Assignment 2

Morgan Yost

February 2, 2016

1 Exercise 3.1

1.1 Problem Description

poisson.m solves the Poisson problem with square $m \times m$ gird and $\Delta x = \Delta y = h$. The problem is set up to solve $u(x, y) = \exp(x + y/2)$ with Dirichlet boundary conditions.

1.1.1 Part A

Test the script by performing a grid refinement study to verify that it is second order accurate.

1.1.2 Part B

Modify the script to work on a rectangular domain $[a_x, b_x] \times [a_y, b_y]$, but still with $\Delta x = \Delta y = h$.

1.1.3 Part C

Further modify the code to allow $\Delta x \neq \Delta y$.

1.2 Problem Solution

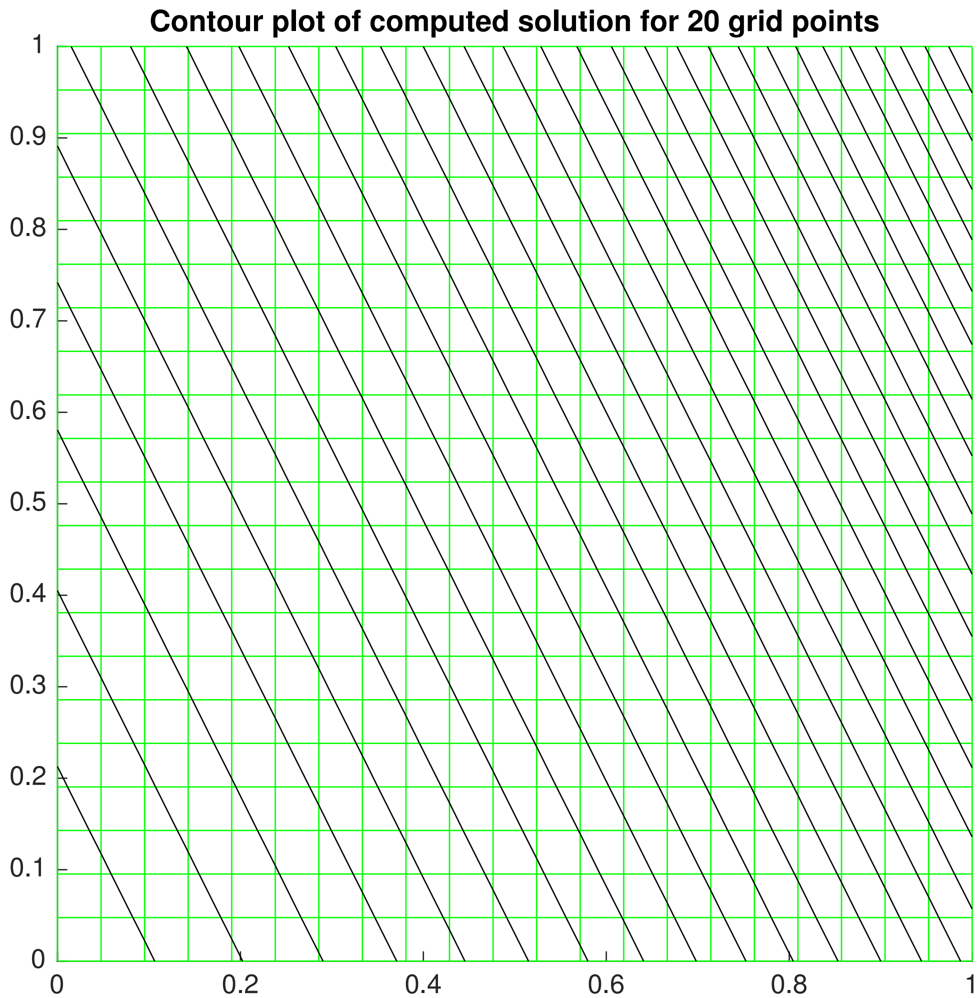
The MATLAB code that outlines the details of the implementation of this problem can be found in Appendix A.

1.2.1 Part A

I was able to perform a grid refinement study by changing the grid size in a for loop. The errors relative to grid size are recorded in the table below:

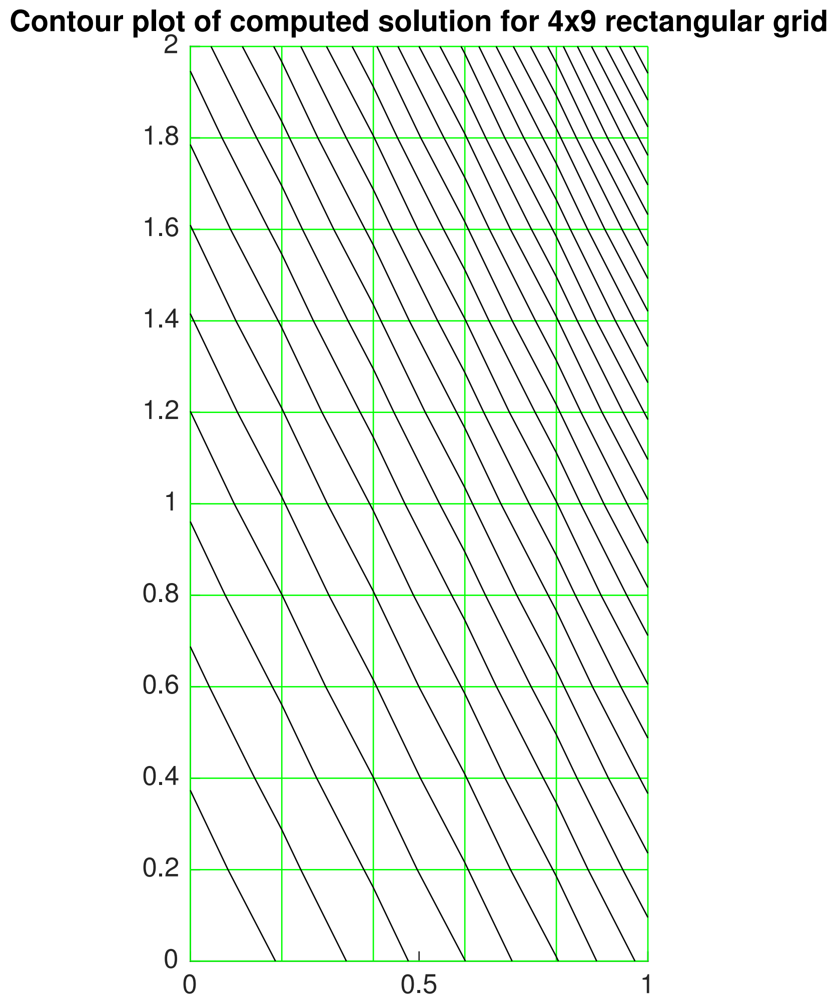
m	error
4	5.50547e-04
12	8.48461e-05
20	3.27323e-05
28	1.71710e-05
36	1.05646e-05
44	7.14325e-06

A sample plot for the 20×20 grid is shown below:



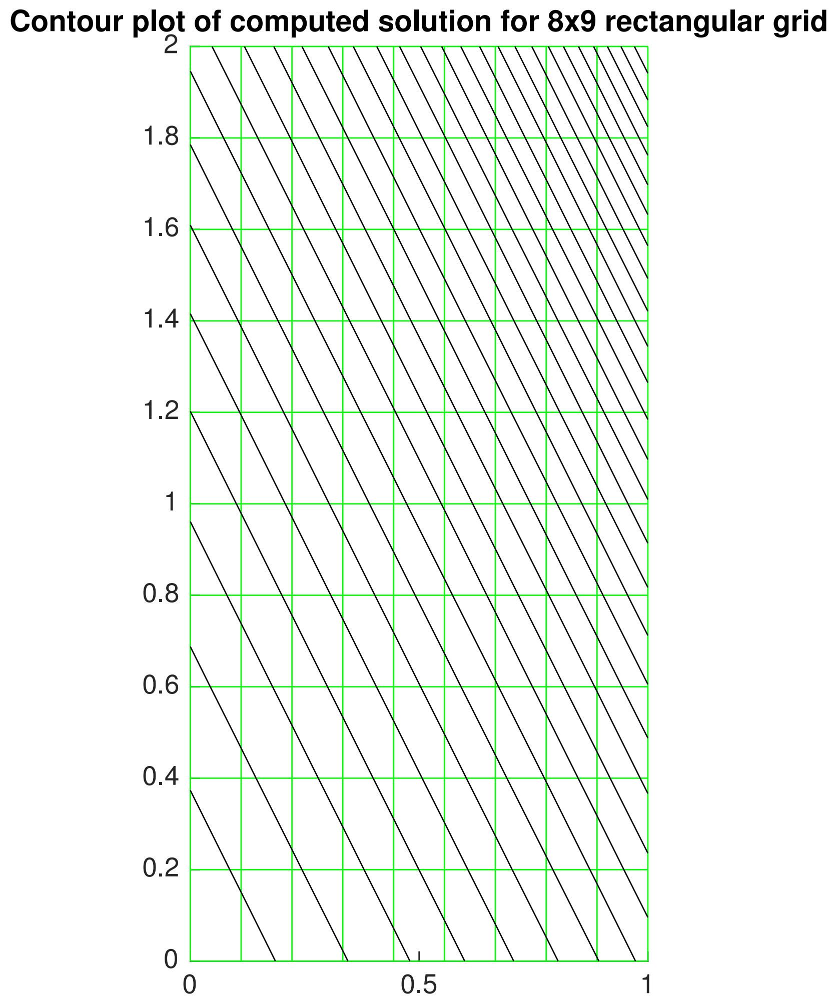
1.2.2 Part B

Modifying the script so that the domain ranges from 0 to 1 in the x and 0 to 2 in the y resulted in an error of $1.1851e-3$ and the following plot:



1.2.3 Part C

Modifying the script so that the domain ranges from 0 to 1 in the x and 0 to 2 in the y with grid spacing in the x of .1111 and .2000 in the y resulted in an error of $4.20526e-4$ and the following plot:



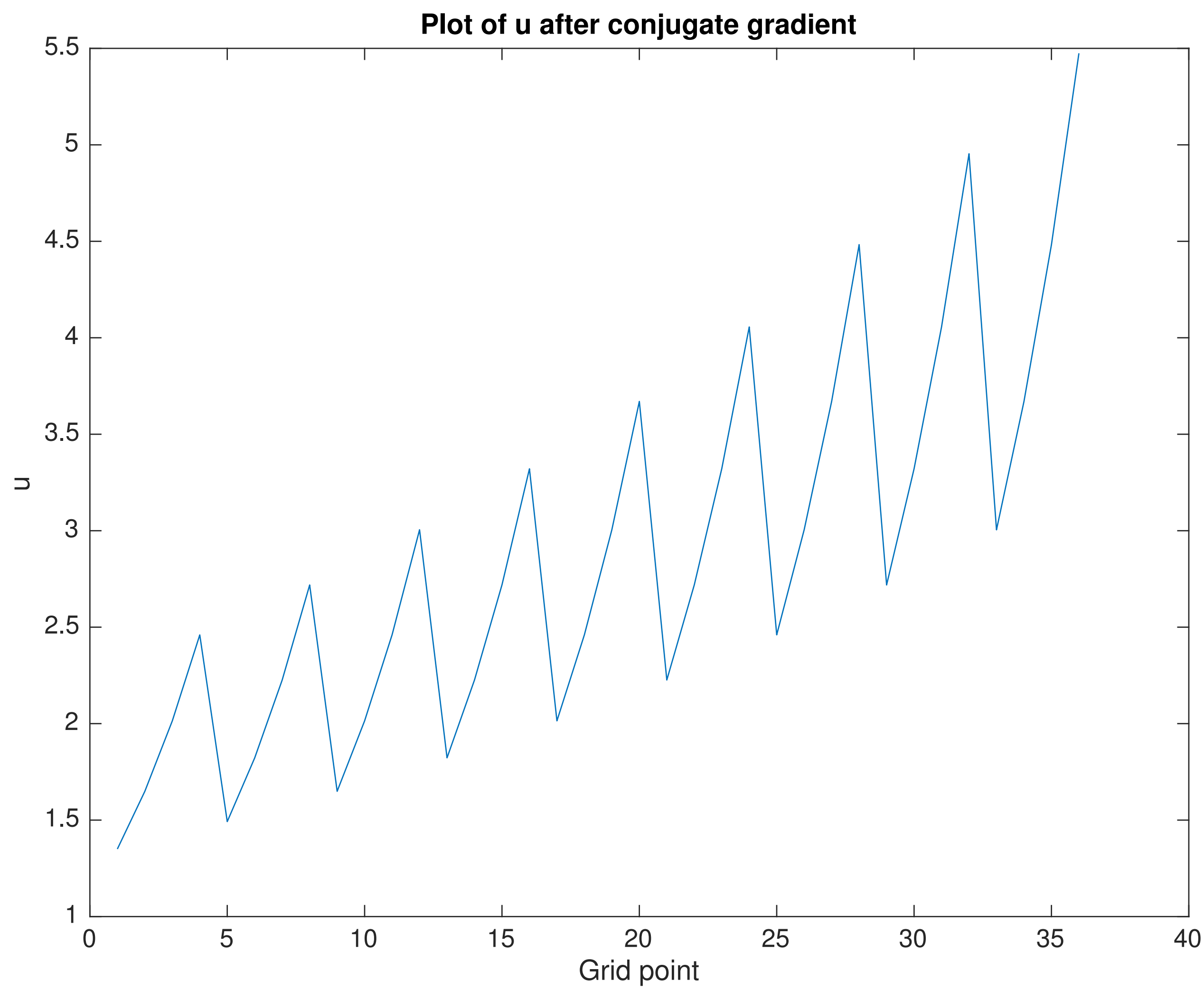
2 Exercise 4.3

2.1 Problem Description

Fix the conjugate_gradient.m file so that it works.

2.2 Problem Solution

I was able to fix the conjugate gradient code by assigning r the correct value before the loop and then updating u within the loop. The details of my implementation can be found in the MATLAB code in Appendix A. I used the A and F from the end of Exercise 3.1 and found u to be the function shown in the figure below.



3 Exercise 5.1

3.1 Problem Description

Prove that the ODE

$$u'(t) = \frac{1}{t^2 + u(t)^2}, \text{for } t \geq 1$$

has a unique solution for all time from any initial condition value $u(1) = \eta$.

3.2 Problem Solution

In order to prove that the ODE has a unique solution, we must show that it is Lipschitz Continuous. This is possible by proving that the gradient of the function with respect to u is finite and bounded as well as continuous.

$$\text{Let } f(u) = u', \text{then } \frac{\partial f}{\partial u} = \frac{-2u(t)}{(t^2 + u(t)^2)^2}$$

For an initial value of $u(1) = \eta$, the partial becomes $\frac{-2\eta}{(t^2+\eta^2)^2}$ which is finite and bounded.

4 Exercise 5.2

4.1 Problem Description

Let $f(u) = \log(u)$, and $u(0) = 2$.

4.1.1 Part A

Determine the best possible Lipschitz constant for this function over $2 \leq u < \infty$.

4.1.2 Part B

Is $f(u)$ Lipschitz continuous over $0 < u < \infty$?

4.1.3 Part C

Consider the initial value problem

$$u'(t) = \log(u(t)),$$
$$u(0) = 2.$$

Explain why we know this problem has a unique solution for all $t \geq 0$.

4.2 Problem Solution

4.2.1 Part A

The best possible Lipschitz constant can be found by $\max_{2 \leq u < \infty}(\frac{\partial f}{\partial u})$.

$$\text{L} \; = \max_{2 \leq u < \infty}\left(\frac{\partial \log(u)}{\partial u}\right) = \max_{2 \leq u < \infty}\left(\frac{1}{\ln(10)2}\right) = .2171$$

4.2.2 Part B

Because

$$\lim_{u \to 0}(\frac{1}{\ln(10)u}) = \infty$$

$f(u)$ is not Lipschitz continuous over $0 < u < \infty$.

4.2.3 Part C

Based on existence and uniqueness theorem in section 5.2.1, if $f(u)$ is Lipschitz continuous over some region $D = |u - \eta| \leq a$, there is a unique solution to the IVP at least up to time $T* = min(t_1, t_0 + a/S)$ where $S = \max_{(u,t) \in D} |f(u,t)|$.

For our initial conditions, $u(0) = 2$, we get a $D = |u - 2| \leq a$ and $L = \frac{1}{\ln(10)*2}$ and $S = \log(a + 2)$. So by the uniqueness and existence theorem, a solution will exist until

$$T* = \frac{a}{\log(a+2)}$$

And since a is arbitrary, we can choose it to maximize the time interval, which yields,

$$T* = \lim_{a \to \infty} \frac{a}{\log(a+2)} = \infty, \text{ when } a = \infty$$

This means, by choosing a to be ∞, we can prove that $f(u)$ has a unique solution for all time greater than or equal to zero.

5 Exercise 5.8

5.1 Problem Description

Consider the following third order initial value problem:

$$v'''(t) + v''(t) + 4v'(t) + 4v(t) = 4t^2 + 8t - 10$$

$$v(0) = -3, v'(0) = -2, v''(0) = 2.$$

5.1.1 Part A

Verfiy the function

$$v(t) = -\sin(2t) + t^2 - 3$$

is a solution to this problem. How do you know it is the unique solution?

5.1.2 Part B

Rewrite the problem as a first order system.

5.1.3 Part C

Use ode113 to solve the problem over $0 \leq t \leq 2$.

5.1.4 Part D

Create a table showing maximum error over acceptable tolerance.

5.1.5 Part E

Repeat part d with the ode45 solver.

5.2 Problem Solution

The implementations of the ODE functions can be found in the MATLAB code that is include in Appendix A.

5.2.1 Part A

If

$$v(t) = -\sin(2t) + t^2 - 3$$

then taking derivatives, we can find that

$$v'(t) = -2\cos(2t) + 2t$$

$$v''(t) = 4\sin(2t) + 2$$

$$v'''(t) = 8\cos(2t)$$

Plugging these values into the original equation, we can verify that $v(t) = -\sin(2t) + t^2 - 3$ is in fact a solution to the differential equation.

$$8\cos(2t) + 4\sin(2t) + 2 + 4(-2\cos(2t) + 2t) + 4(-\sin(2t) + t^2 - 3) = 4t^2 + 8t - 10$$

canceling terms, we see that

$$4t^2 + 8t - 10 = 4t^2 + 8t - 10 \checkmark$$

5.2.2 Part B

The problem written as a first order system of equations is

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} v(t) \\ v'(t) \\ v''(t) \end{bmatrix}$$

and

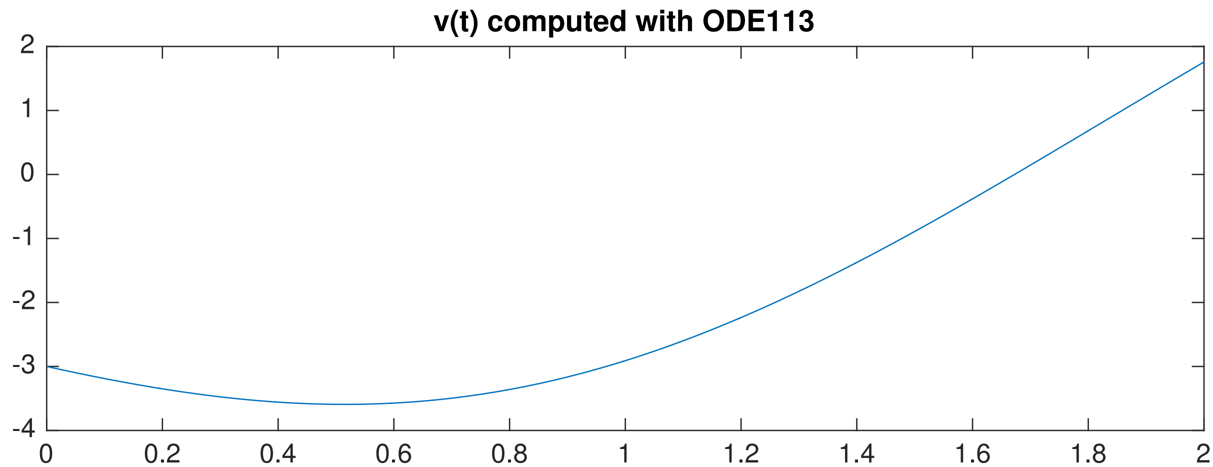
$$\dot{\bar{x}} = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ -x_3 - 4x_2 - 4x_1 + 4t^2 + 8t - 10 \end{bmatrix}$$

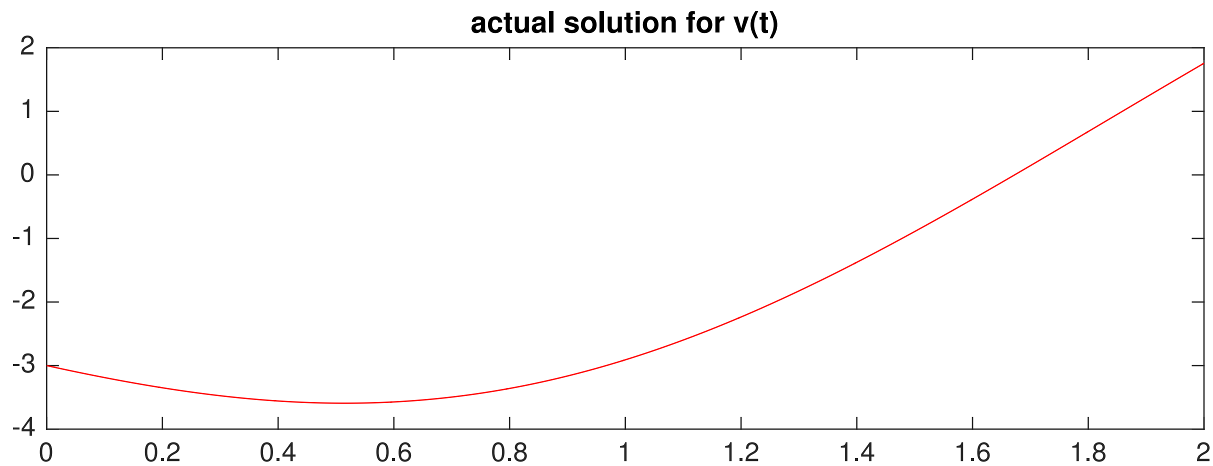
with

$$\bar{x}(0) = \begin{bmatrix} -3 \\ -2 \\ 2 \end{bmatrix}$$

5.2.3 Part C

The solution was computed using ode113 with a tolerance of $1e-3$ and resulted in an error of $6.3383e-4$. The plot of the true and computed solutions is shown in the figure below:





5.2.4 Part D

The table below shows the errors with respect to the tolerances for the ode45 solver:

tol	max error	f evaluations
1.000e-01	6.271e-04	27
1.000e-02	4.875e-04	29
1.000e-03	6.338e-04	33
1.000e-04	1.196e-04	41
1.000e-05	1.996e-05	47
1.000e-06	7.727e-07	63
1.000e-07	2.087e-07	73
1.000e-08	1.283e-08	87
1.000e-09	4.231e-10	115
1.000e-10	6.669e-11	131
1.000e-11	6.143e-12	147
1.000e-12	1.364e-12	157
1.000e-13	5.418e-14	177

5.2.5 Part E

The table below shows the errors with respect to the tolerances for the ode113 solver:

tol	max error	f evaluations
1.000e-01	9.882e-06	67
1.000e-02	1.024e-05	67
1.000e-03	1.044e-05	67
1.000e-04	9.925e-06	67
1.000e-05	5.394e-06	85
1.000e-06	5.069e-07	127
1.000e-07	4.763e-08	199
1.000e-08	4.573e-09	313
1.000e-09	4.398e-10	493
1.000e-10	4.359e-11	781
1.000e-11	4.382e-12	1237
1.000e-12	4.325e-13	1951
1.000e-13	4.396e-14	3091

It can be seen that the ode45 solver had similar errors to the ode113 solver, but that the ode45 solver took many more function evaluations than the ode113 solver.

A MATLAB Code

Table of Contents


```
% poisson2.m  -- solve the Poisson problem u_{xx} + u_{yy} = f(x,y)
% on [a,b] x [a,b].
%
% The 5-point Laplacian is used at interior grid points.
% This system of equations is then solved using backslash.
%
% From  http://www.amath.washington.edu/~rjl/fdmbook/chapter3  (2007)
```

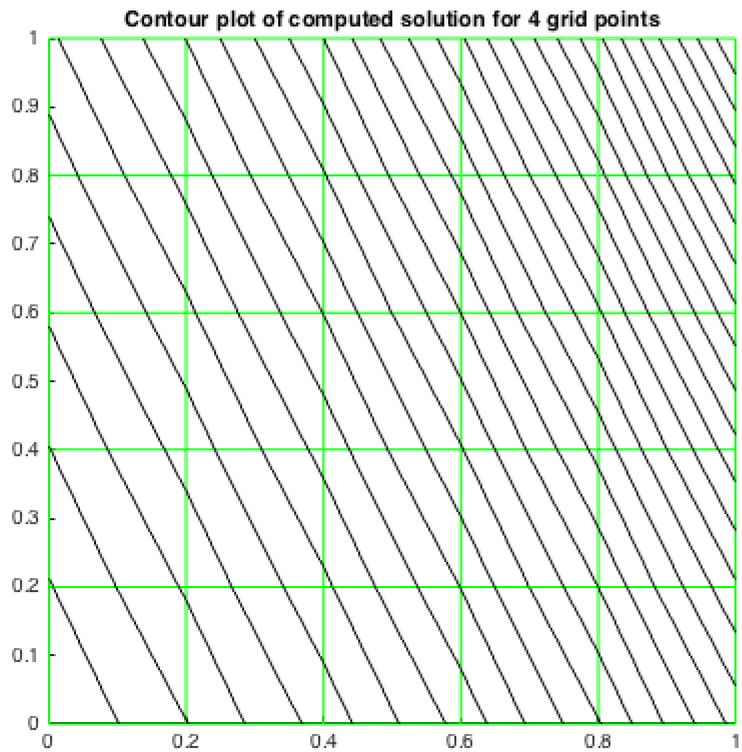
Problem 3.1.a

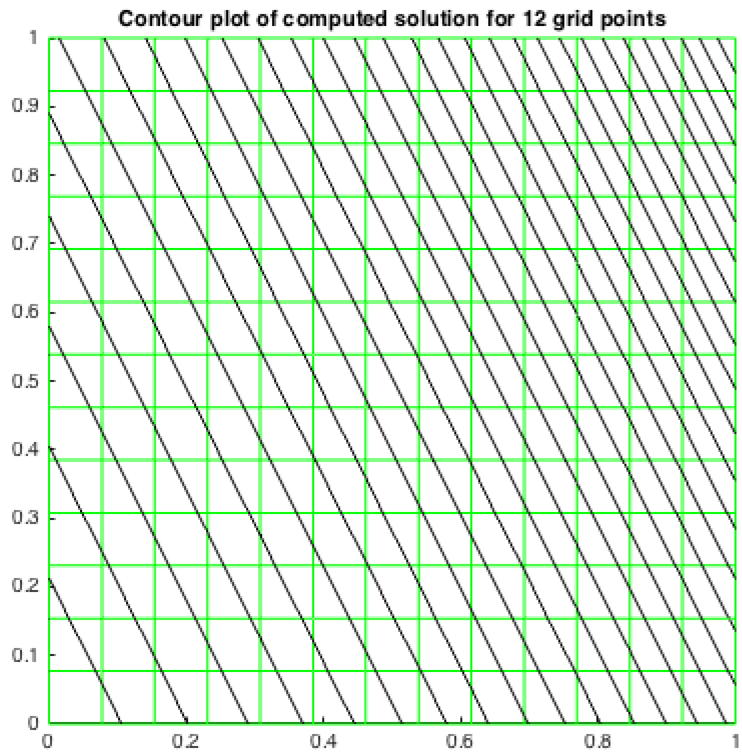
```
clear all
close all
count = 0;
fprintf('m   error\n')
for m = 4:8:48
count = count+1;
a = 0;
b = 1;
h = (b-a)/(m+1);
x = linspace(a,b,m+2);   % grid points x including boundaries
y = linspace(a,b,m+2);   % grid points y including boundaries


[X,Y] = meshgrid(x,y);      % 2d arrays of x,y values
X = X';                     % transpose so that X(i,j),Y(i,j) are
Y = Y';                     % coordinates of (i,j) point

Iint = 2:m+1;              % indices of interior points in x
Jint = 2:m+1;              % indices of interior points in y
Xint = X(Iint,Jint);       % interior points
Yint = Y(Iint,Jint);

f = @(x,y) 1.25*exp(x+y/2);         % f(x,y) function

rhs = f(Xint,Yint);  % evaluate f at interior points for right hand
 side
                           % rhs is modified below for boundary
 conditions.

utrue = exp(X+Y/2);        % true solution for test problem

% set boundary conditions around edges of usoln array:
```

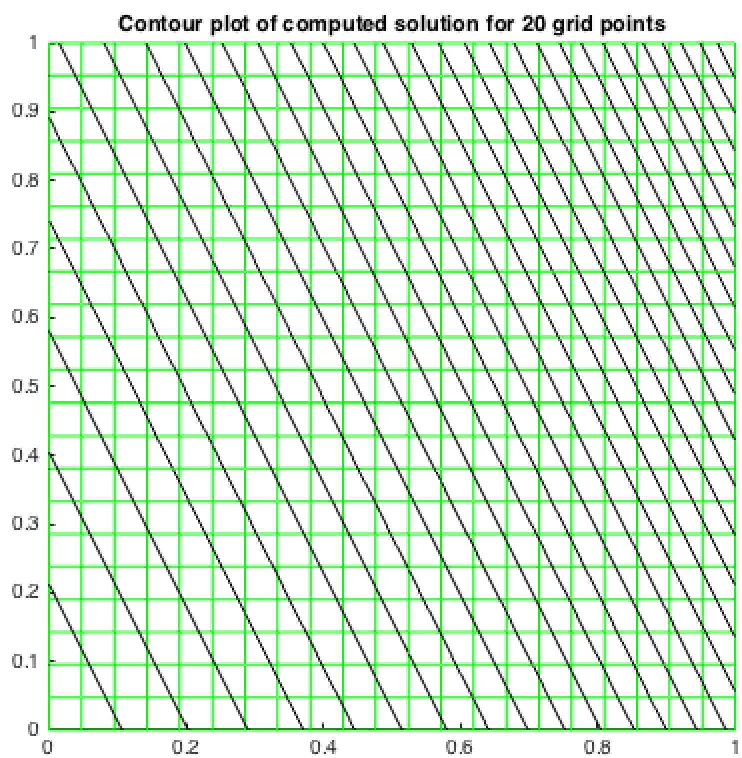
```
usoln = utrue;              % use true solution for this test problem
                            % This sets full array, but only boundary
 values
                            % are used below.  For a problem where
 utrue
                            % is not known, would have to set each
 edge of
                            % usoln to the desired Dirichlet boundary
 values.


% adjust the rhs to include boundary terms:
rhs(:,1) = rhs(:,1) - usoln(Iint,1)/h^2;
rhs(:,m) = rhs(:,m) - usoln(Iint,m+2)/h^2;
rhs(1,:) = rhs(1,:) - usoln(1,Jint)/h^2;
rhs(m,:) = rhs(m,:) - usoln(m+2,Jint)/h^2;


% convert the 2d grid function rhs into a column vector for rhs of
 system:
F = reshape(rhs,m*m,1);

% form matrix A:
I = speye(m);
e = ones(m,1);
T = spdiags([e -4*e e],[-1 0 1],m,m);
S = spdiags([e e],[-1 1],m,m);
A = (kron(I,T) + kron(S,I)) / h^2;


% Solve the linear system:
uvec = A\F;

% reshape vector solution uvec as a grid function and
% insert this interior solution into usoln for plotting purposes:
% (recall boundary conditions in usoln are already set)

usoln(Iint,Jint) = reshape(uvec,m,m);

% assuming true solution is known and stored in utrue:
err = max(max(abs(usoln-utrue)));
%fprintf('grid size: %dx%d\n', m, m);
%fprintf('Error relative to true solution of PDE = %10.5e \n',err)
fprintf(' %d & %10.5e \\\\ \n', m, err);
% plot results:

figure(count)
hold on

% plot grid:
 plot(X,Y,'g');  plot(X',Y','g')

% plot solution:
```

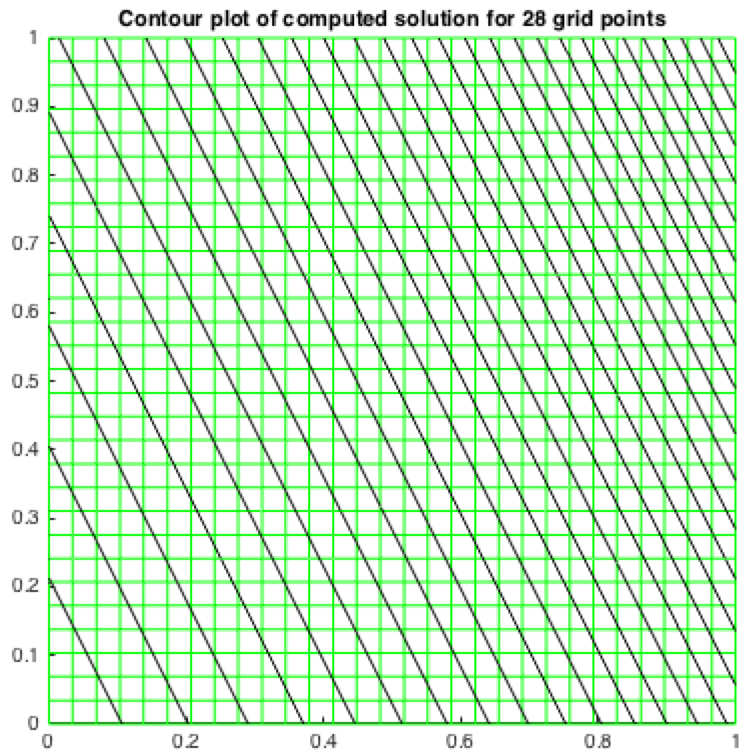
```
contour(X,Y,usoln,30,'k')

axis([a b a b])
daspect([1 1 1])
name = sprintf('Contour plot of computed solution for %d grid points',
 m);
title(name)
hold off
end
```

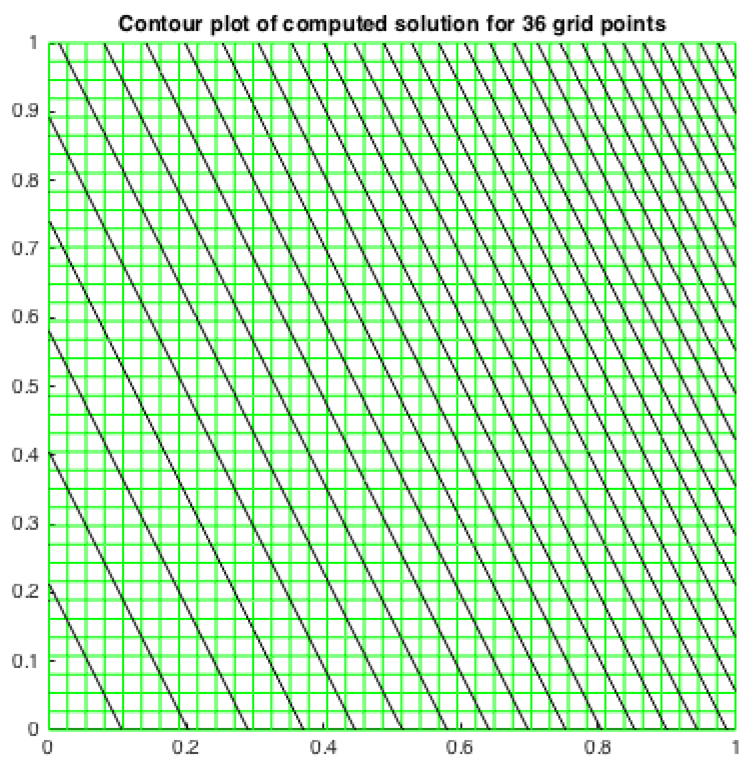
```
m   error
 4 & 5.50547e-04 \\
 12 & 8.48461e-05 \\
 20 & 3.27323e-05 \\
 28 & 1.71710e-05 \\
 36 & 1.05646e-05 \\
 44 & 7.14325e-06 \\
```

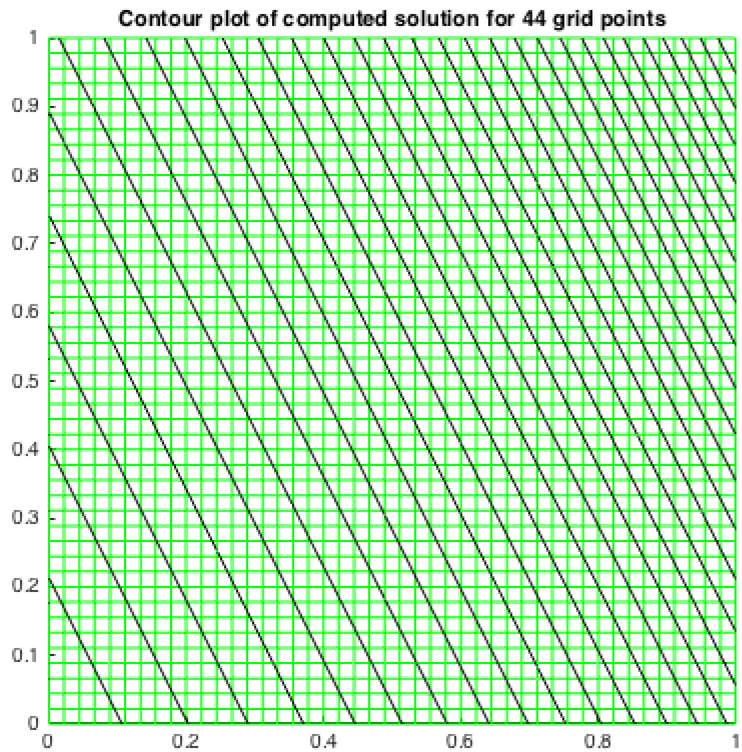



Contour plot of computed solution for 12 grid points
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1


Contour plot of computed solution for 20 grid points
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1


Contour plot of computed solution for 28 grid points
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1


Contour plot of computed solution for 36 grid points
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1

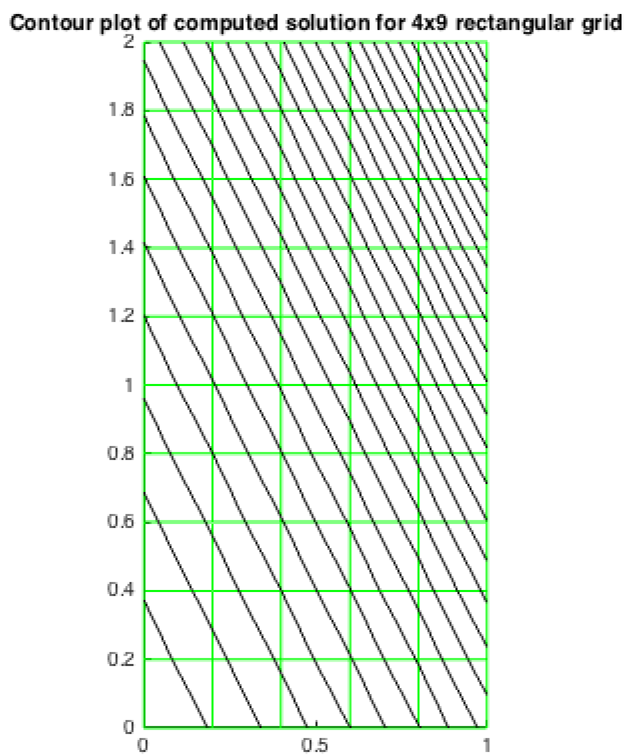


Problem 3.1.b

```
clear all
count = 6;
count = count+1;
m = 4;
ax = 0;
bx = 1;
ay = 0;
by = 2;
h = (bx-ax)/(m+1);
mx = (bx-ax)/h-1;
my = (by-ay)/h-1;

x = linspace(ax, bx, mx+2);  % grid points x including boundaries
y = linspace(ay, by, my+2);   % grid points y including boundaries


[X,Y] = meshgrid(x,y);       % 2d arrays of x,y values
X = X';                      % transpose so that X(i,j),Y(i,j) are
Y = Y';                      % coordinates of (i,j) point

Iint = 2:mx+1;               % indices of interior points in x
Jint = 2:my+1;               % indices of interior points in y
Xint = X(Iint,Jint);        % interior points
```

```
Yint = Y(Iint,Jint);

f = @(x,y) 1.25*exp(x+y/2);          % f(x,y) function

rhs = f(Xint,Yint);  % evaluate f at interior points for right hand
 side
                          % rhs is modified below for boundary
 conditions.

utrue = exp(X+Y/2);         % true solution for test problem

% set boundary conditions around edges of usoln array:

usoln = utrue;               % use true solution for this test problem
                             % This sets full array, but only boundary
 values
                             % are used below.  For a problem where
 utrue
                             % is not known, would have to set each
 edge of
                             % usoln to the desired Dirichlet boundary
 values.


% adjust the rhs to include boundary terms:
rhs(:,1) = rhs(:,1) - usoln(Iint,1)/h^2;
rhs(:,my) = rhs(:,my) - usoln(Iint,my+2)/h^2;
rhs(1,:) = rhs(1,:) - usoln(1,Jint)/h^2;
rhs(mx,:) = rhs(mx,:) - usoln(mx+2,Jint)/h^2;


% convert the 2d grid function rhs into a column vector for rhs of
 system:
F = reshape(rhs,mx*my,1);

% form matrix A:
Ix = speye(mx);
Iy = speye(my);
e = ones(my,1);
T = spdiags([e -4*e e],[-1 0 1],mx,mx);
S = spdiags([e e],[-1 1],my,my);
A = (kron(Iy,T) + kron(S,Ix)) / h^2;


% Solve the linear system:
uvec = A\F;

% reshape vector solution uvec as a grid function and
% insert this interior solution into usoln for plotting purposes:
% (recall boundary conditions in usoln are already set)

usoln(Iint,Jint) = reshape(uvec,mx,my);

% assuming true solution is known and stored in utrue:
```

```
err = max(max(abs(usoln-utrue)));
fprintf('grid size: %dx%d\n', mx, my);
fprintf('Error relative to true solution of PDE = %10.5e \n',err)

% plot results:

figure(count)
hold on

% plot grid:
 plot(X,Y,'g');  plot(X',Y','g')

% plot solution:
contour(X,Y,usoln,30,'k')

axis([ax bx ay by])
daspect([1 1 1])
name = sprintf('Contour plot of computed solution for %dx%d
 rectangular grid', mx, my);
title(name)
hold off

grid size: 4x9
Error relative to true solution of PDE = 1.18510e-03
```

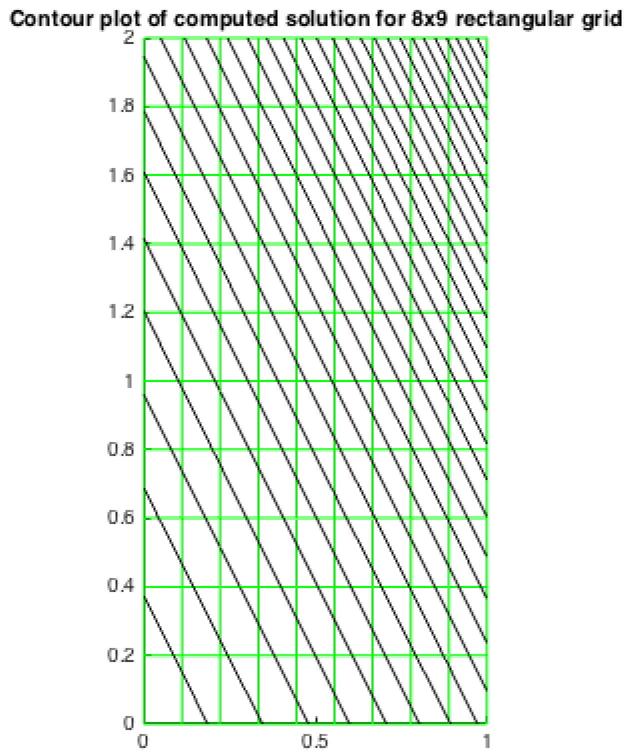


Problem 3.1.c

```
clear all
count = 7;
count = count+1;
mx = 8;
my = 9;
ax = 0;
bx = 1;
ay = 0;
by = 2;
hx = (bx-ax)/(mx+1);
hy = (by-ay)/(my+1);

x = linspace(ax, bx, mx+2);  % grid points x including boundaries
y = linspace(ay, by, my+2);   % grid points y including boundaries


[X,Y] = meshgrid(x,y);      % 2d arrays of x,y values
X = X';                     % transpose so that X(i,j),Y(i,j) are
Y = Y';                     % coordinates of (i,j) point

Iint = 2:mx+1;              % indices of interior points in x
Jint = 2:my+1;              % indices of interior points in y
Xint = X(Iint,Jint);       % interior points
Yint = Y(Iint,Jint);

f = @(x,y) 1.25*exp(x+y/2);         % f(x,y) function

rhs = f(Xint,Yint);  % evaluate f at interior points for right hand
 side
                           % rhs is modified below for boundary
 conditions.

utrue = exp(X+Y/2);        % true solution for test problem

% set boundary conditions around edges of usoln array:

usoln = utrue;              % use true solution for this test problem
                            % This sets full array, but only boundary
 values
                            % are used below.  For a problem where
 utrue
                            % is not known, would have to set each
 edge of
                            % usoln to the desired Dirichlet boundary
 values.


% adjust the rhs to include boundary terms:
rhs(:,1) = rhs(:,1) - usoln(Iint,1)/hy^2;
rhs(:,my) = rhs(:,my) - usoln(Iint,my+2)/hy^2;
rhs(1,:) = rhs(1,:) - usoln(1,Jint)/hx^2;
```

```
rhs(mx,:) = rhs(mx,:) - usoln(mx+2,Jint)/hx^2;


% convert the 2d grid function rhs into a column vector for rhs of
 system:
F = reshape(rhs,mx*my,1);

% form matrix A:
Ix = speye(mx);
Iy = speye(my);
e = ones(my,1);
Tx = spdiags([e -2*e e],[-1 0 1],mx,mx);
Ty = spdiags([0*e -2*e 0*e],[-1 0 1],mx,mx);
S = spdiags([e e],[-1 1],my,my);
A = (kron(Iy,Tx)/hx^2 + kron(Iy,Ty)/hy^2 + kron(S,Ix)/hy^2) ;


% Solve the linear system:
uvec = A\F;

% reshape vector solution uvec as a grid function and
% insert this interior solution into usoln for plotting purposes:
% (recall boundary conditions in usoln are already set)

usoln(Iint,Jint) = reshape(uvec,mx,my);

% assuming true solution is known and stored in utrue:
err = max(max(abs(usoln-utrue)));
fprintf('grid size: %dx%d\n', mx, my);
fprintf('Error relative to true solution of PDE = %10.5e \n',err)

% plot results:

figure(count)
hold on

% plot grid:
 plot(X,Y,'g');  plot(X',Y','g')

% plot solution:
contour(X,Y,usoln,30,'k')

axis([ax bx ay by])
daspect([1 1 1])
name = sprintf('Contour plot of computed solution for %dx%d
 rectangular grid', mx, my);
title(name)
hold off
```

grid size: 8x9
Error relative to true solution of PDE = 4.20526e-04



Published with MATLAB® R2015b

```
%function u = conjugate_gradient(A,f,tol)
%
%   Example:
%   x = conjugate_gradient(A,b,tol)
%
f = F;
tol = 1e-5;
MAXITS = length(f);

u = 0*f;
r = f-A*u;
p = r;
for k = 1:MAXITS
    w = A*p;
    alpha = (r'*r)/(p'*w);
    unew = u+alpha*p;
    rnew = r - alpha*w;
    if( norm(rnew) < tol ),
        fprintf('Converged! its= %7.0f, tol=%10.3e\n', [k tol]);
        return;
    end
    beta = (rnew'*rnew)/(r'*r);
    p = rnew + beta*p;
    r = rnew;
    u = unew;
end
fprintf('Caution: CG went to max iterations without converging!\n');
fprintf('MAXITS = %7.0f, tol =%10.3e\n', [MAXITS tol]);

%end
```

```
Converged! its=      36, tol= 1.000e-05
```

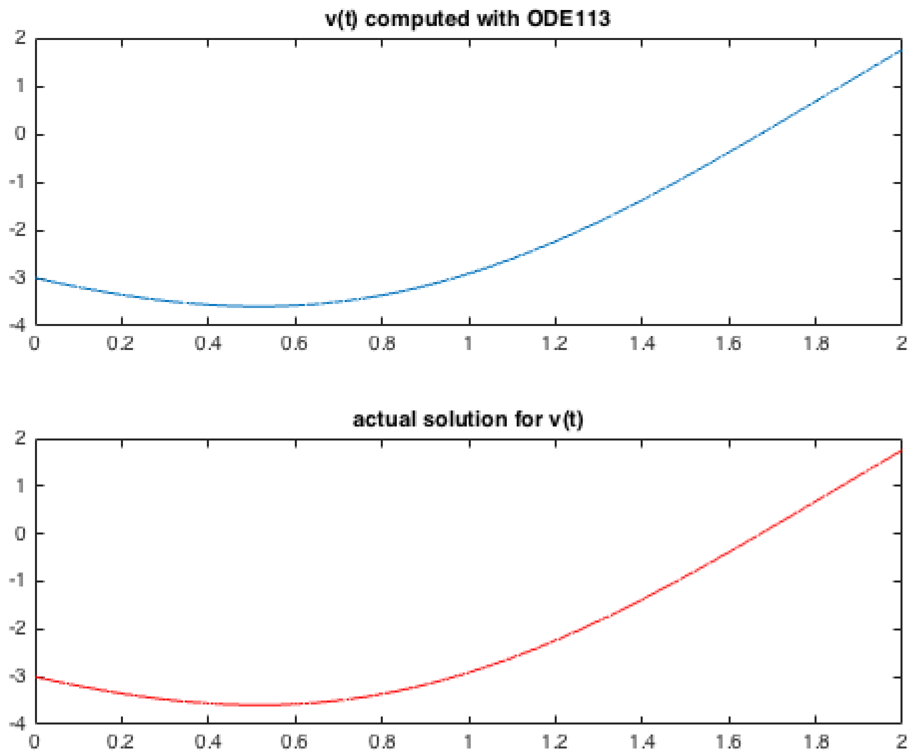
Published with MATLAB® R2015b

Table of Contents


```
% odesampletest
% test odesample for various tolerances
%
% From  http://www.amath.washington.edu/~rjl/fdmbook/chapter5  (2007)
```

Part A

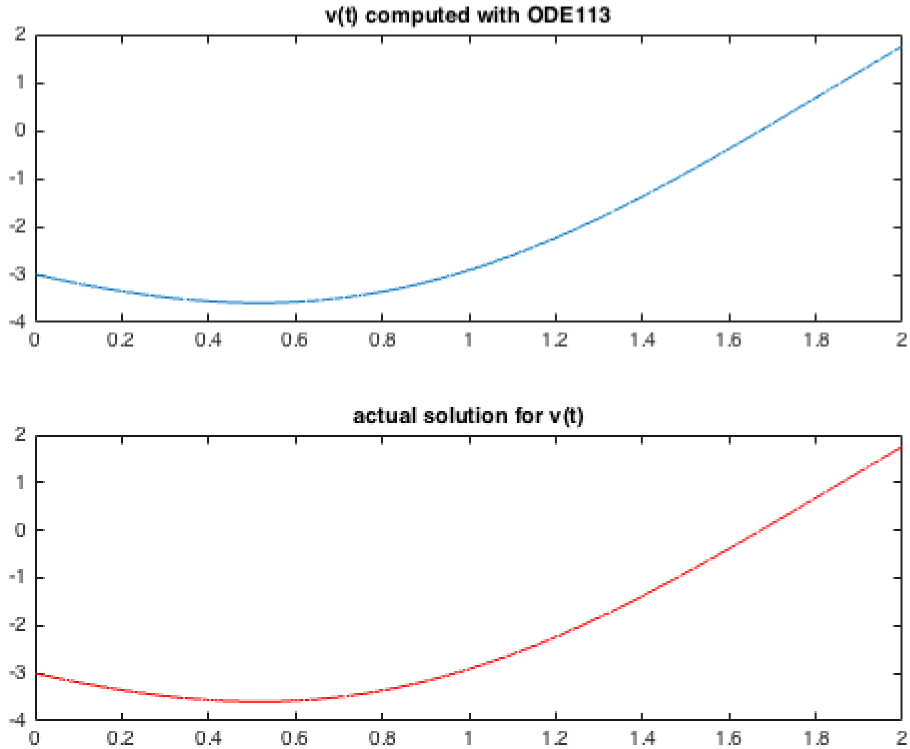
```
ODE113 = 'ode113';
tol = 1e-3;
[error] = Problem5_8_a(tol, 'on', ODE113);
```



Part C

```
close all
```

```
ODE113 = 'ode113';
tol = 1e-3;
err = Problem5_8_a(tol, 'on', ODE113);
```



Part D

```
clear all
ODE45 = ' ode45';
ODE113 = 'ode113';
global fcnevals
fprintf('Results or %s Solver', ODE113)
disp(' ')
disp('       tol       &    max error   &  f evaluations \\')
disp(' ')
for tol = logspace(-1,-13,13)
   %odesample(tol)
   err = Problem5_8_a(tol, 'off', ODE113);
   disp(sprintf('   %12.3e &   %12.3e  & %7i \\\\ ',tol, err,fcnevals))
end
disp(' ')
```

```
Results or ode113 Solver
       tol       &    max error   &  f evaluations \\

     1.000e-01 &      6.271e-04   &      27 \\
     1.000e-02 &      4.875e-04   &      29 \\
```

```
    1.000e-03 &     6.338e-04  &      33 \\
    1.000e-04 &     1.196e-04  &      41 \\
    1.000e-05 &     1.996e-05  &      47 \\
    1.000e-06 &     7.727e-07  &      63 \\
    1.000e-07 &     2.087e-07  &      73 \\
    1.000e-08 &     1.283e-08  &      87 \\
    1.000e-09 &     4.231e-10  &     115 \\
    1.000e-10 &     6.669e-11  &     131 \\
    1.000e-11 &     6.143e-12  &     147 \\
    1.000e-12 &     1.364e-12  &     157 \\
    1.000e-13 &     5.418e-14  &     177 \\
```

Part E

```
fprintf('Results or %s Solver', ODE45)
disp(' ')
disp('       tol      &     max error  &  f evaluations \\')
disp(' ')
for tol = logspace(-1,-13,13)
   %odesample(tol)
   err = Problem5_8_a(tol, 'off', ODE45);
   disp(sprintf('  %12.3e  & %12.3e &  %7i \\\\',tol, err,fcnevals))
end
```

```
Results or  ode45 Solver
       tol      &     max error  &  f evaluations \\

    1.000e-01  &     9.882e-06 &       67 \\
    1.000e-02  &     1.024e-05 &       67 \\
    1.000e-03  &     1.044e-05 &       67 \\
    1.000e-04  &     9.925e-06 &       67 \\
    1.000e-05  &     5.394e-06 &       85 \\
    1.000e-06  &     5.069e-07 &      127 \\
    1.000e-07  &     4.763e-08 &      199 \\
    1.000e-08  &     4.573e-09 &      313 \\
    1.000e-09  &     4.398e-10 &      493 \\
    1.000e-10  &     4.359e-11 &      781 \\
    1.000e-11  &     4.382e-12 &     1237 \\
    1.000e-12  &     4.325e-13 &     1951 \\
    1.000e-13  &     4.396e-14 &     3091 \\
```

Published with MATLAB® R2015b

```
function [error] = Problem5_8_a(tol, figDisp, solver)

% odesample.m
% Sample code for solving a system of ODEs in matlab.
% Solves  v'' = v^2 + (v')^2 - v -1  with v(0)=1, v'(0)=0
% with true solution v(t) = cos(t).
% Rewritten as a first order system.
% From  http://www.amath.washington.edu/~rjl/fdmbook/chapter5  (2007)


global fcnevals

t0 = 0;                       % initial time
u0 = [-3; -2; 2];     % initial data for u(t) as a vector
tfinal = 2;                   % final time
fcnevals = 0;                 % counter for number of function
 evaluations

% solve ode:
options = odeset('AbsTol',tol,'RelTol',tol);
if(solver == 'ode113')
    odesolution = ode113(@f,[t0 tfinal],u0,options);
else %ODE45 default
    odesolution = ode45(@f,[t0 tfinal],u0,options);
end

% plot v = u(1) as a function of t:

figure('Visible', figDisp)
subplot(2, 1, 1)
t = linspace(0, tfinal, 500);
u = deval(odesolution, t);
v = u(1,:);
plot(t,v)
title('v(t) computed with ODE113')

% compare to true solution:
vtrue = -sin(2*t)+t.^2-3;
%hold on
subplot(2, 1 , 2)
plot(t,vtrue,'r')
title('actual solution for v(t)')
%hold off

error = max(abs(v-vtrue));
end


%-----------------------------------

function f = f(t,u)
global fcnevals
```

```
f1 = u(2);
f2 = u(3);
f3 = -u(3)-4*u(2)-4*u(1)+4*t^2+8*t-10;
f = [f1; f2; f3];

fcnevals = fcnevals + 1;
end
```

```
Not enough input arguments.

Error in Problem5_8_a (line 20)
options = odeset('AbsTol',tol,'RelTol',tol);
```

Published with MATLAB® R2015b