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NA 599 056
Homework 2

1. Using the Euler-Explicit, Heun's, Runge-Kutta (RK4), and Adams-Bashforth (AB2) approximation methods, the solution to the following initial value problem was predicted. It is assumed that u is velocity and t is time.

$$\frac{du}{dt} = -u\, tan(t) + sin(2t)$$
$$u(0) = 1$$
$$t \in [0:1]$$

The analytical solution was found to be the following:

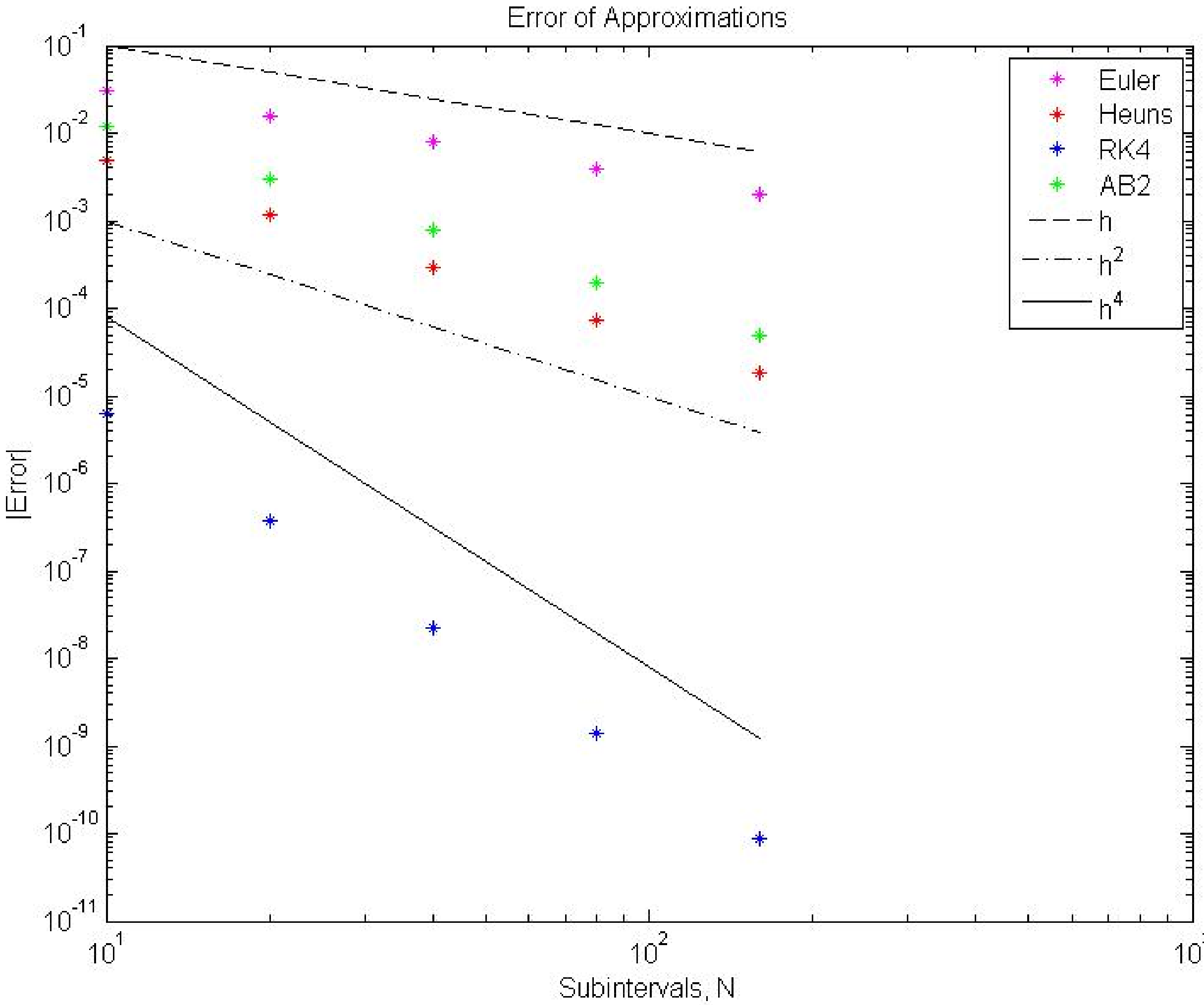
$$u(t) = 3cos(t) - 2cos^2(t)$$

The error between the various methods, for five numbers of subintervals, and the analytical solution at $t = 1$ appears below.

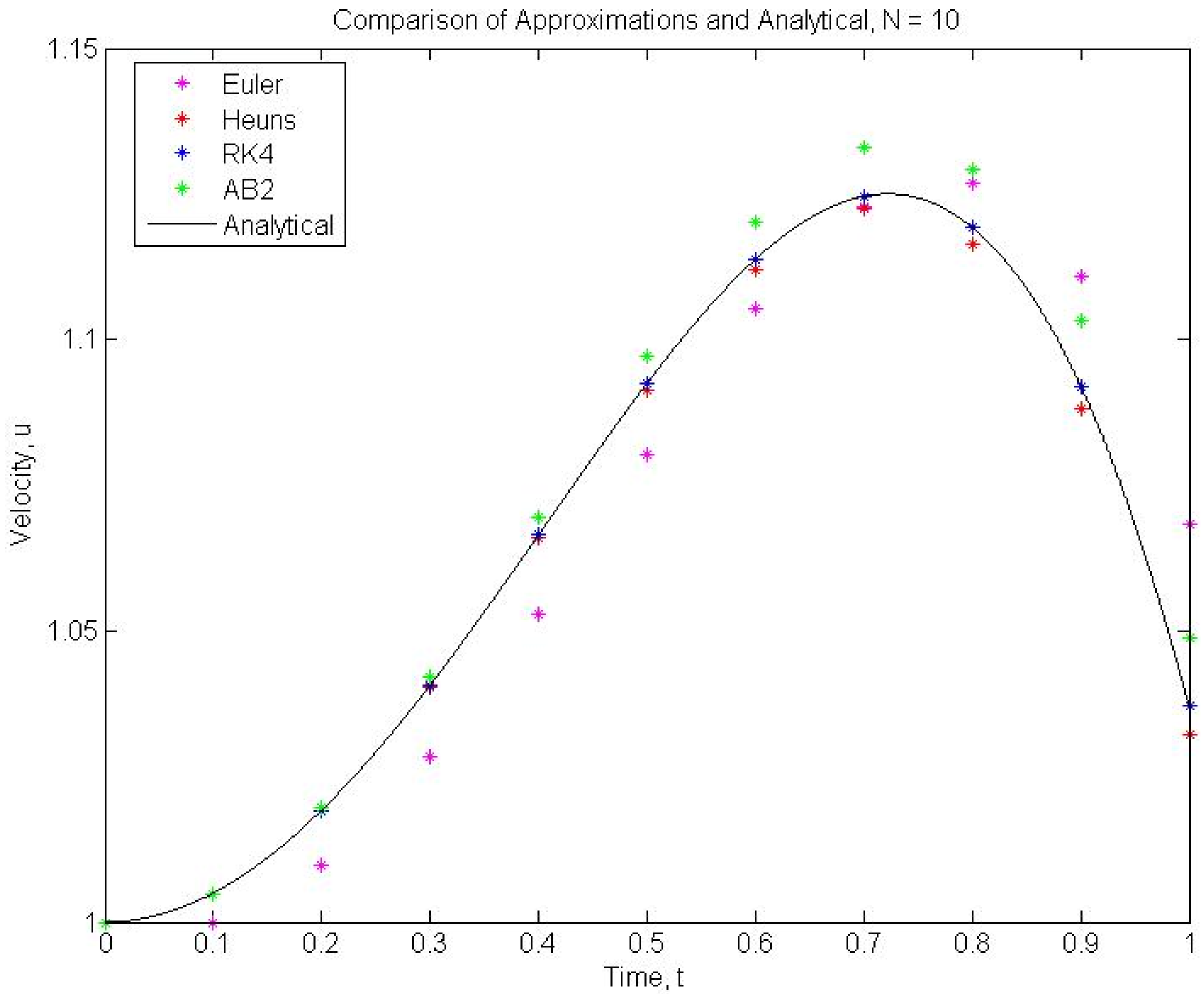
N	Euler	Heun's	RK4	AB2
10	0.031148131608753	0.004934668236809	6.142854180435364e-06	0.011852360078054
20	0.015738218229911	0.001192915082334	3.711523524074778e-07	0.003029257755074
40	0.007898936032542	2.930176320594669e-04	2.272264909564115e-08	7.660482516935740e-04
80	0.003955667081581	7.259883465637884e-05	1.404329985632558e-09	1.926285905606839e-04
160	0.001979234783877	1.806759152334081e-05	8.726086520027820e-11	4.829805354056305e-05

Error between approximations and analytical solution at t=1

The plot below shows the order-of-accuracy of each method. The Euler-Explicit method is first order accurate, Heun's and AB2 are second order, and RK4 is fourth order. It should be noted that RK4 was used as the starter for the AB2 approximations.



2. A comparison of each approximation, using 10 subintervals, and the analytical solution over the given interval is shown in the figure below.



Even with 10 subintervals, the RK4 method provides a very good prediction of this function. However, as mentioned in lecture, this is the most expensive method of the ones compared here.