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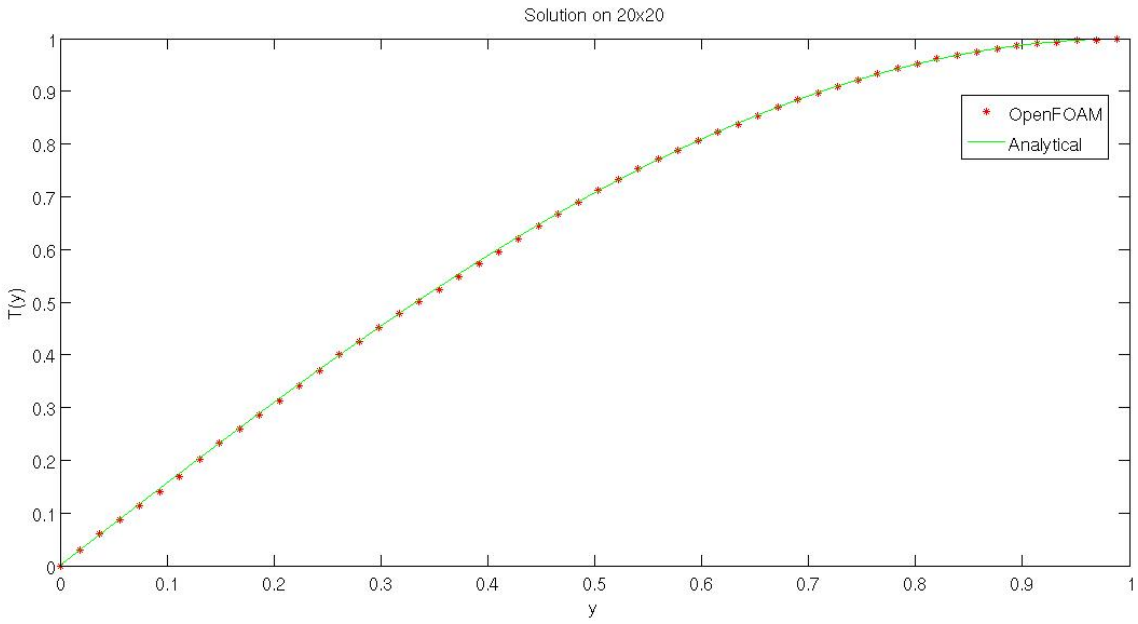
NA 599 056
Homework 4

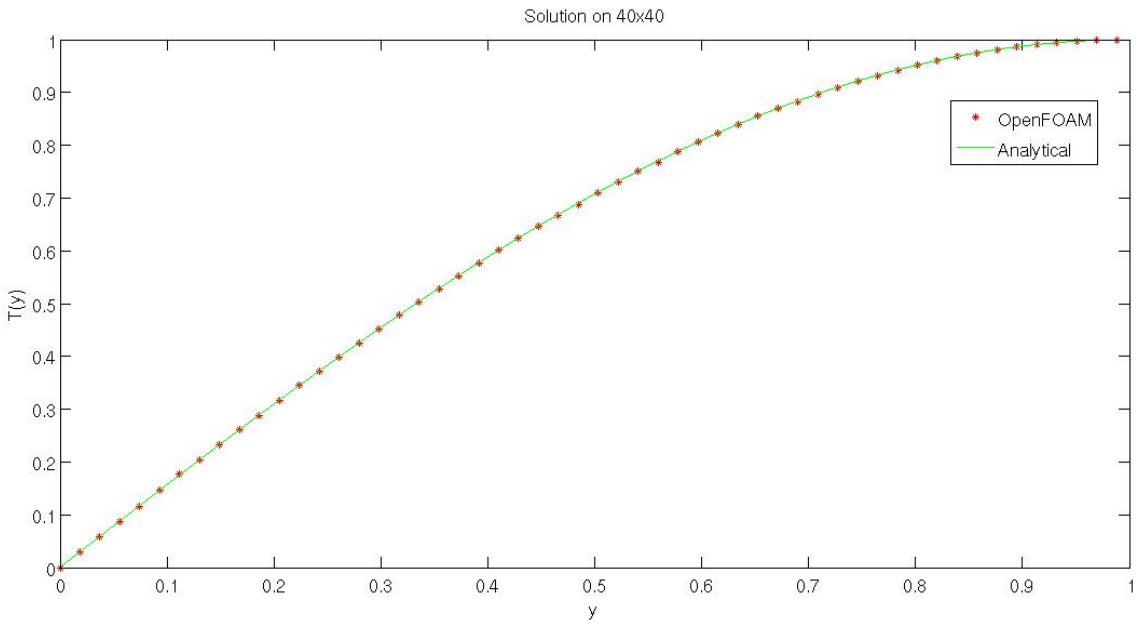
The scalar function below was initialized on two grids using OpenFOAM.

$$T(y) = sin\left(\frac{\pi}{2}y\right) \text{ for } x \in [0{:}1]$$

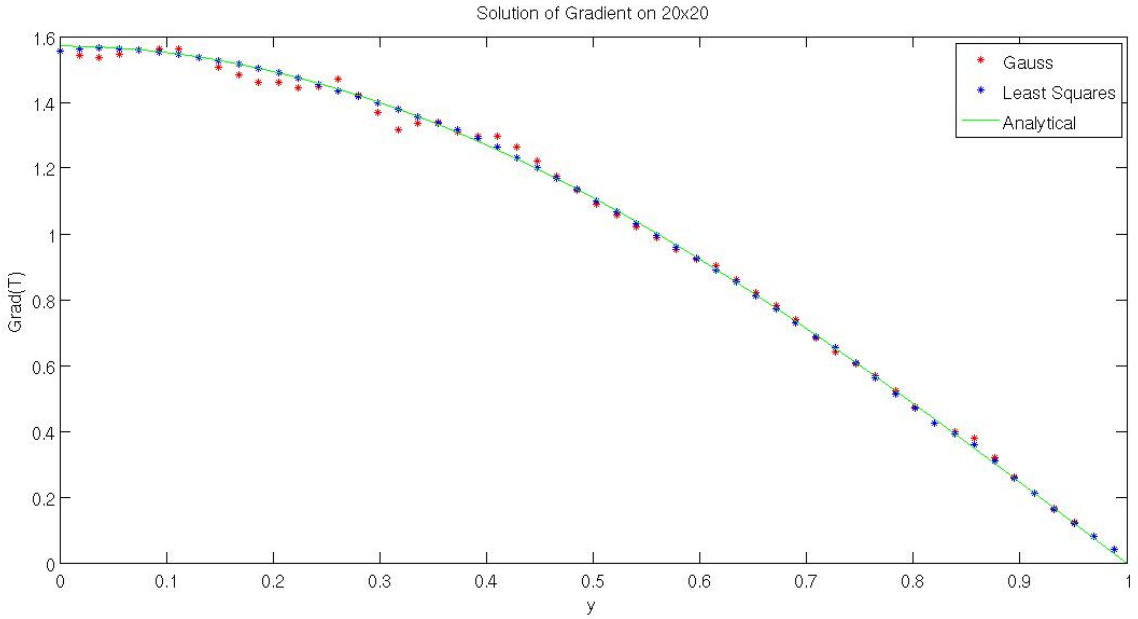
The unstructured grids were 20x20 and 40x40 with 880 and 3480 cells, respectively. The gradient of the function was computed on each grid using a Gaussian approximation scheme and a least squares scheme. Following are plots and discussion concerning the approximation of the function, the approximation of the gradient, and the observed order-of-accuracy.

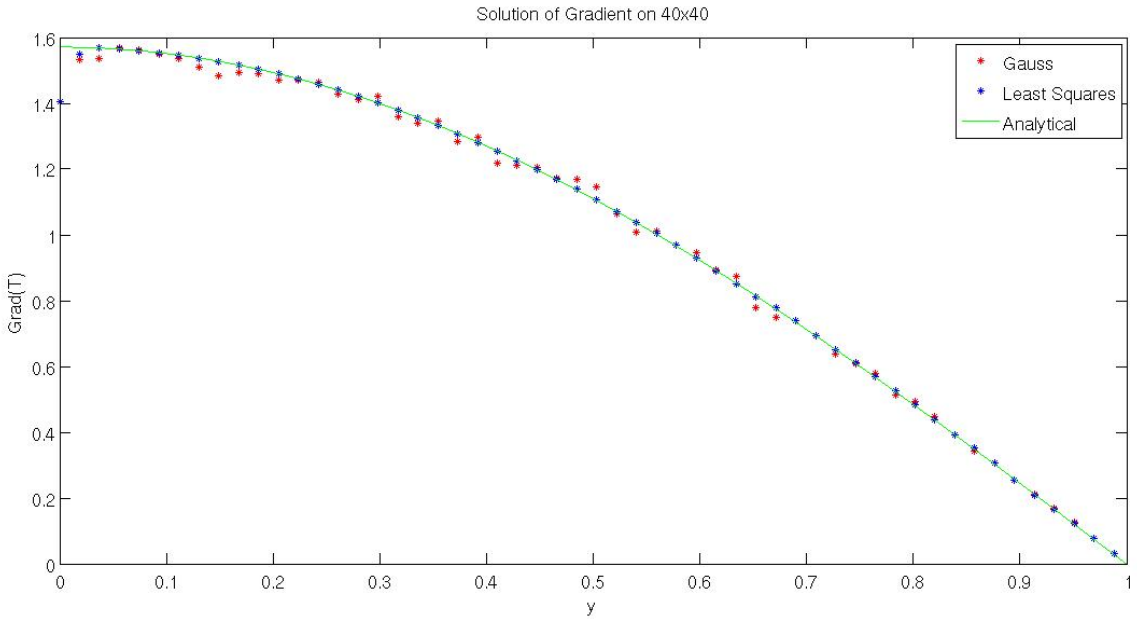
The following two plots show the approximation of the original function $T(y)$ on the 20x20 and 40x40 grids. The function was sampled at 54 uniformly distributed points in the y-direction at x=0.5. The Gauss method and least squares procedure are not identified in these plots because either will produce the same approximation of the function, since they are *gradient* approximations. Both grids seem to approximate the function very well. However, it can be seen that the data points lie closer to the actual function on the 40x40 grid plot.





The next two plots show the approximation of the gradient of the function, $\nabla T(y)$, for both the Gauss and least squares methods, along with the analytical solution of the gradient. Again, the gradient was sampled at 54 uniformly distributed points in the y-direction at x=0.5. Both gradient approximation schemes appear to improve with grid refinement, and the least squares scheme seems to be the better of the two on each grid for this function. In addition, the schemes seem to have difficulty approximating the gradient near the lower boundary (y=0) on the 40x40 grid. One may suggest it is not drastic departure from the analytical solution, but it is definitely a noticeable one.



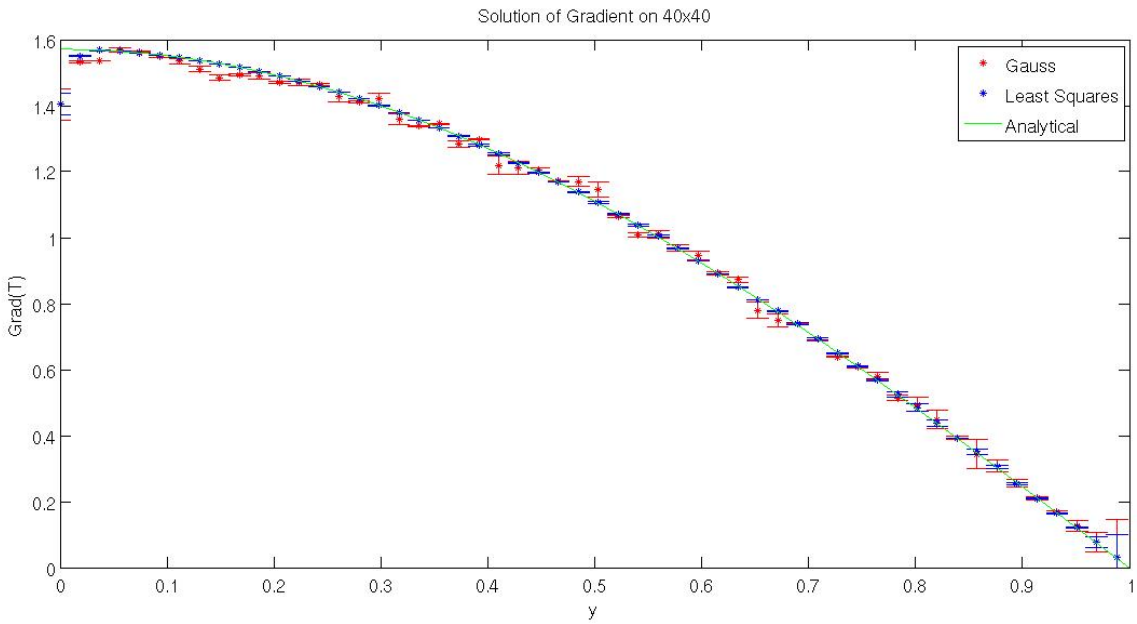


Although the observed order-of-accuracy is a local quantity, allowing one to use a single point for its computation, all 54 points were used in the calculation of the observed order-of-accuracies that appear in the table below.

Number of points	Gradient scheme	Observed order-of-accuracy	Formal order-of-accuracy
54	Gauss	1.8775	2
	Least squares	2.2487	2

As shown, the calculations report an observed order-of-accuracy near the formal order-of-accuracy of 2 for each scheme, but the least squares method is actually greater than 2. However, this is *not* to suggest that this scheme possesses an order-of-accuracy greater than the formal. Instead, it should be noted that the calculation of the observed is sensitive to the number of points sampled, the technique used to sample, and the function being approximated (and possibly other factors, as well).

Lastly, the ASME uncertainty procedure was implemented to calculate the grid convergence index (GCI) of the two methods. The average observed order-of-accuracies from the above table were used in these calculations. Therefore, the size of the error bars on the least squares scheme will be reduced by the presence of the observed order-of-accuracy greater than the formal. The following plot shows the approximate solution to the gradient for each method along with error bars. The analytical solution is also plotted. As stated previously, the least squares procedure is a better method for this approximation. Now, one can also see that the numerical uncertainty of the least squares method is less than that of the Gauss scheme. To conclude, an interesting observation can be made when looking at the data points closest to the upper and lower boundaries (y=0 and y=1). Here, both schemes seem have their first and second largest numerical uncertainties.


Solution of Gradient on 40x40
Gauss
Least Squares
Analytical
Grad(T)
y
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0.1
0.3
0.5
0.7
0.9