Marc Woolliscroft

October 17, 2012

NA 599 056

Homework 3

1. a) DIRICHLET PROBLEM – Convergence with iteration

|  |
| --- |
| 10x10 Discretization |
|  |
|  |
| 160x160 Discretization |
|  |

|  |  |  |  |
| --- | --- | --- | --- |
| Elements per side | No. of iterations | Order of L2 norm for Jacobi | Order of L2 norm for Gauss-Seidel |
| 10 | 200 | 1e-3 | 1e-7 |
| 20 | 800 | 1e-3 | 1e-7 |
| 40 | 6000 | 1e-4 | 1e-9 |
| 80 | 18000 | 1e-5 | 1e-10 |
| 160 | 30000 | 1e-1 | 1e-3 |

*Order of L2 norm for last iteration*

As seen in the previous plots, both the Jacobi and Gauss-Seidel iteration procedures converge for the given Dirichlet boundary condition problem. However, the Gauss-Seidel method converges more rapidly. When the grid is refined, more iterations are necessary to achieve a similar L2 norm of the residual for a more coarse grid. In fact, the finest grid achieved an L2 norm on the order of only 1e-1 for Jacobi and 1e-3 for Gauss-Seidel after 30,000 iterations. A satisfactory L2 norm depends on the problem, user, etc. We are limited by machine accuracy, but one could iterate until the L2 norm for any grid is on the order of 1e-16 (for a machine with double-precision accuracy), it may just take a very, very long time. An order of 1e-6 may be a typical goal.

1. b) Convergence in space

|  |
| --- |
| 10x10 Discretization |
|  |
| 20x20 Discretization |
|  |
| 40x40 Discretization |
|  |
| 80x80 Discretization |
|  |
| 160x160 Discretization |
|  |

Using the iteration information from part 1 a), several grids were used to find a numerical solution. As seen, a finer discretization produces a solution that more accurately represents the continuousness of the problem. In other words, the isotherms or equipotential lines are smoother and get pushed further into the corners of the grid with increasing refinement.

2. a) MIXED PROBLEM – Convergence with iteration

|  |
| --- |
| 10x10 Discretization |
|  |
|  |
| 160x160 Discretization |
|  |

|  |  |  |  |
| --- | --- | --- | --- |
| Elements per side | No. of iterations | Order of L2 norm for 1st-order method | Order of L2 norm for 2nd-order method |
| 10 | 200 | 1e-7 | 1e-8 |
| 20 | 800 | 1e-6 | 1e-7 |
| 40 | 3500 | 1e-7 | 1e-8 |
| 80 | 12000 | 1e-6 | 1e-6 |
| 160 | 30000 | 1e-3 | 1e-3 |

*Order of L2 norm for last iteration*

As seen in the previous plots, the L2 norm of the residual is quite similar for both the 1st and 2nd order methods of approximating the Neumann boundary condition. Both converge successfully for this problem. Again, a greater number of iterations are necessary for a finer grid to achieve a similar L2 norm of coarser grid.

2. b) Convergence in space

|  |
| --- |
| 10x10 Discretization, 1st order method |
|  |
| 10x10 Discretization, 2nd order method |
|  |
| 20x20 Discretization, 1st order method |
|  |
| 20x20 Discretization, 2nd order method |
|  |
| 40x40 Discretization, 1st order method |
|  |
| 40x40 Discretization, 2nd order method |
|  |
| 80x80 Discretization, 1st order method |
|  |
| 80x80 Discretization, 2nd order method |
|  |
| 160x160 Discretization, 1st order method |
|  |
| 160x160 Discretization, 2nd order method |
|  |

Using the iteration information from part 2 a), several grids were used to find a numerical solution. As seen, a finer discretization produces a solution that more accurately represents the continuousness of the problem. In other words, the isotherms or equipotential lines are smoother and get pushed further into the corners of the grid with increasing refinement. The solutions to the 1st and 2nd order methods are quite similar, but a difference on the order of 1e-1 can be seen in the L2 norm (as provided in part 2 a).

Room for improvement: A maximum L2 norm should have been chosen to help dictate the necessary number of iterations for both problems 1 and 2. This is especially true in problem 1 where the L2 norms for the finest grid are on the order of 1e-1 and 1e-3 for Jacobi and Gauss-Seidel, respectively. Unfortunately, there was insufficient time to do this on this assignment.