Standard Code Library

F0RE1GNERS

East China Normal University

September 9102

Contents

一切的开始

宏定义

```cpp
#include<bits/stdc++.h>
using namespace std;

// #define int long long
#define mst(a) memset(a,0,sizeof(a))
#define cf int Tcodeforces, Tcodeforce;cin>>Tcodeforces;for(Tcodeforce = 1; Tcodeforce <= Tcodeforces; Tcodeforce++)
typedef long long ll;
typedef unsigned long long ull;
const ll maxn = 2e5 +7;
const ll maxm = 2e5 +7;
const ll inf = 0x3f3f3f3f;
const ll mod = 1000000007;//1e9+7

signed main()
{
    freopen("D:/c++source file/intxt/in.txt","r",stdin);
    ios :: sync_with_stdio(0);
    cin.tie(0);

    //cerr<<"Time : "<<1000*((double)clock())/(double)CLOCKS_PER_SEC<<"ms";
    return (0);
}

// ----------------------------------------------------------------------------
```

速读

```cpp
//读入整数 可以是负数
inline int read(){
    int x=0,f=1,c=getchar();
    while(!isdigit(c)){if(c=='-')f=-1;c=getchar();}
    while(isdigit(c)){x=(x<<1)+(x<<3)+(c^48);c=getchar();}
    return f==1?x:-x;
}
```

数据结构

线段树

```cpp
#define mid ((l + r) / 2)
#define lson (rt << 1)
#define rson (rt << 1 | 1)
```

- 普适

```cpp
int a[maxn];
struct node {
    int len, val;//长度 数量
    int s, t, pl, el;//第一个 最后一个 前缀上升长度 后缀上升长度
}tree[maxn<<2];

node Merge(node a, node b) {
    node res;
    res.pl = a.pl;
    res.el = b.el;
    res.s = a.s;
    res.t = b.t;
    res.len = a.len + b.len;
    res.val = a.val + b.val;
    if(a.t > b.s) return res;
    if(a.pl == a.len) res.pl = a.len + b.pl;
    if(b.el == b.len) res.el = a.el + b.len;
    res.val += a.el * b.pl;
    return res;
```

```cpp
}

void update(int rt) {
    tree[rt] = Merge(tree[lson], tree[rson]);
}

void build(int l, int r,int rt) {
    if(l == r){
        tree[rt].len = 1;
        tree[rt].val = 1;
        tree[rt].s = tree[rt].t = a[l];
        tree[rt].el = tree[rt].pl = 1;
        return ;
    }
    build(l, mid, lson);
    build(mid+1, r, rson);
    update(rt);
}

void modify(int l, int r, int rt, int x, int y) {
    if(l == r) {
        tree[rt].s = tree[rt].t = y;
        return ;
    }
    if(x <= mid) modify(l, mid, lson, x, y);
    else modify(mid+1, r, rson, x, y);
    update(rt);
}

node query(int l, int r, int rt, int L, int R) {
    if(L <= l && r <= R) return tree[rt];
    node tmp;
    tmp.len = -1;
    if(L <= mid) tmp = query(l ,mid, lson, L, R);
    if(mid < R) {
        if(tmp.len == -1) tmp = query(mid+1, r, rson, L, R);
        else tmp = Merge(tmp, query(mid+1, r, rson, L, R));
    }
    return tmp;
}
```

+ 加法乘法

```cpp
int n, m, md;
int a[maxn];
struct node{
    int val;
    int add, mult;
}tree[maxn<<2];

void buildtree(int l, int r, int rt) {
    if(l == r) {
        tree[rt].val = a[l];
        tree[rt].add = 0;
        tree[rt].mult = 1;
        tree[rt].val %= md;
        return ;
    }

    buildtree(l, mid, lson);
    buildtree(mid+1, r, rson);
    tree[rt].add = 0;
    tree[rt].mult = 1;
    tree[rt].val = tree[lson].val + tree[rson].val;
    tree[rt].val %= md;
    return ;
}
//先乘再加
void pushdown(int l, int r, int rt) {
    tree[lson].val = (tree[lson].val * tree[rt].mult + tree[rt].add * (mid-l+1)) % md;
    tree[lson].add = (tree[lson].add * tree[rt].mult + tree[rt].add) % md;
    tree[lson].mult = (tree[lson].mult * tree[rt].mult) % md;
```

```
    
    tree[rson].val = (tree[rson].val * tree[rt].mult + tree[rt].add * (r-mid)) % md;
    tree[rson].add = (tree[rson].add * tree[rt].mult + tree[rt].add) % md;
    tree[rson].mult = (tree[rson].mult * tree[rt].mult) % md;

    tree[rt].add = 0;
    tree[rt].mult = 1;
    return ;
}

int query(int l, int r, int rt, int L, int R) {
    if(L > r || R < l) return 0;
    if(l >= L && r <= R) {return tree[rt].val;}

    pushdown(l, r, rt);
    return query(l, mid, lson, L, R) + query(mid+1, r, rson, L, R);
}

void add(int l, int r, int rt, int L, int R, int val) {
    if(L > r || R < l) return ;
    if(l >= L && r <= R) {
        tree[rt].add += val;
        tree[rt].val += val * (r-l+1);
        tree[rt].val %= md;
        return ;
    }
    pushdown(l,r,rt);
    add(l, mid, lson, L, R, val);
    add(mid+1, r, rson, L, R, val);
    tree[rt].val = tree[lson].val + tree[rson].val;
        tree[rt].val %= md;
    return ;
}

void mult(int l, int r, int rt, int L, int R, int val) {
    if(L > r || R < l) return ;
    if(l >= L && r <= R) {
        pushdown(l, r, rt);
        tree[rt].mult *= val;
        tree[rt].val *= val;
        tree[rt].val %= md;
        return ;
    }

    pushdown(l, r, rt);
    mult(l, mid, lson, L, R, val);
    mult(mid+1, r, rson, L, R, val);
    tree[rt].val = tree[lson].val + tree[rson].val;
        tree[rt].val %= md;
}
```

树状数组

- 注意：0 是无效下标

```
int tree[maxn];
int n;
int lowbit(int x) {return x & -x;}
void add(int pos, int val)
{
    while(pos <= n)
    {
        tree[pos] += val;
        pos += lowbit(pos);
    }
}
int sum(int pos)
{
    int ans = 0;
    while(pos)
    {
```

```
        ans += tree[pos];
        pos -= lowbit(pos);
    }
    return ans;
}
```

- 区间修改 & 区间查询（单点修改，查询前缀和的前缀和）

```
int tree1[maxn], tree2[maxn];
int n, q;
int lowbit(int x) {return x & -x;}
inline int read(){
    int x=0,f=1,c=getchar();
    while(!isdigit(c)){if(c=='-')f=-1;c=getchar();}
    while(isdigit(c)){x=(x<<1)+(x<<3)+(c^48);c=getchar();}
    return f==1?x:-x;
}
void add(int pos, int val)
{
    int addval = val * pos;
    while(pos <= n)
    {
        tree1[pos] += val;
        tree2[pos] += addval;
        pos += lowbit(pos);
    }
}
int sum1(int pos)
{
    // cout<<"sum1:"<<pos<<"=";
    int ans = 0;
    while(pos)
    {
        ans += tree1[pos];
        pos -= lowbit(pos);
    }
    // cout<<ans<<"\n";
    return ans;
}
int sum2(int pos)
{
    // cout<<"sum2:"<<pos<<"=";
    int ans = 0;
    while(pos)
    {
        ans += tree2[pos];
        pos -= lowbit(pos);
    }
    // cout<<ans<<endl;
    return ans;
}
int sum(int pos){return sum1(pos) * (pos + 1) - sum2(pos);}
void modify(int l, int r, int val)
{
    add(l ,val);
    add(r+1, -val);
}
```

- 二维树状数组单点修改

```
int tree[maxn][maxn];
int xn, yy;
int lowbit(int x) {
    return x & -x;
}
void add(int x, int y, int val) {
    int my = y;

    while (x <= xn) {
        y = my;

        while (y <= yy) {
```

```
            tree[x][y] += val;
            y += lowbit(y);
        }

        x += lowbit(x);
    }
}

int getsum(int x, int y) {
    int ans = 0;
    int my = y;

    while (x) {
        y = my;

        while (y) {
            ans += tree[x][y];
            y -= lowbit(y);
        }

        x -= lowbit(x);
    }

    return ans;
}
int q_get(int x1, int y1, int x2, int y2) {
    int ans = 0;
    ans += getsum(x2, y2);
    ans -= getsum(x1 - 1, y2);
    ans -= getsum(x2, y1 - 1);
    ans += getsum(x1 - 1, y1 - 1);
    return ans;
}
```

- 区间修改二维树状数组

```
int lowbit(int x) {return x & -x;}
void add(int x, int y, int val)
{
    // cout<<x<<" "<<y<<" "<<val<<endl;
    int memoy = y, memox = x;
    while(x <= n)
    {
        y = memoy;
        while(y <= m)
        {
            t1[x][y] += val;
            t2[x][y] += val * memoy;
            t3[x][y] += val * memox;
            t4[x][y] += val * memox * memoy;
            y += lowbit(y);
        }
        x += lowbit(x);
    }
}

int ask(int x, int y)
{
    int ans = 0;
    int memoy = y, memox = x;
    while(x)
    {
        y = memoy;
        while(y)
        {
            ans += (memoy+1)*(memox+1)*t1[x][y];
            ans -= t2[x][y] * (memox + 1);
            ans -= t3[x][y] * (memoy + 1);
            ans += t4[x][y];
            y -= lowbit(y);
        }
        x -= lowbit(x);
```

```
    }
    return ans;
}

void range_add(int xx1, int yy1, int xx2, int yy2, int val)
{
    add(xx1,yy1, val);
    add(xx1, yy2 + 1, -val);
    add(xx2 + 1, yy1, -val);
    add(xx2+1,yy2+1,val);
}

int range_ask(int xx1, int yy1, int xx2, int yy2)
{
    int ans = 0;
    ans += ask(xx1-1,yy1-1);
    ans -= ask(xx1-1,yy2);
    ans -= ask(xx2, yy1-1);
    ans += ask(xx2, yy2);
    return ans;
}
```

并查集

```
int n, m;
int fa[maxn], rk[maxn];
inline void init(){for(int i=0;i<=n;i++){fa[i]=i;rk[i]=1;}}
int find(int x) {return fa[x]==x?x:(fa[x]=find(fa[x]));}
inline void merge(int i, int j)
{
    int x = find(i), y = find(j);
    if(rk[x] <= rk[y]) fa[x] = y;
    else fa[y] = x;
    if(rk[x] == rk[y] && x != y) rk[y]++;
}
```

数学

筛

- 线性筛

```
const LL p_max = 1E6 + 100;
LL pr[p_max], p_sz;
void get_prime() {
    static bool vis[p_max];
    FOR (i, 2, p_max) {
        if (!vis[i]) pr[p_sz++] = i;
        FOR (j, 0, p_sz) {
            if (pr[j] * i >= p_max) break;
            vis[pr[j] * i] = 1;
            if (i % pr[j] == 0) break;
        }
    }
}
```

- 线性筛 + 欧拉函数

```
const LL p_max = 1E5 + 100;
LL phi[p_max];
void get_phi() {
    phi[1] = 1;
    static bool vis[p_max];
    static LL prime[p_max], p_sz, d;
    FOR (i, 2, p_max) {
        if (!vis[i]) {
            prime[p_sz++] = i;
            phi[i] = i - 1;
        }
        for (LL j = 0; j < p_sz && (d = i * prime[j]) < p_max; ++j) {
```

```
            vis[d] = 1;
            if (i % prime[j] == 0) {
                phi[d] = phi[i] * prime[j];
                break;
            }
            else phi[d] = phi[i] * (prime[j] - 1);
        }
    }
}
```

- 线性筛 + 莫比乌斯函数

```
const LL p_max = 1E5 + 100;
LL mu[p_max];
void get_mu() {
    mu[1] = 1;
    static bool vis[p_max];
    static LL prime[p_max], p_sz, d;
    FOR (i, 2, p_max) {
        if (!vis[i]) {
            prime[p_sz++] = i;
            mu[i] = -1;
        }
        for (LL j = 0; j < p_sz && (d = i * prime[j]) < p_max; ++j) {
            vis[d] = 1;
            if (i % prime[j] == 0) {
                mu[d] = 0;
                break;
            }
            else mu[d] = -mu[i];
        }
    }
}
```

扩展欧几里得

- 求 $ax + by = gcd(a, b)$ 的一组解
- 如果 a 和 b 互素，那么 x 是 a 在模 b 下的逆元
- 注意 x 和 y 可能是负数

```
LL ex_gcd(LL a, LL b, LL &x, LL &y) {
    if (b == 0) { x = 1; y = 0; return a; }
    LL ret = ex_gcd(b, a % b, y, x);
    y -= a / b * x;
    return ret;
}
```

- 卡常欧几里得

```
inline int ctz(LL x) { return __builtin_ctzll(x); }
LL gcd(LL a, LL b) {
    if (!a) return b; if (!b) return a;
    int t = ctz(a | b);
    a >>= ctz(a);
    do {
        b >>= ctz(b);
        if (a > b) swap(a, b);
        b -= a;
    } while (b);
    return a << t;
}
```

类欧几里得

- $m = \lfloor \frac{an+b}{c} \rfloor$.
- $f(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor$: 当 $a \geq c$ or $b \geq c$ 时，$f(a,b,c,n) = (\frac{a}{c})n(n+1)/2 + (\frac{b}{c})(n+1) + f(a \bmod c, b \bmod c, c, n)$；否则 $f(a,b,c,n) = nm - f(c, c-b-1, a, m-1)$。
- $g(a,b,c,n) = \sum_{i=0}^{n} i \lfloor \frac{ai+b}{c} \rfloor$: 当 $a \geq c$ or $b \geq c$ 时，$g(a,b,c,n) = (\frac{a}{c})n(n+1)(2n+1)/6 + (\frac{b}{c})n(n+1)/2 + g(a \bmod c, b \bmod c, c, n)$；否则 $g(a,b,c,n) = \frac{1}{2}(n(n+1)m - f(c, c-b-1, a, m-1) - h(c, c-b-1, a, m-1))$。

- $h(a,b,c,n)=\sum_{i=0}^n\lfloor\frac{ai+b}{c}\rfloor^2$：当 $a\geq c$ or $b\geq c$ 时，$h(a,b,c,n)=(\frac{a}{c})^2n(n+1)(2n+1)/6+(\frac{b}{c})^2(n+1)+(\frac{a}{c})(\frac{b}{c})n(n+1)+h(a \bmod c,b \bmod c,c,n)+2(\frac{a}{c})g(a \bmod c,b \bmod c,c,n)+2(\frac{b}{c})f(a \bmod c,b \bmod c,c,n)$；否则 $h(a,b,c,n)=nm(m+1)-2g(c,c-b-1,a,m-1)-2f(c,c-b-1,a,m-1)-f(a,b,c,n)$。

逆元

- 如果 p 不是素数，使用拓展欧几里得
- 前置模板：快速幂 / 扩展欧几里得

```
inline LL get_inv(LL x, LL p) { return bin(x, p - 2, p); }
LL get_inv(LL a, LL M) {
    static LL x, y;
    assert(exgcd(a, M, x, y) == 1);
    return (x % M + M) % M;
}
```

- 预处理 1~n 的逆元

```
LL inv[N];
void inv_init(LL n, LL p) {
    inv[1] = 1;
    FOR (i, 2, n)
        inv[i] = (p - p / i) * inv[p % i] % p;
}
```

- 预处理阶乘及其逆元

```
LL invf[M], fac[M] = {1};
void fac_inv_init(LL n, LL p) {
    FOR (i, 1, n)
        fac[i] = i * fac[i - 1] % p;
    invf[n - 1] = bin(fac[n - 1], p - 2, p);
    FORD (i, n - 2, -1)
        invf[i] = invf[i + 1] * (i + 1) % p;
}
```

组合数

- 如果数较小，模较大时使用逆元
- 前置模板：逆元-预处理阶乘及其逆元

```
inline LL C(LL n, LL m) { // n >= m >= 0
    return n < m || m < 0 ? 0 : fac[n] * invf[m] % MOD * invf[n - m] % MOD;
}
```

- 如果模数较小，数字较大，使用 Lucas 定理
- 前置模板可选 1：求组合数（如果使用阶乘逆元，需 `fac_inv_init(MOD, MOD);`）
- 前置模板可选 2：模数不固定下使用，无法单独使用。

```
LL C(LL n, LL m) { // m >= n >= 0
    if (m - n < n) n = m - n;
    if (n < 0) return 0;
    LL ret = 1;
    FOR (i, 1, n + 1)
        ret = ret * (m - n + i) % MOD * bin(i, MOD - 2, MOD) % MOD;
    return ret;
}
```

```
LL Lucas(LL n, LL m) { // m >= n >= 0
    return m ? C(n % MOD, m % MOD) * Lucas(n / MOD, m / MOD) % MOD : 1;
}
```

- 组合数预处理

```
LL C[M][M];
void init_C(int n) {
    FOR (i, 0, n) {
        C[i][0] = C[i][i] = 1;
        FOR (j, 1, i)
            C[i][j] = (C[i - 1][j] + C[i - 1][j - 1]) % MOD;
```

```
    }
}
```

快速幂

- 如果模数是素数，则可在函数体内加上 `n %= MOD - 1;`（费马小定理）

```
ll FastPowerMod(ll x, ll n)
{
    if(!n) return 1;
    ll t = 1;
    while(n)
    {
        if(n&1) t *= x;
        n >>= 1;
        x *= x;
        x = (x + mod) % mod;
        t = (t + mod) % mod;
    }
    return t;
}
```

质因数分解

- 前置模板：素数筛
- 带指数

```
LL factor[30], f_sz, factor_exp[30];
void get_factor(LL x) {
    f_sz = 0;
    LL t = sqrt(x + 0.5);
    for (LL i = 0; pr[i] <= t; ++i)
        if (x % pr[i] == 0) {
            factor_exp[f_sz] = 0;
            while (x % pr[i] == 0) {
                x /= pr[i];
                ++factor_exp[f_sz];
            }
            factor[f_sz++] = pr[i];
        }
    if (x > 1) {
        factor_exp[f_sz] = 1;
        factor[f_sz++] = x;
    }
}
```

- 不带指数

```
LL factor[30], f_sz;
void get_factor(LL x) {
    f_sz = 0;
    LL t = sqrt(x + 0.5);
    for (LL i = 0; pr[i] <= t; ++i)
        if (x % pr[i] == 0) {
            factor[f_sz++] = pr[i];
            while (x % pr[i] == 0) x /= pr[i];
        }
    if (x > 1) factor[f_sz++] = x;
}
```

原根

- 前置模板：素数筛，快速幂，分解质因数
- 要求 p 为质数

```
LL find_smallest_primitive_root(LL p) {
    get_factor(p - 1);
    FOR (i, 2, p) {
        bool flag = true;
```

```
        FOR (j, 0, f_sz)
            if (bin(i, (p - 1) / factor[j], p) == 1) {
                flag = false;
                break;
            }
        if (flag) return i;
    }
    assert(0); return -1;
}
```

公式

一些数论公式

- 当 $x \geq \phi(p)$ 时有 $a^x \equiv a^{x \bmod \phi(p) + \phi(p)} \pmod p$
- $\mu^2(n) = \sum_{d^2|n} \mu(d)$
- $\sum_{d|n} \varphi(d) = n$
- $\sum_{d|n} 2^{\omega(d)} = \sigma_0(n^2)$，其中 ω 是不同素因子个数
- $\sum_{d|n} \mu^2(d) = 2^{\omega(d)}$

一些数论函数求和的例子

- $\sum_{i=1}^n i[gcd(i,n)=1] = \frac{n\varphi(n)+[n=1]}{2}$
- $\sum_{i=1}^n \sum_{j=1}^m [gcd(i,j)=x] = \sum_d \mu(d) \lfloor \frac{n}{dx} \rfloor \lfloor \frac{m}{dx} \rfloor$
- $\sum_{i=1}^n \sum_{j=1}^m gcd(i,j) = \sum_{i=1}^n \sum_{j=1}^m \sum_{d|gcd(i,j)} \varphi(d) = \sum_d \varphi(d) \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$
- $S(n) = \sum_{i=1}^n \mu(i) = 1 - \sum_{i=1}^n \sum_{d|i,d<i} \mu(d) \overset{t=\frac{i}{d}}{=} 1 - \sum_{t=2}^n S(\lfloor \frac{n}{t} \rfloor)$
  - 利用 $[n=1] = \sum_{d|n} \mu(d)$
- $S(n) = \sum_{i=1}^n \varphi(i) = \sum_{i=1}^n i - \sum_{i=1}^n \sum_{d|i,d<i} \varphi(i) \overset{t=\frac{i}{d}}{=} \frac{i(i+1)}{2} - \sum_{t=2}^n S(\frac{n}{t})$
  - 利用 $n = \sum_{d|n} \varphi(d)$
- $\sum_{i=1}^n \mu^2(i) = \sum_{i=1}^n \sum_{d^2|n} \mu(d) = \sum_{d=1}^{\lfloor \sqrt{n} \rfloor} \mu(d) \lfloor \frac{n}{d^2} \rfloor$
- $\sum_{i=1}^n \sum_{j=1}^n gcd^2(i,j) = \sum_d d^2 \sum_t \mu(t) \lfloor \frac{n}{dt} \rfloor^2$
  $\overset{x=dt}{=} \sum_x \lfloor \frac{n}{x} \rfloor^2 \sum_{d|x} d^2 \mu(\frac{x}{d})$
- $\sum_{i=1}^n \varphi(i) = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n [i \perp j] - 1 = \frac{1}{2} \sum_{i=1}^n \mu(i) \cdot \lfloor \frac{n}{i} \rfloor^2 - 1$

斐波那契数列性质

- $F_{a+b} = F_{a-1} \cdot F_b + F_a \cdot F_{b+1}$
- $F_1 + F_3 + \dots + F_{2n-1} = F_{2n}, F_2 + F_4 + \dots + F_{2n} = F_{2n+1} - 1$
- $\sum_{i=1}^n F_i = F_{n+2} - 1$
- $\sum_{i=1}^n F_i^2 = F_n \cdot F_{n+1}$
- $F_n^2 = (-1)^{n-1} + F_{n-1} \cdot F_{n+1}$
- $gcd(F_a, F_b) = F_{gcd(a,b)}$
- 模 n 周期（皮萨诺周期）
  - $\pi(p^k) = p^{k-1}\pi(p)$
  - $\pi(nm) = lcm(\pi(n), \pi(m)), \forall n \perp m$
  - $\pi(2) = 3, \pi(5) = 20$
  - $\forall p \equiv \pm 1 \pmod{10}, \pi(p) | p - 1$
  - $\forall p \equiv \pm 2 \pmod 5, \pi(p) | 2p + 2$

常见生成函数

- $(1+ax)^n = \sum_{k=0}^n \binom{n}{k} a^k x^k$
- $\dfrac{1-x^{r+1}}{1-x} = \sum_{k=0}^n x^k$

- $\dfrac{1}{1-ax} = \sum_{k=0}^{\infty} a^k x^k$
- $\dfrac{1}{(1-x)^2} = \sum_{k=0}^{\infty} (k+1) x^k$
- $\dfrac{1}{(1-x)^n} = \sum_{k=0}^{\infty} \binom{n+k-1}{k} x^k$
- $e^x = \sum_{k=0}^{\infty} \dfrac{x^k}{k!}$
- $\ln(1+x) = \sum_{k=0}^{\infty} \dfrac{(-1)^{k+1}}{k} x^k$

佩尔方程

若一个丢番图方程具有以下的形式：$x^2 - ny^2 = 1$。且 n 为正整数，则称此二元二次不定方程为**佩尔方程**。

若 n 是完全平方数，则这个方程式只有平凡解 $(\pm 1, 0)$（实际上对任意的 n，$(\pm 1, 0)$ 都是解）。对于其余情况，拉格朗日证明了佩尔方程总有非平凡解。而这些解可由 $\sqrt{n}$ 的连分数求出。

$$x = [a_0; a_1, a_2, a_3] = x = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{\ddots}}}}$$

设 $\frac{p_i}{q_i}$ 是 $\sqrt{n}$ 的连分数表示: $[a_0; a_1, a_2, a_3, ...]$ 的渐近分数列, 由连分数理论知存在 i 使得 (p_i, q_i) 为佩尔方程的解。取其中最小的 i, 将对应的 (p_i, q_i) 称为佩尔方程的基本解, 或最小解, 记作 (x_1, y_1), 则所有的解 (x_i, y_i) 可表示成如下形式: $x_i + y_i\sqrt{n} = (x_1 + y_1\sqrt{n})^i$。或者由以下的递回关系式得到:

$x_{i+1} = x_1 x_i + n y_1 y_i, y_{i+1} = x_1 y_i + y_1 x_i$。

但是：佩尔方程千万不要去推（虽然推起来很有趣，但结果不一定好看，会是两个式子）。记住佩尔方程结果的形式通常是 $a_n = ka_{n-1} - a_{n-2}$（a_{n-2} 前的系数通常是 -1）。暴力 / 凑出两个基础解之后加上一个 0，容易解出 k 并验证。

Burnside & Polya

- $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$

注：X^g 是 g 下的不动点数量，也就是说有多少种东西用 g 作用之后可以保持不变。

- $|Y^X/G| = \frac{1}{|G|} \sum_{g \in G} m^{c(g)}$

注：用 m 种颜色染色，然后对于某一种置换 g，有 $c(g)$ 个置换环，为了保证置换后颜色仍然相同，每个置换环必须染成同色。

皮克定理

$2S = 2a + b - 2$

- S 多边形面积
- a 多边形内部点数
- b 多边形边上点数

莫比乌斯反演

- $g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(\frac{n}{d})$
- $f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu(\frac{d}{n}) f(d)$

低阶等幂求和

- $\sum_{i=1}^{n} i^1 = \frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n$
- $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$
- $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$

- $\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n$
- $\sum_{i=1}^{n} i^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12} = \frac{1}{6}n^6 + \frac{1}{2}n^5 + \frac{5}{12}n^4 - \frac{1}{12}n^2$

一些组合公式

- 错排公式：$D_1 = 0, D_2 = 1, D_n = (n-1)(D_{n-1} + D_{n-2}) = n!(\frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!}) = \lfloor \frac{n!}{e} + 0.5 \rfloor$
- 卡塔兰数（n 对括号合法方案数，n 个结点二叉树个数，$n \times n$ 方格中对角线下方的单调路径数，凸 $n+2$ 边形的三角形划分数，n 个元素的合法出栈序列数）：$C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$

中国剩余定理

- 无解返回 -1
- 前置模板：扩展欧几里得

```
LL CRT(LL *m, LL *r, LL n) {
    if (!n) return 0;
    LL M = m[0], R = r[0], x, y, d;
    FOR (i, 1, n) {
        d = ex_gcd(M, m[i], x, y);
        if ((r[i] - R) % d) return -1;
        x = (r[i] - R) / d * x % (m[i] / d);
        // 防爆 LL
        // x = mul((r[i] - R) / d, x, m[i] / d);
        R += x * M;
        M = M / d * m[i];
        R %= M;
    }
    return R >= 0 ? R : R + M;
}
```

博弈

- Nim 游戏：每轮从若干堆石子中的一堆取走若干颗。先手必胜条件为石子数量异或和非零。
- 阶梯 Nim 游戏：可以选择阶梯上某一堆中的若干颗向下推动一级，直到全部推下去。先手必胜条件是奇数阶梯的异或和非零（对于偶数阶梯的操作可以模仿）。
- Anti-SG：无法操作者胜。先手必胜的条件是：
  - SG 不为 0 且某个单一游戏的 SG 大于 1 。
  - SG 为 0 且没有单一游戏的 SG 大于 1。
- Every-SG：对所有单一游戏都要操作。先手必胜的条件是单一游戏中的最大 step 为奇数。
  - 对于终止状态 step 为 0
  - 对于 SG 为 0 的状态，step 是最大后继 step +1
  - 对于 SG 非 0 的状态，step 是最小后继 step +1
- 树上删边：叶子 SG 为 0，非叶子结点为所有子结点的 SG 值加 1 后的异或和。

尝试：

- 打表找规律
- 寻找一类必胜态（如对称局面）
- 直接博弈 dp

图论

LCA

- 倍增

```
void dfs(int u, int fa) {
    pa[u][0] = fa; dep[u] = dep[fa] + 1;
    FOR (i, 1, SP) pa[u][i] = pa[pa[u][i - 1]][i - 1];
    for (int& v: G[u]) {
        if (v == fa) continue;
        dfs(v, u);
    }
```

```
}

int lca(int u, int v) {
    if (dep[u] < dep[v]) swap(u, v);
    int t = dep[u] - dep[v];
    FOR (i, 0, SP) if (t & (1 << i)) u = pa[u][i];
    FORD (i, SP - 1, -1) {
        int uu = pa[u][i], vv = pa[v][i];
        if (uu != vv) { u = uu; v = vv; }
    }
    return u == v ? u : pa[u][0];
}
```

欧拉路径

```
int S[N << 1], top;
Edge edges[N << 1];
set<int> G[N];

void DFS(int u) {
    S[top++] = u;
    for (int eid: G[u]) {
        int v = edges[eid].get_other(u);
        G[u].erase(eid);
        G[v].erase(eid);
        DFS(v);
        return;
    }
}

void fleury(int start) {
    int u = start;
    top = 0; path.clear();
    S[top++] = u;
    while (top) {
        u = S[--top];
        if (!G[u].empty())
            DFS(u);
        else path.push_back(u);
    }
}
```

强连通分量与 2-SAT

```
int n, m;
vector<int> G[N], rG[N], vs;
int used[N], cmp[N];

void add_edge(int from, int to) {
    G[from].push_back(to);
    rG[to].push_back(from);
}

void dfs(int v) {
    used[v] = true;
    for (int u: G[v]) {
        if (!used[u])
            dfs(u);
    }
    vs.push_back(v);
}

void rdfs(int v, int k) {
    used[v] = true;
    cmp[v] = k;
    for (int u: rG[v])
        if (!used[u])
            rdfs(u, k);
}

```

```
int scc() {
    memset(used, 0, sizeof(used));
    vs.clear();
    for (int v = 0; v < n; ++v)
        if (!used[v]) dfs(v);
    memset(used, 0, sizeof(used));
    int k = 0;
    for (int i = (int) vs.size() - 1; i >= 0; --i)
        if (!used[vs[i]]) rdfs(vs[i], k++);
    return k;
}

int main() {
    cin >> n >> m;
    n *= 2;
    for (int i = 0; i < m; ++i) {
        int a, b; cin >> a >> b;
        add_edge(a - 1, (b - 1) ^ 1);
        add_edge(b - 1, (a - 1) ^ 1);
    }
    scc();
    for (int i = 0; i < n; i += 2) {
        if (cmp[i] == cmp[i + 1]) {
            puts("NIE");
            return 0;
        }
    }
    for (int i = 0; i < n; i += 2) {
        if (cmp[i] > cmp[i + 1]) printf("%d\n", i + 1);
        else printf("%d\n", i + 2);
    }
}
```

拓扑排序

```
vector<int> toporder(int n) {
    vector<int> orders;
    queue<int> q;
    for (int i = 0; i < n; i++)
        if (!deg[i]) {
            q.push(i);
            orders.push_back(i);
        }
    while (!q.empty()) {
        int u = q.front(); q.pop();
        for (int v: G[u])
            if (!--deg[v]) {
                q.push(v);
                orders.push_back(v);
            }
    }
    return orders;
}
```

Tarjan

割点

- 判断割点
- 注意原图可能不连通

```
int dfn[N], low[N], clk;
void init() { clk = 0; memset(dfn, 0, sizeof dfn); }
void tarjan(int u, int fa) {
    low[u] = dfn[u] = ++clk;
    int cc = fa != -1;
    for (int& v: G[u]) {
        if (v == fa) continue;
        if (!dfn[v]) {
            tarjan(v, u);
```

```
            low[u] = min(low[u], low[v]);
            cc += low[v] >= dfn[u];
        } else low[u] = min(low[u], dfn[v]);
    }
    if (cc > 1) // ...
}
```

桥

- 注意原图不连通和重边

```
int dfn[N], low[N], clk;
void init() { memset(dfn, 0, sizeof dfn); clk = 0; }
void tarjan(int u, int fa) {
    low[u] = dfn[u] = ++clk;
    int _fst = 0;
    for (E& e: G[u]) {
        int v = e.to; if (v == fa && ++_fst == 1) continue;
        if (!dfn[v]) {
            tarjan(v, u);
            if (low[v] > dfn[u]) // ...
            low[u] = min(low[u], low[v]);
        } else low[u] = min(low[u], dfn[v]);
    }
}
```

强连通分量缩点

```
int low[N], dfn[N], clk, B, bl[N];
vector<int> bcc[N];
void init() { B = clk = 0; memset(dfn, 0, sizeof dfn); }
void tarjan(int u) {
    static int st[N], p;
    static bool in[N];
    dfn[u] = low[u] = ++clk;
    st[p++] = u; in[u] = true;
    for (int& v: G[u]) {
        if (!dfn[v]) {
            tarjan(v);
            low[u] = min(low[u], low[v]);
        } else if (in[v]) low[u] = min(low[u], dfn[v]);
    }
    if (dfn[u] == low[u]) {
        while (1) {
            int x = st[--p]; in[x] = false;
            bl[x] = B; bcc[B].push_back(x);
            if (x == u) break;
        }
        ++B;
    }
}
```

点双连通分量 / 广义圆方树

- 数组开两倍
- 一条边也被计入点双了（适合拿来建圆方树），可以用点数 <= 边数过滤

```
struct E { int to, nxt; } e[N];
int hd[N], ecnt;
void addedge(int u, int v) {
    e[ecnt] = {v, hd[u]};
    hd[u] = ecnt++;
}
int low[N], dfn[N], clk, B, bno[N];
vector<int> bc[N], be[N];
bool vise[N];
void init() {
    memset(vise, 0, sizeof vise);
    memset(hd, -1, sizeof hd);
    memset(dfn, 0, sizeof dfn);
```

```
    memset(bno, -1, sizeof bno);
    B = clk = ecnt = 0;
}

void tarjan(int u, int feid) {
    static int st[N], p;
    static auto add = [&](int x) {
        if (bno[x] != B) { bno[x] = B; bc[B].push_back(x); }
    };
    low[u] = dfn[u] = ++clk;
    for (int i = hd[u]; ~i; i = e[i].nxt) {
        if ((feid ^ i) == 1) continue;
        if (!vise[i]) { st[p++] = i; vise[i] = vise[i ^ 1] = true; }
        int v = e[i].to;
        if (!dfn[v]) {
            tarjan(v, i);
            low[u] = min(low[u], low[v]);
            if (low[v] >= dfn[u]) {
                bc[B].clear(); be[B].clear();
                while (1) {
                    int eid = st[--p];
                    add(e[eid].to); add(e[eid ^ 1].to);
                    be[B].push_back(eid);
                    if ((eid ^ i) <= 1) break;
                }
                ++B;
            }
        } else low[u] = min(low[u], dfn[v]);
    }
}
```

计算几何

字符串

manacher

```
int RL[N];
void manacher(int* a, int n) { // "abc" => "#a#b#a#"
    int r = 0, p = 0;
    FOR (i, 0, n) {
        if (i < r) RL[i] = min(RL[2 * p - i], r - i);
        else RL[i] = 1;
        while (i - RL[i] >= 0 && i + RL[i] < n && a[i - RL[i]] == a[i + RL[i]])
            RL[i]++;
        if (RL[i] + i - 1 > r) { r = RL[i] + i - 1; p = i; }
    }
    FOR (i, 0, n) --RL[i];
}
```

哈希

内置了自动双哈希开关（小心 TLE）。

```
#include <bits/stdc++.h>
using namespace std;

#define ENABLE_DOUBLE_HASH

typedef long long LL;
typedef unsigned long long ULL;

const int x = 135;
const int N = 4e5 + 10;
const int p1 = 1e9 + 7, p2 = 1e9 + 9;
ULL xp1[N], xp2[N], xp[N];

void init_xp() {
```

```cpp
    xp1[0] = xp2[0] = xp[0] = 1;
    for (int i = 1; i < N; ++i) {
        xp1[i] = xp1[i - 1] * x % p1;
        xp2[i] = xp2[i - 1] * x % p2;
        xp[i] = xp[i - 1] * x;
    }
}

struct String {
    char s[N];
    int length, subsize;
    bool sorted;
    ULL h[N], hl[N];

    ULL hash() {
        length = strlen(s);
        ULL res1 = 0, res2 = 0;
        h[length] = 0;  // ATTENTION!
        for (int j = length - 1; j >= 0; --j) {
        #ifdef ENABLE_DOUBLE_HASH
            res1 = (res1 * x + s[j]) % p1;
            res2 = (res2 * x + s[j]) % p2;
            h[j] = (res1 << 32) | res2;
        #else
            res1 = res1 * x + s[j];
            h[j] = res1;
        #endif
            // printf("%llu\n", h[j]);
        }
        return h[0];
    }

    // 获取子串哈希，左闭右开区间
    ULL get_substring_hash(int left, int right) const {
        int len = right - left;
    #ifdef ENABLE_DOUBLE_HASH
        // get hash of s[left...right-1]
        unsigned int mask32 = ~(0u);
        ULL left1 = h[left] >> 32, right1 = h[right] >> 32;
        ULL left2 = h[left] & mask32, right2 = h[right] & mask32;
        return (((left1 - right1 * xp1[len] % p1 + p1) % p1) << 32) |
               (((left2 - right2 * xp2[len] % p2 + p2) % p2));
    #else
        return h[left] - h[right] * xp[len];
    #endif
    }

    void get_all_subs_hash(int sublen) {
        subsize = length - sublen + 1;
        for (int i = 0; i < subsize; ++i)
            hl[i] = get_substring_hash(i, i + sublen);
        sorted = 0;
    }

    void sort_substring_hash() {
        sort(hl, hl + subsize);
        sorted = 1;
    }

    bool match(ULL key) const {
        if (!sorted) assert (0);
        if (!subsize) return false;
        return binary_search(hl, hl + subsize, key);
    }

    void init(const char *t) {
        length = strlen(t);
        strcpy(s, t);
    }
};
```

```cpp
int LCP(const String &a, const String &b, int ai, int bi) {
    // Find LCP of a[ai...] and b[bi...]
    int l = 0, r = min(a.length - ai, b.length - bi);
    while (l < r) {
        int mid = (l + r + 1) / 2;
        if (a.get_substring_hash(ai, ai + mid) == b.get_substring_hash(bi, bi + mid))
            l = mid;
        else r = mid - 1;
    }
    return l;
}

int check(int ans) {
    if (T.length < ans) return 1;
    T.get_all_subs_hash(ans); T.sort_substring_hash();
    for (int i = 0; i < S.length - ans + 1; ++i)
        if (!T.match(S.get_substring_hash(i, i + ans)))
            return 1;
    return 0;
}

int main() {
    init_xp();  // DON'T FORGET TO DO THIS!

    for (int tt = 1; tt <= kases; ++tt) {
        scanf("%d", &n); scanf("%s", str);
        S.init(str);
        S.hash(); T.hash();
    }
}
```

KMP

- 前缀函数（每一个前缀的最长 border）

```cpp
void get_pi(int a[], char s[], int n) {
    int j = a[0] = 0;
    FOR (i, 1, n) {
        while (j && s[i] != s[j]) j = a[j - 1];
        a[i] = j += s[i] == s[j];
    }
}
```

- Z 函数（每一个后缀和该字符串的 LCP 长度）

```cpp
void get_z(int a[], char s[], int n) {
    int l = 0, r = 0; a[0] = n;
    FOR (i, 1, n) {
        a[i] = i > r ? 0 : min(r - i + 1, a[i - l]);
        while (i + a[i] < n && s[a[i]] == s[i + a[i]]) ++a[i];
        if (i + a[i] - 1 > r) { l = i; r = i + a[i] - 1; }
    }
}
```

Trie

```cpp
namespace trie {
    int t[N][26], sz, ed[N];
    void init() { sz = 2; memset(ed, 0, sizeof ed); }
    int _new() { memset(t[sz], 0, sizeof t[sz]); return sz++; }
    void ins(char* s, int p) {
        int u = 1;
        FOR (i, 0, strlen(s)) {
            int c = s[i] - 'a';
            if (!t[u][c]) t[u][c] = _new();
            u = t[u][c];
        }
        ed[u] = p;
    }
}
```

AC 自动机

```cpp
const int N = 1e6 + 100, M = 26;

int mp(char ch) { return ch - 'a'; }

struct ACA {
    int ch[N][M], danger[N], fail[N];
    int sz;
    void init() {
        sz = 1;
        memset(ch[0], 0, sizeof ch[0]);
        memset(danger, 0, sizeof danger);
    }
    void insert(const string &s, int m) {
        int n = s.size(); int u = 0, c;
        FOR (i, 0, n) {
            c = mp(s[i]);
            if (!ch[u][c]) {
                memset(ch[sz], 0, sizeof ch[sz]);
                danger[sz] = 0; ch[u][c] = sz++;
            }
            u = ch[u][c];
        }
        danger[u] |= 1 << m;
    }
    void build() {
        queue<int> Q;
        fail[0] = 0;
        for (int c = 0, u; c < M; c++) {
            u = ch[0][c];
            if (u) { Q.push(u); fail[u] = 0; }
        }
        while (!Q.empty()) {
            int r = Q.front(); Q.pop();
            danger[r] |= danger[fail[r]];
            for (int c = 0, u; c < M; c++) {
                u = ch[r][c];
                if (!u) {
                    ch[r][c] = ch[fail[r]][c];
                    continue;
                }
                fail[u] = ch[fail[r]][c];
                Q.push(u);
            }
        }
    }
} ac;

char s[N];

int main() {
    int n; scanf("%d", &n);
    ac.init();
    while (n--) {
        scanf("%s", s);
        ac.insert(s, 0);
    }
    ac.build();

    scanf("%s", s);
    int u = 0; n = strlen(s);
    FOR (i, 0, n) {
        u = ac.ch[u][mp(s[i])];
        if (ac.danger[u]) {
            puts("YES");
            return 0;
        }
    }
    puts("NO");
    return 0;
```

```
}
```

杂项