# Standard Code Library

F0RE1GNERS

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## 一切的开始

#### 宏定义

```
#include<bits/stdc++.h>
   using namespace std;
   // #define int long long
    #define mst(a) memset(a,0,sizeof(a))
   \#define\ cf\ int\ Tcodeforces; Tcodeforces; for (Tcodeforce = 1;\ Tcodeforce <=\ Tcodeforces;\ Tcodeforce++)
   typedef long long ll;
   typedef unsigned long long ull;
   const ll maxn = 2e5 +7;
   const ll maxm = 2e5 +7;
   const ll inf = 0x3f3f3f3f;
11
   const ll mod = 1000000007;//1e9+7
12
13
   signed main()
14
15
        freopen("D:/c++source file/intxt/in.txt","r",stdin);
16
17
        ios :: sync_with_stdio(0);
        cin.tie(0);
18
19
20
        //cerr<<"Time : "<<1000*((double)clock())/(double)CLOCKS_PER_SEC<<"ms";</pre>
        return (0);
21
23
24
    谏读
    //读入整数 可以是负数
    inline int read(){
2
        int x=0,f=1,c=getchar();
        while(!isdigit(c)){if(c=='-')f=-1;c=getchar();}
        while(isdigit(c))\{x=(x<<1)+(x<<3)+(c^48); c=getchar();\}
        return f==1?x:-x;
   }
    ## 杂类输入
       • getline
   char s[];
   int len;
   cin.getline(s, len);
    数据结构
   线段树
    #define mid ((l + r) / 2)
    #define lson (rt << 1)</pre>
    #define rson (rt << 1 | 1)
       ● 普适
   int a[maxn];
   \textbf{struct node} \ \{
        int len, val;//长度 数量
        int s, t, pl, el;//第一个 最后一个 前缀上升长度 后缀上升长度
   }tree[maxn<<2];</pre>
   node Merge(node a, node b) {
       node res;
       res.pl = a.pl;
       res.el = b.el;
10
       res.s = a.s;
       res.t = b.t;
12
```

```
res.len = a.len + b.len;
13
14
        res.val = a.val + b.val;
        if(a.t > b.s) return res;
15
        if(a.pl == a.len) res.pl = a.len + b.pl;
16
17
        if(b.el == b.len) res.el = a.el + b.len;
        res.val += a.el * b.pl;
18
        return res;
19
    }
20
21
    void update(int rt) {
22
        tree[rt] = Merge(tree[lson], tree[rson]);
23
24
25
    void build(int l, int r,int rt) {
26
        if(l == r){
27
             tree[rt].len = 1;
28
29
            tree[rt].val = 1;
            tree[rt].s = tree[rt].t = a[l];
30
31
             tree[rt].el = tree[rt].pl = 1;
            return ;
32
33
        build(l, mid, lson);
34
        build(mid+1, r, rson);
35
        update(rt);
37
    }
38
    void modify(int l, int r, int rt, int x, int y) {
39
        if(l == r) {
40
41
            tree[rt].s = tree[rt].t = y;
            return ;
42
43
        if(x <= mid) modify(l, mid, lson, x, y);</pre>
44
45
        else modify(mid+1, r, rson, x, y);
        update(rt);
    }
47
48
    node query(int l, int r, int rt, int L, int R) {
49
        if(L <= l && r <= R) return tree[rt];</pre>
50
51
        node tmp;
        tmp.len = -1;
52
53
        if(L <= mid) tmp = query(l ,mid, lson, L, R);</pre>
        if(mid < R) {
54
             if(tmp.len == -1) tmp = query(mid+1, r, rson, L, R);
55
56
             else tmp = Merge(tmp, query(mid+1, r, rson, L, R));
57
        }
58
        return tmp;
    }
59
    + 加法乘法
    int n, m, md;
    int a[maxn];
    struct node{
        int val;
        int add, mult;
    }tree[maxn<<2];</pre>
    void buildtree(int l, int r, int rt) {
        if(l == r) {
             tree[rt].val = a[l];
10
11
             tree[rt].add = 0;
             tree[rt].mult = 1;
12
             tree[rt].val %= md;
13
14
             return ;
        }
15
16
        buildtree(l, mid, lson);
17
        buildtree(mid+1, r, rson);
18
19
        tree[rt].add = 0;
20
        tree[rt].mult = 1;
21
        tree[rt].val = tree[lson].val + tree[rson].val;
        tree[rt].val %= md;
22
```

```
return :
23
24
   }
    //先乘再加
25
    void pushdown(int l, int r, int rt) {
26
27
        tree[lson].val = (tree[lson].val * tree[rt].mult + tree[rt].add * (mid-l+1)) % md;
        tree[lson].add = (tree[lson].add * tree[rt].mult + tree[rt].add) % md;
28
        tree[lson].mult = (tree[lson].mult * tree[rt].mult) % md;
29
30
        tree[rson].val = (tree[rson].val * tree[rt].mult + tree[rt].add * (r-mid)) % md;
31
32
        tree[rson].add = (tree[rson].add * tree[rt].mult + tree[rt].add) % md;
        tree[rson].mult = (tree[rson].mult * tree[rt].mult) % md;
33
34
        tree[rt].add = 0;
35
        tree[rt].mult = 1;
36
        return ;
37
   }
38
39
    int query(int l, int r, int rt, int L, int R) {
40
41
        if(L > r || R < l) return 0;
        if(l >= L && r <= R) {return tree[rt].val;}</pre>
42
43
44
        pushdown(l, r, rt);
45
        return query(l, mid, lson, L, R) + query(mid+1, r, rson, L, R);
   }
47
48
    void add(int l, int r, int rt, int L, int R, int val) {
        if(L > r || R < l) return ;</pre>
49
        if(l >= L && r <= R) {
50
51
            tree[rt].add += val;
            tree[rt].val += val * (r-l+1);
52
            tree[rt].val %= md;
53
            return ;
54
55
        }
56
        pushdown(l,r,rt);
        add(l, mid, lson, L, R, val);
57
58
        add(mid+1, r, rson, L, R, val);
        tree[rt].val = tree[lson].val + tree[rson].val;
59
            tree[rt].val %= md;
60
61
        return ;
   }
62
63
    void mult(int l, int r, int rt, int L, int R, int val) {
64
        if(L > r || R < l) return ;</pre>
65
66
        if(l >= L && r <= R) {
            pushdown(l, r, rt);
67
68
            tree[rt].mult *= val;
            tree[rt].val *= val;
69
            tree[rt].val %= md;
            return ;
71
72
        }
73
        pushdown(l, r, rt);
74
        mult(l, mid, lson, L, R, val);
        mult(mid+1, r, rson, L, R, val);
76
77
        tree[rt].val = tree[lson].val + tree[rson].val;
            tree[rt].val %= md;
78
   }
79
    树状数组
       ● 注意: 0 是无效下标
   int tree[maxn];
    int n;
2
    int lowbit(int x) {return x & -x;}
    void add(int pos, int val)
        while(pos <= n)</pre>
            tree[pos] += val;
            pos += lowbit(pos);
```

```
}
10
11
    }
    int sum(int pos)
12
13
14
        int ans = 0;
        while(pos)
15
16
            ans += tree[pos];
17
            pos -= lowbit(pos);
18
19
        return ans;
20
21
    }
        • 区间修改 & 区间查询(单点修改,查询前缀和的前缀和)
    int tree1[maxn], tree2[maxn];
1
    int n, q;
2
    int lowbit(int x) {return x & -x;}
    inline int read(){
        int x=0,f=1,c=getchar();
        while(!isdigit(c)){if(c=='-')f=-1;c=getchar();}
        while(isdigit(c))\{x=(x<<1)+(x<<3)+(c^48); c=getchar();\}
        return f==1?x:-x;
    }
    void add(int pos, int val)
    {
11
12
        int addval = val * pos;
        while(pos <= n)</pre>
13
14
15
            tree1[pos] += val;
            tree2[pos] += addval;
16
17
            pos += lowbit(pos);
        }
18
19
    }
    int sum1(int pos)
20
    {
21
22
        // cout<<"sum1:"<<pos<<"=";
        int ans = 0;
23
        while(pos)
24
25
        {
26
            ans += tree1[pos];
27
            pos -= lowbit(pos);
28
29
        // cout<<ans<<"\n";
        return ans;
30
31
    int sum2(int pos)
32
33
        // cout<<"sum2:"<<pos<<"=";
34
        int ans = 0;
35
        while(pos)
36
37
            ans += tree2[pos];
38
39
            pos -= lowbit(pos);
        }
40
41
        // cout<<ans<<endl;</pre>
        return ans;
42
43
    }
    int sum(int pos){return sum1(pos) * (pos + 1) - sum2(pos);}
44
    void modify(int l, int r, int val)
45
46
    {
        add(l ,val);
47
        add(r+1, -val);
48
    }
49
        • 二维树状数组单点修改
    int tree[maxn][maxn];
    int xn, yy;
    int lowbit(int x) {
        return x & -x;
    }
```

```
void add(int x, int y, int val) {
7
        int my = y;
8
        while (x \le xn) {
10
            y = my;
11
12
             while (y <= yy) {
                tree[x][y] += val;
13
                 y += lowbit(y);
14
15
16
17
            x += lowbit(x);
        }
18
    }
19
20
    int getsum(int x, int y) {
21
22
        int ans = 0;
        int my = y;
23
24
        while (x) {
25
26
            y = my;
27
28
            while (y) {
                 ans += tree[x][y];
                 y -= lowbit(y);
30
31
32
            x -= lowbit(x);
33
34
        }
35
        return ans;
36
37
38
    int q_get(int x1, int y1, int x2, int y2) {
39
        int ans = 0;
        ans += getsum(x2, y2);
40
        ans -= getsum(x1 - 1, y2);
41
        ans -= getsum(x2, y1 - 1);
42
        ans += getsum(x1 - 1, y1 - 1);
43
44
        return ans;
    }
45
        • 区间修改区间查询二维树状数组
    int lowbit(int x) {return x & -x;}
    void add(int x, int y, int val)
2
3
    {
         // cout<<x<<" "<<y<<" "<<val<<endl;
        int memoy = y, memox = x;
5
        while(x <= n)
            y = memoy;
8
            while(y <= m)</pre>
             {
10
11
                 t1[x][y] += val;
                 t2[x][y] += val * memoy;
12
13
                 t3[x][y] += val * memox;
                 t4[x][y] += val * memox * memoy;
14
15
                 y += lowbit(y);
17
             x += lowbit(x);
18
    }
19
20
    int ask(int x, int y)
21
22
    {
23
        int ans = 0;
        int memoy = y, memox = x;
24
        while(x)
25
26
            y = memoy;
27
            \mbox{while}(\mbox{y})
28
             {
29
```

```
ans += (memoy+1)*(memox+1)*t1[x][y];
30
                ans -= t2[x][y] * (memox + 1);
31
                ans -= t3[x][y] * (memoy + 1);
32
                ans += t4[x][y];
33
                y -= lowbit(y);
34
            }
35
            x -= lowbit(x);
36
        }
37
        return ans;
38
39
   }
40
41
   void range_add(int xx1, int yy1, int xx2, int yy2, int val)
42
        add(xx1,yy1, val);
43
        add(xx1, yy2 + 1, -val);
44
        add(xx2 + 1, yy1, -val);
45
46
        add(xx2+1,yy2+1,val);
   }
47
48
   int range_ask(int xx1, int yy1, int xx2, int yy2)
49
50
   {
51
        int ans = 0;
        ans += ask(xx1-1,yy1-1);
52
        ans -= ask(xx1-1,yy2);
        ans -= ask(xx2, yy1-1);
54
        ans += ask(xx2, yy2);
55
56
        return ans;
   }
57
    并查集
   int n, m;
   int fa[maxn], rk[maxn];
   inline void init(){for(int i=0;i<=n;i++){fa[i]=i;rk[i]=1;}}</pre>
   int find(int x) {return fa[x]==x?x:(fa[x]=find(fa[x]));}
   inline void merge(int i, int j)
5
        int x = find(i), y = find(j);
        if(rk[x] <= rk[y]) fa[x] = y;
        else fa[y] = x;
10
        if(rk[x] == rk[y] \&\& x != y) rk[y] ++;
   }
11
    数学
       gcd
   ll Gcd(ll a,ll b){return b==0?a:Gcd(b,a%b);}
    素数筛
       线性筛
   visit[] 第 i 个数是不是质数,1 表示质数,prime[] 质数集合 2 3 5 7,k 质数数量, 筛到 maxn
   bool visit[maxn];
2
    int k, prime[maxn];
   void initPrime()//init
4
        visit[0] = visit[1] = 1;
        for(int i = 2; i < maxn; i++)</pre>
7
            if(!visit[i]) prime[k++] = i;
            for(int j = 0; j < k && i*prime[j]<maxn; j++)//遍历素数数组
11
            {
                visit[i * prime[j]] = 1;
12
                if(i % prime[j] == 0) break;
13
            }
```

```
15 }
16 }
```

• 线性筛+欧拉函数

欧拉函数(Euler's totient function),即  $\varphi(n)$ ,表示的是小于等于 n 和 n 互质的数的个数。存在 phi[i] 里。

```
//phi(i):= 小于等于 i 的与 i 互质的数的个数
   const int p_max = 1e5 + 100;
   int phi[p_max];
    void get_phi() {
        static bool vis[p_max];
        static int prime[p_max], p_sz, d;
        vis[0] = vis[1] = 1; phi[1] = 1;
        for(int i = 2; i < p_max; i++) {</pre>
            if (!vis[i]) {
                prime[p_sz++] = i;
                phi[i] = i - 1;
11
            for (int j = 0; j < p_sz && (d = i * prime[j]) < p_max; ++j) {</pre>
13
                 vis[d] = 1;
14
15
                 if (i % prime[j] == 0) {
                    phi[d] = phi[i] * prime[j];
16
18
19
                else phi[d] = phi[i] * (prime[j] - 1);
            }
20
        }
21
   }
```

#### 扩展欧几里得

- $\vec{x} ax + by = gcd(a, b)$  的一组解
- 如果 a 和 b 互素,那么 x 是 a 在模 b 下的逆元
- 注意 x 和 y 可能是负数

```
//ax+by=qcd(a,b) 是否有解 有解解就是 x
    bool ex_gcd(int a, int b, int& x, int& y) {
        if(b == 0) {
            x = 1;
            y = 0;
            return a;
        int d = ex_gcd(b, a%b, x, y);
        int temp = x;
        x = y;
10
        y = temp - a/b * y;
11
12
        return d;
13
    //同余方程 ax+by=c 即 ax=c(mod b) 是否有解
15
    bool CongruenceEquation(int a, int b,int c, int& x, int& y) {
        int d = ex_gcd(a,b,x,y);
16
17
        if(c%d != 0) return 0;
        int k = c / d;
18
        x \star = k;
19
        y *= k;
20
        return 1;
   }
22
```

#### 类欧几里得

- $m = \lfloor \frac{an+b}{c} \rfloor$ .
- $f(a,b,c,n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor$ : 当  $a \geq c$  or  $b \geq c$  时, $f(a,b,c,n) = (\frac{a}{c})n(n+1)/2 + (\frac{b}{c})(n+1) + f(a \bmod c,b \bmod c,c,n)$ ; 否则 f(a,b,c,n) = nm f(c,c-b-1,a,m-1)。
- $g(a,b,c,n) = \sum_{i=0}^{n} i \lfloor \frac{ai+b}{c} \rfloor$ : 当  $a \geq c$  or  $b \geq c$  时, $g(a,b,c,n) = (\frac{a}{c})n(n+1)(2n+1)/6 + (\frac{b}{c})n(n+1)/2 + g(a \mod c,b \mod c,c,n)$ ; 否则  $g(a,b,c,n) = \frac{1}{2}(n(n+1)m-f(c,c-b-1,a,m-1)-h(c,c-b-1,a,m-1))$ 。
- $h(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor^2$ : 当  $a \geq c$  or  $b \geq c$  时, $h(a,b,c,n) = (\frac{a}{c})^2 n(n+1)(2n+1)/6 + (\frac{b}{c})^2 (n+1) + (\frac{a}{c})(\frac{b}{c})n(n+1) + h(a \bmod c, b \bmod c, c, n) + 2(\frac{a}{c})g(a \bmod c, b \bmod c, c, n) + 2(\frac{b}{c})f(a \bmod c, b \bmod c, c, n)$ ; 否则

```
h(a,b,c,n) = nm(m+1) - 2g(c,c-b-1,a,m-1) - 2f(c,c-b-1,a,m-1) - f(a,b,c,n)
```

#### 逆元

- 如果 p 不是素数, 使用拓展欧几里得
- 前置模板: 快速幂 / 扩展欧几里得

```
int n;
    ll mod = (int)(1e9+7);
    inline ll qpow(ll base, ll rk) {
        ll ans = 1;
        ll now = (base % mod + mod) % mod;
        for(; rk; rk >>= 1) {
            if(rk & 1) {ans *= now; ans %= mod;}
            now *= now; now %= mod;
        }
        return ans;
   }
11
12
    //费马小定理求单个数逆元 mod 必须素数
13
   ll get_inv(ll x) {return qpow(x, mod - 2);}
14
   int inv_n[maxn];
16
17
    //线性求 n 个数的逆元
   void init_n() {
18
        inv_n[1] = 1;
19
        for(int i = 2; i <= n; i++) {</pre>
            inv_n[i] = 1ll * (mod - mod / i) * inv_n[mod % i] % mod;
21
22
   }
23
24
   ll a[maxn], inv[maxn], s[maxn], sv[maxn];//原数组 逆元 前缀积 逆元前缀积
   //线性求任意 n 个数的逆元
26
27
   void init_any() {
        s[0] = 1;
28
        for(int i = 1; i <= n; i++) s[i] = s[i-1] * a[i] % mod;</pre>
29
        sv[n] = qpow(s[n], mod-2);
30
        for(int i = n; i >= 1; i--) sv[i-1] = sv[i] * a[i] % mod;
31
        for(int i = 1; i <= n; i++) inv[i] = sv[i] * s[i-1] % mod;</pre>
32
   }
33
       ● 预处理阶乘及其逆元
    LL invf[M], fac[M] = {1};
1
    void fac_inv_init(LL n, LL p) {
        FOR (i, 1, n)
           fac[i] = i * fac[i - 1] % p;
        invf[n - 1] = bin(fac[n - 1], p - 2, p);
        FORD (i, n - 2, -1)
           invf[i] = invf[i + 1] * (i + 1) % p;
   }
```

#### 组合数

- 如果数较小,模较大时使用逆元
- 前置模板: 逆元-预处理阶乘及其逆元

```
inline LL C(LL n, LL m) { // n >= m >= 0
return n < m || m < 0 ? 0 : fac[n] * invf[m] % MOD * invf[n - m] % MOD;
}</pre>
```

#### 快速幂

● 如果模数是素数,则可在函数体内加上 n %= MOD - 1; (费马小定理)

```
while(n)
            if(n\&1) t *= x;
            n >>= 1;
            x *= x;
            x = (x + mod) \% mod;
            t = (t + mod) \% mod;
        return t;
13
```

## 质因数分解

- 前置模板:素数筛
- 带指数

```
LL factor[30], f_sz, factor_exp[30];
    void get_factor(LL x) {
        f_sz = 0;
        LL t = sqrt(x + 0.5);
        for (LL i = 0; pr[i] <= t; ++i)</pre>
            if (x % pr[i] == 0) {
                factor_exp[f_sz] = 0;
                while (x % pr[i] == 0) {
                    x /= pr[i];
                    ++factor_exp[f_sz];
                factor[f_sz++] = pr[i];
12
        if (x > 1) {
           factor_exp[f_sz] = 1;
            factor[f_sz++] = x;
17
   }
```

## 公式

#### 调和级数部分和

$$S = \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{n}S = \ln(n) + eulr + \frac{1}{2n}elur = 0.57721\,56649\,01532\,86060$$

#### 一些数论公式

- 当  $x \ge \phi(p)$  时有  $a^x \equiv a^{x \bmod \phi(p) + \phi(p)} \pmod{p}$
- $\mu^2(n) = \sum_{d^2|n} \mu(d)$
- $\sum_{d|n} \varphi(d) = n$
- $\sum_{d|n}^{-1} 2^{\omega(d)} = \sigma_0(n^2)$ ,其中 $\omega$ 是不同素因子个数
- $\bullet \ \textstyle \sum_{d|n} \mu^2(d) = 2^{\omega(d)}$

#### 一些数论函数求和的例子

- $\begin{array}{l} \bullet \ \, \sum_{i=1}^n i[gcd(i,n)=1] = \frac{n\varphi(n)+[n=1]}{2} \\ \bullet \ \, \sum_{i=1}^n \sum_{j=1}^m [gcd(i,j)=x] = \sum_d \mu(d) \lfloor \frac{n}{dx} \rfloor \lfloor \frac{m}{dx} \rfloor \\ \bullet \ \, \sum_{i=1}^n \sum_{j=1}^m gcd(i,j) = \sum_{i=1}^n \sum_{j=1}^m \sum_{d|gcd(i,j)} \varphi(d) = \sum_d \varphi(d) \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor \end{array}$
- $S(n) = \sum_{i=1}^n \mu(i) = 1 \sum_{i=1}^n \sum_{d|i,d < i} \mu(d) \stackrel{t = \frac{i}{d}}{=} 1 \sum_{t=2}^n S(\lfloor \frac{n}{t} \rfloor)$  利用  $[n=1] = \sum_{d|n} \mu(d)$
- $S(n) = \sum_{i=1}^{n} \varphi(i) = \sum_{i=1}^{n} i \sum_{i=1}^{n} \sum_{d|i,d < i} \varphi(i) \stackrel{t = \frac{i}{d}}{=} \frac{i(i+1)}{2} \sum_{t=2}^{n} S(\frac{n}{t})$  利用  $n = \sum_{d|n} \varphi(d)$
- $\sum_{i=1}^n \mu^2(i) = \sum_{i=1}^n \sum_{d^2 \mid n} \mu(d) = \sum_{d=1}^{\lfloor \sqrt{n} \rfloor} \mu(d) \lfloor \frac{n}{d^2} \rfloor$

```
\begin{array}{l} \bullet \; \sum_{i=1}^n \sum_{j=1}^n gcd^2(i,j) = \sum_d d^2 \sum_t \mu(t) \lfloor \frac{n}{dt} \rfloor^2 \\ \stackrel{x=dt}{=} \sum_x \lfloor \frac{n}{x} \rfloor^2 \sum_{d \mid x} d^2 \mu(\frac{x}{d}) \\ \bullet \; \sum_{i=1}^n \varphi(i) = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n [i \perp j] - 1 = \frac{1}{2} \sum_{i=1}^n \mu(i) \cdot \lfloor \frac{n}{i} \rfloor^2 - 1 \end{array}
```

#### 斐波那契数列性质

```
• F_{a+b} = F_{a-1} \cdot F_b + F_a \cdot F_{b+1}

• F_1 + F_3 + \dots + F_{2n-1} = F_{2n}, F_2 + F_4 + \dots + F_{2n} = F_{2n+1} - 1

• \sum_{i=1}^n F_i = F_{n+2} - 1

• \sum_{i=1}^n F_i^2 = F_n \cdot F_{n+1}

• F_n^2 = (-1)^{n-1} + F_{n-1} \cdot F_{n+1}

• \gcd(F_a, F_b) = F_{\gcd(a,b)}

• \notin n \text{ flip} \ (bc 诺语期)

• \pi(p^k) = p^{k-1}\pi(p)

• \pi(nm) = lcm(\pi(n), \pi(m)), \forall n \perp m

• \pi(2) = 3, \pi(5) = 20

• \forall p \equiv \pm 1 \ (\text{mod } 10), \pi(p) | p - 1

• \forall p \equiv \pm 2 \ (\text{mod } 5), \pi(p) | 2p + 2
```

#### 一些组合公式

- 错排公式:  $D_1=0, D_2=1, D_n=(n-1)(D_{n-1}+D_{n-2})=n!(\frac{1}{2!}-\frac{1}{3!}+\cdots+(-1)^n\frac{1}{n!})=\lfloor\frac{n!}{e}+0.5\rfloor$
- 卡塔兰数 (n 对括号合法方案数,n 个结点二叉树个数, $n\times n$  方格中对角线下方的单调路径数,凸 n+2 边形的三角形划分数,n 个元素的合法出栈序列数):  $C_n=\frac{1}{n+1}\binom{2n}{n}=\frac{(2n)!}{(n+1)!n!}$

#### 中国剩余定理

- 无解返回 -1
- 前置模板: 扩展欧几里得

```
LL CRT(LL *m, LL *r, LL n) {
        if (!n) return 0;
        LL M = m[0], R = r[0], x, y, d;
        FOR (i, 1, n) {
           d = ex_gcd(M, m[i], x, y);
           if ((r[i] - R) % d) return -1;
            x = (r[i] - R) / d * x % (m[i] / d);
            // 防爆 LL
            // x = mul((r[i] - R) / d, x, m[i] / d);
           R += x * M;
            M = M / d * m[i];
           R %= M;
       }
13
        return R >= 0 ? R : R + M;
14
   }
```

#### 博弈

- Nim 游戏: 每轮从若干堆石子中的一堆取走若干颗。先手必胜条件为石子数量异或和非零。
- 阶梯 Nim 游戏:可以选择阶梯上某一堆中的若干颗向下推动一级,直到全部推下去。先手必胜条件是奇数阶梯的异或和非零(对于偶数阶梯的操作可以模仿)。
- Anti-SG: 无法操作者胜。先手必胜的条件是:
  - SG 不为 0 且某个单一游戏的 SG 大于 1。
  - SG为0且没有单一游戏的SG大于1。
- Every-SG: 对所有单一游戏都要操作。先手必胜的条件是单一游戏中的最大 step 为奇数。
  - 对于终止状态 step 为 0
  - 对于 SG 为 0 的状态, step 是最大后继 step +1
  - 对于 SG 非 0 的状态, step 是最小后继 step +1
- 树上删边:叶子 SG 为 0,非叶子结点为所有子结点的 SG 值加 1 后的异或和。

#### 尝试:

- 打表找规律
- 寻找一类必胜态(如对称局面)
- 直接博弈 dp

## 图论

#### LCA

● 倍增

```
int N, M, S;
    int fa[maxn][31], dep[maxn];
    //fa[i][j]: 第 i 个点的第 2^j 个祖先
    vector<int> G[maxn];
    void dfs(int root,int fno)
7
        fa[root][0] = fno;
        dep[root] = dep[fno] + 1;
        //初始化 fa
        for(int i = 1; i < 31; ++i)</pre>
11
12
            fa[root][i] = fa[fa[root][i-1]][i-1];
13
        }
14
15
        //遍历
        for(auto y : G[root])
16
17
             if(y == fno) continue;
18
19
             dfs(y, root);
20
    }
21
22
    int lca(int x, int y)
23
24
    {
        if(dep[x] > dep[y]) swap(x,y);
25
        int tem = dep[y] - dep[x];
26
        for(int i = 0; tem; ++i, tem >>= 1)
27
28
             if(tem&1) y = fa[y][i];
29
30
        if(y == x) return x;
31
32
        for(int i = 30; i >= 0 && y != x; --i)
33
             if(fa[x][i] != fa[y][i])
35
             {
36
                 x = fa[x][i];
37
                 y = fa[y][i];
38
        return fa[x][0];
40
41
    }
    void dfs(int u, int fa) {
        pa[u][0] = fa; dep[u] = dep[fa] + 1;
        FOR (i, 1, SP) pa[u][i] = pa[pa[u][i - 1]][i - 1];
        for (int& v: G[u]) {
             if (v == fa) continue;
             dfs(v, u);
        }
    }
    int lca(int u, int v) {
   if (dep[u] < dep[v]) swap(u, v);</pre>
10
11
        int t = dep[u] - dep[v];
12
        FOR (i, 0, SP) if (t & (1 << i)) u = pa[u][i];
13
        FORD (i, SP - 1, -1) {
14
             int uu = pa[u][i], vv = pa[v][i];
15
            if (uu != vv) { u = uu; v = vv; }
```

```
17
18
        return u == v ? u : pa[u][0];
    }
19
    欧拉路径
    //欧拉回路 1 有向图 2 无向图
1
    int tsk, n, m;
    int head[maxn], nxt[maxm], to[maxm], tote;
    int ind[maxn], outd[maxn];
    void addedge(int u, int v)
    {
        nxt[++tote] = head[u];
        head[u] = tote;
        to[tote] = v;
10
    int getegs(int rt)
11
12
    {
        return ind[rt] + outd[rt];
13
14
15
16
    int fa[maxn], rk[maxn];
    vector<int> path;
17
    int skt[maxm], top = 0;
18
    inline void init(){for(int i=0;i<=n;i++){fa[i]=i;rk[i]=1;}}</pre>
    int find(int x) {return fa[x]==x?x:(fa[x]=find(fa[x]));}
20
    inline void merge(int i, int j)
21
22
    {
        int x = find(i), y = find(j);
23
24
        if(rk[x] <= rk[y]) fa[x] = y;
        else fa[y] = x;
25
        if(rk[x] == rk[y] \&\& x != y) rk[y]++;
    }
27
29
    int vis[maxm];
    void dfs1(int u)
30
31
        for(int &e = head[u]; e; e = nxt[e])
32
33
             if(vis[e>>1]) continue;int t = e;
34
35
             vis[t>>1] = 1;dfs1(to[t]);
             path.push_back(t&1?-(t>>1):t>>1);
36
37
    }
38
39
    void task1()
40
41
        tote = 1;
42
43
        mst(vis);
        for(int i = 1; i <= m; i++)</pre>
44
45
46
             int u, v;
47
             cin>>u>>v;
48
             addedge(u,v);
             addedge(v,u);
49
             ind[v]++;outd[u]++;
             merge(u,v);
51
52
53
        int ori = -1, f = 1;
        for(int i = 1; i <= n; i++)</pre>
54
55
56
             int qwq = getegs(i);
             if(qwq == 0) continue;
             if(qwq & 1) f = 0;
58
             if(ori == -1) ori = i;
59
60
            if(find(ori) != find(i)) f = 0;
61
        if(f==0) {cout<<"NO"<<endl; return;}</pre>
62
        else cout<<"YES"<<endl;</pre>
63
        if(ori != -1)
64
65
        {
```

```
dfs1(ori);
67
              for(int i = path.size() - 1; i >= 0; i--)
68
              {
                   cout<<path[i]<<" ";</pre>
69
              }
              cout<<endl;</pre>
71
72
     }
73
74
     void dfs2(int rt)
75
76
77
         for(int &e = head[rt]; e; e = nxt[e])
78
              if(vis[e]) continue;int t = e;
79
80
              vis[e] = 1;
              dfs2(to[e]);
81
82
              skt[++top] = t;
83
84
     }
85
     void task2()
86
87
88
          tote = 0;
89
         for(int i = 1; i <= m; i++)</pre>
90
91
              int u, v;
92
              cin>>u>>v;
              addedge(u,v);
93
              merge(u,v);
              ind[v]++;outd[u]++;
95
96
         int ori = -1, f = 1;
97
98
         for(int i = 1; i <= n; i++)</pre>
99
              int qwq = getegs(i);
100
101
              if(qwq == 0) continue;
              if(ind[i] != outd[i]) f = 0;
102
              if(ori == -1) ori = i;
103
              if(find(ori) != find(i)) f = 0;
104
105
         if(f == 0) {cout<<"NO"<<endl;return;}</pre>
         else cout<<"YES"<<endl;</pre>
107
          if(ori != -1)
108
109
              dfs2(ori);
110
111
              for(int i = m; i >= 1; i--) cout<<skt[i]<<" ";</pre>
              cout<<endl;</pre>
112
113
              // for(int i = path.size() - 1; i >= 0; i--)
              // cout<<path[i]<<" ";
114
              // cout<<endl;</pre>
115
         }
116
     }
117
     int S[N << 1], top;</pre>
     Edge edges[N << 1];</pre>
     set<int> G[N];
     void DFS(int u) {
         S[top++] = u;
         for (int eid: G[u]) {
              int v = edges[eid].get_other(u);
              G[u].erase(eid);
              G[v].erase(eid);
10
              DFS(v);
12
              return;
         }
13
14
     }
15
     void fleury(int start) {
16
         int u = start;
17
          top = 0; path.clear();
18
         S[top++] = u;
19
```

```
while (top) {
20
21
            u = S[--top];
            if (!G[u].empty())
22
                DFS(u);
23
24
            else path.push_back(u);
        }
25
   }
26
    强连通分量与 2-SAT
   //2-sat
2
   int n, m;//n 人 1~n(1&2 同党) m 关系
3
    int oth(int x) {return x%2?x+1:x-1;}//另一个人
   vector<int> e[maxn];//存边
    int dfn[maxn], low[maxn], dfncnt;
    int tj_stack[maxn], in_stack[maxn], tp;
    int scc[maxn], scc_cnt;
    void tarjan(int rt)
    {
11
12
        dfn[rt] = low[rt] = ++dfncnt;
        tj_stack[++tp] = rt, in_stack[rt] = 1;
13
        for(int i = 0; i < e[rt].size(); i++)</pre>
14
15
            int y = e[rt][i];
16
17
            if(!dfn[y])
18
            {
                 tarjan(y);
19
                 low[rt] = min(low[rt], low[y]);
20
21
22
            else if(in_stack[y]) low[rt] = min(low[rt], dfn[y]);
23
        if(dfn[rt] == low[rt])
25
             ++scc_cnt;
26
27
            while(tj_stack[tp] != rt)
28
29
                 scc[tj_stack[tp]] = scc_cnt;
                 in_stack[tj_stack[tp]] = 0;
30
31
                 tp--;
32
            scc[tj_stack[tp]] = scc_cnt;
33
            in_stack[tj_stack[tp]] = 0;
            tp--;
35
36
   }
37
38
    void init()
39
40
    {
        mst(dfn);
41
42
        mst(low);
        dfncnt=0;
43
44
        tp = 0;
        for(int i = 1; i <= n; i++) e[i].clear();</pre>
45
46
        mst(scc);
        scc_cnt = 0;
47
        mst(tj_stack);
48
49
        mst(in_stack);
   }
50
52
    int main()
        // freopen("D:/c++source file/intxt/in.txt","r",stdin);
54
        ios :: sync_with_stdio(0);
55
56
        cin.tie(0);
57
        while(cin>>n>>m)
59
            n *= 2;
60
            init();
61
```

```
62
63
             for(int i = 1; i <= m; i++)</pre>
64
65
                 int u, v;
                 cin>>u>>v;
                 e[u].push_back(oth(v));
67
68
                 e[v].push_back(oth(u));
69
             for(int i = 1; i <= n; i++)</pre>
70
71
             {
                 if(!dfn[i]) tarjan(i);
72
73
             }
             int f = 0;
74
             for(int i = 1; i <= n; i+=2)</pre>
75
76
             {
                 if(scc[i] == scc[oth(i)])
77
78
                 {
                      cout<<"NIE"<<endl;</pre>
79
                      f = 1;
                      break;
81
                 }
82
83
             if(f) continue;
84
             for(int i = 1; i <= n; i+=2)</pre>
             {
86
87
                 cout<<(scc[i] > scc[oth(i)] ? oth(i) : i)<<endl;</pre>
             }
88
        }
89
90
91
        //cerr<<"Time : "<<1000*((double)clock())/(double)CLOCKS_PER_SEC<<"ms";
92
        return (0);
93
94
        先 tarjan 缩点 然后检查行不行 然后直接输出
95
        */
96
    }
    int n, m;
    vector<int> G[N], rG[N], vs;
    int used[N], cmp[N];
5
    void add_edge(int from, int to) {
        G[from].push_back(to);
        rG[to].push_back(from);
    void dfs(int v) {
10
        used[v] = true;
11
        for (int u: G[v]) {
12
13
             if (!used[u])
                 dfs(u);
14
15
16
        vs.push_back(v);
    }
17
18
    void rdfs(int v, int k) {
19
20
        used[v] = true;
        cmp[v] = k;
21
        for (int u: rG[v])
22
             if (!used[u])
23
                 rdfs(u, k);
24
25
    }
26
    int scc() {
        memset(used, 0, sizeof(used));
28
        vs.clear();
29
30
        for (int v = 0; v < n; ++v)
             if (!used[v]) dfs(v);
31
        memset(used, 0, sizeof(used));
32
        int k = 0;
33
        for (int i = (int) vs.size() - 1; i >= 0; --i)
34
             if (!used[vs[i]]) rdfs(vs[i], k++);
35
```

```
return k;
36
37
   }
38
    int main() {
39
        cin >> n >> m;
        n *= 2;
41
42
        for (int i = 0; i < m; ++i) {</pre>
            int a, b; cin >> a >> b;
43
            add_edge(a - 1, (b - 1) ^ 1);
44
            add_edge(b - 1, (a - 1) ^ 1);
45
        }
46
47
        scc();
        for (int i = 0; i < n; i += 2) {</pre>
48
            if (cmp[i] == cmp[i + 1]) {
49
                puts("NIE");
50
                return 0;
51
52
            }
53
        for (int i = 0; i < n; i += 2) {</pre>
            if (cmp[i] > cmp[i + 1]) printf("%d\n", i + 1);
55
            else printf("%d\n", i + 2);
56
57
58
   }
    拓扑排序
    vector<int> G[MAXN]; // vector 实现的邻接表
                          // 标志数组
    int c[MAXN];
2
    vector<int> topo;
                          // 拓扑排序后的节点
3
    bool dfs(int u) {
5
      c[u] = -1;
      for (int v : G[u]) {
        if (c[v] < 0)
         return false;
        else if (!c[v])
10
          if (!dfs(v)) return false;
11
12
13
      c[u] = 1;
      topo.push_back(u);
14
15
      return true;
   }
16
17
18
    bool toposort() {
      topo.clear();
19
      memset(c, 0, sizeof(c));
20
      for (int u = 0; u < n; u++)
21
        if (!c[u])
22
23
          if (!dfs(u)) return false;
      reverse(topo.begin(), topo.end());
24
      return true;
25
   }
26
    vector<int> toporder(int n) {
        vector<int> orders;
2
        queue<int> q;
3
        for (int i = 0; i < n; i++)</pre>
4
            if (!deg[i]) {
                q.push(i);
                orders.push_back(i);
            }
        while (!q.empty()) {
            int u = q.front(); q.pop();
10
            for (int v: G[u])
11
                if (!--deg[v]) {
                     q.push(v);
13
14
                     orders.push_back(v);
                }
15
16
        return orders;
   }
18
```

## Tarjan

#### 割点

• 判断割点

```
• 注意原图可能不连通
   int dfn[N], low[N], clk;
   void init() { clk = 0; memset(dfn, 0, sizeof dfn); }
void tarjan(int u, int fa) {
        low[u] = dfn[u] = ++clk;
        int cc = fa != -1;
        for (int& v: G[u]) {
            if (v == fa) continue;
            if (!dfn[v]) {
                tarjan(v, u);
                low[u] = min(low[u], low[v]);
11
                cc += low[v] >= dfn[u];
            } else low[u] = min(low[u], dfn[v]);
12
13
        if (cc > 1) // ...
14
   }
    桥
       ● 注意原图不连通和重边
   int dfn[N], low[N], clk;
   void init() { memset(dfn, 0, sizeof dfn); clk = 0; }
   void tarjan(int u, int fa) {
        low[u] = dfn[u] = ++clk;
        int _fst = 0;
        for (E& e: G[u]) {
            int v = e.to; if (v == fa && ++_fst == 1) continue;
            if (!dfn[v]) {
                tarjan(v, u);
                if (low[v] > dfn[u]) // ...
                low[u] = min(low[u], low[v]);
11
            } else low[u] = min(low[u], dfn[v]);
12
        }
13
   }
14
    强连通分量缩点
   int n, m;
1
   int val_ori[maxn];
   vector<int> e_ori[maxn], e_scc[maxn];
   int dfn[maxn], low[maxn], dfncnt, tj_stack[maxn], in_stack[maxn], tp;
    int scc[maxn], scc_cnt, val_scc[maxn];//属于哪个 scc 一共多少 scc 缩点后权值和
   //编号从 1 开始
   void tarjan(int rt)
   {
        dfn[rt] = low[rt] = ++dfncnt;
10
11
        tj_stack[++tp] = rt, in_stack[rt] = 1;
        for(auto y : e_ori[rt])
12
13
            if(!dfn[y])
14
15
            {
                tarjan(y);
                low[rt] = min(low[rt], low[y]);
17
            else if(in_stack[y]) low[rt] = min(low[rt], dfn[y]);
19
20
        if(dfn[rt] == low[rt])
21
22
            ++scc_cnt;
24
            while(tj_stack[tp] != rt)
25
            {
26
                scc[tj_stack[tp]] = scc_cnt;
                in_stack[tj_stack[tp]] = 0;
27
```

```
val_scc[scc_cnt] += val_ori[tj_stack[tp]];
28
29
            }
30
            scc[tj_stack[tp]] = scc_cnt;
31
            in_stack[tj_stack[tp]] = 0;
             val_scc[scc_cnt] += val_ori[tj_stack[tp]];
33
34
             tp--;
        }
35
    }
36
37
    int dp[maxn];//当前节点出发最大权值
38
39
    void search_dp(int rt)
40
    {
        dp[rt] = val_scc[rt];
41
42
        int maxv = 0;
        for(auto y : e_scc[rt])
43
44
             if(!dp[y]) search_dp(y);
45
46
             maxv = max(maxv, dp[y]);
47
        dp[rt] = dp[rt] + maxv;
48
49
    }
50
    int main()
52
    {
53
        // freopen("D:/c++source file/intxt/in.txt","r",stdin);
54
        ios :: sync_with_stdio(0);
        cin.tie(0);
55
        cin>>n>>m;
        for(int i = 1; i <= n; i++) cin>>val_ori[i];
57
        for(int i = 1; i <= m; i++)</pre>
58
59
             int u, v;
60
61
             cin>>u>>v;
             e_ori[u].push_back(v);
62
63
        for(int i = 1; i <= n; i++)</pre>
64
65
             if(!dfn[i])
67
             {
68
                 tarjan(i);
69
70
        for(int i = 1; i <= n; i++)</pre>
71
72
73
             for(auto y : e_ori[i])
74
75
                 if(scc[i] != scc[y])//注意 tarjan 完了逆拓扑序建反图
76
                 {
77
                      e_scc[scc[i]].push_back(scc[y]);
78
            }
79
        }
        int ans = 0;
81
82
        for(int i = 1; i <= scc_cnt; i++)</pre>
83
             if(!dp[i])
84
85
                 search_dp(i);
86
                 ans = max(dp[i], ans);
87
88
89
        }
        cout<<ans<<endl;</pre>
91
92
        // cerr<<"Time : "<<1000*((double)clock())/(double)CLOCKS_PER_SEC<<"ms";</pre>
        return (0);
93
    int low[N], dfn[N], clk, B, bl[N];
    vector<int> bcc[N];
2
    void init() { B = clk = 0; memset(dfn, 0, sizeof dfn); }
3
    void tarjan(int u) {
```

```
static int st[N], p;
6
        static bool in[N];
        dfn[u] = low[u] = ++clk;
        st[p++] = u; in[u] = true;
        for (int& v: G[u]) {
             if (!dfn[v]) {
10
11
                 tarjan(v);
                 low[u] = min(low[u], low[v]);
12
            } else if (in[v]) low[u] = min(low[u], dfn[v]);
13
14
        if (dfn[u] == low[u]) {
15
16
             while (1) {
                int x = st[--p]; in[x] = false;
17
                 bl[x] = B; bcc[B].push_back(x);
18
                 if (x == u) break;
19
20
             }
21
             ++B;
        }
22
    }
    最小生成树及
    //唯一
    struct edge{
        int from, to, val;
    }e[maxn];
    int n, m, ans;
    int fa[maxn];
    bool cmp(edge a, edge b){return a.val < b.val;}</pre>
    void init()
    {
        for(int i = 0; i <= n; i++)</pre>
10
11
             fa[i] = i;
13
        }
        ans = 0;
14
15
    int find(int x) {return fa[x]==x?x:(fa[x] = find(fa[x]));}
16
17
    bool uniKruskal()
18
19
        int sum1 = 0, sum2 = 0;//已使用的 可能使用的
        int p = 0;//相同指针
20
        int flag = 0, num = 0;
21
22
        for(int i = 1; i <= m + 1; i++)</pre>
23
        {
             if(p<i)</pre>
24
25
             {
                 if(sum1 != sum2)
26
27
                     flag = 1;
28
                     break;
29
30
                 sum1 = 0, sum2 = 0;
31
                 for(int j = i; j <= m+1; j++)</pre>
32
33
34
                     if(e[j].val != e[i].val)
35
                     {
                          p = j-1;
36
37
                          break;
38
                     if(find(e[j].from) != find(e[j].to))
39
                      ++sum2;
40
                 }
42
             if(i>m) break;
43
44
            int x = find(e[i].from);
             int y = find(e[i].to);
45
             if(x != y \&\& num != n-1)
47
             {
                 num++;
48
49
                 sum1++;
```

```
// merge(x, y);
50
51
                 fa[x] = fa[y];
                 ans += e[i].val;
52
            }
53
54
        if(flag) return false;
55
56
        else return true;
    }
57
58
    int main()
59
60
    {
        freopen("D:/c++source file/intxt/in.txt","r",stdin);
61
        // ios :: sync_with_stdio(0);
62
        // cin.tie(0);
63
        cf
64
65
66
             cin>>n>>m;
            init();
67
             for(int i = 1; i <= m; i++) cin>>e[i].from>>e[i].to>>e[i].val;
69
             sort(e+1,e+1+m, cmp);
             if(uniKruskal()) cout<<ans<<endl;</pre>
70
71
             else cout<<"Not Unique!"<<endl;</pre>
72
        }
73
74
75
        //cerr<<"Time : "<<1000*((double)clock())/(double)CLOCKS_PER_SEC<<"ms";
76
        return (0);
    }
77
    Dijk 最短路
    namespace Dijk
1
2
    {
        const int maxm = 5e5;
3
        const int inff = 2147483647;
        int n, m, s;
        int dis[maxn], vis[maxn];//距离 是否在集合里
        \textbf{struct edge} \{
             int v, w;
             edge(int b, int c):v(b),w(c){}
10
        };
11
        struct node{
             int dis, u;
12
13
             bool operator > (const node a) const {return dis>a.dis;}
14
        };
        vector<edge> e[maxm];
15
        struct Dijkstra
16
17
             void init()//输入数据 重置
18
             {
19
                 cin>>n>>m>>s;
20
                 for(int i = 0; i <= m; i++) e[i].clear();</pre>
21
                 for(int i = 0; i <= n; i++) dis[i] = inff;</pre>
22
23
                 memset(vis, 0, sizeof(vis));
                 for(int i = 1; i <= m; i++)</pre>
24
25
                     int a, b, c;
26
                     cin>>a>>b>>c;
27
28
                     e[a].push_back(edge(b,c));
                 }
29
             void work()//Dijk
31
             {
33
                 dis[s] = 0;
                 priority_queue<node, vector<node>, greater<node> > q;
34
35
                 q.push({0, s});
                 while(!q.empty())
36
37
                     int x = q.top().u;q.pop();
38
                     if(vis[x]) continue;
39
40
                     vis[x] = 1;
```

```
for(int i = 0; i < e[x].size(); i++)</pre>
41
42
                           int to = e[x][i].v, w = e[x][i].w;
43
                           if(dis[to] > dis[x] + w)
44
45
                               dis[to] = dis[x] + w;
46
47
                                q.push({dis[to], to});
                           }
48
                      }
49
                 }
50
             }
51
52
         };
    }
53
54
    int main()
55
56
    {
         // freopen("D:/c++source file/intxt/in.txt","r",stdin);
57
         ios :: sync_with_stdio(0);
58
59
         cin.tie(0);
         Dijk::Dijkstra a;
60
         a.init();
61
62
         a.work();
63
         for(int i = 1; i <= Dijk::n; i++)</pre>
             cout<<Dijk::dis[i]<<' ';</pre>
65
66
         }
67
68
69
         //cerr<<"Time : "<<1000*((double)clock())/(double)CLOCKS_PER_SEC<<"ms";</pre>
         return (0);
70
    }
71
```

## 计算几何

## 字符串

## manacher

```
namespace Manacher {
1
           //记得改 maxn 要开两倍
           char Manacher_s[maxn];
3
           int d[maxn];//变换后的字符串回文半径
           int maxlen;//最长回文串长度
           void manacher(char s[], int n) {
                 memset(d,0,sizeof(d));
                 int cnt = 0;
                 Manacher_s[cnt++] = '#';
                 for(int i = 0; i < n; i++) {Manacher_s[cnt++]=s[i]; Manacher_s[cnt++]='#';}</pre>
10
                 n = strlen(Manacher_s);
11
                 int r = 0, p = 0;
12
                 for(int i = 0; i < n; i++) {</pre>
13
                        if(i < r) d[i] = min(d[2*p-i], r-i);</pre>
14
                        else d[i] = 1;
15
                        \label{lem:while} \textbf{while}(i-d[i]>=0\&\&i+d[i]<n\&\&Manacher\_s[i-d[i]]==Manacher\_s[i+d[i]]) \ d[i]++;
                        if(d[i]+i-1>r) {r=d[i]+i-1;p=i;}
17
18
                 for(int i = 0; i < n; i++) d[i]--;</pre>
19
                 maxlen = 0;
20
                 for(int i = 0; i < n; i++) {</pre>
21
                        if(i&1) maxlen = max(maxlen, (d[i]>>1)*2+1);
22
                        else maxlen = max(maxlen, d[i]);
                 }
24
          }
25
26
    }
```

#### 哈希

内置了自动双哈希开关(小心 TLE)。

```
#include <bits/stdc++.h>
    using namespace std;
2
    #define ENABLE_DOUBLE_HASH
    typedef long long LL;
    typedef unsigned long long ULL;
    const int x = 135;
    const int N = 4e5 + 10;
    const int p1 = 1e9 + 7, p2 = 1e9 + 9;
11
12
    ULL xp1[N], xp2[N], xp[N];
13
    void init_xp() {
14
        xp1[0] = xp2[0] = xp[0] = 1;
15
        for (int i = 1; i < N; ++i) {</pre>
16
17
            xp1[i] = xp1[i - 1] * x % p1;
            xp2[i] = xp2[i - 1] * x % p2;
18
19
            xp[i] = xp[i - 1] * x;
        }
20
    }
21
22
    struct String {
23
        char s[N];
        int length, subsize;
25
        bool sorted;
26
        ULL h[N], hl[N];
27
28
29
        ULL hash() {
            length = strlen(s);
30
            ULL res1 = 0, res2 = 0;
31
            h[length] = 0; // ATTENTION!
32
            for (int j = length - 1; j >= 0; --j) {
33
34
            #ifdef ENABLE_DOUBLE_HASH
                 res1 = (res1 * x + s[j]) % p1;
35
                 res2 = (res2 * x + s[j]) % p2;
36
                 h[j] = (res1 << 32) | res2;
37
38
39
                 res1 = res1 * x + s[j];
                 h[j] = res1;
40
41
            #endif
                // printf("%llu\n", h[j]);
42
43
44
            return h[0];
45
46
        // 获取子串哈希, 左闭右开区间
47
        ULL get_substring_hash(int left, int right) const {
            int len = right - left;
49
50
        #ifdef ENABLE_DOUBLE_HASH
51
            // get hash of s[left...right-1]
            unsigned int mask32 = ~(0u);
52
            ULL left1 = h[left] >> 32, right1 = h[right] >> 32;
            ULL left2 = h[left] & mask32, right2 = h[right] & mask32;
54
55
            return (((left1 - right1 * xp1[len] % p1 + p1) % p1) << 32) |</pre>
                    (((left2 - right2 * xp2[len] % p2 + p2) % p2));
56
57
58
            return h[left] - h[right] * xp[len];
59
        #endif
60
61
        void get_all_subs_hash(int sublen) {
62
63
            subsize = length - sublen + 1;
             for (int i = 0; i < subsize; ++i)</pre>
64
65
                 hl[i] = get_substring_hash(i, i + sublen);
66
            sorted = 0:
67
68
        void sort_substring_hash() {
69
70
            sort(hl, hl + subsize);
            sorted = 1;
71
```

```
}
72
73
         bool match(ULL key) const {
74
             if (!sorted) assert (0);
75
             if (!subsize) return false;
             return binary_search(hl, hl + subsize, key);
77
78
79
         void init(const char *t) {
80
81
             length = strlen(t);
             strcpy(s, t);
82
83
84
    };
85
    int LCP(const String &a, const String &b, int ai, int bi) {
86
         // Find LCP of a[ai...] and b[bi...]
87
88
         int l = 0, r = min(a.length - ai, b.length - bi);
         while (l < r) {</pre>
89
             int mid = (l + r + 1) / 2;
             if (a.get_substring_hash(ai, ai + mid) == b.get_substring_hash(bi, bi + mid))
91
92
                 l = mid;
             else r = mid - 1;
93
94
         }
         return l;
95
    }
96
97
    int check(int ans) {
98
         if (T.length < ans) return 1;</pre>
99
100
         T.get_all_subs_hash(ans); T.sort_substring_hash();
         for (int i = 0; i < S.length - ans + 1; ++i)</pre>
101
             if (!T.match(S.get_substring_hash(i, i + ans)))
102
                 return 1;
103
         return 0;
104
105
    }
106
    int main() {
107
         init_xp(); // DON'T FORGET TO DO THIS!
108
109
110
         for (int tt = 1; tt <= kases; ++tt) {</pre>
             scanf("%d", &n); scanf("%s", str);
111
112
             S.init(str);
             S.hash(); T.hash();
113
114
115
    }
    KMP
    namespace Kmp {
1
2
         //下标从 ⊙ 开始
         int nxt[maxn];//前缀函数值
3
         int cnt = 0;//可以匹配的点的数量
4
         vector<int> res;//可以匹配的点的下标
         void kmp(int a[], char s[], int n) {
             int j = a[0] = 0;
             for(int i = 1; i < n; i++) {</pre>
                 while(j && s[i] != s[j]) j = a[j-1];
                 a[i] = j += s[i] == s[j];
10
11
12
         }
         void compare(char s[], char mode[], int la, int lb) {
13
14
             res.clear(); memset(nxt, 0, sizeof(nxt));
             kmp(nxt, mode, lb);
15
             int j = 0;
             for(int i = 0; i < la; i++) {</pre>
17
                 while(j && s[i] != mode[j]) j = nxt[j-1];
18
19
                 if(s[i] == mode[j]) j++;
                 if(j == lb) res.push_back(i-j+1);
20
             }
21
             cnt = res.size();
22
23
    };
```

```
● 前缀函数(每一个前缀的最长 border)
   void get_pi(int a[], char s[], int n) {
1
        int j = a[0] = 0;
2
        FOR (i, 1, n) {
3
            while (j && s[i] != s[j]) j = a[j - 1];
            a[i] = j += s[i] == s[j];
   }
       ● Z函数(每一个后缀和该字符串的 LCP 长度)
   void get_z(int a[], char s[], int n) {
1
        int l = 0, r = 0; a[0] = n;
2
        FOR (i, 1, n) {
            a[i] = i > r ? 0 : min(r - i + 1, a[i - l]);
4
            while (i + a[i] < n && s[a[i]] == s[i + a[i]]) ++a[i];</pre>
            if (i + a[i] - 1 > r) { l = i; r = i + a[i] - 1; }
7
   }
   Trie
   namespace trie {
1
        int t[N][26], sz, ed[N];
        void init() { sz = 2; memset(ed, 0, sizeof ed); }
3
        int _new() { memset(t[sz], 0, sizeof t[sz]); return sz++; }
        void ins(char* s, int p) {
            int u = 1;
            FOR (i, 0, strlen(s)) {
                int c = s[i] - 'a';
                if (!t[u][c]) t[u][c] = _new();
10
                u = t[u][c];
11
            ed[u] = p;
12
13
        }
14
   }
    AC 自动机
   const int N = 1e6 + 100, M = 26;
   int mp(char ch) { return ch - 'a'; }
4
    struct ACA {
       int ch[N][M], danger[N], fail[N];
        int sz;
        void init() {
8
            sz = 1;
            memset(ch[\theta], \theta, sizeof ch[\theta]);
10
            memset(danger, 0, sizeof danger);
11
12
        void insert(const string &s, int m) {
13
14
            int n = s.size(); int u = 0, c;
15
            FOR (i, 0, n) {
                c = mp(s[i]);
16
                if (!ch[u][c]) {
17
                    memset(ch[sz], 0, sizeof ch[sz]);
18
19
                    danger[sz] = 0; ch[u][c] = sz++;
                }
20
                u = ch[u][c];
21
22
            danger[u] |= 1 << m;
23
24
        void build() {
25
            queue<int> Q;
26
            fail[0] = 0;
27
28
            for (int c = 0, u; c < M; c++) {
29
                u = ch[0][c];
                if (u) { Q.push(u); fail[u] = 0; }
30
            }
31
```

```
while (!Q.empty()) {
32
33
                 int r = Q.front(); Q.pop();
                 danger[r] |= danger[fail[r]];
34
                 for (int c = 0, u; c < M; c++) {</pre>
35
                     u = ch[r][c];
                     if (!u) {
37
                          ch[r][c] = ch[fail[r]][c];
38
39
                          continue;
40
                     fail[u] = ch[fail[r]][c];
41
                     Q.push(u);
42
                 }
43
             }
44
45
        }
    } ac;
46
47
    char s[N];
48
49
    int main() {
        int n; scanf("%d", &n);
51
        ac.init();
52
        while (n--) {
53
            scanf("%s", s);
54
             ac.insert(s, 0);
        }
56
57
        ac.build();
58
        scanf("%s", s);
59
60
        int u = 0; n = strlen(s);
        FOR (i, 0, n) {
61
62
             u = ac.ch[u][mp(s[i])];
             if (ac.danger[u]) {
63
64
                 puts("YES");
                 return 0;
65
             }
66
67
        puts("NO");
68
        return 0;
69
    }
70
```

杂项