Assignment

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3COE20

Q1. mean = θ_1 (Normal distribution)

variance = θ_2

max likelihood estimate function

$$L(\theta_1, \theta_2 \mid x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_2)^2}{2\theta_2}}$$

$$\ln L(\theta_1, \theta_2 \mid x_1 x_2 \ldots x_n) = -\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i \theta_j)^2$$

for θ_1

$$\frac{\partial}{\partial \theta_1} \ln L\left(\prod\right) = \frac{n}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)$$

R.H.S $= 0$

$$\therefore \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\sum (x_i - \theta_1) = 0$$

$$\hat{\theta}_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

So MLE of θ_1 $\longrightarrow$ sample mean for θ_2

$$\frac{\partial}{\partial \theta_2} \ln L(\qquad) = -\frac{n}{2\theta_2} + \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

RHS = 0

$$-\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2$$

$$\hat{\theta}_2 = \frac{1}{n} \Sigma (x_i - \hat{\theta}_1)^2$$

So MLE for $\theta_2 \longrightarrow$ sample variance

Q2. Bernoulli distribution

parameter $\rightarrow \theta \in \Theta = (0,1)$ unknown

$\rightarrow m$ (known +ve $\mathbb{Z}$)

MLE

$$L(\theta \mid x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(X_i = x_i / \theta)$$

$$P(X_i = x_i / \theta) = \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking log

$$\ln L(\theta \mid x_1, x_2 \ldots x_n) = \sum_{x_i=1}^{n} \ln (\theta^{x_i} (1-\theta)^{m-x_i})$$

$$= \sum_{i=1}^{n} (x_i \ln \theta \; (m - x_i) \ln (1-\theta))$$

$$\frac{d}{d\theta}(\qquad) = 0$$

$$\sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right) = 0$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = mn - \sum_{i=1}^{n} \frac{x_i}{1-\theta}$$

$$\therefore \theta = \sum_{i=1}^{n} \frac{x_i}{n \cdot m}$$

So max likelihood estimate of θ is :

$$\hat{\theta}_{MLE} = \sum_{i=1}^{n} \frac{x_i}{n \cdot m}$$