CHAPTER 7

Section 7.1

1.

a. $z_{\alpha/2} = 2.81$ implies that $\alpha/2 = 1 - \Phi(2.81) = .0025$, so $\alpha = .005$ and the confidence level is $100(1-\alpha)\% = 99.5\%$.

b. $z_{\alpha/2} = 1.44$ implies that $\alpha = 2[1 - \Phi(1.44)] = .15$, and the confidence level is $100(1-\alpha)\% = 85\%$.

c. 99.7% confidence implies that $\alpha = .003$, $\alpha/2 = .0015$, and $z_{.0015} = 2.96$. (Look for cumulative area equal to $1 - .0015 = .9985$ in the main body of table A.3.) Or, just use $z \approx 3$ by the empirical rule.

d. 75% confidence implies $\alpha = .25$, $\alpha/2 = .125$, and $z_{.125} = 1.15$.

3.

a. A 90% confidence interval will be narrower. The z critical value for a 90% confidence level is 1.645, smaller than the z of 1.96 for the 95% confidence level, thus producing a narrower interval.

b. Not a correct statement. Once and interval has been created from a sample, the mean μ is either enclosed by it, or not. We have 95% confidence in the general procedure, under repeated and independent sampling.

c. Not a correct statement. The interval is an estimate for the population mean, not a boundary for population values.

d. Not a correct statement. In theory, if the process were repeated an infinite number of times, 95% of the intervals would contain the population mean μ. We *expect* 95 out of 100 intervals will contain μ, but we don't know this to be true.

5.

a. $4.85 \pm \dfrac{(1.96)(.75)}{\sqrt{20}} = 4.85 \pm .33 = (4.52, 5.18)$.

b. $z_{\alpha/2} = z.01 = 2.33$, so the interval is $4.56 \pm \dfrac{(2.33)(.75)}{\sqrt{16}} = (4.12, 5.00)$.

c. $n = \left[\dfrac{2(1.96)(.75)}{.40}\right]^2 = 54.02 \nearrow 55$.

d. Width $w = 2(.2) = .4$, so $n = \left[\dfrac{2(2.58)(.75)}{.4}\right]^2 = 93.61 \nearrow 94$.

17. $\bar{x} - z_{.01}\frac{s}{\sqrt{n}} = 135.39 - 2.33\frac{4.59}{\sqrt{153}} = 135.39 - .865 = 134.53$. We are 99% confident that the true average ultimate tensile strength is greater than 134.53.

19. $\hat{p} = \frac{201}{356} = .5646$; We calculate a 95% confidence interval for the proportion of all dies that pass the probe:

$$\frac{.5646 + \frac{(1.96)^2}{2(356)} \pm 1.96\sqrt{\frac{(.5646)(.4354)}{356} + \frac{(1.96)^2}{4(356)^2}}}{1 + \frac{(1.96)^2}{356}} = \frac{.5700 \pm .0518}{1.01079} = (.513, .615)$$

The simpler CI formula (7.11) gives $.5646 \pm 1.96\sqrt{\frac{.5646(.4354)}{356}} = (.513, .616)$, which is almost identical.

21. For a one-sided bound, we need $z_\alpha = z_{.05} = 1.645$; $\hat{p} = \frac{250}{1000} = .25$; and $\tilde{p} = \frac{.25 + 1.645^2 / 2000}{1 + 1.645^2 / 1000} = .2507$. The resulting 95% upper confidence bound for p, the true proportion of such consumers who never apply for a rebate, is $.2507 + \frac{1.645\sqrt{(.25)(.75)/1000 + (1.645)^2/(4\cdot1000^2)}}{1 + (1.645)^2/1000} = .2507 + .0225 = .2732$.

Yes, there is compelling evidence the true proportion is less than 1/3 (.3333), since we are 95% confident this true proportion is less than .2732.

23.

a. With such a large sample size, we can use the "simplified" CI formula (7.11). With $\hat{p} = .25$, $n = 2003$, and $z_{\alpha/2} = z_{.005} = 2.576$, the 99% confidence interval for p is

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = .25 \pm 2.576\sqrt{\frac{(.25)(.75)}{2003}} = .25 \pm .025 = (.225, .275).$$

b. Using the "simplified" formula for sample size and $\hat{p} = \hat{q} = .5$,

$$n = \frac{4z^2\hat{p}\hat{q}}{w^2} = \frac{4(2.576)^2(.5)(.5)}{(.05)^2} = 2654.31$$

So, a sample of size at least 2655 is required. (We use $\hat{p} = \hat{q} = .5$ here, rather than the values from the sample data, so that our CI has the desired width irrespective of what the true value of p might be. See the textbook discussion toward the end of Section 7.2.)

25.

a. $$n = \frac{2(1.96)^2(.25) - (1.96)^2(.01) \pm \sqrt{4(1.96)^4(.25)(.25 - .01) + .01(1.96)^4}}{.01} \approx 381$$

b. $$n = \frac{2(1.96)^2\left(\tfrac{1}{3}\cdot\tfrac{2}{3}\right) - (1.96)^2(.01) \pm \sqrt{4(1.96)^4\left(\tfrac{1}{3}\cdot\tfrac{2}{3}\right)\left(\tfrac{1}{3}\cdot\tfrac{2}{3} - .01\right) + .01(1.96)^4}}{.01} \approx 339$$

c. With df = $n - 1 = 16$, the critical value for a 95% CI is $t_{.025,16} = 2.120$, and the interval is $438.29 \pm (2.120)\left(\frac{15.14}{\sqrt{17}}\right) = 438.29 \pm 7.785 = (430.51, 446.08)$. Since 440 is within the interval, 440 is a plausible value for the true mean. 450, however, is not, since it lies outside the interval.

35. $n = 15$, $\bar{x} = 25.0$, $s = 3.5$; $t_{.025,14} = 2.145$

a. A 95% CI for the mean: $25.0 \pm 2.145\frac{3.5}{\sqrt{15}} = (23.06, 26.94)$.

b. A 95% prediction interval: $25.0 \pm 2.145(3.5)\sqrt{1+\frac{1}{15}} = (17.25, 32.75)$. The prediction interval is about 4 times wider than the confidence interval.

37.

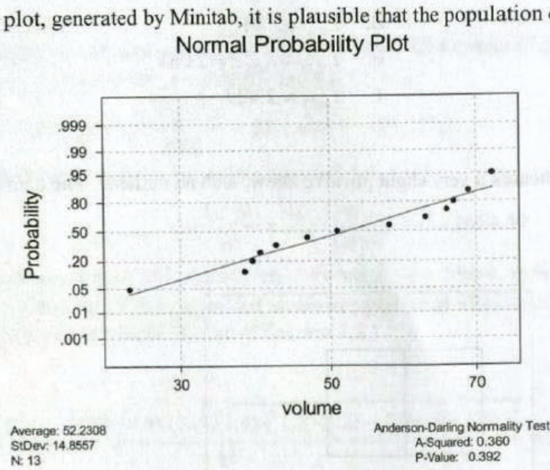
a. A 95% CI : $.9255 \pm 2.093(.0181) = .9255 \pm .0379 \Rightarrow (.8876, .9634)$

b. A 95% P.I. : $.9255 \pm 2.093(.0809)\sqrt{1+\frac{1}{20}} = .9255 \pm .1735 \Rightarrow (.7520, 1.0990)$

c. A tolerance interval is requested, with $k = 99$, confidence level 95%, and $n = 20$. The tolerance critical value, from Table A.6, is 3.615. The interval is $.9255 \pm 3.615(.0809) \Rightarrow (.6330, 1.2180)$.

39.

a. Based on the plot, generated by Minitab, it is plausible that the population distribution is normal.



b. We require a tolerance interval. From table A.6, with 95% confidence, $k = 95$, and n=13, the tolerance critical value is 3.081. $\bar{x} \pm 3.081s = 52.231 \pm 3.081(14.856) = 52.231 \pm 45.771 \Rightarrow (6.460, 98.002)$.

c. A prediction interval, with $t_{.025,12} = 2.179$:
$52.231 \pm 2.179(14.856)\sqrt{1+\frac{1}{13}} = 52.231 \pm 33.593 \Rightarrow (18.638, 85.824)$