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NA 620 – HW 3

Question 1 – Flow around a Cylinder

The flow around a cylinder was examined using the Gauss Linear and Least Squares gradient schemes. An example of the flow around a cylinder can be seen in *Figure 1.* The flow field was sampled at 45 degrees from the x-axis. The magnitude of the velocity along this line was calculated using *Equation 1.* Then, the analytical solution was calculated using *Equation 2.* *Table 1* presents a summary of the variables used for both equations.

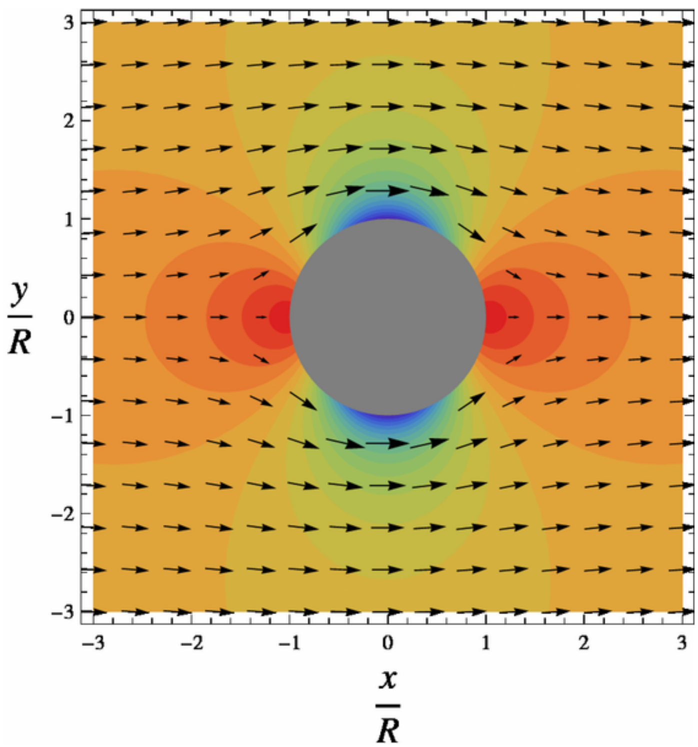

Figure 1 – Example of flow around a cylinder

Variable	Meaning	Value
U	Free stream velocity	1.0
$gradTx$	Horizontal velocity component	Varies (OpenFOAM output)
$gradTy$	Vertical velocity component	Varies (OpenFOAM output)
r	Arc length	Varies (OpenFOAM output)
R	Cylinder radius	0.5
θ	Arc angle	45° from horizontal

Table 1 – Summary of equation variables

$$|U| = \sqrt{(gradTx)^2 + (gradTy)^2} \qquad \textit{Eqn. 1}$$

$$|U| = \sqrt{\left(U\left(r - \frac{R^2}{r^2}\right)cos\theta\right)^2 + \left(-U\left(r + \frac{R^2}{r^2}\right)sin\theta\right)^2} \qquad \textit{Eqn. 2}$$

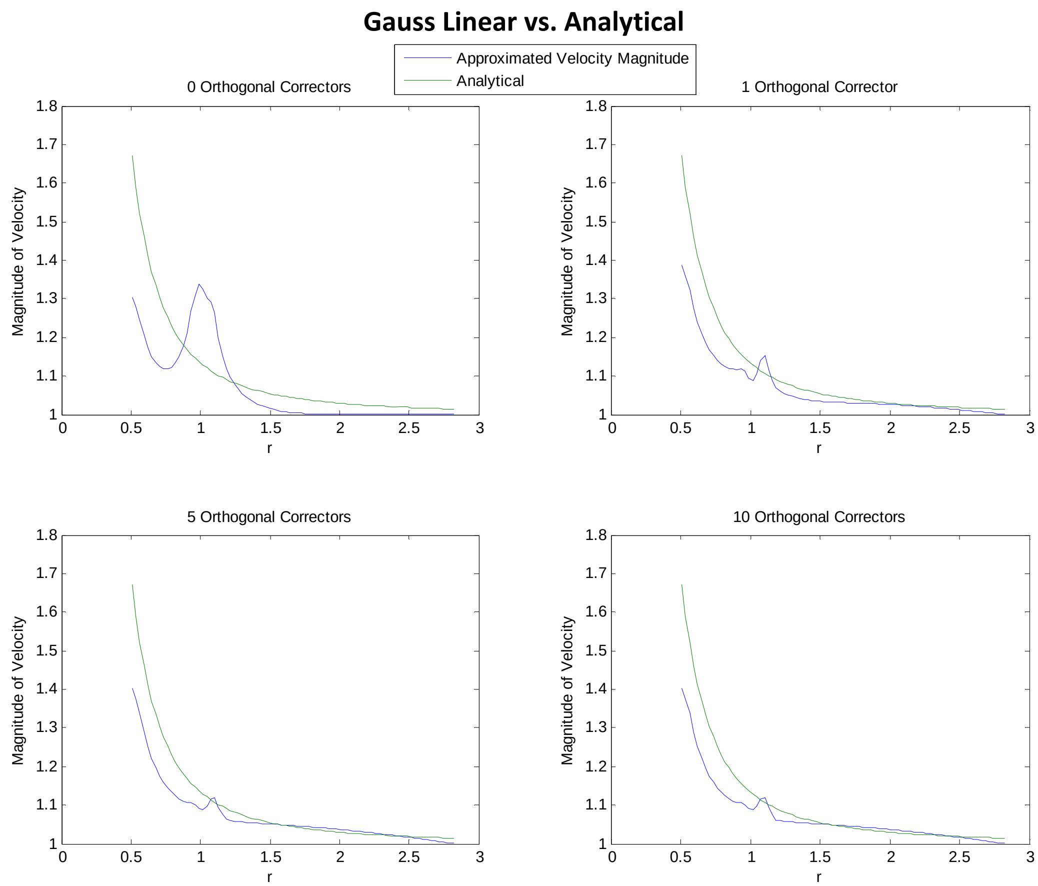

Figure 2 – Comparison of Gauss Linear to analytical solution with 0, 1, 5, and 10 orthogonal correctors

As seen in *Figure 2*, the Gauss Linear scheme seems to achieve a converged solution with five non-orthogonal correctors, as the solution appears to be the same with ten correctors. The solution improves with the use of non-orthogonal correctors, but it still produces a hump near r = 1.1 regardless of the number of correctors. A point within this hump will be used for a portion of the ASME uncertainty analysis.

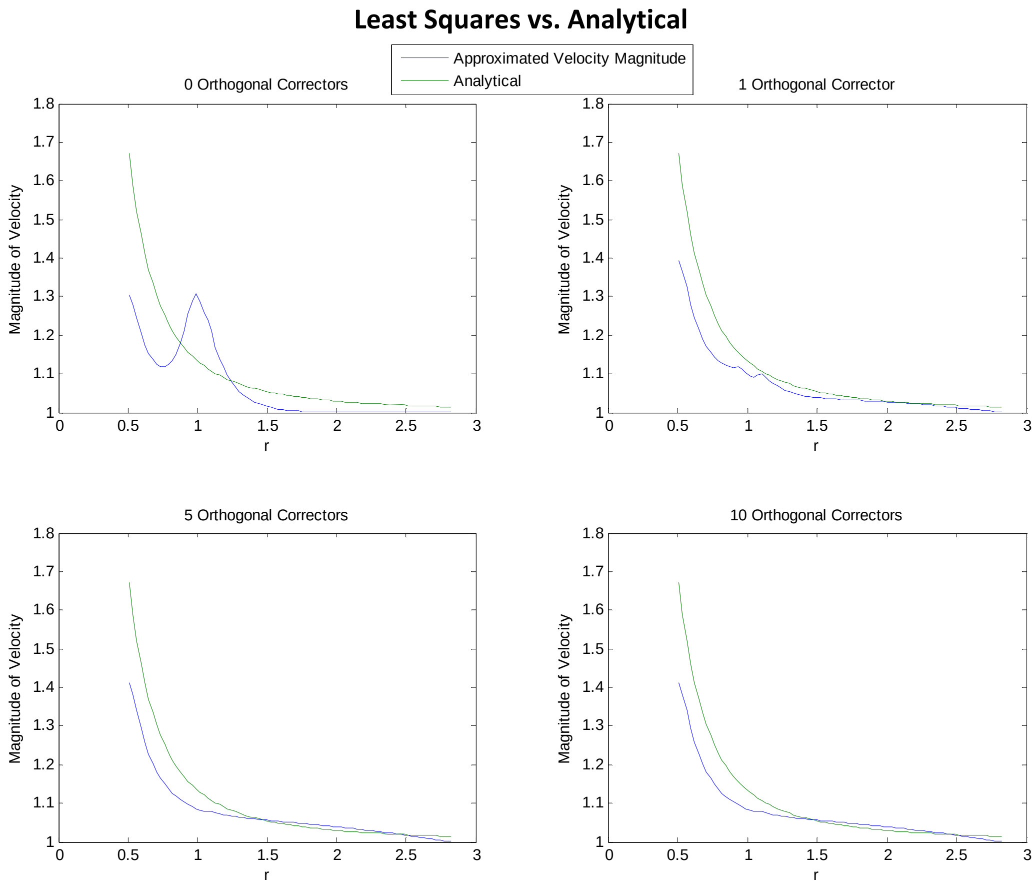

Figure 3 – Comparison of Least Squares to analytical solution with 0, 1, 5, and 10 orthogonal correctors

As seen in *Figure 3*, the Least Squares scheme seems to achieve a converged solution with five non-orthogonal correctors, as the solution appears to be the same with ten correctors. The solution improves with the use of non-orthogonal correctors, and the hump seen using the Gauss Linear scheme disappears with five correctors. Because of this smoother, more analytically-representative solution, the Least Squares scheme seems more accurate than the Gauss Linear scheme.

An ASME uncertainty analysis was performed on three different meshes for the both schemes. *Table 2* displays a summary of meshes. $\Delta A_{average}$ is reported because certain cells are blocks whereas the ones near the cylinder use arcs for four edges. Therefore, the areas of the cells vary.

	$\Delta A_{average}$	N_i
Mesh 3 – Coarse	1.52E-02	500
Mesh 2 – Base	3.80E-03	2000
Mesh 1 – Fine	9.51E-04	8000

Table 2 – Summary of Meshes

		ASME Criteria
N_1, N_2, N_3	8000, 2000, 500	$N_1 > N_2 > N_3$
r_{21}	2.0	$r_{21} > 1.3$
r_{32}	2.0	$r_{32} > 1.3$

Table 3 – Cell ratios and ASME Criteria

As seen in *Table 3, t*he necessary ASME criteria were satisfied. To perform the analysis, three points along the sample line were chosen as well as an average of all possible sample points. An explanation for the reason behind this selection is presented in *Table 4*. *Tables 5* and *6* show the observed order of accuracy and the numerical uncertainty for the Gauss Linear and Least Squares schemes using ten non-orthogonal correctors at the locations previously mentioned.

Sample	Reasoning
r = 0.51	Near cylinder edge
r = 1.10	Within hump region
r = 2.72	Far from cylinder
Average	All possible points considered

Table 4 – Explanation of ASME uncertainty sample selection

Gauss Linear Scheme

	ASME Notation	r = 0.51	r = 1.10	r = 2.72	Average
U for Mesh 1	ϕ_1	1.4411	1.0700	1.0058	1.0748
U for Mesh 2	ϕ_2	1.4046	1.1188	1.0057	1.0737
U for Mesh 3	ϕ_3	1.3360	1.0811	1.0059	1.0710
	ε_{32}	-0.0685	-0.0377	0.0002	-0.0027
	ε_{21}	-0.0366	0.0488	-0.0001	-0.0011
	s	1.0000	-1.0000	-1.0000	0.5366
Order of Accuracy	p	0.9063	0.3745	0.8467	0.9907
	q(p)	0.0000	0.0000	0.0000	0.0000
	ϕ^{21}_{ext}	1.4829	0.9053	1.0060	1.0795
	ϕ^{32}_{ext}	1.4829	1.2459	1.0055	1.0822
App. Rel. Error	e^{21}_{a}	2.537%	4.562%	0.010%	0.352%
App. Rel. Error	e^{32}_{a}	4.879%	3.366%	0.018%	0.585%
Extrap. Rel. Error	e^{21}_{ext}	2.820%	18.193%	0.013%	0.932%
Extrap. Rel. Error	e^{32}_{ext}	5.285%	10.198%	0.023%	0.939%
Numerical Uncertainty	GCI^{21}_{fine}	3.627%	19.241%	0.016%	1.151%
Numerical Uncertainty	GCI^{32}_{coarse}	6.975%	14.194%	0.028%	1.224%

Table 5 – Order of Accuracy and Numerical Uncertainty for the Gauss Linear Scheme

Least Squares Scheme

	ASME Notation	r = 0.51	r = 1.10	r = 2.72	Average
U for Mesh 1	ϕ_1	1.4449	1.0762	1.0060	1.0752
U for Mesh 2	ϕ_2	1.4113	1.0774	1.0057	1.0747
U for Mesh 3	ϕ_3	1.3474	1.0745	1.0057	1.0731
	ε_{32}	-0.0638	-0.0030	0.0001	-0.0016
	ε_{21}	-0.0336	0.0013	-0.0003	-0.0005
	s	1.0000	-1.0000	-1.0000	0.6829
Order of Accuracy	p	0.9239	1.2367	2.5873	2.2319
	q(p)	0.0000	0.0000	0.0000	0.0000
	ϕ^{21}_{ext}	1.4824	1.0752	1.0060	1.0729
	ϕ^{32}_{ext}	1.4824	1.0796	1.0056	1.0761
App. Rel. Error	e^{21}_{a}	2.329%	0.117%	0.030%	0.072%
App. Rel. Error	e^{32}_{a}	4.523%	0.275%	0.005%	0.229%
Extrap. Rel. Error	e^{21}_{ext}	2.529%	0.086%	0.006%	0.369%
Extrap. Rel. Error	e^{32}_{ext}	4.799%	0.202%	0.001%	0.402%
Numerical Uncertainty	GCI^{21}_{fine}	3.244%	0.108%	0.007%	0.424%
Numerical Uncertainty	GCI^{32}_{coarse}	6.301%	0.253%	0.001%	0.508%

Table 6 – Order of Accuracy and Numerical Uncertainty for the Least Squares Scheme

As seen when comparing the orders of accuracy and the numerical uncertainties, both schemes achieve similar near the edge of the cylinder. Reasonable numerical uncertainties of 3-4% and orders of accuracy of approximately 0.9 are obtained in this region. The difference between the two schemes can be seen well when examining the hump region of the solutions. The Gauss Linear scheme, which still produces a hump with correctors, achieves a relatively poor numerical uncertainty of nearly 20% and an order of accuracy of only 0.4. On the other hand, the Least Squares scheme becomes smooth with the correctors and achieves values of about 0.1% and 1.2 for the uncertainty and accuracy, respectively. Both schemes produce relatively high accuracy and low uncertainty for the region far from the cylinder. However, the Least Squares scheme is more accurate with an order of 2.5 compared to 0.8 for the Gauss Linear. It should be noted that since the average accuracies and uncertainties consider all points along the sample line, they may not accurately represent the accuracies of the schemes. These averages are heavily dependent on the size of the cylinder relative to the size of the grid, or the amount of free stream fluid. Since the schemes are quite accurate in the free stream, their average accuracies would be over-represented if more of this free stream flow be sampled.

Question 2 – Flow within a Boundary Layer

A boundary layer profile was modeled using the Gauss Linear and Least Squares gradient schemes on an unstructured 20 x 20 grid. The profile was modeled by using a velocity, T, where $T = y^{1/7}$. The vertical gradient component of the velocity was examined against the analytical values obtained using $\nabla T_y = \frac{1}{7} y^{-6/7}$. *Figure 4* displays comparisons of the two schemes against the analytical solution. *Figure 4* was obtained using a vertical sample line through the center of the sample space.

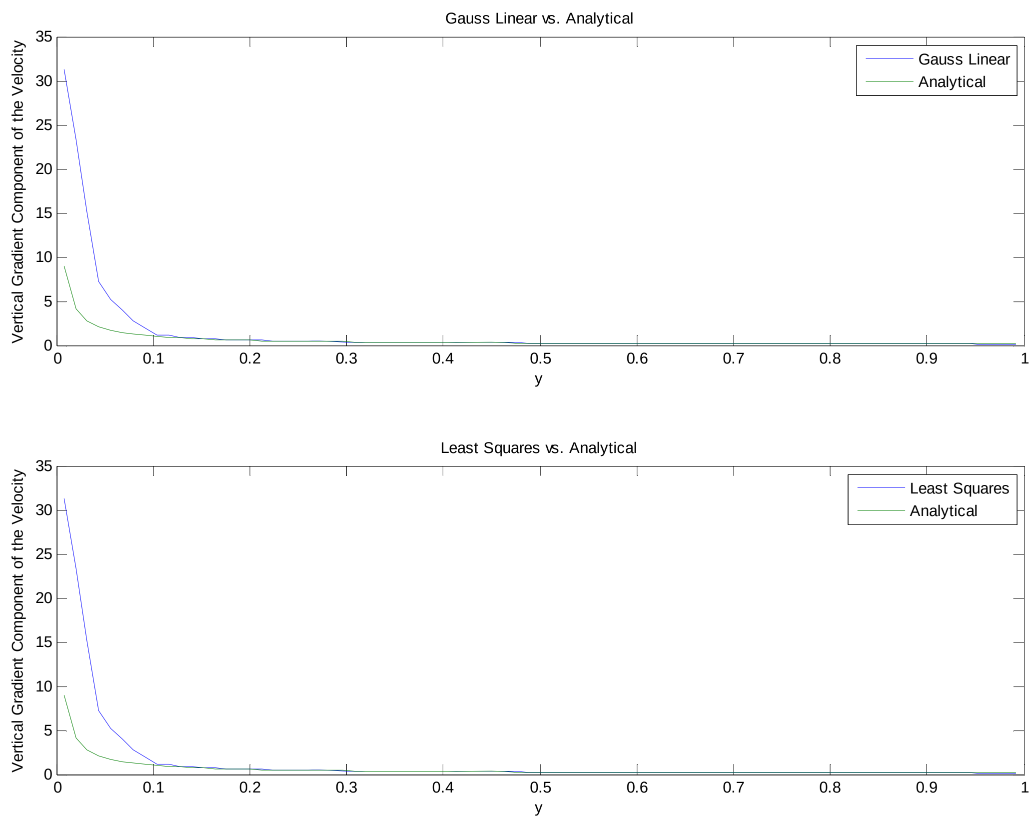

Figure 4 – Comparison of Gauss Linear and Least Squares vs. analytical solution

Both schemes appear to produce very similar solutions. However, they over-predict the analytical solution near the boundary where y=0. *Figures 5* and *6* show contour plots of the differences between both schemes and the analytical solution. The over-prediction near the boundary of the schemes' solutions can be seen as the error in this region is the greatest.

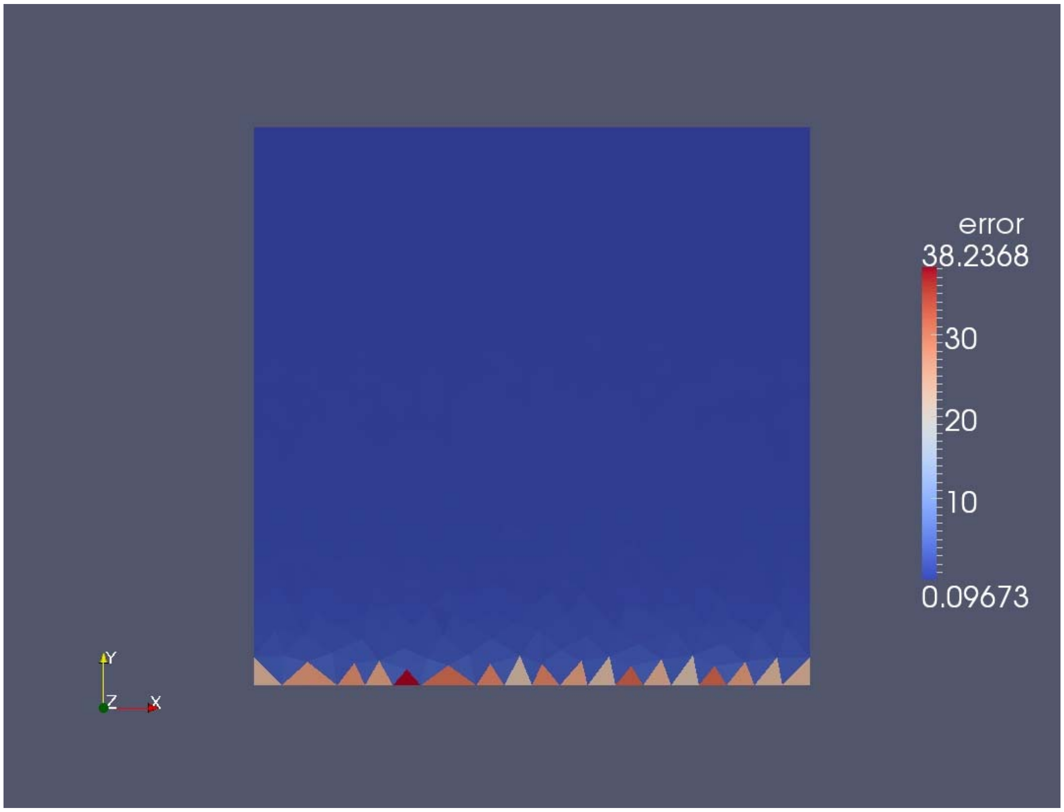

Figure 5 – Difference between Gauss Linear and analytical (Gauss Linear – analytical)

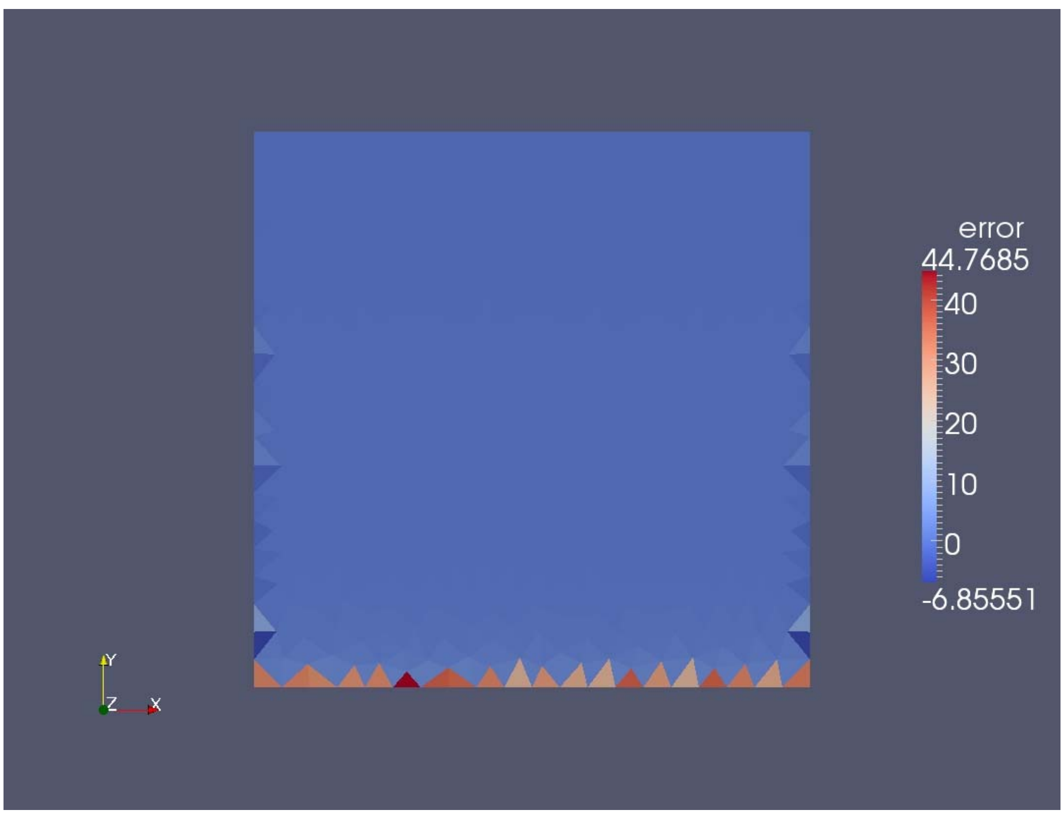

Figure 6 – Difference between Least Squares and analytical (Least Squares – analytical)