Moyuan Huang

January 15, 2017

THEORY1 — LaTeX

Problem 1. Theory Problem 1: Sum of squares of Fibonacci numbers

For convenience, let us denote $\Sigma_{i=1}^{n} F_i^2 = S_n$. The goal is to proof on induction on $n \geq 1$ that

$$\Sigma_{i=1}^{n} F_i^2 = F_n F_{n+1} \tag{1}$$

(a) let $n = 1$, then

$$S_1 = F_1^2 = 1 = F_1 F_2$$

(b) Suppose that Eq. 1 holds when $n = k$. Equivalently,

$$S_k = F_k F_{k+1}$$

(c) When $n = k + 1$,

$$\begin{aligned} S_{k+1} = & \quad \Sigma_{i=1}^{k+1} F_i^2 \\ = & \quad S_k + F_{k+1}^2 \\ = & \quad F_k F_{k+1} + F_{k+1}^2 \\ = & \quad (F_k + F_{k+1}) F_{k+1} \\ = & \quad F_{k+2} F_{k+1} \end{aligned}$$

This means that Eq. 1 still holds when $n = k + 1$.

(d) By induction, $\Sigma_{i=1}^{n} F_i^2 = F_n F_{n+1}$ for $n \geq 1$.

Submitted by Moyuan Huang on January 15, 2017.