Assignment 2

Varun Shankar

February 10, 2019

Instructions

Please type up your replies, generate a PDF, and send me the output. I highly recommend you use LaTeX to typeset your assignments. While there is a bit of a learning curve, it is worth learning. If not, consider using Lyx, a front-end to LaTeX. If neither of those work, feel free to use Word or Writer to type up your assignments. Submit your PDF and code in a single zip file via Canvas. Make sure it can be run on at least a couple of machines, and make sure the code is well commented so I know you know what you're doing.

Problems

1. (a) Generate an $O(h^2)$ (second-order) approximation to the *first* derivative of some function $f(x)$ using 3 points, x_0, x_1 and x_2 using the polynomial interpolation approach. Give the finite difference formulas for each of the points x_0, x_1, and x_2. Assume they are equally spaced, with spacing h.

   (b) Using the error term for polynomial interpolation, show that the error above is $O(h^2)$.

   (c) Write Matlab code that uses your approximation to compute the derivative of any function f. The code should be a function called FirstDer(). FirstDer takes in two inputs: the first is the set of all points (X) at which we want the derivative, and the second is the set of data/function values at those points ($Y = f|_X = f(x_k), k = 0, \ldots, N$). So, you have FirstDer(X,Y). The output is the list of values of the first derivative at each of the points computed by your formula from part a. The function f should be defined as an anonymous function in a file called DriverDer, which calls FirstDer() with the appropriate inputs, collects the outputs, computes errors, and plots them. Assume the points in X are sorted in ascending order.
   HINT: Loop over x, grab 3 points at a time, apply your formula from part a. Pay careful attention to the first and last points in your list.

   (d) Plot the relative error of the approximation to the second derivative as a function of increasing number of points. Does this match your derivation from part b?

2. (a) Generate an approximation to $\int_a^b f(x)dx$ using a quadratic polynomial interpolant to f. Assume you have 3 equispaced points, including the end points a and b; that is, let $x_0 = a, x_1 = (a+b)/2$ and $x_2 = b$ be the 3 points.

   (b) Derive the corresponding error term for this quadrature rule.

   (c) What is the problem with using equally-spaced points to generate quadrature rules as the number of points increases? How do you alleviate this problem?