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September 16, 2012

NA 599 056

Homework 1

1. A quadrature rule, specifically the midpoint rule, was used to numerically approximate the following integral:

The midpoint rule approximation is implemented using the following formula:

In the formula above, and are the lower and upper bounds of the integral, respectively. Six different values for the number of subintervals, , were used. These and their corresponding approximations appear in *Table 1.*

|  |  |
| --- | --- |
| **Subintervals, N** | **Im** |
| 2 | 0.754597943772199 |
| 4 | 0.748747131891009 |
| 8 | 0.747303578730748 |
| 16 | 0.746943912516367 |
| 32 | 0.746854072623361 |
| 64 | 0.746831617445408 |

*Table 1 – Midpoint rule integral approximations*

MATLAB was used to write a script to compute the approximations, and format long was used to ensure that roundoff error was negligible.

2. In an attempt to increase the order of the numerical estimates, Richardson’s extrapolation was used by combining two (N and 2N) midpoint rule estimates. The equation used for Richardson’s extrapolation appears below, where is the step size, .

The results from the Richardson’s extrapolation are presented in *Table 2.*

|  |  |
| --- | --- |
| **Subintervals, N** | **IRE** |
| 2, 4 | 0.746796861263946 |
| 4, 8 | 0.746822394343994 |
| 8, 16 | 0.746824023778240 |
| 16, 32 | 0.746824125992360 |
| 32, 64 | 0.746824132386090 |

*Table 2 – Richardson’s extrapolation integral approximations*

3. In order to assess the error in the numerical approximations, a “correct” value, , of the integral was obtained with the following:

The built-in erf() function in MATLAB was used to evaluate . Still, this integral evaluation is not exactly correct. It contains roundoff error, since the double precision limits the value to 15 decimal places. However, this value was considered to be the truth.

The error between the midpoint rule and Richardson’s extrapolation approximations and the truth was obtained by taking the absolute value of the difference between an approximation and the truth. This is summarized in *Table 3*, where the Richardson’s extrapolation error corresponds to the larger of the two subintervals required for the calculation*.* These, along with the step size raised to integer powers, were plotted against the number of subintervals using log-log scaling.

|  |  |  |
| --- | --- | --- |
| **N** | **Im Error** | **IRE Error** |
| 2 | 0.007773810959772 |  |
| 4 | 0.001922999078582 | 2.727154848114477e-05 |
| 8 | 4.794459183209421e-04 | 1.738468432876950e-06 |
| 16 | 1.197797039399484e-04 | 1.090341870124689e-07 |
| 32 | 2.993981093446507e-05 | 6.820067399715413e-09 |
| 64 | 7.484632980836459e-06 | 4.263369657309113e-10 |

*Table 3 – Error of approximations*

As seen in the plot on the following page, the midpoint rule shows second-order accuracy, since its error decreases at the same rate as h2. On the other hand, Richardson’s extrapolation appears to be fourth-order accurate because its error decreases at the same rate as h4. This was expected to be only third-order accurate, but fourth-order accuracy is still possible if the third-order error term in the expansion of the integral be small. Indeed, a quick calculation for the C3 term in the expansion showed that it was on the order of 10-10. The plots of h2 and h4 were shifted vertically to conveniently appear near the two approximations. Therefore, the slopes of h2 and h4 are the only information one should take from the figure.

