43.

a. Let μ and σ denote the unknown mean and standard deviation. The given information provides $.05 = P(X < 39.12) = \Phi\left(\frac{39.12-\mu}{\sigma}\right) \Rightarrow \frac{39.12-\mu}{\sigma} \approx -1.645 \Rightarrow 39.12-\mu = -1.645\sigma$ and

$$.10 = P(X > 73.24) = 1-\Phi\left(\frac{73.24-\mu}{\sigma}\right) \Rightarrow \frac{73.24-\mu}{\sigma} = \Phi^{-1}(.9) \approx 1.28 \Rightarrow 73.24-\mu = 1.28\sigma.$$

Subtract the top equation from the bottom one to get $34.12 = 2.925\sigma$, or $\sigma \approx 11.665$ mph. Then, substitute back into either equation to get $\mu \approx 58.309$ mph.

b. $P(50 \le X \le 65) = \Phi(.57) - \Phi(-.72) = .7157 - .2358 = .4799$.

c. $P(X > 70) = 1 - \Phi(1.00) = 1 - .8413 = .1587$.

45. With $\mu = .500$ inches, the acceptable range for the diameter is between .496 and .504 inches, so unacceptable bearings will have diameters smaller than .496 or larger than .504.
The new distribution has $\mu = .499$ and $\sigma = .002$.

$$P(X < .496 \text{ or } X > .504) = P\left(Z < \frac{.496-.499}{.002}\right) + P\left(Z > \frac{.504-.499}{.002}\right) = P(Z < -1.5) + P(Z > 2.5) =$$

$\Phi(-1.5) + [1 - \Phi(2.5)] = .073$. 7.3% of the bearings will be unacceptable.

47. The stated condition implies that 99% of the area under the normal curve with $\mu = 12$ and $\sigma = 3.5$ is to the left of $c - 1$, so $c - 1$ is the 99th percentile of the distribution. Since the 99th percentile of the standard normal distribution is $z = 2.33$, $c - 1 = \mu + 2.33\sigma = 20.155$, and $c = 21.155$.

49.

a. $P(X > 4000) = P\left(Z > \frac{4000-3432}{482}\right) = P(Z > 1.18) = 1 - \Phi(1.18) = 1 - .8810 = .1190$;

$$P(3000 < X < 4000) = P\left(\frac{3000-3432}{482} < Z < \frac{4000-3432}{482}\right) = \Phi(1.18) - \Phi(-.90) = .8810 - .1841 = .6969.$$

b. $P(X < 2000 \text{ or } X > 5000) = P\left(Z < \frac{2000-3432}{482}\right) + P\left(Z > \frac{5000-3432}{482}\right)$

$= \Phi(-2.97) + [1 - \Phi(3.25)] = .0015 + .0006 = .0021$.

c. We will use the conversion 1 lb = 454 g, then 7 lbs = 3178 grams, and we wish to find $P(X > 3178) = P\left(Z > \frac{3178-3432}{482}\right) = 1 - \Phi(-.53) = .7019$.

d. We need the top .0005 and the bottom .0005 of the distribution. Using the z table, both .9995 and .0005 have multiple z values, so we will use a middle value, ±3.295. Then $3432 \pm 3.295(482) = 1844$ and 5020. The most extreme .1% of all birth weights are less than 1844 g and more than 5020 g.

e. Converting to pounds yields a mean of 7.5595 lbs and a standard deviation of 1.0608 lbs. Then $P(X > 7) = P\left(Z > \frac{7-7.5595}{1.0608}\right) = 1 - \Phi(-.53) = .7019$. This yields the same answer as in part **c**.

51. $P(|X - \mu| \ge \sigma) = 1 - P(|X - \mu| < \sigma) = 1 - P(\mu - \sigma < X < \mu + \sigma) = 1 - P(-1 \le Z \le 1) = .3174$.
Similarly, $P(|X - \mu| \ge 2\sigma) = 1 - P(-2 \le Z \le 2) = .0456$ and $P(|X - \mu| \ge 3\sigma) = .0026$.
These are considerably less than the bounds 1, .25, and .11 given by Chebyshev.

Section 4.4

59.

a. $E(X) = \frac{1}{\lambda} = 1$.

b. $\sigma = \frac{1}{\lambda} = 1$.

c. $P(X \le 4) = 1 - e^{-(1)(4)} = 1 - e^{-4} = .982$.

d. $P(2 \le X \le 5) = (1 - e^{-(1)(5)}) - (1 - e^{-(1)(2)}) = e^{-2} - e^{-5} = .129$.

61. Note that a mean value of 2.725 for the exponential distribution implies $\lambda = \frac{1}{2.725}$. Let X denote the duration of a rainfall event.

a. $P(X \ge 2) = 1 - P(X < 2) = 1 - P(X \le 2) = 1 - F(2; \lambda) = 1 - [1 - e^{-(1/2.725)(2)}] = e^{-2/2.725} = .4800$; $P(X \le 3) = F(3; \lambda) = 1 - e^{-(1/2.725)(3)} = .6674$; $P(2 \le X \le 3) = .6674 - .4800 = .1874$.

b. For this exponential distribution, $\sigma = \mu = 2.725$, so $P(X > \mu + 2\sigma) =$ $P(X > 2.725 + 2(2.725)) = P(X > 8.175) = 1 - F(8.175; \lambda) = e^{-(1/2.725)(8.175)} = e^{-3} = .0498$. On the other hand, $P(X < \mu - \sigma) = P(X < 2.725 - 2.725) = P(X < 0) = 0$, since an exponential random variable is non-negative.

63.

a. If a customer's calls are typically short, the first calling plan makes more sense. If a customer's calls are somewhat longer, then the second plan makes more sense, viz. 99¢ is less than 20min(10¢/min) = \$2 for the first 20 minutes under the first (flat-rate) plan.

b. $h_1(X) = 10X$, while $h_2(X) = 99$ for $X \le 20$ and $99 + 10(X - 20)$ for $X > 20$. With $\mu = 1/\lambda$ for the exponential distribution, it's obvious that $E[h_1(X)] = 10E[X] = 10\mu$. On the other hand,

$$E[h_2(X)] = 99 + 10\int_{20}^{\infty}(x-20)\lambda e^{-\lambda x}dx = 99 + \frac{10}{\lambda}e^{-20\lambda} = 99 + 10\mu e^{-20/\mu}.$$

When $\mu = 10$, $E[h_1(X)] = 100¢ = \$1.00$ while $E[h_2(X)] = 99 + 100e^{-2} \approx \$1.13$.
When $\mu = 15$, $E[h_1(X)] = 150¢ = \$1.50$ while $E[h_2(X)] = 99 + 150e^{-4/3} \approx \$1.39$.
As predicted, the first plan is better when expected call length is lower, and the second plan is better when expected call length is somewhat higher.

65.

a. From the mean and sd equations for the gamma distribution, $\alpha\beta = 37.5$ and $\alpha\beta^2 = (21.6)2 = 466.56$. Take the quotient to get $\beta = 466.56/37.5 = 12.4416$. Then, $\alpha = 37.5/\beta = 37.5/12.4416 = 3.01408\ldots$.

b. $P(X > 50) = 1 - P(X \le 50) = 1 - F(50/12.4416; 3.014) = 1 - F(4.0187; 3.014)$. If we approximate this by $1 - F(4; 3)$, Table A.4 gives $1 - .762 = .238$. Software gives the more precise answer of .237.

c. $P(50 \le X \le 75) = F(75/12.4416; 3.014) - F(50/12.4416; 3.014) = F(6.026; 3.014) - F(4.0187; 3.014) =$ $F(6; 3) - F(4; 3) = .938 - .762 = .176$.

Section 4.5

73.

a. $P(X \le 250) = F(250; 2.5, 200) = 1 - e^{-(250/200)^{2.5}} = 1 - e^{-1.75} = .8257.$
$P(X < 250) = P(X \le 250) = .8257.$
$P(X > 300) = 1 - F(300; 2.5, 200) = e^{-(1.5)^{2.5}} = .0636.$

b. $P(100 \le X \le 250) = F(250; 2.5, 200) - F(100; 2.5, 200) = .8257 - .162 = .6637.$

c. The question is asking for the median, $\tilde{\mu}$. Solve $F(\tilde{\mu}) = .5$: $.5 = 1 - e^{-(\tilde{\mu}/200)^{2.5}} \Rightarrow$
$e^{-(\tilde{\mu}/200)^{2.5}} = .5 \Rightarrow (\tilde{\mu}/200)^{2.5} = -\ln(.5) \Rightarrow \tilde{\mu} = 200(-\ln(.5))^{1/2.5} = 172.727$ hours.

75. Using the substitution $y = \left(\frac{x}{\beta}\right)^{\alpha} = \frac{x^{\alpha}}{\beta^{\alpha}}$. Then $dy = \frac{\alpha x^{\alpha-1}}{\beta^{\alpha}}dx$, and $\mu = \int_0^{\infty} x \cdot \frac{\alpha}{\beta^{\alpha}} x^{\alpha-1} e^{-(x/\beta)^{\alpha}} dx =$

$\int_0^{\infty} (\beta^{\alpha} y)^{1/\alpha} \cdot e^{-y} dy = \beta \int_0^{\infty} y^{1/\alpha} e^{-y} dy = \beta \cdot \Gamma\left(1 + \frac{1}{\alpha}\right)$ by definition of the gamma function.

77.

a. $E(X) = e^{\mu + \sigma^2/2} = e^{4.82} = 123.97.$
$V(X) = \left(e^{2(4.5)+.8^2}\right) \cdot \left(e^{.8^2} - 1\right) = 13{,}776.53 \Rightarrow \sigma = 117.373.$

b. $P(X \le 100) = \Phi\left(\frac{\ln(100) - 4.5}{.8}\right) = \Phi(0.13) = .5517.$

c. $P(X \ge 200) = 1 - P(X < 200) = 1 - \Phi\left(\frac{\ln(200) - 4.5}{.8}\right) = 1 - \Phi(1.00) = 1 - .8413 = .1587.$ Since X is continuous, $P(X > 200) = .1587$ as well.

79. Notice that μ_X and σ_X are the mean and standard deviation of the lognormal variable X in this example; they are not the parameters μ and σ which usually refer to the mean and standard deviation of $\ln(X)$. We're given $\mu_X = 10{,}281$ and $\sigma_X/\mu_X = .40$, from which $\sigma_X = .40\mu_X = 4112.4$.

a. To find the mean and standard deviation of $\ln(X)$, set the lognormal mean and variance equal to the appropriate quantities: $10{,}281 = E(X) = e^{\mu + \sigma^2/2}$ and $(4112.4)^2 = V(X) = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$. Square the first equation: $(10{,}281)^2 = e^{2\mu + \sigma^2}$. Now divide the variance by this amount:

$$\frac{(4112.4)^2}{(10{,}281)^2} = \frac{e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)}{e^{2\mu+\sigma^2}} \Rightarrow e^{\sigma^2} - 1 = (.40)^2 = .16 \Rightarrow \sigma = \sqrt{\ln(1.16)} = .38525$$

That's the standard deviation of $\ln(X)$. Use this in the formula for $E(X)$ to solve for μ:
$10{,}281 = e^{\mu + (.38525)^2/2} = e^{\mu + .07421} \Rightarrow \mu = 9.164$. That's $E(\ln(X))$.

b. $P(X \le 15{,}000) = P\left(Z \le \frac{\ln(15{,}000) - 9.164}{.38525}\right) = P(Z \le 1.17) = \Phi(1.17) = .8790.$

c. $P(X \geq \mu_X) = P(X \geq 10{,}281) = P\left(Z \geq \dfrac{\ln(10{,}281) - 9.164}{.38525}\right) = P(Z \geq .19) = 1 - \Phi(0.19) = .4247$. Even though the normal distribution is symmetric, the lognormal distribution is not a symmetric distribution. (See the lognormal graphs in the textbook.) So, the mean and the median of X aren't the same and, in particular, the probability X exceeds its own mean doesn't equal .5.

d. One way to check is to determine whether $P(X < 17{,}000) = .95$; this would mean 17,000 is indeed the 95th percentile. However, we find that $P(X < 17{,}000) = \Phi\left(\dfrac{\ln(17{,}000) - 9.164}{.38525}\right) = \Phi(1.50) = .9332$, so 17,000 is not the 95th percentile of this distribution (it's the 93.32%ile).

81.

a. $V(X) = e^{2(2.05)+.06}(e^{.06} - 1) = 3.96 \Rightarrow \text{SD}(X) = 1.99$ months.

b. $P(X > 12) = 1 - P(X \leq 12) = 1 - P\left(Z \leq \dfrac{\ln(12) - 2.05}{\sqrt{.06}}\right) = 1 - \Phi(1.78) = .0375$.

c. The mean of X is $E(X) = e^{2.05+.06/2} = 8.00$ months, so $P(\mu_X - \sigma_X < X < \mu_X + \sigma_X) = P(6.01 < X < 9.99) =$
$\Phi\left(\dfrac{\ln(9.99) - 2.05}{\sqrt{.06}}\right) - \Phi\left(\dfrac{\ln(6.01) - 2.05}{\sqrt{.06}}\right) = \Phi(1.03) - \Phi(-1.05) = .8485 - .1469 = .7016$.

d. $.5 = F(x) = \Phi\left(\dfrac{\ln(x) - 2.05}{\sqrt{.06}}\right) \Rightarrow \dfrac{\ln(x) - 2.05}{\sqrt{.06}} = \Phi^{-1}(.5) = 0 \Rightarrow \ln(x) - 2.05 = 0 \Rightarrow$ the median is given by $x = e^{2.05} = 7.77$ months.

e. Similarly, $\dfrac{\ln(\eta_{.99}) - 2.05}{\sqrt{.06}} = \Phi^{-1}(.99) = 2.33 \Rightarrow \eta_{.99} = e^{2.62} = 13.75$ months.

f. The probability of exceeding 8 months is $P(X > 8) = 1 - \Phi\left(\dfrac{\ln(8) - 2.05}{\sqrt{.06}}\right) = 1 - \Phi(.12) = .4522$, so the expected number that will exceed 8 months out of $n = 10$ is just $10(.4522) = 4.522$.

83. Since the standard beta distribution lies on (0, 1), the point of symmetry must be ½, so we require that $f\left(\tfrac{1}{2} - \mu\right) = f\left(\tfrac{1}{2} + \mu\right)$. Cancelling out the constants, this implies $\left(\tfrac{1}{2} - \mu\right)^{\alpha-1}\left(\tfrac{1}{2} + \mu\right)^{\beta-1} = \left(\tfrac{1}{2} + \mu\right)^{\alpha-1}\left(\tfrac{1}{2} - \mu\right)^{\beta-1}$, which (by matching exponents on both sides) in turn implies that $\alpha = \beta$.

Alternatively, symmetry about ½ requires μ = ½, so $\dfrac{\alpha}{\alpha + \beta} = .5$. Solving for α gives $\alpha = \beta$.