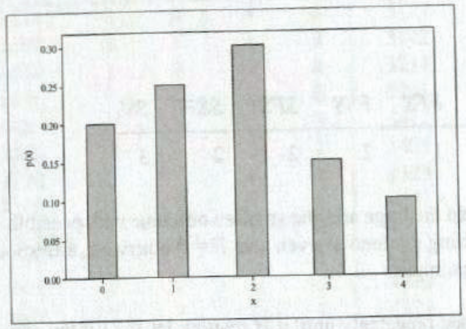
Section 3.2

11.

a.



b. $P(X \geq 2) = p(2) + p(3) + p(4) = .30 + .15 + .10 = .55$, while $P(X > 2) = .15 + .10 = .25$.

c. $P(1 \leq X \leq 3) = p(1) + p(2) + p(3) = .25 + .30 + .15 = .70$.

d. Who knows? (This is just a little joke by the author.)

13.

a. $P(X \leq 3) = p(0) + p(1) + p(2) + p(3) = .10+.15+.20+.25 = .70$.

b. $P(X < 3) = P(X \leq 2) = p(0) + p(1) + p(2) = .45$.

c. $P(X \geq 3) = p(3) + p(4) + p(5) + p(6) = .55$.

d. $P(2 \leq X \leq 5) = p(2) + p(3) + p(4) + p(5) = .71$.

e. The number of lines <u>not</u> in use is $6 - X$, and $P(2 \leq 6 - X \leq 4) = P(-4 \leq -X \leq -2) =$ $P(2 \leq X \leq 4) = p(2) + p(3) + p(4) = .65$.

f. $P(6 - X \geq 4) = P(X \leq 2) = .10 + .15 + .20 = .45$.

15.

a. (1,2) (1,3) (1,4) (1,5) (2,3) (2,4) (2,5) (3,4) (3,5) (4,5)

b. X can only take on the values 0, 1, 2. $p(0) = P(X = 0) = P(\{(3,4)\ (3,5)\ (4,5)\}) = 3/10 = .3$; $p(2) = P(X = 2) = P(\{(1,2)\}) = 1/10 = .1$; $p(1) = P(X = 1) = 1 - [p(0) + p(2)] = .60$; and otherwise $p(x) = 0$.

c. $F(0) = P(X \leq 0) = P(X = 0) = .30$;
$F(1) = P(X \leq 1) = P(X = 0 \text{ or } 1) = .30 + .60 = .90$;
$F(2) = P(X \leq 2) = 1$.

Therefore, the complete cdf of X is

$$F(x) = \begin{cases} 0 & x < 0 \\ .30 & 0 \le x < 1 \\ .90 & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

17.

a. $p(2) = P(Y = 2) = P(\text{first 2 batteries are acceptable}) = P(AA) = (.9)(.9) = .81$.

b. $p(3) = P(Y = 3) = P(UAA \text{ or } AUA) = (.1)(.9)^2 + (.1)(.9)^2 = 2[(.1)(.9)^2] = .162$.

c. The fifth battery must be an A, and exactly one of the first four must also be an A. Thus, $p(5) = P(AUUUA \text{ or } UAUUA \text{ or } UUAUA \text{ or } UUUAA) = 4[(.1)^3(.9)^2] = .00324$.

d. $p(y) = P(\text{the } y^{\text{th}} \text{ is an } A \text{ and so is exactly one of the first } y - 1) = (y - 1)(.1)^{y-2}(.9)^2$, for $y = 2, 3, 4, 5, \ldots$

19. $p(0) = P(Y = 0) = P(\text{both arrive on Wed}) = (.3)(.3) = .09$;
$p(1) = P(Y = 1) = P((\text{W,Th}) \text{ or } (\text{Th,W}) \text{ or } (\text{Th,Th})) = (.3)(.4) + (.4)(.3) + (.4)(.4) = .40$;
$p(2) = P(Y = 2) = P((\text{W,F}) \text{ or } (\text{Th,F}) \text{ or } (\text{F,W}) \text{ or } (\text{F,Th}) \text{ or } (\text{F,F})) = .32$;
$p(3) = 1 - [.09 + .40 + .32] = .19$.

21.

a. First, $1 + 1/x > 1$ for all $x = 1, \ldots, 9$, so $\log(1 + 1/x) > 0$. Next, check that the probabilities sum to 1: $\sum_{x=1}^{9} \log_{10}(1+1/x) = \sum_{x=1}^{9} \log_{10}\left(\frac{x+1}{x}\right) = \log_{10}\left(\frac{2}{1}\right) + \log_{10}\left(\frac{3}{2}\right) + \cdots + \log_{10}\left(\frac{10}{9}\right)$; using properties of logs, this equals $\log_{10}\left(\frac{2}{1} \times \frac{3}{2} \times \cdots \times \frac{10}{9}\right) = \log_{10}(10) = 1$.

b. Using the formula $p(x) = \log_{10}(1 + 1/x)$ gives the following values: $p(1) = .301$, $p(2) = .176$, $p(3) = .125$, $p(4) = .097$, $p(5) = .079$, $p(6) = .067$, $p(7) = .058$, $p(8) = .051$, $p(9) = .046$. The distribution specified by Benford's Law is <u>not</u> uniform on these nine digits; rather, lower digits (such as 1 and 2) are much more likely to be the lead digit of a number than higher digits (such as 8 and 9).

c. The jumps in $F(x)$ occur at $0, \ldots, 8$. We display the cumulative probabilities here: $F(1) = .301$, $F(2) = .477$, $F(3) = .602$, $F(4) = .699$, $F(5) = .778$, $F(6) = .845$, $F(7) = .903$, $F(8) = .954$, $F(9) = 1$. So, $F(x) = 0$ for $x < 1$; $F(x) = .301$ for $1 \le x < 2$;
$F(x) = .477$ for $2 \le x < 3$; etc.

d. $P(X \le 3) = F(3) = .602$; $P(X \ge 5) = 1 - P(X < 5) = 1 - P(X \le 4) = 1 - F(4) = 1 - .699 = .301$.

23.

a. $p(2) = P(X = 2) = F(3) - F(2) = .39 - .19 = .20$.

b. $P(X > 3) = 1 - P(X \le 3) = 1 - F(3) = 1 - .67 = .33$.

c. $P(2 \le X \le 5) = F(5) - F(2–1) = F(5) - F(1) = .92 - .19 = .78$.

d. $P(2 < X < 5) = P(2 < X \le 4) = F(4) - F(2) = .92 - .39 = .53$.

33.

a. $E(X^2) = \sum_{x=0}^{1} x^2 \cdot p(x) = 0^2(1-p) + 1^2(p) = p.$

b. $V(X) = E(X^2) - [E(X)]^2 = p - [p]^2 = p(1-p).$

c. $E(X^{79}) = 0^{79}(1-p) + 1^{79}(p) = p$. In fact, $E(X^n) = p$ for any non-negative power n.

35. Let $h_3(X)$ and $h_4(X)$ equal the net revenue (sales revenue minus order cost) for 3 and 4 copies purchased, respectively. If 3 magazines are ordered ($6 spent), net revenue is $\$4 - \$6 = -\$2$ if $X = 1$, $2(\$4) - \$6 = \$2$ if $X = 2$, $3(\$4) - \$6 = \$6$ if $X = 3$, and also $6 if $X = 4$, 5, or 6 (since that additional demand simply isn't met. The values of $h_4(X)$ can be deduced similarly. *B*oth distributions are summarized below.

x	1	2	3	4	5	6
$h_3(x)$	–2	2	6	6	6	6
$h_4(x)$	–4	0	4	8	8	8
$p(x)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$

Using the table, $E[h_3(X)] = \sum_{x=1}^{6} h_3(x) \cdot p(x) = (-2)(\frac{1}{15}) + \ldots + (6)(\frac{2}{15}) = \$4.93.$

Similarly, $E[h_4(X)] = \sum_{x=1}^{6} h_4(x) \cdot p(x) = (-4)(\frac{1}{15}) + \ldots + (8)(\frac{2}{15}) = \$5.33.$

Therefore, ordering 4 copies gives slightly higher revenue, on the average.

37. Using the hint, $E(X) = \sum_{x=1}^{n} x \cdot \left(\frac{1}{n}\right) = \frac{1}{n}\sum_{x=1}^{n} x = \frac{1}{n}\left[\frac{n(n+1)}{2}\right] = \frac{n+1}{2}$. Similarly,

$E(X^2) = \sum_{x=1}^{n} x^2 \cdot \left(\frac{1}{n}\right) = \frac{1}{n}\sum_{x=1}^{n} x^2 = \frac{1}{n}\left[\frac{n(n+1)(2n+1)}{6}\right] = \frac{(n+1)(2n+1)}{6}$, so

$V(X) = \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = \frac{n^2-1}{12}.$

39. From the table, $E(X) = \sum xp(x) = 2.3$, $E(X^2) = 6.1$, and $V(X) = 6.1 - (2.3)^2 = .81$. Each lot weighs 5 lbs, so the number of pounds left $= 100 - 5X$. Thus the expected weight left is $E(100 - 5X) = 100 - 5E(X) =$ 88.5 lbs, and the variance of the weight left is $V(100 - 5X) = V(-5X) = (-5)^2V(X) = 25V(X) = 20.25$.

41. Use the hint: $V(aX + b) = E[((aX+b) - E(aX+b))^2] = \sum [ax + b - E(aX+b)]^2 p(x) =$

$\sum [ax + b - (a\mu + b)]^2 p(x) = \sum [ax - a\mu]^2 p(x) = a^2 \sum (x-\mu)^2 p(x) = a^2 V(X).$

43. With $a = 1$ and $b = -c$, $E(X - c) = E(aX + b) = a\,E(X) + b = E(X) - c$.
When $c = \mu$, $E(X - \mu) = E(X) - \mu = \mu - \mu = 0$; i.e., the expected deviation from the mean is zero.

45. $a \le X \le b$ means that $a \le x \le b$ for all x in the range of X. Hence $ap(x) \le xp(x) \le bp(x)$ for all x, and

$$\sum ap(x) \le \sum xp(x) \le \sum bp(x)$$
$$a\sum p(x) \le \sum xp(x) \le b\sum p(x)$$
$$a \cdot 1 \le E(X) \le b \cdot 1$$
$$a \le E(X) \le b$$

Section 3.4

47.

a. $B(4;15,.7) = .001$.

b. $b(4;15,.7) = B(4;15,.7) - B(3;15,.7) = .001 - .000 = .001$.

c. Now $p = .3$ (multiple vehicles). $b(6;15,.3) = B(6;15,.3) - B(5;15,.3) = .869 - .722 = .147$.

d. $P(2 \le X \le 4) = B(4;15,.7) - B(1;15,.7) = .001$.

e. $P(2 \le X) = 1 - P(X \le 1) = 1 - B(1;15,.7) = 1 - .000 = 1$.

f. The information that 11 accidents involved multiple vehicles is redundant (since $n = 15$ and $x = 4$). So, this is actually identical to **b**, and the answer is .001.

49. Let X be the number of "seconds," so $X \sim \text{Bin}(6, .10)$.

a. $P(X=1) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{6}{1} (.1)^1 (.9)^5 = .3543$.

b. $P(X \ge 2) = 1 - [P(X=0) + P(X=1)] = 1 - \left[\binom{6}{0}(.1)^0(.9)^6 + \binom{6}{1}(.1)^1(.9)^5\right] = 1 - [.5314 + .3543] =$ .1143.

c. Either 4 or 5 goblets must be selected.

Select 4 goblets with zero defects: $P(X=0) = \binom{4}{0}(.1)^0(.9)^4 = .6561$.

Select 4 goblets, one of which has a defect, and the 5th is good: $\left[\binom{4}{1}(.1)^1(.9)^3\right] \times .9 = .26244$

So, the desired probability is $.6561 + .26244 = .91854$.

51. Let X be the number of faxes, so $X \sim \text{Bin}(25, .25)$.

a. $E(X) = np = 25(.25) = 6.25$.

b. $V(X) = np(1-p) = 25(.25)(.75) = 4.6875$, so $SD(X) = 2.165$.

c. $P(X > 6.25 + 2(2.165)) = P(X > 10.58) = 1 - P(X \le 10.58) = 1 - P(X \le 10) = 1 - B(10;25,.25) = .030$.

53. Let "success" = has at least one citation and define X = number of individuals with at least one citation. Then $X \sim \text{Bin}(n = 15, p = .4)$.

   a. If at least 10 have no citations (failure), then at most 5 have had at least one (success): $P(X \le 5) = B(5;15,.40) = .403$.

   b. Half of 15 is 7.5, so less than half means 7 or fewer: $P(X \le 7) = B(7;15,.40) = .787$.

   c. $P(5 \le X \le 10) = P(X \le 10) - P(X \le 4) = .991 - .217 = .774$.

55. Let "success" correspond to a telephone that is submitted for service while under warranty and must be replaced. Then $p = P(\text{success}) = P(\text{replaced} \mid \text{submitted}) \cdot P(\text{submitted}) = (.40)(.20) = .08$. Thus X, the number among the company's 10 phones that must be replaced, has a binomial distribution with $n = 10$ and $p = .08$, so $P(X = 2) = \binom{10}{2}(.08)^2(.92)^8 = .1478$.

57. Let X = the number of flashlights that work, and let event B = {battery has acceptable voltage}. Then $P(\text{flashlight works}) = P(\text{both batteries work}) = P(B)P(B) = (.9)(.9) = .81$. We have assumed here that the batteries' voltage levels are independent.
   Finally, $X \sim \text{Bin}(10, .81)$, so $P(X \ge 9) = P(X = 9) + P(X = 10) = .285 + .122 = .407$.

59. In this example, $X \sim \text{Bin}(25, p)$ with p unknown.

   a. $P(\text{rejecting claim when } p = .8) = P(X \le 15 \text{ when } p = .8) = B(15; 25, .8) = .017$.

   b. $P(\underline{\text{not}} \text{ rejecting claim when p} = .7) = P(X > 15 \text{ when } p = .7) = 1 - P(X \le 15 \text{ when } p = .7) =$
   $= 1 - B(15; 25, .7) = 1 - .189 = .811$.
   For $p = .6$, this probability is $= 1 - B(15; 25, .6) = 1 - .575 = .425$.

   c. The probability of rejecting the claim when $p = .8$ becomes $B(14; 25, .8) = .006$, smaller than in **a** above. However, the probabilities of **b** above increase to .902 and .586, respectively. So, by changing 15 to 14, we're making it less likely that we will reject the claim when it's true (p really is $\ge .8$), but more likely that we'll "fail" to reject the claim when it's false (p really is $< .8$).

61. If topic A is chosen, then $n = 2$. When $n = 2$, $P(\text{at least half received}) = P(X \ge 1) = 1 - P(X = 0) =$
   $1 - \binom{2}{0}(.9)^0(.1)^2 = .99$.

   If topic B is chosen, then $n = 4$. When $n = 4$, $P(\text{at least half received}) = P(X \ge 2) = 1 - P(X \le 1) =$
   $1 - \left[\binom{4}{0}(.9)^0(.1)^4 + \binom{4}{1}(.9)^1(.1)^3\right] = .9963$.
   Thus topic B should be chosen if $p = .9$.

   However, if $p = .5$, then the probabilities are .75 for A and .6875 for B (using the same method as above), so now A should be chosen.

63.

   a. $b(x; n, 1-p) = \binom{n}{x}(1-p)^x(p)^{n-x} = \binom{n}{n-x}(p)^{n-x}(1-p)^x = b(n-x; n, p)$.
   Conceptually, $P(x \text{ S's when } P(\text{S}) = 1 - p) = P(n-x \text{ F's when } P(\text{F}) = p)$, since the two events are identical, but the labels S and F are arbitrary and so can be interchanged (if $P(\text{S})$ and $P(\text{F})$ are also interchanged), yielding $P(n-x \text{ S's when } P(\text{S}) = 1 - p)$ as desired.

b. Use the conceptual idea from **a:** $B(x; n, 1-p) = P(\underline{\text{at most}}\ x \text{ S's when } P(\text{S}) = 1-p) =$
$P(\underline{\text{at least}}\ n\text{–}x \text{ F's when } P(\text{F}) = p)$, since these are the same event
$= P(\text{at least } n\text{–}x \text{ S's when } P(\text{S}) = p)$, since the S and F labels are arbitrary
$= 1 - P(\text{at most } n\text{–}x\text{–}1 \text{ S's when } P(\text{S}) = p) = 1 - B(n\text{–}x\text{–}1; n, p)$.

c. Whenever $p > .5$, $(1-p) < .5$ so probabilities involving X can be calculated using the results **a** and **b** in combination with tables giving probabilities only for $p \le .5$.

65.

a. Although there are three payment methods, we are only concerned with S = uses a debit card and F = does not use a debit card. Thus we can use the binomial distribution. So, if X = the number of customers who use a debit card, $X \sim \text{Bin}(n = 100, p = .2)$. From this,
$E(X) = np = 100(.2) = 20$, and $V(X) = npq = 100(.2)(1–.2) = 16$.

b. With S = doesn't pay with cash, $n = 100$ and $p = .7$, so $\mu = np = 100(.7) = 70$, and $V = 21$.

67. When $n = 20$ and $p = .5$, $\mu = 10$ and $\sigma = 2.236$, so $2\sigma = 4.472$ and $3\sigma = 6.708$.
The inequality $|X - 10| \ge 4.472$ is satisfied if either $X \le 5$ or $X \ge 15$, or
$P(|X-\mu| \ge 2\sigma) = P(X \le 5 \text{ or } X \ge 15) = .021 + .021 = .042$. The inequality $|X - 10| \ge 6.708$ is satisfied if either $X \le 3$ or $X \ge 17$, so $P(|X-\mu| \ge 3\sigma) = P(X \le 3 \text{ or } X \ge 17) = .001 + .001 = .002$.

Section 3.5

69. According to the problem description, X is hypergeometric with $n = 6$, $N = 12$, and $M = 7$.

a. $P(X=4) = \dfrac{\binom{7}{4}\binom{5}{2}}{\binom{12}{6}} = \dfrac{350}{924} = .379$. $P(X \le 4) = 1 - P(X > 4) = 1 - [P(X=5) + P(X=6)] =$

$$1 - \left[\frac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}} + \frac{\binom{7}{6}\binom{5}{0}}{\binom{12}{6}}\right] = 1 - [.114 + .007] = 1 - .121 = .879.$$

b. $E(X) = n \cdot \dfrac{M}{N} = 6 \cdot \dfrac{7}{12} = 3.5$; $V(X) = \left(\dfrac{12-6}{12-1}\right)6\left(\dfrac{7}{12}\right)\left(1-\dfrac{7}{12}\right) = 0.795$; $\sigma = 0.892$. So,
$P(X > \mu + \sigma) = P(X > 3.5 + 0.892) = P(X > 4.392) = P(X = 5 \text{ or } 6) = .121$ (from part **a**).

c. We can approximate the hypergeometric distribution with the binomial if the population size and the number of successes are large. Here, $n = 15$ and $M/N = 40/400 = .1$, so
$h(x;15, 40, 400) \approx b(x;15, .10)$. Using this approximation, $P(X \le 5) \approx B(5; 15, .10) = .998$ from the binomial tables. (This agrees with the exact answer to 3 decimal places.)