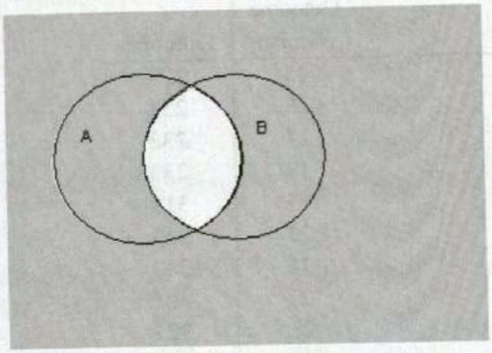
b. In the diagram below, the shaded area represents $(A \cap B)'$. Using the right-hand diagram from (a), the <u>union</u> of A' and B' is represented by the areas that have either shading <u>or</u> stripes (or both). Both of the diagrams display the same area.



Section 2.2

11.

a. .07.

b. .15 + .10 + .05 = .30.

c. Let A = the selected individual owns shares in a stock fund. Then $P(A) = .18 + .25 = .43$. The desired probability, that a selected customer does <u>not</u> shares in a stock fund, equals $P(A') = 1 - P(A) = 1 - .43 = .57$. This could also be calculated by adding the probabilities for all the funds that are not stocks.

13.

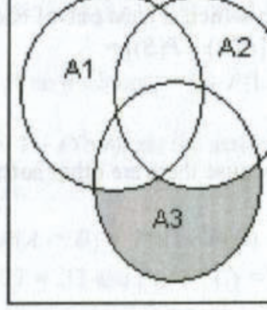
a. $A_1 \cup A_2$ = "awarded either #1 or #2 (or both)": from the addition rule,
$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = .22 + .25 - .11 = .36$.

b. $A_1' \cap A_2'$ = "awarded neither #1 or #2": using the hint and part (a),
$P(A_1' \cap A_2') = P((A_1 \cup A_2)') = 1 - P(A_1 \cup A_2) = 1 - .36 = .64$.

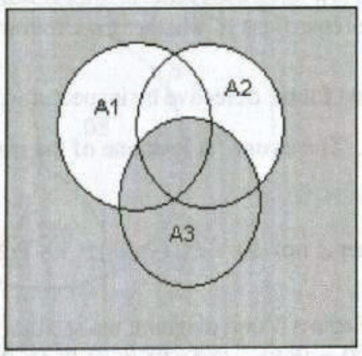
c. $A_1 \cup A_2 \cup A_3$ = "awarded at least one of these three projects": using the addition rule for 3 events,
$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) =$
$.22 + .25 + .28 - .11 - .05 - .07 + .01 = .53$.

d. $A_1' \cap A_2' \cap A_3'$ = "awarded none of the three projects":
$P(A_1' \cap A_2' \cap A_3') = 1 - P(\text{awarded at least one}) = 1 - .53 = .47$.

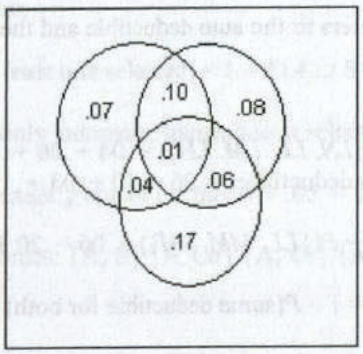
e. $A_1' \cap A_2' \cap A_3$ = "awarded #3 but neither #1 nor #2": from a Venn diagram,
$P(A_1' \cap A_2' \cap A_3) = P(A_3) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) =$
$.28 - .05 - .07 + .01 = .17$. The last term addresses the "double counting" of the two subtractions.



f. $(A_1' \cap A_2') \cup A_3$ = "awarded neither of #1 and #2, or awarded #3": from a Venn diagram,
$P((A_1' \cap A_2') \cup A_3) = P(\text{none awarded}) + P(A_3) = .47$ (from **d**) $+ .28 = 75$.



Alternatively, answers to **a-f** can be obtained from probabilities on the accompanying Venn diagram:



Section 2.3

29.

a. There are 26 letters, so allowing repeats there are $(26)(26) = (26)^2 = 676$ possible 2-letter domain names. Add in the 10 digits, and there are 36 characters available, so allowing repeats there are $(36)(36) = (36)^2 = 1296$ possible 2-character domain names.

b. By the same logic as part **a**, the answers are $(26)^3 = 17{,}576$ and $(36)^3 = 46{,}656$.

c. Continuing, $(26)^4 = 456{,}976$; $(36)^4 = 1{,}679{,}616$.

d. P(4-character sequence is already owned) = 1 – P(4-character sequence still available) = 1 – $97{,}786/(36)^4 = .942$.

31.

a. Use the Fundamental Counting Principle: $(9)(5) = 45$.

b. By the same reasoning, there are $(9)(5)(32) = 1440$ such sequences, so such a policy could be carried out for 1440 successive nights, or almost 4 years, without repeating exactly the same program.

33.

a. Since there are 15 players and 9 positions, and order matters in a line-up (catcher, pitcher, shortstop, etc. are different positions), the number of possibilities is $P_{9,15} = (15)(14)\ldots(7)$ or $15!/(15–9)! =$ 1,816,214,440.

b. For each of the starting line-ups in part (a), there are 9! possible batting orders. So, multiply the answer from (a) by 9! to get $(1{,}816{,}214{,}440)(362{,}880) = 659{,}067{,}881{,}472{,}000$.

c. Order still matters: There are $P_{3,5} = 60$ ways to choose three left-handers for the outfield and $P_{6,10} =$ 151,200 ways to choose six right-handers for the other positions. The total number of possibilities is = $(60)(151{,}200) = 9{,}072{,}000$.

35.

a. There are $\binom{10}{5} = 252$ ways to select 5 workers from the day shift. In other words, of all the ways to select 5 workers from among the 24 available, 252 such selections result in 5 day-shift workers. Since the grand total number of possible selections is $\binom{24}{5} = 42504$, the probability of randomly selecting 5 day-shift workers (and, hence, no swing or graveyard workers) is $252/42504 = .00593$.

b. Similar to **a**, there are $\binom{8}{5} = 56$ ways to select 5 swing-shift workers and $\binom{6}{5} = 6$ ways to select 5 graveyard-shift workers. So, there are $252 + 56 + 6 = 314$ ways to pick 5 workers from the same shift. The probability of this randomly occurring is $314/42504 = .00739$.

c. P(at least two shifts represented) = 1 – P(all from same shift) = 1 – .00739 = .99261.

d. Rather than consider many different options (choose 1, choose 2, etc.), re-frame the problem this way: at least 6 draws are required to get a 23W bulb iff a random sample of five bulbs fails to produce a 23W bulb. Since there are 11 non-23W bulbs, the chance of getting no 23W bulbs in a sample of size 5 is $\binom{11}{5}/\binom{15}{5}$ = 462/3003 = .154.

41.

a. (10)(10)(10)(10) = 10^4 = 10,000. These are the strings 0000 through 9999.

b. Count the number of prohibited sequences. There are (i) 10 with all digits identical (0000, 1111, …, 9999); (ii) 14 with sequential digits (0123, 1234, 2345, 3456, 4567, 5678, 6789, and 7890, plus these same seven descending); (iii) 100 beginning with 19 (1900 through 1999). That's a total of 10 + 14 + 100 = 124 impermissible sequences, so there are a total of 10,000 – 124 = 9876 permissible sequences. The chance of randomly selecting one is just $\frac{9876}{10{,}000} = .9876$.

c. All PINs of the form 8xx1 are legitimate, so there are (10)(10) = 100 such PINs. With someone randomly selecting 3 such PINs, the chance of guessing the correct sequence is 3/100 = .03.

d. Of all the PINs of the form 1xx1, eleven is prohibited: 1111, and the ten of the form 19x1. That leaves 89 possibilities, so the chances of correctly guessing the PIN in 3 tries is 3/89 = .0337.

43. There are $\binom{52}{5}$ = 2,598,960 five-card hands. The number of 10-high straights is (4)(4)(4)(4)(4) = 4^5 = 1024 (any of four 6s, any of four 7s, etc.). So, $P(\text{10 high straight}) = \frac{1024}{2{,}598{,}960} = .000394$. Next, there ten "types of straight: A2345, 23456, …, 910JQK, 10JQKA. So, $P(\text{straight}) = 10 \times \frac{1024}{2{,}598{,}960} = .00394$. Finally, there are only 40 straight flushes: each of the ten sequences above in each of the 4 suits makes (10)(4) = 40. So, $P(\text{straight flush}) = \frac{40}{2{,}598{,}960} = .00001539$.

Section 2.4

45.

a. $P(A) = .106 + .141 + .200 = .447$, $P(C) = .215 + .200 + .065 + .020 = .500$, and $P(A \cap C) = .200$.

b. $P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{.200}{.500} = .400$. If we know that the individual came from ethnic group 3, the probability that he has Type A blood is .40. $P(C|A) = \frac{P(A \cap C)}{P(A)} = \frac{.200}{.447} = .447$. If a person has Type A blood, the probability that he is from ethnic group 3 is .447.

c. Define D = "ethnic group 1 selected." We are asked for $P(D|B')$. From the table, $P(D \cap B') = .082 + .106 + .004 = .192$ and $P(B') = 1 - P(B) = 1 - [.008 + .018 + .065] = .909$. So, the desired probability is $P(D|B') = \frac{P(D \cap B')}{P(B')} = \frac{.192}{.909} = .211$.