Kalman Filter SoC estimation for Li-Ion batteries

P. Spagnol, S. Rossi, S. M. Savaresi

Abstract—State-of-Charge (SoC) estimation is a key factor for correct and safe battery management, particularly for the development of the Battery Management System (BMS) [1]. The paper deals with this problem for the currently most promising technology in the battery filed: Lithium-Ion batteries. An electric model of the cell is identified and verified, in order to apply Kalman Filter Theory and design an algorithm for estimating SoC. Particularly the aim of the algorithm is to reject measurement noise and parametric uncertainties and be applicable to different cells of the same manufacturer and technology. In this purpose, design criterions that speed up the convergence time or make the estimation robust to noise measurements are presented. Results on two different cells of the same manufacturer and typology are shown, focusing on the different behaviors of the estimation due to different design choices.

I. INTRODUCTION

ITHIUM-ION batteries represent nowadays the choice Lof energy carrier for many electric application: from small electronics [2] to Electric Vehicles (EVs) [3]. High power and high energy capabilities as well as the room for improvements bring manufacturers and researchers to invest significant resources on lithium-ion batteries. There are several topics related to battery applications that still have to be solved, particularly if we consider the relatively new technology we are taking into account. Among all the research activities, SoC estimation plays a key role: in fact SoC knowledge allows to correctly and safely manage the battery. A unique SoC definition does not exist; however SoC is usually considered the ratio of the available capacity of a cell and its maximum attainable capacity. Therefore the SoC of a battery is an abstract energetic concept more than an actual physical variable. It is not available as a measurement; thus it has to be estimated. Different methodologies and techniques has been developed during the years in order to obtain more and more accurate estimations [4]. Coulomb counting, Open-Circuit-Voltage (OCV) estimations and impedance spectroscopy are the most common techniques. However, each of them has some drawbacks that make the methodology unfeasible for practical purposes. Coulomb Counting is based on the definition of SoC (presented in Section IV). However noise

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P. Spagnol and S. M. Savaresi are with DEI, Politecnico di Milano, Via Ponzio 34/5 - 20133 Milano Italy (phone: +39 02 23994021; e-mail: spagnol@ elet.polimi.it; savaresi@elet.polimi.it).

S. Rossi, is with Eldor Corporation SpA Via E.Fermi 93 22030 Orsenigo (CO) - Italy (e-mail: Stefano.rossi@eldor.it).

in the current measurement (typical in practical application) introduces errors and drifts in the estimation. OCV methods are based on the relationship between the SoC and the voltage when the battery is disconnected from any load and is fully relaxed. It requires battery model and accurate voltage measurement (normally not available). Finally, impedance spectroscopy requires expensive instrumentation and the interruption of battery working operation.

In this work we try to overcome all these drawbacks applying Kalman Filter Theory. There are several works in literature attempting to estimate SoC using Kalman Filter. However the majority of them are based on electrochemical models [5], or *ad hoc* models [1], [6] instead of simple electric models [7]. An algorithm that can be seen as a special Kalman Filter applied on electric models is presented in [8]. Mixing the model representation (common for OCV estimation) and Coulomb Counting methodologies (that are based on the definition of SoC), we came up with an algorithm that is able to reject measurement noise and at the same time parametric errors in the model. A study of the project criterion in order to make the estimation faster or more accurate is also provided.

In Section II the laboratory equipment used to characterize and perform the experiment is described. In particular its capabilities and its limits are presented.

Section III is a brief summary of the cell under test: a Lithium – Ion cobalt based battery.

Battery electric model, test design for identification and validation and the methodology followed to face these tasks are the topics of Section IV.

In Section V, starting from the model identified, we present step by step the project of Kalman Filter algorithm: in particular we focus the attention on the design criterion in order to have accurate estimation or speed up the convergence of the SoC estimation. Results of experiments on the cells under test are provided.

II. LABORATORY EQUIPMENT

A. Test bench

A fully controllable test bench is necessary to obtain data sets useful for:

- characterize cells under test;
- indentify and validate battery models;
- design and test the SoC estimation algorithm.

Therefore, a measurement chain has been setup and it is made up by:

Fully programmable Power Supply (up to 80 V, 60 A);

- Fully programmable Load (up to 80V, 60A);
- Acquisition board;
- Current sensors.

As for the current sensors, two different technologies have been applied in the measurement chain:

- Shunt resistance (0.2Ω) for reference measurements and model identification;
- External closed-loop Hall-effect sensor (30 mA accuracy, 15 A full scale) for SoC algorithm validation.

Power Supply and Load are controlled in mutual exclusion, so that it is possible to generate arbitrary current profiles.

III. CELL UNDER TEST

A. Boston Power Sonata® 4400

Table 1 [9] shows the main characteristics of the cell under test. As for the chemistry, Boston-Power currently uses cobalt and manganese on the cathode with graphite on the anode.

Nominal capacity		4400 mAh
Nominal voltage		3.7 V
Standard charging method	Constant current (CC)	3.1 A (0.7 C) to 4.2 V
	Constant voltage (CV)	4.2 V to 50 mA
Cell weight		92 g (typical)
Operating temperature	Charge	-10 to 60 °C
	Discharge	-40 to 70 °C

C [1/hour] is the C-rate normalization defined as $C = I/S_{C,0}$ (1).

- *I* is the current flowing in the battery [A];
- $S_{C,0}$ is the nominal capacity [Ah].

Manufacturer declares 1300-1400 cycles before the death of the battery under standard fully discharge (at 0.5 C until 2.75 V voltage cut-off) and fully standard charge at 23° C.

In this work we are performing all the experiments at room temperature, so temperature effects are not considered.

IV. CELL DYNAMIC MODEL

A. Model structure

The aim of this Section is to present the model used to describe the dynamics of the lithium-ion cells and to implement the SoC estimator based on Kalman filter.

A mathematical model of a lithium-ion cell can be developed according to different frameworks: it can be based on chemical modeling, equivalent electrical circuit modeling, or black-box modeling.

Depending on the aim of the battery model, a suitable framework can be selected. Since we are interested in a model with a small computational effort and based on simple electrical components, the best choice is within the electrical modeling framework.

Therefore, a 2nd order equivalent circuit model (so called Randle model) adapted from [8] has been chosen (Fig. 1).

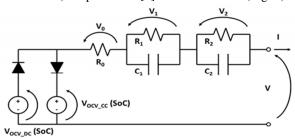


Fig. 1. Electrical circuit representation

Accordingly to Fig. 1, the model output is described by the following equation:

$$v(t) = v(SoC) - (v_0(t) + v_1(t) + v_2(t))$$
 (2).

where

$$v_{OCV}(SoC) = \begin{cases} v_{OCV_CC}(SoC) & if \quad i(t) < -\bar{\iota} \\ v_{OCV_DC}(SoC) & if \quad i(t) > +\bar{\iota} \end{cases} \tag{3}.$$

and

$$SoC(t) = SoC_0 - \int i(t)dt / (3600 S_{C,a})$$
 (4).

- v(t) is the voltage measured at the battery poles [V];
- $v_{OCV}(SoC)$ is the Open-Circuit-Voltage [V];
- v_{OCV_CC}(SoC) and v_{OCV_DC}(SoC) are the OCVs identified in charge and discharge to take into account hysteresis phenomena [V];
- $v_0(t)$, $v_1(t)$, $v_2(t)$ are the voltages of the electric model accordingly to Fig. 1 [V];
- *i(t)* is the current flowing in the battery (measurable) [A]:
- $\bar{\iota}$ is the value of the current used to discriminate charge and discharge phases, taking into account the noise affecting the measurement [A];
- *SoC(t)* is the SoC of the battery at a specific time;
- SoC_0 is the initial value of SoC;
- $S_{c,a}$ is the actual capacity of the battery [Ah].

Therefore the model can be re-written, in the Laplace-transform domain, as:

$$v(t) = v(SoC) - \left(R_0 + \frac{R_1}{(1+sR_1C_1)} + \frac{R_2}{1+sR_2C_2}\right)i(t) =$$

$$= v_{OCV}(SoC) - \left(R_0 + \frac{K_p(1+sT_2)}{(1+sT_{p1})(1+sT_{p2})}\right)i(t) =$$

$$= v_{OCV}(SoC) - R_0i(t) - G(s)i(t).$$
(5).

- R_0 , R_1 , R_2 , are the resistance parameters of the electric model according to Fig. 1 [Ω];
- C_1 , C_2 are the capacitors parameters of the electric model according to Fig. 1 [F];
- K_p , T_z , T_{p1} , T_{p2} are the gain and zero and poles time constants of the transfer function G(s).

B. Model identification

1) Open-Circuit-Voltage Map estimation

OCV can be represented as a function of SoC. By definition, the OCV is the voltage measured at the poles of

the battery when it is disconnected from any kind of load and all the internal chemical processes are completely relaxed. Unfortunately the relaxation of the lithium-ion battery chemistry, like most of the other battery chemistries, lasts many hours; therefore it is practically impossible to identify the real OCV. For practical purposes, OCV is evaluated measuring battery voltage after shorter resting periods. It can be experimentally proved that 15 minutes of resting allows the battery under test to reach the 96% of the full resting voltage value, so called OCV.

We designed two different current profiles (one in charge and one in discharge) with the same characteristics in order to model OCV as a piecewise linear function of SoC.

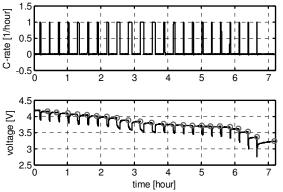
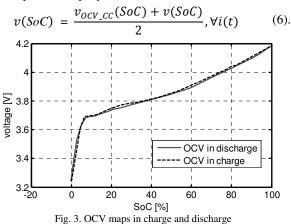


Fig. 2. Current and voltage measured for OCV and parameter identification in discharge

Both profiles are composed by 1C current steps (from a fully charge or discharge battery), characterized by different durations: smaller at high and low SoC and bigger at medium SoC. This is to get more evaluations in the regions where the maps are critically non-linear. Current and voltage measured during discharge are shown in Fig. 2. The circles underline where the evaluation of OCV takes place: these values correspond to the 96% of OCV.

The OCV maps in charge and discharge (Fig. 3) do not present the typical hysteresis [10] for the battery under test. This consideration allows us to use the average value of the two maps and simplify (2), so that:



2) Parameters estimation

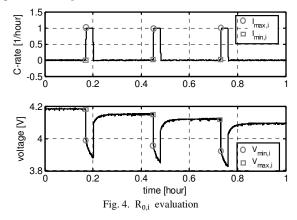
From the same experiment and considering (5) it is

possible to identify all the parameters of the model shown in Fig. 1.

As for R_0 , we evaluated the immediate effects of the current jumps on voltage variation neglecting the relaxation behavior as shown in [11]:

$$R_{0,i} = \frac{v_{min,i} - v_{max,i}}{i_{max,i} - i_{min,i}}$$
 (7).

where the meaning of $v_{min,i}, v_{max,i}, i_{max,i}, i_{min,i}$ is explained in Fig. 4.



Performing this evaluation for both charge and discharge we did not find differences in the resistance values for the battery under test; thus, the average value of $R_{0,i}$, represents R_0 .

$$R_0 = \sum_{i=1}^n R_{0,i} / n \tag{8}$$

As for the other terms enclosed in the transfer function G(s), it is possible to manipulate (5) in order to identify the gain and time constants.

$$\left(-v(t) + v_{OCV}(SoC) - R_0i(t)\right)/i(t) = G(s) \tag{9}.$$

This parameter estimation gives the following results:

ELECTRIC MODEL IDENTIFIED PARAMETERS		
R_0	44.1 mΩ	
R_1	$18.6~\mathrm{m}\Omega$	
R_2	$4.0~\mathrm{m}\Omega$	
C_1	69176 F	
C_2	138 F	

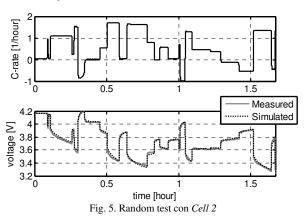
Standard deviation of the error between measured and simulated output considering the whole identification experiment is 24.6mV. However, considering the range between 100%-10% SoC (linear behavior), standard deviation of the error decreases to 10.3mV.

C. Model validation

In order to validate the model a random current profile (from -1C to 2C) has been applied to the batteries, performing a globally charge depleting behavior from a fully charged battery to 10% SoC. This profile has been tested on the cell used for the identification process (*Cell 1*) and on

another cell of the same type but never used before (*Cell 2*). We show the result for the *Cell 2* in Fig. 5. Standard deviation of the error between the simulated and measured voltage during the whole experiment is 18.8mV.

Summarizing, the model developed is able to catch the behavior in the linear SoC range (100% - 10%) with great accuracy. As expected the performance slightly decreases validating the model on a different cell because of parametric dispersion. However the model is still accurate as shown in Fig. 5.



V. STATE OF CHARGE ESTIMATION BASED ON KALMAN FILTER

A. Objectives

As detailed in the Introduction, SoC and its estimation are key factors for designing and developing a valuable Battery Management System suitable for a particular application. SoC is not directly measurable; thus, its estimation needs to be robust with respect to:

- The initial SoC (not correctly set);
- Measurement noise and offset (typical in application not performed in a controlled environment);
- Parametric dispersion between cells of the same manufacturer.

Kalman Filter scheme presented in this work is able to comply with these requirements with a simple structure of the observer.

B. State Space Model

In order to design the Kalman Filter observer we need to write the system in the State Space form.

Considering Fig. 1 and equation (5), we can write the system as:

$$\begin{pmatrix} \dot{v_1} \\ \dot{v_2} \\ SoC \end{pmatrix} = \begin{pmatrix} -\frac{1}{R_1 C_1} & 0 & 0 \\ 0 & -\frac{1}{R_2 C_2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ SoC \end{pmatrix} + \begin{pmatrix} \frac{1}{C_1} \\ \frac{1}{C_2} \\ -\frac{1}{C} \end{pmatrix} i$$
 (10).

$$v = -v_1 - v_2 + v_{OCV}(SoC) - R_0i$$

However, in order to apply the linear Kalman filter theory,

it is possible to linearize the $v_{\it OCV}(\it SoC)$ in the 10% - 100% SoC range, thus:

$$v_{OCV}(SoC) = v_{OCV_0} + k_e SoC + d(SoC)$$
 (11).

where:

- v_{OCV_0} is the intercept value of OCV at 0% SoC;
- k_e is the linear regression coefficient;
- d(SoC) is the residual due to the non linearities: it can be represented as an exogenous disturbance.

Therefore:

$$v = (-1 \quad -1 \quad k_e) \begin{pmatrix} v_1 \\ v_2 \\ SoC \end{pmatrix} - R_0 i$$
 (12).

The system is in the form:

$$\dot{x} = Ax + Bu
y = Cx + Du$$
(13).

Where:

$$\dot{x} = \begin{pmatrix} v_1 \\ v_2 \\ SoC \end{pmatrix} \quad y = v$$

$$A = \begin{pmatrix} -\frac{1}{R_1 C_1} & 0 & 0 \\ 0 & -\frac{1}{R_2 C_2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} \frac{1}{C_1} \\ \frac{1}{C_2} \\ -\frac{1}{S_{C,a}} \end{pmatrix} \qquad (14).$$

$$C = (-1 \quad -1 \quad k_e) \qquad D = -R_0$$

Determinant of the observability matrix is different from zero, therefore the matrix has maximum rank. Therefore, the system in (14) is observable.

C. Observer design criterion

Considering a generic system in the following form:

$$\dot{x} = Ax + Bu + \eta_x$$

$$y = Cx + Du + \eta_y$$
(15).

where η_x and η_y are Gaussian white noise affecting the state and the output of the system, Kalman Filter Theory is based on the following hypothesis:

- system described by (15) is stochastic;
- η_x and η_y are white Gaussian noise with zero mean and variances $Var(\eta_x(t)) = Q$, $Var(\eta_y(t)) = R$ and covariance $Cov(\eta_x(t), \eta_y(t)) = N$;
- initial state is a Gaussian vector with covariance $Cov(x(0)) = P_0$ and mean $\mathbb{E}(x(0)) = x_0$;

If all the hypothesis over-mentioned are verified it is possible to consider the following observer:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(t)(y - \hat{y})
\hat{y} = C\hat{x} + Du$$
(16).

where L(t) is a time variant gain that minimizes the covariance of the estimation error. However in order to simplify the observer, we can consider the steady state gain \bar{L} solving the algebraic Riccati equation. In fact if:

- (A, C) is observable;
- (A, B_q) is reachable where $Q = B_q^T B_q$; the differential Riccati equation asymptotically converges

to a single solution positive definite and the steady state predictor based on the gain \bar{L} is stable.

The problem of choosing the value of Q, R, N matrixes is not trivial: in fact they can be seen as the degree of uncertainty affecting the overall system. Thus, they are the sum of two different contributions:

- noise affecting current and voltage measurements;
- model parameters uncertainty due to non-linearity or not perfect parameter identification (for instance using Cell 2) or initialization.

Thus, considering that parametric uncertainty and measurement noise are uncorrelated:

$$Q = Q_p + Q_n$$
 $R = R_p + R_n$ $N = N_p + N_n$ (17).

Where the subscript p identifies the variance matrices due to parametric uncertainty and n the ones due to measurement noise.

The following sections aim to describe the approach in order to design Q, R, N matrixes.

D. Parametric uncertainty

Let the real system (13) be a perturbation of the identified model:

$$\dot{x}_{id} = A_{id} x_{id} + B_{id} u
y_{id} = C_{id} x_{id} + D_{id} u$$
(18).

where x_{id} and y_{id} are the state and output of the identified model, and A_{id} , B_{id} , C_{id} , D_{id} are the identified state space matrixes.

Therefore, considering (18) and (13) there is an error

$$e = (x - x_{id}) \tag{19}.$$

also when no other exogenous disturbances are present. Thus, it is always possible to add to the identified system (18) noise terms due to the parametric uncertainty:

$$\dot{x}_{idr} = A_{id}x_{idr} + B_{id}u + \eta_{x_{idr}} y_{idr} = C_{id}x_{idr} + D_{id}u + \eta_{y_{idr}}$$
 (20).

where x_{idr} and y_{idr} are the state and the output of the identified system plus the disturbance terms. The error between (20) and (18) is:

$$e_{idr} = (x_{idr} - x_{id}) (21).$$

If we are able to match the variances of e and e_{idr} selecting properly Q_p, R_p, N_p matrixes, it is possible to design the observer on the identified system where the parametric uncertainties are treated as an exogenous disturbance.

Considering the experiments and the model identification (Table 2) it is possible to neglect the contribution of uncertainty due to A_{id} and C_{id} , focusing on B_{id} and D_{id} . In fact the high value of currents and the uncertainty of the term $S_{C,a}$ makes those contributions dominant over the other for SoC estimation.

For the previous considerations, we can represent the real system in (13) as the identified system plus an equivalent noise term:

$$\dot{x} = Ax + B_{id}u + (B - B_{id})u y = Cx + D_{id}u + (D - D_{id})u$$
 (22).

Considering the covariance matrix V as:

$$V = \begin{pmatrix} Q & N \\ N & R \end{pmatrix} \tag{23}.$$

the covariance matrix due to parametric uncertainties V_p is:

$$V_p = H_p H_p^T Var(i(t))$$
 (24).

where:

$$H_p = \begin{pmatrix} B - B_{id} \\ D - D_{id} \end{pmatrix} \tag{25}.$$

and the current i(t) is a Gaussian variable with zero mean (charge and discharge events) and:

$$Var(i(t)) = i_{MAX}^2/9$$
 (26).

Where i_{MAX} is the maximum value of the current permitted by the manufacturer (15A). Thus, the standard deviation is $i_{MAX}/3$ and $\pm 3\sigma$ cover all the possible currents.

Finally, we can add R_p the variance of the error introduced by the linearization of the OCV(SoC) map Var(d(SoC)).

E. Measurement noise

Let the system in (13) be perfectly identified. However this system is corrupted by measurement noise:

$$u = u_{nom} + \xi \tag{27}.$$

Thus, (13) can be written as:

$$\dot{x} = Ax + Bu_{nom} + B\xi$$

$$y = Cx + Du_{nom} + D\xi$$
(28).

Covariance matrix due to measurement noise V_n is:

$$V_n = H_n H_n^T Var(\xi(t))$$
 (29).

where:

$$H_n = \begin{pmatrix} B \\ D \end{pmatrix} \tag{30}.$$

and $\xi(t)$ is a white Gaussian noise with zero mean superposed to the input and its variance $Var(\xi(t))$ can be evaluated from some experiments.

F. Simulation results

Simulation results are performed considering the gain \overline{L} designed for the linearized system; however the observer is based on equations (10) in order to take into account the non-linearity between v_{OCV} and (SoC).

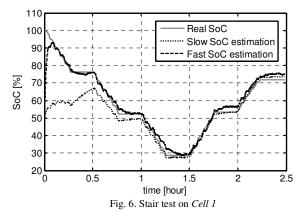
In order to see how the Kalman Filter works, we used the current measurement from the Hall sensors adding white Gaussian noise with zero mean and variance of 1A offline. Moreover in order to test the robustness of the scheme, a constant offset of 0.1A is added to the input. We introduced also a white Gaussian noise with zero mean and variance of 10mV to the measured voltage.

For the project of the observer we decided the following parametric uncertainty:

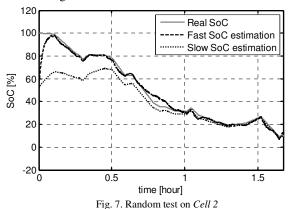
- 80% on identified value of C_1 and C_2 ;
- 80% on the identified value of R_0 ;
- High uncertainty on $S_{C,a}$;

The simulation are based on the data collected during stair current tests and the random test previously described. However we initialized the observer with an incorrect SoC. Results for stair tests on *Cell 1* are shown in Fig. 6. Both the experiments start with an incorrect SoC initialization $(SoC_0 = 50\%)$. The differences between the estimations are in the project of matrix V_p , particularly the convergence is faster, giving a different *weight* to the term that depends on $S_{C,a}$ in H_p – thus increasing its variance. The greater the value of that element in H_p , the faster the estimation. However the accuracy decreases. For the following results, the *weight* is ten times greater for the fast estimator than for the slow one.

Slow estimation converges after one hour. However its standard deviation - after it reaches the real SoC - is 1.00%. On the other hand fast SoC estimation converges in 10 minutes, but the estimation is noisy, with a standard deviation of 3.88% SoC.



As for the *Cell 2*, considering the same filter design, the result for the random test - starting from $SoC_0 = 50\%$ - are shown in Fig. 7.



VI. CONCLUSION AND FUTURE WORK

This paper presents a methodology in order to design an observer based on Kalman Filter Theory for SoC estimation.

A model of the battery, its identification and validation are presented, showing good simulation results with respect to the real data acquired in laboratory, also considering datasets from a different cell of the same manufacturer. An observer based on Kalman Filter Theory is designed, focusing particularly on two different uncertainties:

parametric uncertainty (spread of parameters among different cells) and measurement noise (typical in real applications). Particularly the results show a good accuracy in estimating the SoC. Furthermore, different observer show a *trade-off* problem: fast convergence estimator penalizes the accuracy of the estimation and, on the contrary, high accurate estimator penalizes the rate of convergence.

The methodology can be extended on cell of different manufacturer or different technologies. Moreover, it would be interesting to take into account also temperature effects in the electric model and for the SoC estimation.

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