**HW 2 Even Solutions**

**Required problems:** Ch 2: 22, 26, 48, 56

**22:** Let $A$ be the event that the motorist must stop at the first signal, and $B$ be the event that the motorist must stop at the second signal. We are given $\mathrm{P}(A) = 0.4, \mathrm{P}(B) = 0.5, \mathrm{P}(A \cup B) = 0.7$.

We can use the inclusion-exclusion principle to help fill out the Venn Diagram.

$$\begin{aligned} \mathrm{P}(A \cup B) &= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B) \\ 0.7 &= 0.4 + 0.5 - \mathrm{P}(A \cap B) \\ \mathrm{P}(A \cap B) &= 0.2 \end{aligned}$$

From the Venn Diagram, the answers to all the parts are direct

**a.** $\mathrm{P}(A \cap B) = 0.2$

**b.** $\mathrm{P}(A \cap B') = 0.2$

**c.** $\mathrm{P}(A \cap B') + \mathrm{P}(A' \cap B) = 0.2 + 0.3 = 0.5$

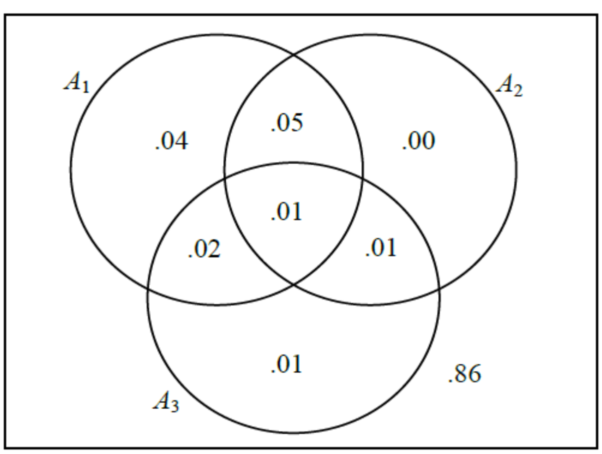
**26.** These questions can be solved algebraically, or with the Venn diagram below.

**a.** $P(A_1') = 1 - P(A_1) = 1 - .12 = .88$.

**b.** The addition rule says $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Solving for the intersection ("and") probability, you get $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = .12 + .07 - .13 = .06$.

**c.** A Venn diagram shows that $P(A \cap B') = P(A) - P(A \cap B)$. Applying that here with $A = A_1 \cap A_2$ and $B = A_3$, you get $P([A_1 \cap A_2] \cap A_3') = P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) = .06 - .01 = .05$.

**d.** The event "at most two defects" is the complement of "all three defects," so the answer is just 1 – $P(A_1 \cap A_2 \cap A_3)$ = 1 – .01 = .99.



**48.**

**a.** $P(A_2 \mid A_1) = \dfrac{P(A_2 \cap A_1)}{P(A_1)} = \dfrac{.06}{.12} = .50$. The numerator comes from Exercise 26.

**b.** $P(A_1 \cap A_2 \cap A_3 \mid A_1) = \dfrac{P([A_1 \cap A_2 \cap A_3] \cap A_1)}{P(A_1)} = \dfrac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \dfrac{.01}{.12} = .0833$. The numerator simplifies because $A_1 \cap A_2 \cap A_3$ is a subset of $A_1$, so their intersection is just the smaller event.

**c.** For this example, you definitely need a Venn diagram. The seven pieces of the partition inside the three circles have probabilities .04, .05, .00, .02, .01, .01, and .01. Those add to .14 (so the chance of no defects is .86).
Let $E$ = "exactly one defect." From the Venn diagram, $P(E) = .04 + .00 + .01 = .05$. From the addition above, $P$(at least one defect) = $P(A_1 \cup A_2 \cup A_3) = .14$. Finally, the answer to the question is $P(E \mid A_1 \cup A_2 \cup A_3) = \dfrac{P(E \cap [A_1 \cup A_2 \cup A_3])}{P(A_1 \cup A_2 \cup A_3)} = \dfrac{P(E)}{P(A_1 \cup A_2 \cup A_3)} = \dfrac{.05}{.14} = .3571$. The numerator simplifies because $E$ is a subset of $A_1 \cup A_2 \cup A_3$.

**d.** $P(A_3' \mid A_1 \cap A_2) = \dfrac{P(A_3' \cap [A_1 \cap A_2])}{P(A_1 \cap A_2)} = \dfrac{.05}{.06} = .8333$. The numerator is Exercise 26(c), while the denominator is Exercise 26(b).

**56.** $$P(A \mid B) + P(A' \mid B) = \frac{P(A \cap B)}{P(B)} + \frac{P(A' \cap B)}{P(B)} = \frac{P(A \cap B) + P(A' \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$