c(t)$ is the impulse response of the channel. The blockdiagram of an ASK transmission system that uses $p_x(t)$ directly at f_c is shown below.



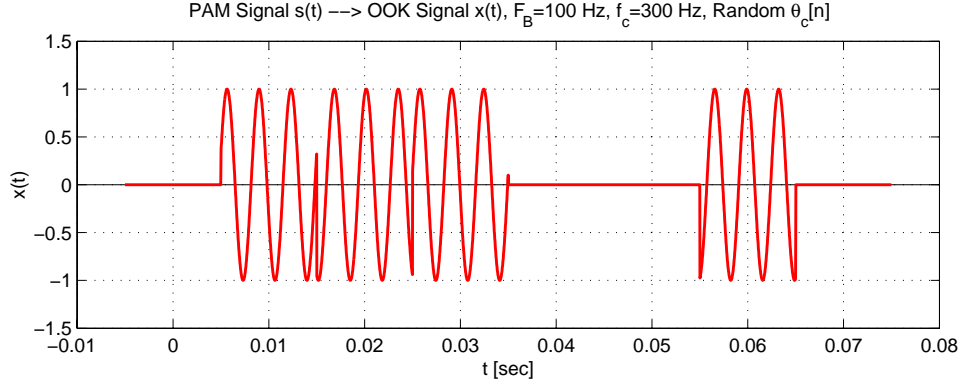
The simplest method to achieve wireless data transmission is to turn an oscillator with frequency f_c on or off, depending on whether a one or a zero is transmitted. Using the morse code for on-off keying, this is how Marconi demonstrated in 1901 that transatlantic wireless communication was possible. What is likely to happen, however, if the oscillator at f_c is turned on and off, is that the carrier phase θ_c takes on random values between 0 and 2π . A model for this is shown in the following blockdiagram.



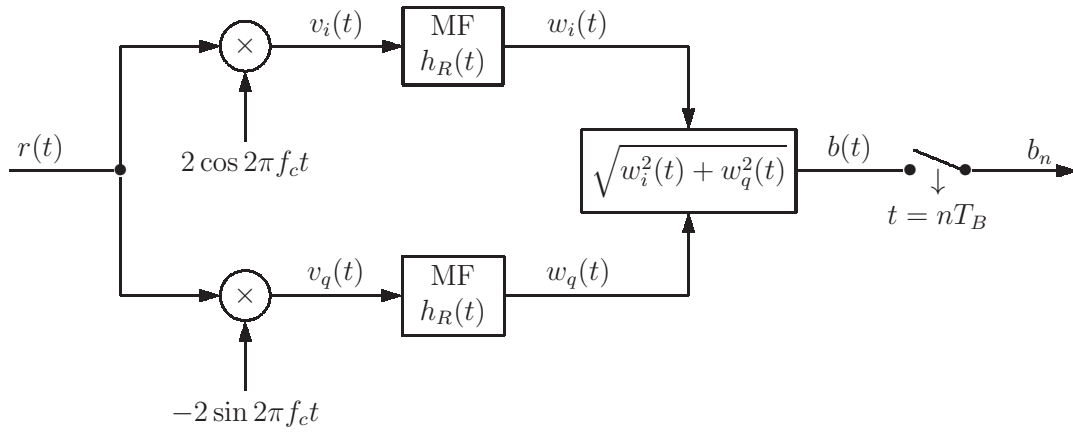
Here it is assumed that for each new symbol a_n a random carrier phase value $\theta_c[n]$ is chosen (even if the oscillator is not turned off between two consecutive ones). Thus

$$\begin{aligned}
 x(t) &= A_c \sum_{n=-\infty}^{\infty} a_n p(t - nT_B) \cos(2\pi f_c t + \theta_c[n]) \\
 &= A_c \left[\sum_{n=-\infty}^{\infty} a_n \cos \theta_c[n] p(t - nT_B) \cos 2\pi f_c t - \sum_{n=-\infty}^{\infty} a_n \sin \theta_c[n] p(t - nT_B) \sin 2\pi f_c t \right],
 \end{aligned}$$

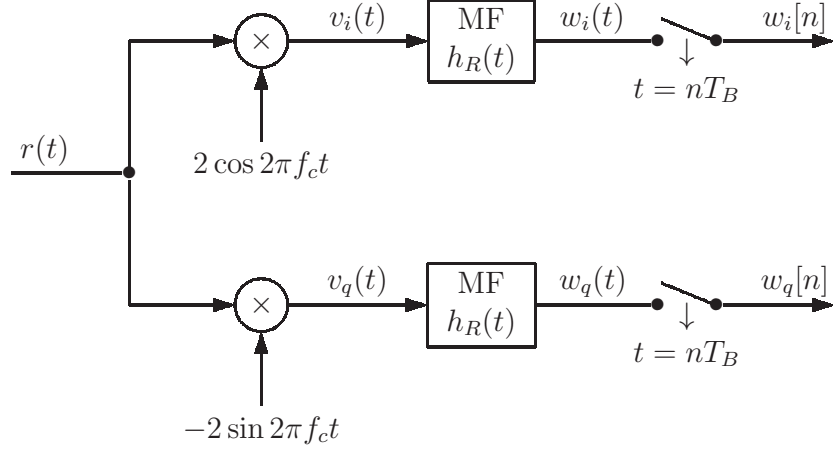
where $\theta_c[n] \in [0, 2\pi)$. Because the transmitted carrier phase of such a signal is not constant, the resulting ASK signal will be called **non-coherent ASK**. An example when $\{a_n\} = \{0, 1, 1, 1, 0, 0, 1, 0\}$, $F_B = 100$ baud, and $f_c = 300$ Hz is shown in the next graph.



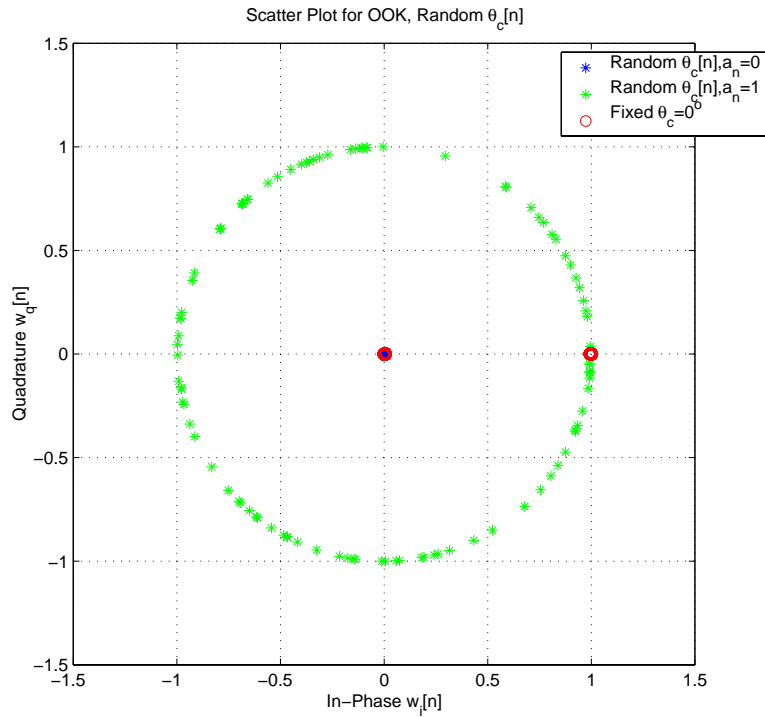
To demodulate a signal with (unknown) random phase, a **matched filter envelope detector (MFED)**, such as the one shown in the following blockdiagram, can be used.



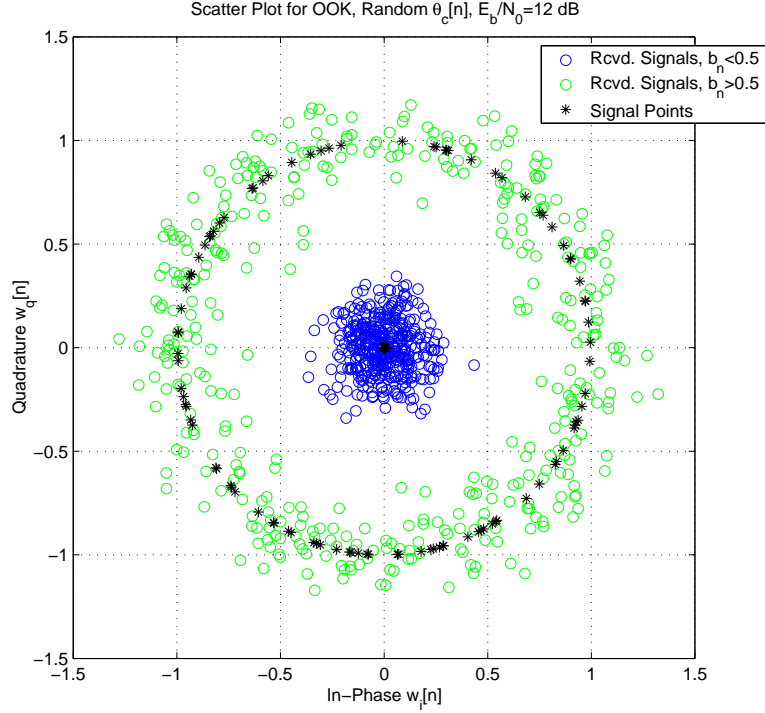
In the process of computing $b(t) = \sqrt{w_i^2(t) + w_q^2(t)}$, the dependence on the carrier phase (and in fact the dependence on the exact carrier frequency f_c) drops out. The interesting observation to be made is that, because of the orthogonality of $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$, the in-phase component $w_i(t)$ and the quadrature component $w_q(t)$ span a 2-dimensional space. Upon sampling at times $t = nT_B$, as shown in the blockdiagram below, this becomes a **2-dimensional signal space** spanned by $w_i[n]$ (real part in complex plane) and $w_q[n]$ (imaginary part in complex plane).



Thus, coherent ASK uses a one-dimensional signal space and noncoherent ASK uses a two-dimensional signal space. This is shown for OOK in the following figure which is called a **scatter plot**.



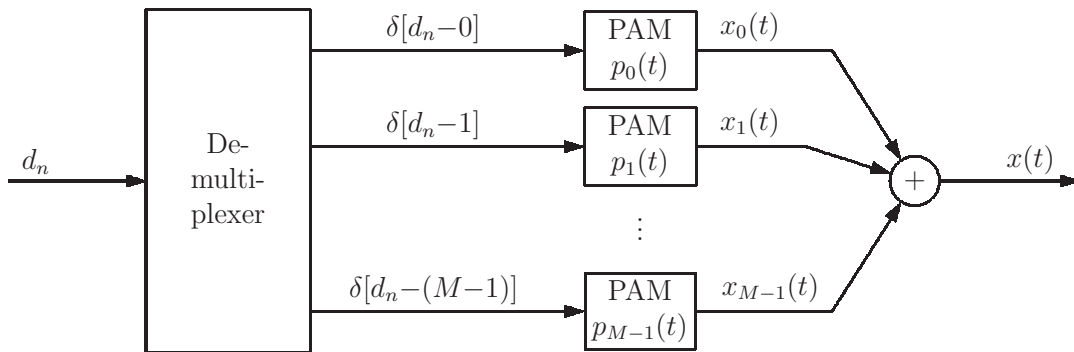
Note that for noncoherent ASK the signal “points” are actually points on concentric circles. If the received signal is noisy, then the noise manifests itself as deviations of these points from the concentric circles, as shown for OOK with SNR $E_b/\mathcal{N}_0 = 12$ dB in the scatter plot below.



The black stars are the signal points of the noncoherent ASK signal at the transmitter. The blue and green circles are the received noisy signal points with blue indicating a decision for a received 0 and green indicating a decision for a received 1. As the SNR decreases, the received signal points deviate more and more from their nominal positions at the origin or on the circle with radius 1, and the probability of symbol error, $P_s(\mathcal{E})$, approaches 0.5.

1.2 Frequency Shift Keying

Rather than using a digital DT sequence to change the amplitude of a carrier, the frequency of the carrier can be changed by the data sequence to be transmitted. The resulting digital bandpass signaling method is called **frequency shift keying (FSK)**. One way to implement M -ary FSK is shown in the following block diagram.



An M -ary DT sequence d_n , with $d_n \in \{0, 1, \dots, M-1\}$, is first demultiplexed into M binary sequences, the first one having 1's in all positions where d_n is zero and 0's everywhere else, the second one having 1's in all positions where d_n is one and 0's everywhere else, etc. The M sequences are then converted to OOK signals of the form

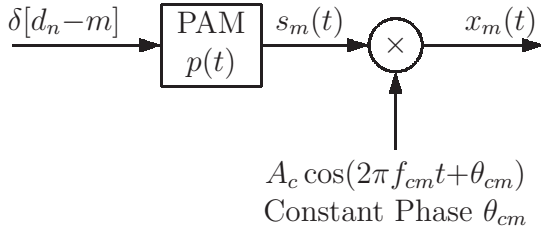
$$x_m(t) = \sum_{n=-\infty}^{\infty} \delta[d_n - m] p_m(t - nT_B), \quad m = 0, 1, \dots, M-1, \quad \delta[k] = \begin{cases} 1, & k = 0, \\ 0, & \text{otherwise,} \end{cases}$$

and added up to produce the FSK signal $x(t)$ as

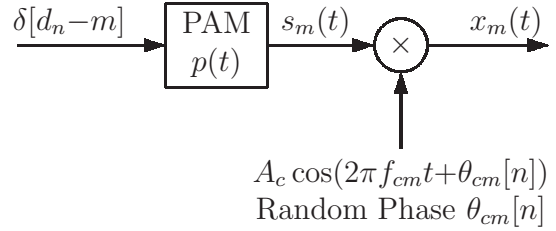
$$x(t) = \sum_{m=0}^{M-1} x_m(t) = \sum_{m=0}^{M-1} \sum_{n=-\infty}^{\infty} \delta[d_n - m] p_m(t - nT_B).$$

Depending on whether the individual OOK signals are coherent or non-coherent, one can distinguish between **coherent FSK** and **non-coherent FSK**. The blockdiagrams below show how to implement the individual bandpass PAM functions $p_m(t)$, $m = 0, 1, \dots, M-1$, in each case.

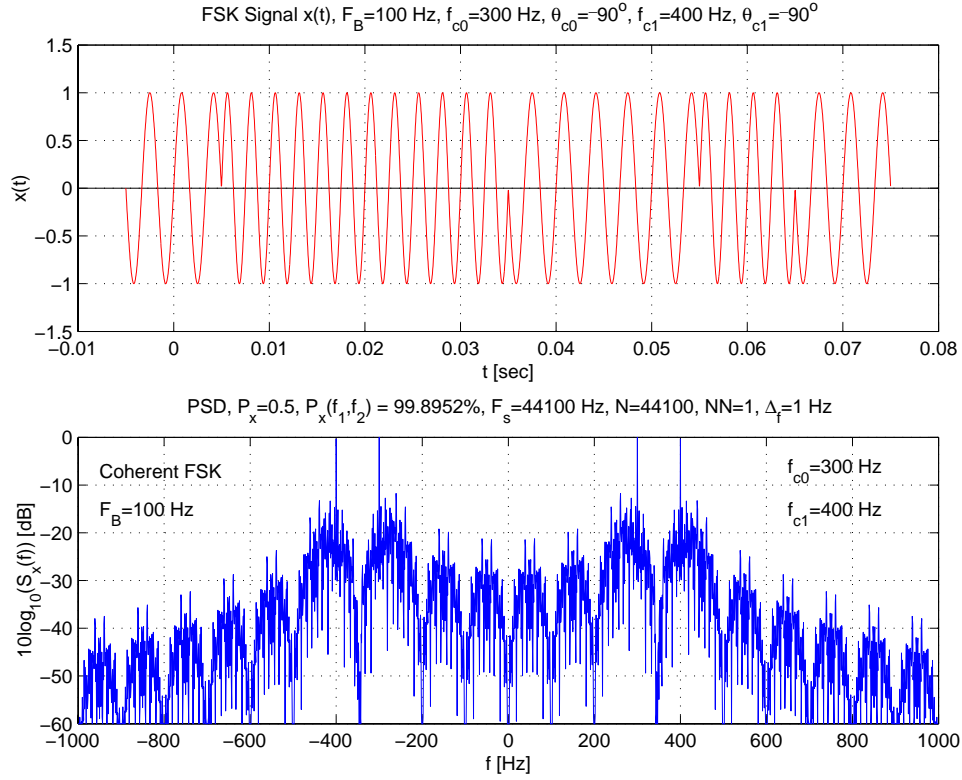
PAM $p_m(t)$ for Coherent FSK



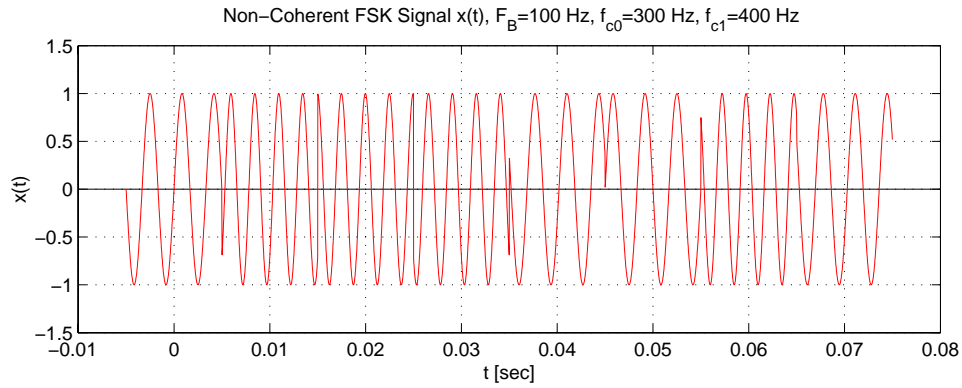
PAM $p_m(t)$ for Non-Coherent FSK



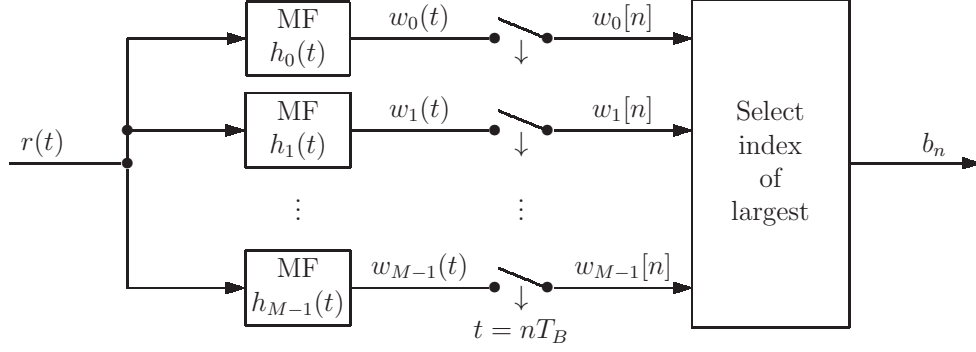
Thus, for coherent FSK $p_m(t) = A_c p(t) \cos(2\pi f_{cm}t + \theta_{cm})$, and for non-coherent FSK $p_m(t) = A_c p(t) \cos(2\pi f_{cm}t + \theta_{cm}[n])$. Note that, when the FSK signal is made up from the outputs of individual oscillators, $x(t)$ may contain phase jumps at the symbol boundaries, as can be seen in the following graph that shows an example of coherent binary FSK with rectangular $p(t)$, $F_B = 100$ baud, $f_{c1} = 300$ Hz, $\theta_{c1} = -90^\circ$, $f_{c2} = 400$ Hz, $\theta_{c2} = -90^\circ$.



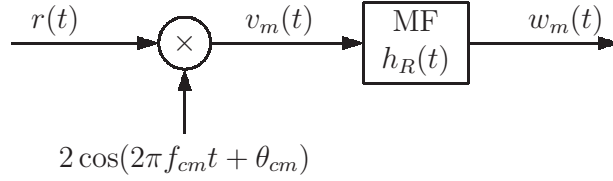
The upper graph shows $x(t)$ in the time domain for $\{d_n\} = \{0, 1, 1, 1, 0, 0, 1, 0\}$, and the lower graph shows the PSD $S_x(f)$ when d_n is a random binary signal with equally likely 0's and 1's. The next graph shows $x(t)$ for the same data and parameters, but using a non-coherent FSK transmitter.



The block diagram below shows the basic structure of a receiver for M -ary FSK.



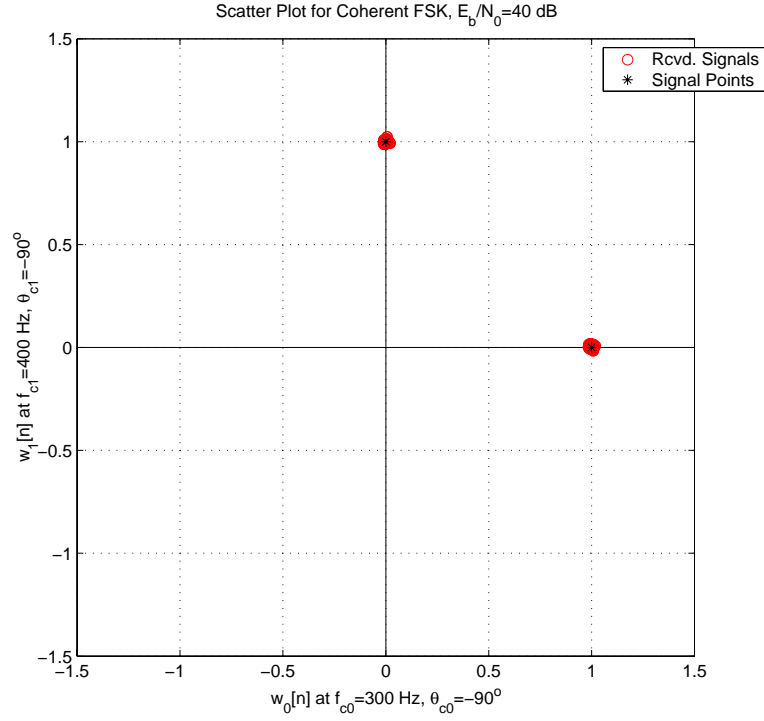
There is a matched filter at each of the M FSK frequencies f_{c0}, \dots, f_{cM-1} . Coherent FSK can be received with coherent matched filters. One way to implement a coherent MF when f_{cm} and θ_{cm} are fixed and known is shown in the following blockdiagram.



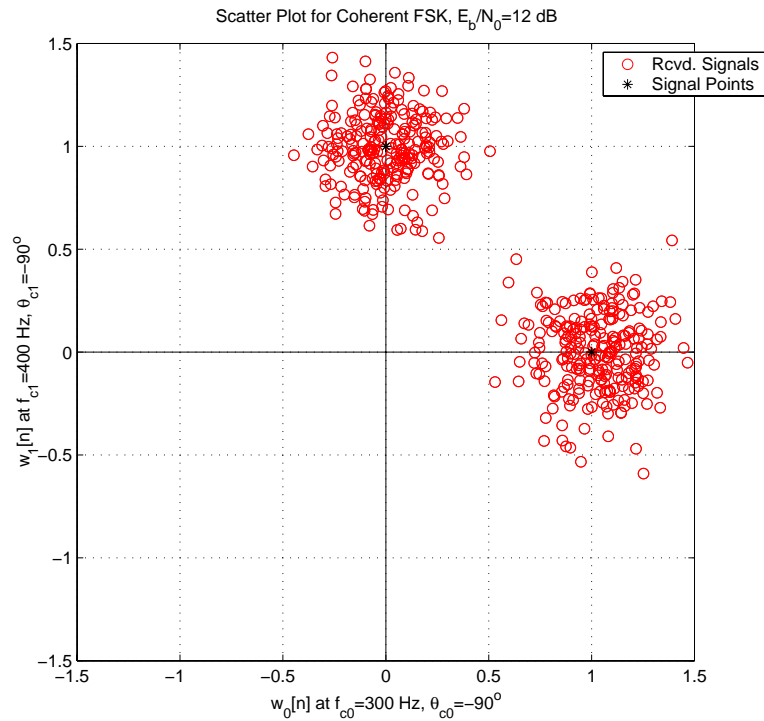
An interesting question is how close together any two adjacent frequencies, say f_{cl} and $f_{cm} = f_{cl} + \Delta_f$, should be chosen. By making Δ_f small, the total bandwidth used by M -ary FSK can be reduced, but if Δ_f is too small, then the normalized distance between symbols using different frequencies becomes too small and thus the probability of error increases. As a general criterion, one usually tries to make all M symbols in M -ary FSK **orthogonal**, i.e., assuming $p(t)$ is a rectangular pulse of width T_B , one selects Δ_f in $f_{cm} = f_{cl} + \Delta_f$ such that

$$\int_{(n-1/2)T_B}^{(n+1/2)T_B} \cos(2\pi f_{cl}t + \theta_{cl}) \cos(2\pi f_{cm}t + \theta_{cm}) dt = 0, \quad \text{for all integers } n.$$

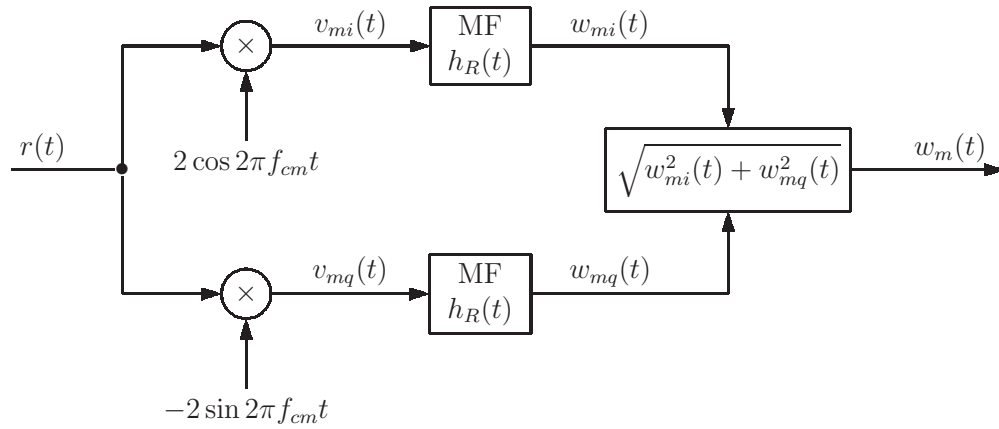
The scatter plot in the following figure shows $w_0[n]$ versus $w_1[n]$ for coherent binary FSK satisfying the orthogonality condition.



For the graph above a FSK signal with high SNR (40 dB) was used so that the nominal signal points (black *) and the received signal points (red o) coincide. The graph below shows how this changes when the SNR is reduced to $E_b/N_0 = 12$ dB.

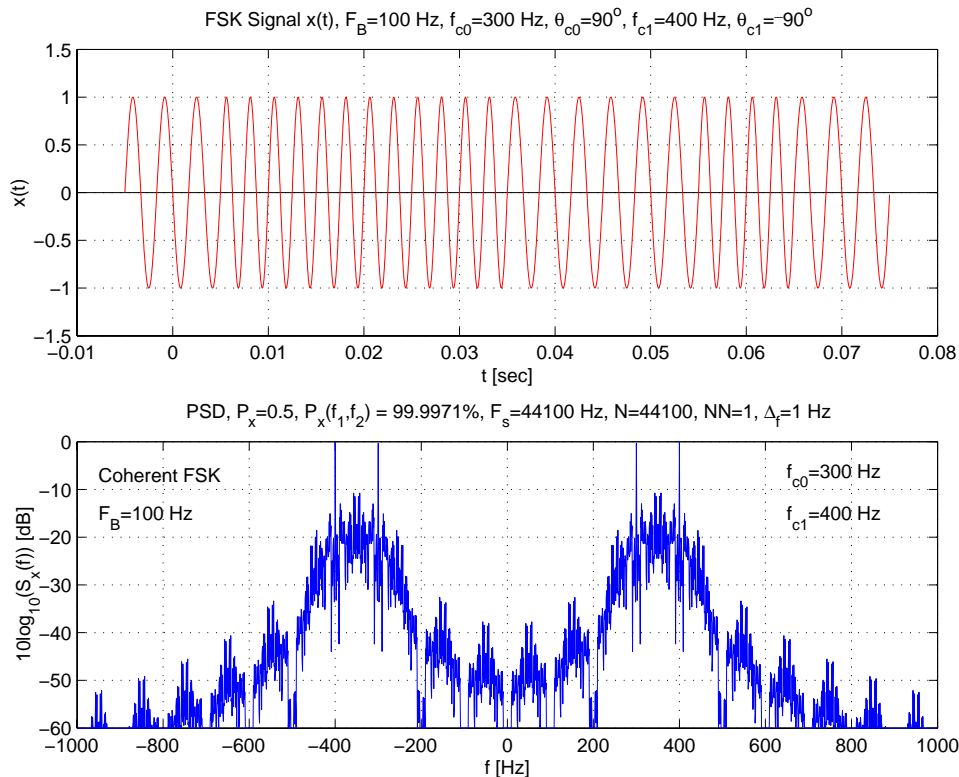


If the carrier phase is random and/or unknown to the receiver, then matched filter envelope detectors (MFED) of the form shown in the blockdiagram below need to be used.



In this case, a scatter plot of binary FSK would have to be drawn in 4-dimensional space. Looking at just $w_{0i}[n]$ versus $w_{0q}[n]$, or $w_{1i}[n]$ versus $w_{1q}[n]$ yields signal “points” that either lie at the origin or on a circle, centered at the origin.

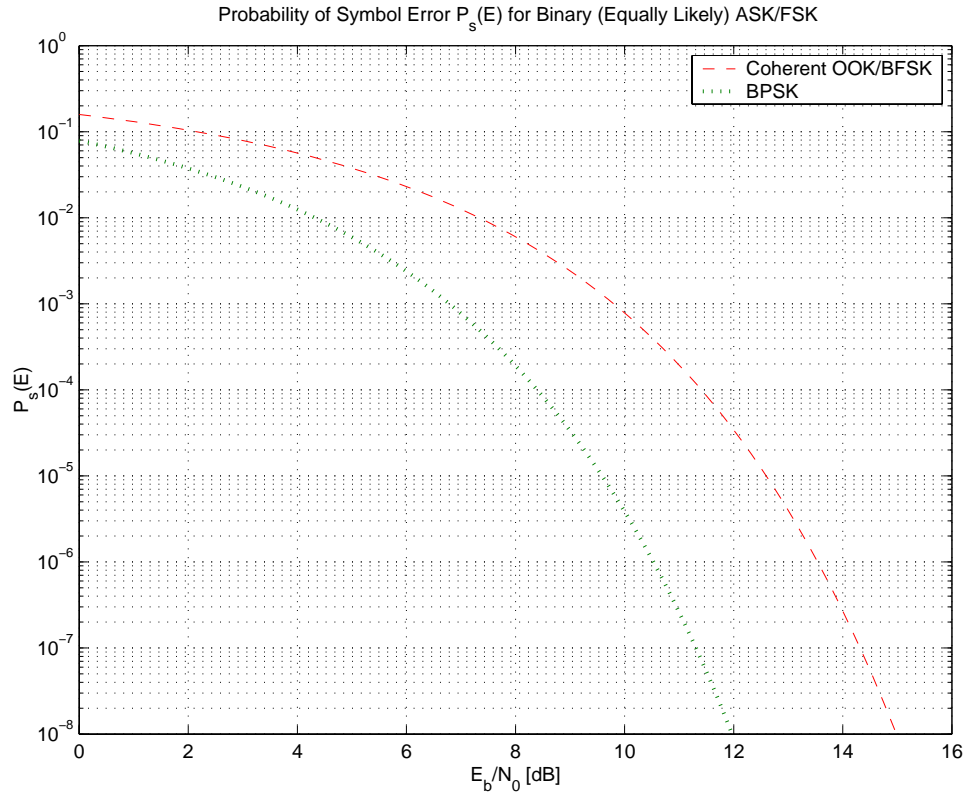
One more version of FSK that is actually preferred in practice is **continuous phase FSK (CPFSK)**. This can be obtained from coherent FSK by choosing the frequencies f_{cm} and the phases θ_{cm} in such a way that phase jumps are avoided at the symbol boundaries. An example of binary CPFSK and its PSD is shown in the following two graphs.



As can be seen in the lower graph, the absence of phase jumps in $x(t)$ yields a PSD $S_x(f)$ that decreases rapidly for frequencies below f_{c0} and above f_{cM-1} (in the M -ary case), which reduces interference between communication systems that use adjacent bands.

1.3 Probability of Error

The graph below shows the probability $P_s(\mathcal{E})$ of error for OOK, BPSK and BFSK when coherent receivers are used.



In general a non-coherent receiver will have a larger $P_s(\mathcal{E})$ for a given E_b/N_0 . The reason for this is that a coherent OOK/ASK receiver at some carrier frequency f_c (or f_{ci} for the i -th FSK frequency) only needs to look at the in-phase component of the demodulation, whereas a non-coherent receiver needs to look at both the in-phase and the quadrature components. Thus, a coherent receiver only picks up noise from the in-phase channel, whereas a non-coherent receiver picks up noise from both the in-phase and the quadrature channels.

1.4 Digital Modulation in GNU Radio

– To be completed –

2 Lab Experiments

E1. Amplitude Shift Keying. (a) Start a new Python module called `keyfun.py` and use your `pam12` function from the `pamfun` module as a building block to complete the ASK transmitter function `askxmtr` whose header is shown below.

```

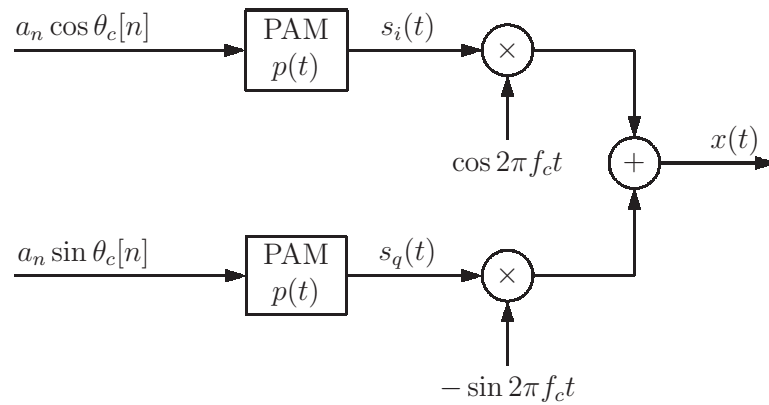
# File: keyfun.py
# Functions for amplitude/frequency/phase shift keying
# ASK, FSK, PSK and hybrid APSK
from pylab import *
import ecen4652 as ecen
import pamfun
import filtfun

def askxmtr(sig_an,Fs,ptype,pparms,xtype,fcparms):
    """
    Amplitude Shift Keying (ASK) Transmitter for
    Coherent ('coh') and Non-coherent ('noncoh') ASK Signals
    >>>> sig_xt,sig_st = askxmtr(sig_an,Fs,ptype,pparms,xtype,fcparms) <<<<<
    where sig_xt: waveform from class sigWave
        sig_xt.signal(): transmitted ASK signal, sampling rate Fs
            x(t) = s(t)*cos(2*pi*fc*t+(pi/180)*thetac)
        sig_xt.timeAxis(): time axis for x(t), starts at t=-TB/2
        sig_st: waveform from class sigWave
        sig_st.signal(): baseband PAM signal s(t) for 'coh'
        sig_st.signal(): st = sit + 1j*sqt for 'noncoh'
        sit: PAM signal of an*cos(pi/180*thetacn)
        sqt: PAM signal of an*sin(pi/180*thetacn)
        xtype: Transmitter type from list {'coh','noncoh'}
        sig_an: sequence from class sigSequ
        sig_an.signal() = [an] for {'coh'}
        sig_an.signal() = [[an],[thetacn]] for {'noncoh'}
        an: N-symbol DT input sequence a_n, 0<=n<N
        thetacn: N-symbol DT sequence theta_c[n] in degrees,
            used instead of thetac for {'noncoh'} ASK
        sig_an.get_FB(): baud rate of a_n (and theta_c[n]), TB=1/FB
        Fs: sampling rate of x(t), s(t)
        ptype: pulse type from list
            ['man','rcf','rect','rrcf','sinc','tri']
        pparms = [] for {'man','rect','tri'}
        pparms = [k, alpha] for {'rcf','rrcf'}
        pparms = [k, beta] for {'sinc'}
        k: "tail" truncation parameter for {'rcf','rrcf','sinc'}
            (truncates at -k*TB and k*TB)
        alpha: Rolloff parameter for {'rcf','rrcf'}, 0<=alpha<=1
        beta: Kaiser window parameter for {'sinc'}
        fcparms = [fc, thetac] for {'coh'}
        fcparms = [fc] for {'noncoh'}
        fc: carrier frequency in Hz
        thetac: carrier phase in deg (0: cos, -90: sin)
    """

```

To generate noncoherent ASK signals with a random carrier sequence $\theta_c[n]$, use in-phase

and quadrature PAM signals $s_i(t)$ and $s_q(t)$ and QAM modulation as shown in the following blockdiagram.



Test your transmitter by generating a short (about 10 symbol times) random coherent OOK signal and a short random noncoherent OOK signal and displaying them in the time domain. Use $F_s = 44100$ Hz, $F_B = 100$ baud, $f_c = 300$ Hz, and a rectangular pulse $p(t)$ of width $T_B = 1/F_B$. Generate a uniformly distributed random phase for noncoherent OOK using the `rand` function.

(b) In the `keyfun.py` module, complete the following ASK receiver function, called `askrcvr`. The goal is to be able to use it to receive coherent and noncoherent ASK signals, and to make scatter plots.


```

def askrcvr(sig_rt,rtype,fcparms,FBparms,ptype,pparms):
    """
    Amplitude Shift Keying (ASK) Receiver for
    Coherent ('coh') and Non-coherent ('noncoh') ASK Signals
    >>>> sig_bn,sig_bt,sig_wt,ixn =
            askrcvr(sig_rt,rtype,fcparms,FBparms,ptype,pparms) <<<<
    where sig_bn: sequence from class sigSequ
            sig_bn.signal(): received DT sequence b[n]
            sig_bt: waveform from class sigWave
            sig_bt.signal(): received 'CT' PAM signal b(t)
            sig_wt: waveform from class sigWave
            sig_wt.signal(): wt = wit + 1j*wqt
            wit:      in-phase component of b(t)
            wqt:      quadrature component of b(t)
            ixn:      sampling time indexes for b(t)->b[n], w(t)->w[n]
            sig_rt: waveform from class sigWave
            sig_rt.signal(): received (noisy) ASK signal r(t)
            sig_rt.timeAxis(): time axis for r(t)
            rtype:    receiver type from list ['coh','noncoh']
            fcparms = [fc, thetac] for {'coh'}
            fcparms = [fc]         for {'noncoh'}
            fc:       carrier frequency in Hz
            thetac:   carrier phase in deg (0: cos, -90: sin)
            FBparms = [FB, dly]
            FB:       baud rate of PAM signal, TB=1/FB
            dly:      sampling delay for b(t)->b[n], fraction of TB
                     sampling times are t=n*TB+t0 where t0=dly*TB
            ptype:    pulse type from list
                     ['man','rcf','rect','rrcf','sinc','tri']
            pparms = [] for 'man','rect','tri'
            pparms = [k, alpha] for {'rcf','rrcf'}
            pparms = [k, beta] for {'sinc'}
            k:        "tail" truncation parameter for {'rcf','rrcf','sinc'}
                     (truncates at -k*TB and k*TB)
            alpha:    Rolloff parameter for {'rcf','rrcf'}, 0<=alpha<=1
            beta:     Kaiser window parameter for {'sinc'}

    """

```

Use the `pamrcvr10` function in module `pamfun` as part of the ASK receiver. Test `askrcvr` together with `askxmtr` using random (unipolar) binary data and the parameters given in part (a).

(c) Let $F_s = 44100$ Hz, $F_B = 100$ baud, $f_c = 2100$ Hz, and let $p(t)$ be a rectangular pulse of width T_B . Use random binary data to produce (i) a coherent OOK signal, (ii) a noncoherent OOK signal, and (iii) a BPSK signal, each of duration 2 sec. Plot and compare the PSDs for all three cases. Then use the `wt` and `wn=wt[ixn]` outputs of the `askrcvr` function to make scatter plots for the three signals and compare them. Are there any interesting spectral lines

if you look at the PSDs of the squared ASK signals? How do things change if you use a triangular pulse $p(t)$ (of total width $2T_B$ from $-T_B$ to $+T_B$) instead of the rectangular pulse?

(d) The wav files `asksig901.wav`, `asksig902.wav` and `asksig903.wav` are binary ASK signals which contain 8-bit, LSB-first, ASCII signals. Analyze the three signals and extract the text messages. **Hint:** The signals in the wav files have been scaled so that their amplitude (including noise) is less than or equal to one. Use eye diagrams and/or scatter plots to determine the proper threshold below which the receiver decides that a 0 was received and above which it decides that a 1 was received.

E2. Frequency Shift Keying. (a) For the Python module `keyfun.py`, write a function called `fskxmtr` which implements an M -ary FSK transmitter for either coherent, noncoherent, or continuous phase FSK. The header of this function is shown below.

```

def fskxmtr(M,dnthcn,FB,Fs,ptype,pparms,xtype,fcparms):
    """
    M-ary Frequency Shift Keying (FSK) Transmitter for
    Choherent ('coh'), Non-coherent ('noncoh'), and
    Continuous Phase ('cpfsk') FSK Signals
    >>>> sig_xt = fskxmtr(M,sig_dn,Fs,ptype,pparms,xtype,fcparms) <<<<
    where sig_xt: waveform from class sigWave
        sig_xt.signal(): transmitted FSK signal, sampling rate Fs
        sig_xt.timeAxis(): time axis for x(t), starts at t=-TB/2
        M: number of distinct symbol values in d[n]
        xtype: Transmitter type from set {'coh','noncoh','cpfsk'}
        sig_dn: sequence from class sigSequ
        sig_dn.signal() = [dn] for ['coh','cpfsk']
        sig_dn.signal() = [[dn],[thetacn]] for ['noncoh']
        dn: M-ary (0,1,...,M-1) N-symbol DT input sequence d_n
        thetacn: N-symbol DT sequence theta_c[n] in degrees,
                used instead of thetac0..thetacM-1 for {'noncoh'} FSK
        sig_dn.get_FB(): baud rate of d_n (and theta_c[n]), TB=1/FB
        Fs: sampling rate of x(t)
        ptype: pulse type from set
                {'man','rcf','rect','rrcf','sinc','tri'}
        pparms = [] for {'man','rect','tri'}
        pparms = [k alpha] for {'rcf','rrcf'}
        pparms = [k beta] for {'sinc'}
        k: "tail" truncation parameter for {'rcf','rrcf','sinc'}
            (truncates p(t) to -k*TB <= t < k*TB)
        alpha: Rolloff parameter for {'rcf','rrcf'}, 0<=alpha<=1
        beta: Kaiser window parameter for {'sinc'}
        fcparms = [[fc0,fc1,...,fcM-1],[thetac0,thetac1,...,thetacM-1]]
                for {'coh'}
        fcparms = [fc0,fc1,...,fcM-1] for {'noncoh'}
        fcparms = [fc, deltaf] for {'cpfsk'}
        fc0,fc1,...,fcM-1: FSK (carrier) frequencies
                        for {'coh','noncoh'}
        thetac0,thetac1,...,thetacM-1: FSK (carrier) phases in deg
                        (0: cos, -90: sin) for {'coh'}
        fc: carrier frequency for {'cpfsk'}
        deltaf: frequency spacing for {'cpfsk'}
                for dn=0 -> fc, dn=1 -> fc+deltaf,
                dn=2 -> fc+2*deltaf, etc
    """

```

Test `fskxmtr` by recreating the three (time domain) sample graphs for $F_B = 100$ baud, $f_{c0} = 300$ Hz and $f_{c1} = 400$ Hz, which were given in the introduction for binary coherent FSK, noncoherent FSK, and CPFSK. Use $d_n = \{0, 1, 1, 1, 0, 0, 1, 0\}$ and (for the second graph) $\theta_c[n] = \{270^\circ, 225^\circ, 4^\circ, 135^\circ, 250^\circ, 90^\circ, 40^\circ, 240^\circ\}$.

(b) In the `keyfun.py` module, implement the FSK receiver function `fskrcvr` whose header

is given below.

```

def fskrxvr(M,tt,rt,rtype,fcparms,FBparms,ptype,pparms):
    """
    M-ary Frequency Shift Keying (FSK) Receiver for
    Coherent ('coh'), Non-coherent ('noncoh'), and
    Phase Detector ('phdet') FSK Reception
    >>>> sig_bn,sig_wt,ixn =
        fskrxvr(M,sig_rt,rtype,fcparms,FBparms,ptype,pparms) <<<<
    where sig_bn: sequence from class sigSequ
        sig_bn.signal(): received DT sequence b[n]
        sig_wt: waveform from class sigWave
        sig_wt.signal(): wt = [[w0it+1j*w0qt],[w1it+1j*w1qt],...,
                                [wM-1it+1j*wM-1qt]]

        wmit:      m-th in-phase matched filter output
        wmqt:      m-th quadrature matched filter output
        ixn:       sampling time indexes for b(t)->b[n], w(t)->w[n]
        M:         number of distinct FSK frequencies
        sig_rt: waveform from class sigwave
        sig_rt.signal(): received (noisy) FSK signal r(t)
        sig_rt.timeAxis(): time axis for r(t)
        rtype:     receiver type from list {'coh','noncoh','phdet'}
        fcparms = [[fc0,fc1,...,fcM-1],[thetac0,thetac1,...,thetacM-1]]
                   for {'coh'}
        fcparms = [fc0,fc1,...,fcM-1] for {'noncoh'}
        fcparms = [fc, deltaf]         for {'phdet'}
        fc0,fc1,...,fcM-1: FSK (carrier) frequencies
                           for {'coh','noncoh'}
        thetac0,thetac1,...,thetacM-1: FSK (carrier) phases in deg
                                       (0: cos, -90: sin) for {'coh'}
        fc:        carrier frequency for {'cpfsk'}
        deltaf:     frequency spacing for {'cpfsk'}
                   for dn=0 -> fc, dn=1 -> fc+deltaf,
                   dn=2 -> fc+2*deltaf, etc
        FBparms = [FB, dly]
        FB:        baud rate of PAM signal, TB=1/FB
        dly:       sampling delay for wm(t)->wm[n], fraction of TB
                   sampling times are t=n*TB+t0 where t0=dly*TB
        ptype:     pulse type from list
                   {'man','rcf','rect','rrcf','sinc','tri'}
        pparms = [] for {'man','rect','tri'}
        pparms = [k, alpha] for {'rcf','rrcf'}
        pparms = [k, beta] for {'sinc'}
        k:         "tail" truncation parameter for {'rcf','rrcf','sinc'}
                   (truncates at -k*TB and k*TB)
        alpha:     Rolloff parameter for {'rcf','rrcf'}, 0<=alpha<=1
        beta:      Kaiser window parameter for {'sinc'}

    """

```

Depending on the choice of the `rtype`, the receiver should perform demodulation using either M coherent MFs, M MFEDs, or using a phase detector followed by differentiation and a MF. Test all receiver modes with the signals that you generated in (a) when you were testing `fskxmtr`.

(c) Generate a coherent binary FSK signal from random data with equally likely 0's and 1's, using a rectangular $p(t)$, $F_B = 100$ baud, $f_{c0} = 300$ Hz, $\theta_{c0} = 0^\circ$, $f_{c1} = 400$ Hz, and $\theta_{c1} = 0^\circ$. Use a coherent demodulator to produce a scatter plot of $w_0[n]$ versus $w_1[n]$. Change the phase θ_{c1} from 0° to 180° and check whether the signals transmitted at f_{c0} and at f_{c1} remain orthogonal in the signal space spanned by $w_0[n]$ and $w_1[n]$. Is it possible to reduce the frequency spacing $\Delta_f = f_{c1} - f_{c0}$ to a value less than F_B while maintaining orthogonality? Try changing f_{c1} to 350 Hz and vary the phase θ_{c1} again from 0° to 180° .

(d) Use uniformly distributed random M -ary data of length about 2 sec to generate PSD plots of $M = 2$ and $M = 4$ coherent, noncoherent, and continuous-phase FSK (CPFSK) signals with rectangular $p(t)$. Determine the -40 dB bandwidth in all cases. Use $F_B = 100$ baud, $f_{c0} = 2100$ Hz, and $f_{cm} = f_{c0} + mF_B$. For coherent FSK set $\theta_{cm} = 0$ for all m . For CPFSK choose the phases θ_{cm} such that there are no phase jumps.

(e) The wav files `fsksig901.wav` and `fsksig902.wav` contain binary FSK signals made from 8-bit, LSB-first, ASCII encoded characters. Analyze the two signals and extract the text messages.

(f) The binary GNU Radio file `digMod_905.bin` contains several (complex-valued) binary communication signals. Use the GNU Radio Companion to find the signals recorded in the file and determine their properties such as the type of modulation and the carrier frequencies. Each of the signals contains an ASCII coded (MSB first) message. Try to demodulate the signals and extract the messages.

E3. ASK and FSK Signals in GNU Radio. –To be completed–