Assignment 1

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Instructions

Please type up your replies, generate a PDF, and send me the output. I highly recommend you use LaTeX to typeset your assignments. While there is a bit of a learning curve, it is worth learning. If not, consider using Lyx, a front-end to LaTeX. If neither of those work, feel free to use Word or Writer to type up your assignments. Submit your PDF and code in a single zip file via Canvas. Make sure it can be run on at least a couple of machines, and make sure the code is well commented so I know you know what you're doing.

NOTE:

- You will be graded on the quality of your write-up; the more clearly you explain things, the more points you get. Writing brusque text with no clarity will be severely penalized. Attempt to sound like a textbook!
- Make sure to clearly mark the question number and sub-question number.
- Plagiarism in the text and the code will not be tolerated. If you *must* use someone else's code for some part of your assignment, do so and cite it. You will not get a full score, but you will at least get partial credit.

Problems

1. You have been given code for generating polynomial interpolants by inverting the Vandermonde matrix, all using equispaced points on $[-1, 1]$. You will be interpolating the Runge function $f(x) = \frac{1}{1+25x^2}$.

   (a) Write code to build the Vandermonde matrix on Chebyshev extrema.

   (b) Generate a plot of condition number of the Vandermonde matrix vs number of nodes for both types of points (equispaced and Chebyshev).

   (c) Generate a plot of the polynomial interpolants for $N = 9$ and $N = 50$ for both types of points.

   (d) Build a polynomial interpolant at Chebyshev extrema (instead of equispaced points) using the Vandermonde matrix. Evaluate this interpolant at $10,000$ equispaced nodes. Produce a plot of total time for interpolation at increasing numbers of nodes and evaluation at 10,000 equispaced nodes (measured using Matlab's functions) vs the accuracy (measured as the ℓ_2 relative error in the polynomial interpolant).

   (e) You have also been provided code to perform barycentric Lagrange interpolation at Chebyshev extrema. Produce a plot of total time vs accuracy (as above) at $10,000$ equispaced nodes. How does it compare to the Vandermonde approach?

   (f) Matlab has a built-in function called polyfit to do polynomial interpolation. How does polyfit compare in terms of accuracy and computational cost to the barycentric approach? How does it compare to the Vandermonde approach? Produce plots similar to those above to justify your answers.

Note that the relative ℓ_2 error is measured as $e_{\ell_2} = \frac{\|\boldsymbol{f} - \mathbf{p}\|_2}{\|\boldsymbol{f}\|_2}$, where $\boldsymbol{f} = [f(x_0), \ldots, f(x_{10000})]^T$ and $\mathbf{p} = [p(x_0), \ldots, p(x_{10000})]^T$.