7. Let μ_1 denote the true mean course GPA for all courses taught by full-time faculty, and let μ_2 denote the true mean course GPA for all courses taught by part-time faculty. The hypotheses of interest are H_0: $\mu_1 = \mu_2$ versus H_a: $\mu_1 \neq \mu_2$; or, equivalently, H_0: $\mu_1 - \mu_2 = 0$ v. H_a: $\mu_1 - \mu_2 \neq 0$.

The large-sample test statistic is $z = \dfrac{(\bar{x}-\bar{y})-\Delta_0}{\sqrt{\dfrac{s_1^2}{m}+\dfrac{s_2^2}{n}}} = \dfrac{(2.7186-2.8639)-0}{\sqrt{\dfrac{(.63342)^2}{125}+\dfrac{(.49241)^2}{88}}} = -1.88$. The corresponding two-tailed P-value is $P(|Z| \geq |-1.88|) = 2[1 - \Phi(1.88)] = .0602$.
Since the P-value exceeds $\alpha = .01$, we fail to reject H_0. At the .01 significance level, there is insufficient evidence to conclude that the true mean course GPAs differ for these two populations of faculty.

9.

a. Point estimate $\bar{x}-\bar{y} = 19.9-13.7 = 6.2$. It appears that there could be a difference.

b. H_0: $\mu_1 - \mu_2 = 0$, H_a: $\mu_1 - \mu_2 \neq 0$, $z = \dfrac{(19.9-13.7)}{\sqrt{\dfrac{39.1^2}{60}+\dfrac{15.8^2}{60}}} = \dfrac{6.2}{5.44} = 1.14$, and the P-value $= 2[P(Z > 1.14)] = 2(.1271) = .2542$. The P-value is larger than any reasonable α, so we do not reject H_0. There is no statistically significant difference.

c. No. With a normal distribution, we would expect most of the data to be within 2 standard deviations of the mean, and the distribution should be symmetric. Two sd's above the mean is 98.1, but the distribution stops at zero on the left. The distribution is positively skewed.

d. We will calculate a 95% confidence interval for μ, the true average length of stays for patients given the treatment. $19.9 \pm 1.96\dfrac{39.1}{\sqrt{60}} = 19.9 \pm 9.9 = (10.0, 21.8)$.

11. $(\bar{x}-\bar{y}) \pm z_{\alpha/2}\sqrt{\dfrac{s_1^2}{m}+\dfrac{s_2^2}{n}} = (\bar{x}-\bar{y}) \pm z_{\alpha/2}\sqrt{(SE_1)^2+(SE_2)^2}$. Using $\alpha = .05$ and $z_{\alpha/2} = 1.96$ yields $(5.5-3.8) \pm 1.96\sqrt{(0.3)^2+(0.2)^2} = (0.99, 2.41)$. We are 95% confident that the true average blood lead level for male workers is between 0.99 and 2.41 higher than the corresponding average for female workers.

13. $\sigma_1 = \sigma_2 = .05$, $d = .04$, $\alpha = .01$, $\beta = .05$, and the test is one-tailed $\Rightarrow$

$$n = \frac{(.0025+.0025)(2.33+1.645)^2}{.0016} = 49.38, \text{ so use } n = 50.$$

15.

a. As either m or n increases, SD decreases, so $\dfrac{\mu_1-\mu_2-\Delta_0}{SD}$ increases (the numerator is positive), so $\left(z_\alpha - \dfrac{\mu_1-\mu_2-\Delta_0}{SD}\right)$ decreases, so $\beta = \Phi\left(z_\alpha - \dfrac{\mu_1-\mu_2-\Delta_0}{SD}\right)$ decreases.

b. As β decreases, z_β increases, and since z_β is the numerator of n, n increases also.

Section 9.2

17.

a. $\nu = \dfrac{\left(5^2/10+6^2/10\right)^2}{\left(5^2/10\right)^2/9+\left(6^2/10\right)^2/9} = \dfrac{37.21}{.694+1.44} = 17.43 \approx 17.$

b. $\nu = \dfrac{\left(5^2/10+6^2/15\right)^2}{\left(5^2/10\right)^2/9+\left(6^2/15\right)^2/14} = \dfrac{24.01}{.694+.411} = 21.7 \approx 21.$

c. $\nu = \dfrac{\left(2^2/10+6^2/15\right)^2}{\left(2^2/10\right)^2/9+\left(6^2/15\right)^2/14} = \dfrac{7.84}{.018+.411} = 18.27 \approx 18.$

d. $\nu = \dfrac{\left(5^2/12+6^2/24\right)^2}{\left(5^2/12\right)^2/11+\left(6^2/24\right)^2/23} = \dfrac{12.84}{.395+.098} = 26.05 \approx 26.$

19. For the given hypotheses, the test statistic is $t = \dfrac{115.7-129.3+10}{\sqrt{\frac{5.03^2}{6}+\frac{5.38^2}{6}}} = \dfrac{-3.6}{3.007} = -1.20$, and the df is

$\nu = \dfrac{(4.2168+4.8241)^2}{\dfrac{(4.2168)^2}{5}+\dfrac{(4.8241)^2}{5}} = 9.96$, so use df = 9. The P-value is $P(T \le -1.20 \text{ when } T \sim t_9) \approx .130$.

Since .130 > .01, we don't reject H_0.

21. Let μ_1 = the true average gap detection threshold for normal subjects, and μ_2 = the corresponding value for CTS subjects. The relevant hypotheses are H_0: $\mu_1 - \mu_2 = 0$ v. H_a: $\mu_1 - \mu_2 < 0$, and the test statistic is

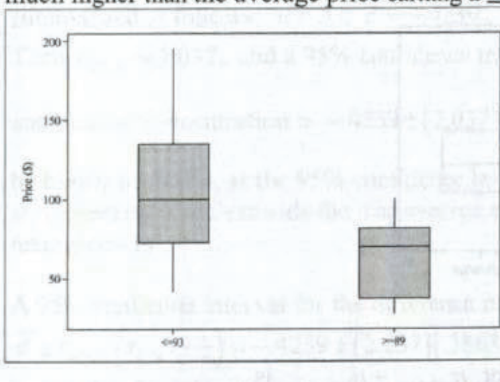
$t = \dfrac{1.71-2.53}{\sqrt{.0351125+.07569}} = \dfrac{-.82}{.3329} = -2.46$. Using df $\nu = \dfrac{(.0351125+.07569)^2}{\dfrac{(.0351125)^2}{7}+\dfrac{(.07569)^2}{9}} = 15.1$, or 15, the P-value

is $P(T \le -2.46 \text{ when } T \sim t_{15}) \approx .013$. Since .013 > .01, we fail to reject H_0 at the α = .01 level. We have insufficient evidence to claim that the true average gap detection threshold for CTS subjects exceeds that for normal subjects.

25.

a. Normal probability plots of both samples (not shown) exhibit substantial linear patterns, suggesting that the normality assumption is reasonable for both populations of prices.

b. The comparative boxplots below suggest that the average price for a wine earning a ≥ 93 rating is much higher than the average price earning a ≤ 89 rating.



c. From the data provided, $\bar{x} = 110.8$, $\bar{y} = 61.7$, $s_1 = 48.7$, $s_2 = 23.8$, and $\nu \approx 15$. The resulting 95% CI for the difference of population means is $(110.8 - 61.7) \pm t_{.025,15}\sqrt{\frac{48.7^2}{12} + \frac{23.8^2}{14}} = (16.1, 82.0)$. That is, we are 95% confident that wines rated ≥ 93 cost, on average, between \$16.10 and \$82.00 more than wines rated ≤ 89. Since the CI does not include 0, this certainly contradicts the claim that price and quality are unrelated.

27.

a. Let's construct a 99% CI for μ_{AN}, the true mean intermuscular adipose tissue (IAT) under the described AN protocol. Assuming the data comes from a normal population, the CI is given by $\bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} = .52 \pm t_{.005,15}\frac{.26}{\sqrt{16}} = .52 \pm 2.947\frac{.26}{\sqrt{16}} = (.33, .71)$. We are 99% confident that the true mean IAT under the AN protocol is between .33 kg and .71 kg.

b. Let's construct a 99% CI for $\mu_{AN} - \mu_C$, the difference between true mean AN IAT and true mean control IAT. Assuming the data come from normal populations, the CI is given by

$$(\bar{x} - \bar{y}) \pm t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}} = (.52 - .35) \pm t_{.005,21}\sqrt{\frac{(.26)^2}{16} + \frac{(.15)^2}{8}} = .17 \pm 2.831\sqrt{\frac{(.26)^2}{16} + \frac{(.15)^2}{8}} = (-.07, .41).$$

Since this CI includes zero, it's plausible that the difference between the two true means is zero (i.e., $\mu_{AN} - \mu_C = 0$). [Note: the df calculation $\nu = 21$ comes from applying the formula in the textbook.]

29. Let μ_1 = the true average compression strength for strawberry drink and let μ_2 = the true average compression strength for cola. A lower tailed test is appropriate. We test H_0: $\mu_1 - \mu_2 = 0$ v. H_a: $\mu_1 - \mu_2 < 0$. The test statistic is $t = \frac{-14}{\sqrt{29.4 + 15}} = -2.10$; $\nu = \frac{(44.4)^2}{\frac{(29.4)^2}{14} + \frac{(15)^2}{14}} = \frac{1971.36}{77.8114} = 25.3$, so use df=25.

The P-value $\approx P(t < -2.10) = .023$. This P-value indicates strong support for the alternative hypothesis. The data does suggest that the extra carbonation of cola results in a higher average compression strength.

41.

a. Let μ_D denote the true mean change in total cholesterol under the aripiprazole regimen. A 95% CI for μ_D, using the "large-sample" method, is $\bar{d} \pm z_{\alpha/2}\frac{s_D}{\sqrt{n}} = 3.75 \pm 1.96(3.878) = (-3.85, 11.35)$.

b. Now let μ_D denote the true mean change in total cholesterol under the quetiapine regimen. The hypotheses are H_0: $\mu_D = 0$ versus H_a: $\mu_D > 0$. Assuming the distribution of cholesterol changes under this regimen is normal, we may apply a paired t test:

$$t = \frac{\bar{d} - \Delta_0}{s_D/\sqrt{n}} = \frac{9.05 - 0}{4.256} = 2.126 \Rightarrow P\text{-value} = P(T_{35} \geq 2.126) \approx P(T_{35} \geq 2.1) = .02.$$

Our conclusion depends on our significance level. At the $\alpha = .05$ level, there is evidence that the true mean change in total cholesterol under the quetiapine regimen is positive (i.e., there's been an increase); however, we do not have sufficient evidence to draw that conclusion at the $\alpha = .01$ level.

c. Using the "large-sample" procedure again, the 95% CI is $\bar{d} \pm 1.96\frac{s_D}{\sqrt{n}} = \bar{d} \pm 1.96SE(\bar{d})$. If this equals (7.38, 9.69), then midpoint $= \bar{d} = 8.535$ and width $= 2(1.96\,SE(\bar{d})) = 9.69 - 7.38 = 2.31 \Rightarrow$ $SE(\bar{d}) = \frac{2.31}{2(1.96)} = .59$. Now, use these values to construct a 99% CI (again, using a "large-sample" z method): $\bar{d} \pm 2.576SE(\bar{d}) = 8.535 \pm 2.576(.59) = 8.535 \pm 1.52 = (7.02, 10.06)$.

43.

a. Although there is a "jump" in the middle of the Normal Probability plot, the data follow a reasonably straight path, so there is no strong reason for doubting the normality of the population of differences.

b. A 95% lower confidence bound for the population mean difference is:

$$\bar{d} - t_{.05,14}\left(\frac{s_d}{\sqrt{n}}\right) = -38.60 - (1.761)\left(\frac{23.18}{\sqrt{15}}\right) = -38.60 - 10.54 = -49.14.$$ We are 95% confident that the true mean difference between age at onset of Cushing's disease symptoms and age at diagnosis is greater than -49.14.

c. A 95% upper confidence bound for the population mean difference is $38.60 + 10.54 = 49.14$.

45.

a. Yes, it's quite plausible that the population distribution of differences is normal, since the accompanying normal probability plot of the differences is quite linear.