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NA 620 – HW 1

Calculation of Wave Resistance

Length (m)	4
Beam (m)	0.4
Draft (m)	0.25
Wetted Surface (m^2)	2.3796

Table 1 – Wigley Hull Principal Dimensions

To compute the wave resistance of the Wigley hull, the hull was discretized into a set of points on the centerplane. The parabolic equation of the hull surface was used to develop a representation of the 3D hull on a 2D surface. Initially, the length of the hull was divided into 30 evenly spaced points, and the draft was divided into 10 evenly spaced points. Eventually, the number of points was increased to create finer meshes around the hull. This is presented in *Table 2.*

Iteration	N_x	N_z
1	30	10
2	40	20
3	60	30

Table 2 – Mesh Refinements

Figure 1 shows the curvature of the hull represented on a 2D surface using the final and finest mesh tested with N_x = 60, N_z = 30.

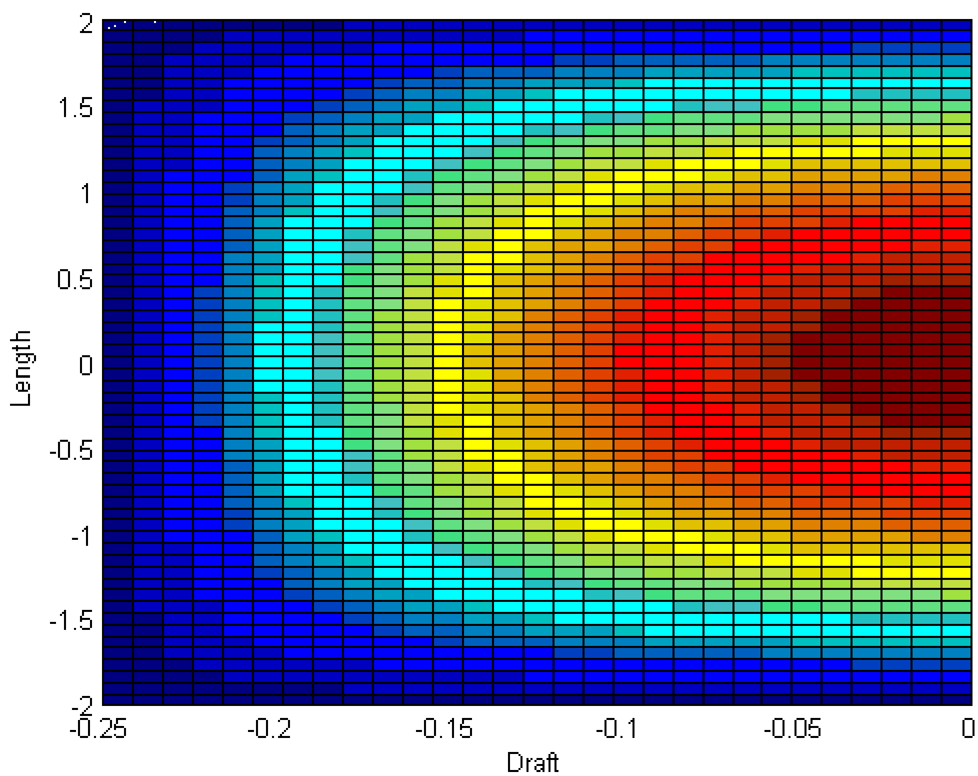

Figure 1 – Curvature of the Wigley Hull

Following the development of the hull surface, the slope of the hull was determined at each point on the centerplane. The representation of these slopes is shown in *Figure 2.*

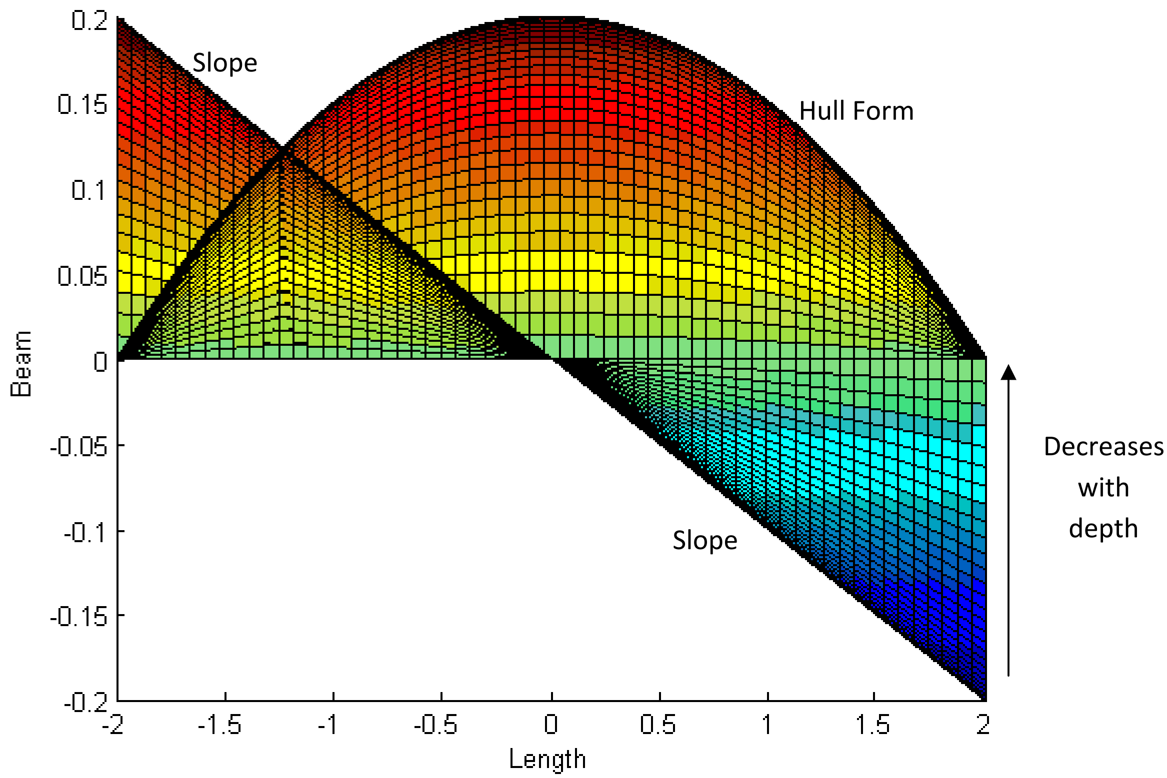

Figure 2 – Slope of the Wigley Hull

The slope is shown as a surface because as a point on the hull approaches the draft of 0.25 meters, the slope at that point approaches zero.

Then, knowing that secant(π/2) is infinity, θ was discretized from zero to (π/2 – δ) where δ was a small value used to prevent the secant function from growing to infinity while integrating to find the wave resistance. Initially, a value of δ = π/10 was used, but δ = π/8 was seen to produce better results; therefore δ = π/8 was used in all results found in this report.

With all the necessary variables discretized, the process of computing the wave resistance started with an examination of the wave resistance integral shown in *Equation 1.*

$$R_w = \rho\pi U_\infty^2 \int_0^{\frac{\pi}{2}} (P^2 + Q^2) \cdot cos^3(\theta) d\theta \qquad \textit{Eqn. 1}$$

After identifying the wave resistance as dependent upon P and Q, these functions were then examined as shown in *Equation 2* and *3.*

$$P = \frac{2}{\pi} \nu \cdot sec^3(\theta) \int_{-L/2}^{L/2} \Gamma(x,\theta) \cos(\nu x sec(\theta)) dx \qquad \textit{Eqn. 2}$$

$$Q = \frac{2}{\pi}\nu \cdot sec^3(\theta) \int_{-L/2}^{L/2} \Gamma(x,\theta)\sin(\nu x sec(\theta))dx \qquad \textit{Eqn. 3}$$

Lastly, with these functions dependent upon Γ, this function was examined as shown in *Equation 4*, where η_x is the slope of the hull.

$$\Gamma(x,\theta) = \int_{-T}^{0} \eta_x \cdot e^{\nu sec^2(\theta) z} dzdx \qquad \textit{Eqn. 4}$$

Due to the smoothness of the hull, Simpson's method of integration was used for each integral presented in *Equations 1-4*.

After computing the predicted wave resistance, *Equation 5* was used to compute the coefficient of the wave resistance.

$$C_w = \frac{R_w}{\frac{1}{2} \cdot \rho \cdot U^2 \cdot S} \qquad \textit{Eqn. 5}$$

Then, the IITC – 1957 friction line formula was used to estimate the coefficient of friction. The coefficient of the wave resistance and the coefficient of friction were summed to obtain the coefficient of total resistance. These steps are shown in *Equations 6* and *7*.

$$C_f = \frac{0.075}{log_{10}(Re-2)^2} \qquad \textit{Eqn. 6}$$

$$C_T = C_f + C_w \qquad \textit{Eqn. 7}$$

The predicted coefficient of total resistance was compared to the Wigley experimental results for Froude numbers from 0.1 to 0.4. Both the fixed and free to sink and trim cases were considered. This is seen in *Figures 3* and *4.*

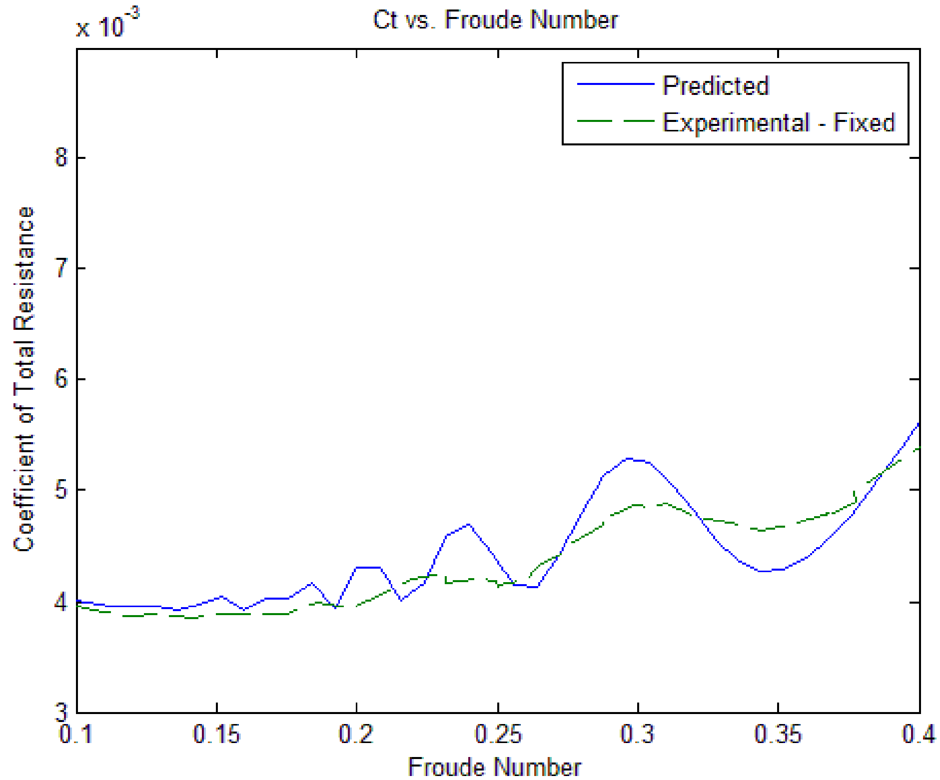

Figure 3 – Predicted C_T and Experimental C_T with Fixed Trim and Sinkage vs. Froude number

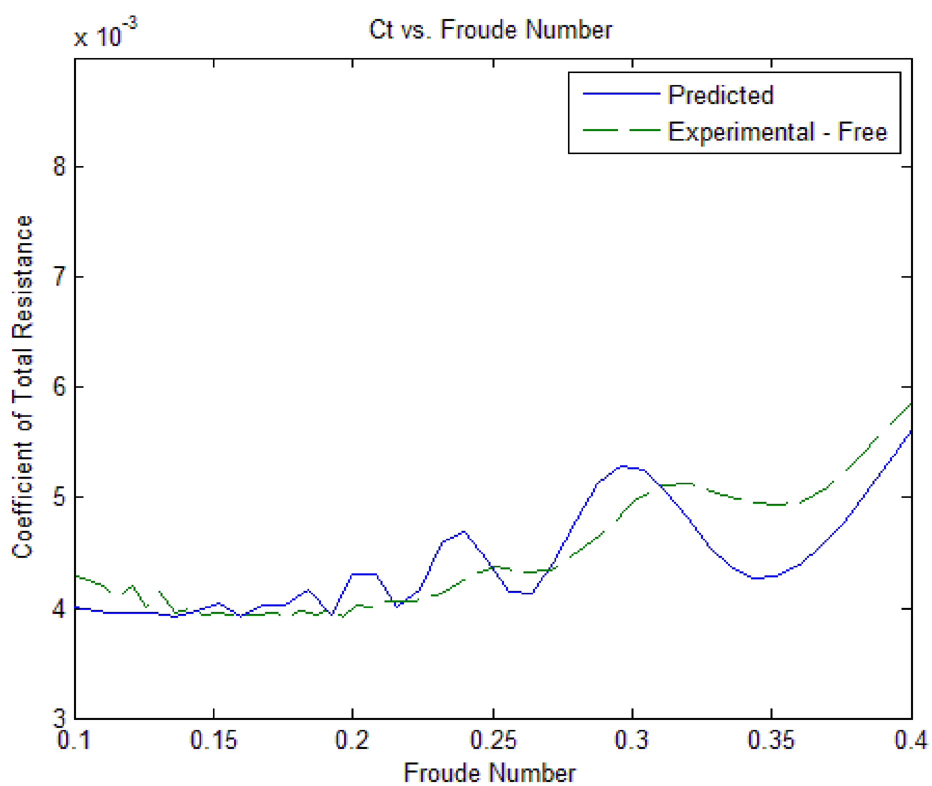

Figure 4 – Predicted C_T and Experimental C_T with Free Trim and Sinkage vs. Froude number

Overall, the general shape of the predicted coefficient of total resistance behaves much like the experimental data – increasing as the Froude number increases. However, the compilation of error can be seen in the predicted data as they oscillate, with increasing Froude number, more and more away from the experimental data. Also, predicted data for Froude numbers less than 0.1 may be unreliable since ν grows to infinity as speed approaches zero.

Uncertainty and Error

An uncertainty analysis was performed in accordance with the ASME guidelines. The wave resistances from the three finest meshes were compared. Since the wave resistance changes with speed, four Froude numbers and their corresponding wave resistances were used in the analysis.

	N_x	N_z	ΔA_i	N_i	h_i
Mesh 3 – Coarse	30	10	3.33E-03	300	0.0577
Mesh 2 – Base	40	20	1.25E-03	800	0.0354
Mesh 1 – Fine	60	30	5.56E-04	1800	0.0236

Table 3 – Summary of Meshes

As seen, the ASME guideline of $h_1 < h_2 < h_3$ was satisfied.

		ASME
N_1, N_2, N_3	1800, 800, 300	$N_1 > N_2 > N_3$
r_{21}	1.5	$r_{21} > 1.3$
r_{32}	1.6330	$r_{32} > 1.3$

Table 4 – Cell ratios and ASME Criteria

Again, the necessary ASME criteria were satisfied. *Table 5* shows the observed order of accuracy and numerical uncertainty for the meshes at various Froude numbers and other parameters used to calculate these values. It seems as though the numerical estimations through the use of meshes converge as the Froude number increases. This is partly caused by ν being large with low speed. Also, the discretization of θ produces error due to the inability to numerically evaluate secant(π/2).

	ASME Notation	Fr = 0.1	Fr = 0.2	Fr = 0.3	Fr = 0.4
R_w for Mesh 1 (N)	ϕ_1	0.0205	1.6036	8.80005	19.5608
R_w for Mesh 2 (N)	ϕ_2	0.98155	1.514	8.7036	19.4734
R_w for Mesh 3 (N)	ϕ_3	2.91435	1.3998	8.57355	19.3345
	ε_{32}	1.9328	-0.1142	-0.1301	-0.1389
	ε_{21}	0.9611	-0.0896	-0.0965	-0.0874
	s	1.0000	1.0000	1.0000	1.0000
Order of Accuracy	p	1.1262	0.1168	0.2422	0.6069
	q(p)	-0.2421	-0.1952	-0.2007	-0.2172
	ϕ^{21}_{ext}	-1.6400	3.4505	9.7346	19.8740
App. Rel. Error	e^{21}_{a}	4,688.05 %	5.59 %	1.10 %	0.45 %
Extrap. Rel. Error	e^{21}_{ext}	101.25 %	53.53 %	9.60 %	1.58 %
Numerical Uncertainty	GCI^{21}_{fine}	10,124.99 %	143.97 %	13.28 %	2.00 %

Table 5 – Order of Accuracy and Numerical Uncertainty

```
clear all

load ctFixed.txt;
load ctFree.txt;

FrctFixed = ctFixed(:,1);
ctFixedData = ctFixed(:,2);
FrctFree = ctFree(:,1);
ctFreeData = ctFree(:,2);

L = 4;
B = 0.4;
T = 0.25;
S = 2.3796;
g = 9.81;
Fr = linspace(0,0.4,51);
U = Fr.*sqrt(g*L);
Re = (U.*L)/(1.124*10^(-6));      % 1.124*10^(-6) is kinematic viscosity at 15.5 deg.↙
celsius
nu = 1./(L*Fr.^2);
rho = 1025;

Nz = 30;
Deltaz = T/(Nz-1);

Nx = 60;
Deltax = L/(Nx-1);

x = linspace(-2,2,Nx);
z = linspace(-0.25,0,Nz);

for i = 1:length(x)
    for j = 1:length(z)
y(i,j) = B/2*(1-(z(j)/T)^2)*(1-(2*x(i)/L)^2);
dydx(i,j) = 4*B*x(i)/(L^2)*((z(j))^2/(T^2)-1);
    end
end

surface(z,x,y)
xlabel('Draft');
ylabel('Length');
zlabel('Beam');
pause

surface(z,x,dydx)
pause

Ntheta = 40;
Delta = pi/8;
Deltatheta = (pi/2-Delta)/(Ntheta - 1);
Theta = linspace(0,pi/2-Delta,Ntheta);
```

```
for n = 1:length(Fr)
    for i = 1:Nx
        for k = 1:Ntheta
            for j = 1:Nz
                if j == 1
                    gamma(i,k) = Deltaz*(0.5*dydx(i,j)*exp(nu(n)*(sec(Theta(k)))^2*z↙
(j)));
                elseif j == length(z)
                    gamma(i,k) = gamma(i,k) + Deltaz*(0.5*dydx(i,j)*exp(nu(n)*(sec(Theta↙
(k)))^2*z(j)));
                else
                    gamma(i,k) = gamma(i,k) + Deltaz*(dydx(i,j)*exp(nu(n)*(sec(Theta↙
(k)))^2*z(j)));
                end
            end
        end
    end

    for k = 1:Ntheta
        for i = 1:Nx
            if i == 1
                P(k) = 0.5*gamma(i,k)*cos(nu(n)*x(i)*sec(Theta(k)))*2/pi*nu(n)↙
*Deltax*sec(Theta(k))^3;
                Q(k) = 0.5*gamma(i,k)*sin(nu(n)*x(i)*sec(Theta(k)))*2/pi*nu(n)↙
*Deltax*sec(Theta(k))^3;
            elseif i == length(x)
                P(k) = P(k) + (0.5*gamma(i,k)*cos(nu(n)*x(i)*sec(Theta(k))))*2/pi*nu(n)↙
*Deltax*sec(Theta(k))^3;
                Q(k) = Q(k) + (0.5*gamma(i,k)*sin(nu(n)*x(i)*sec(Theta(k))))*2/pi*nu(n)↙
*Deltax*sec(Theta(k))^3;
            else
                P(k) = P(k) + (gamma(i,k)*cos(nu(n)*x(i)*sec(Theta(k))))*2/pi*nu(n)↙
*Deltax*sec(Theta(k))^3;
                Q(k) = Q(k) + (gamma(i,k)*sin(nu(n)*x(i)*sec(Theta(k))))*2/pi*nu(n)↙
*Deltax*sec(Theta(k))^3;
            end
        end

        if k == 1
            Rw(n) = rho*pi*Deltatheta*U(n)^2*0.5*(P(k)^2 + Q(k)^2)*cos(Theta(k))^3;
        elseif k == Ntheta
            Rw(n) = Rw(n) + rho*pi*Deltatheta*U(n)^2*0.5*(P(k)^2 + Q(k)^2)*cos(Theta(k))↙
^3;
        else
            Rw(n) = Rw(n) + rho*pi*Deltatheta*U(n)^2*(P(k)^2 + Q(k)^2)*cos(Theta(k))^3;
        end
    end

    Cw(n) = Rw(n)/(0.5*rho*U(n)^2*S);
    Cf(n) = 0.075/(log10(Re(n)) - 2)^2;
```

```
    Ct(n) = Cf(n) + Cw(n);

    Rf(n) = Cf(n)*0.5*rho*U(n)^2*S;
    Rt(n) = Rw(n) + Rf(n);

end

% subplot(1,2,1)
plot(Fr,Ct,FrctFixed,ctFixedData,'--')
axis([0.1 0.4 0.003 0.009])
legend('Predicted','Experimental - Fixed')
xlabel('Froude Number')
ylabel('Coefficient of Total Resistance')
title('Ct vs. Froude Number')

pause

% subplot(1,2,2)
plot(Fr,Ct,FrctFree,ctFreeData,'--')
axis([0.1 0.4 0.003 0.009])
legend('Predicted','Experimental - Free')
xlabel('Froude Number')
ylabel('Coefficient of Total Resistance')
title('Ct vs. Froude Number')
```

$$implicit := \left\{p = \left(\frac{1}{\ln(1.5)}\right)\cdot \mathrm{abs}\left(\ln\left(\mathrm{abs}\left(\frac{1.9328}{0.9611}\right)\right) + q\right), q = \ln\left(\frac{(1.5^p - 1)}{(1.632993162^p - 1)}\right)\right\}$$

$$\left\{p = 2.466303462\ |0.6986465458 + q|, q = \ln\left(\frac{1.5^p - 1}{1.632993162^p - 1}\right)\right\} \quad (1)$$

$fsolve(implicit)$

$$\{p = 1.126102190, q = -0.2420513994\} \quad (2)$$

$$implicit2 := \left\{p = \left(\frac{1}{\ln(1.5)}\right)\cdot \mathrm{abs}\left(\ln\left(\mathrm{abs}\left(\frac{0.1142}{0.0896}\right)\right) + q\right), q = \ln\left(\frac{(1.5^p - 1)}{(1.632993162^p - 1)}\right)\right\}$$

$$\left\{p = 2.466303462\ |0.2425959769 + q|, q = \ln\left(\frac{1.5^p - 1}{1.632993162^p - 1}\right)\right\} \quad (3)$$

$fsolve(implicit2)$

$$\{p = 0.1168377903, q = -0.1952223296\} \quad (4)$$

$$implicit3 := \left\{p = \left(\frac{1}{\ln(1.5)}\right)\cdot \mathrm{abs}\left(\ln\left(\mathrm{abs}\left(\frac{0.1301}{0.0965}\right)\right) + q\right), q = \ln\left(\frac{(1.5^p - 1)}{(1.632993162^p - 1)}\right)\right\}$$

$$\left\{p = 2.466303462\ |0.2987603768 + q|, q = \ln\left(\frac{1.5^p - 1}{1.632993162^p - 1}\right)\right\} \quad (5)$$

$fsolve(implicit3)$

$$\{p = 0.2419040456, q = -0.2006767268\} \quad (6)$$

$$implicit4 := \left\{p = \left(\frac{1}{\ln(1.5)}\right)\cdot \mathrm{abs}\left(\ln\left(\mathrm{abs}\left(\frac{0.1389}{0.0874}\right)\right) + q\right), q = \ln\left(\frac{(1.5^p - 1)}{(1.632993162^p - 1)}\right)\right\}$$

$$\left\{p = 2.466303462\ |0.4632589669 + q|, q = \ln\left(\frac{1.5^p - 1}{1.632993162^p - 1}\right)\right\} \quad (7)$$

$fsolve(implicit4)$

$$\{p = 0.6069472537, q = -0.2171630330\} \quad (8)$$