HW 4 Even Solutions

Required problems: Ch 3: 48, 58ab, 62, 68, 72, 80a, 110

48. $X \sim \text{Bin}(25, .05)$

a. $P(X \le 3) = B(3;25,.05) = .966$, while $P(X < 3) = P(X \le 2) = B(2;25,.05) = .873$.

b. $P(X \ge 4) = 1 - P(X \le 3) = 1 - B(3;25,.05) = .1 - .966 = .034$.

c. $P(1 \le X \le 3) = P(X \le 3) - P(X \le 0) = .966 - .277 = .689$.

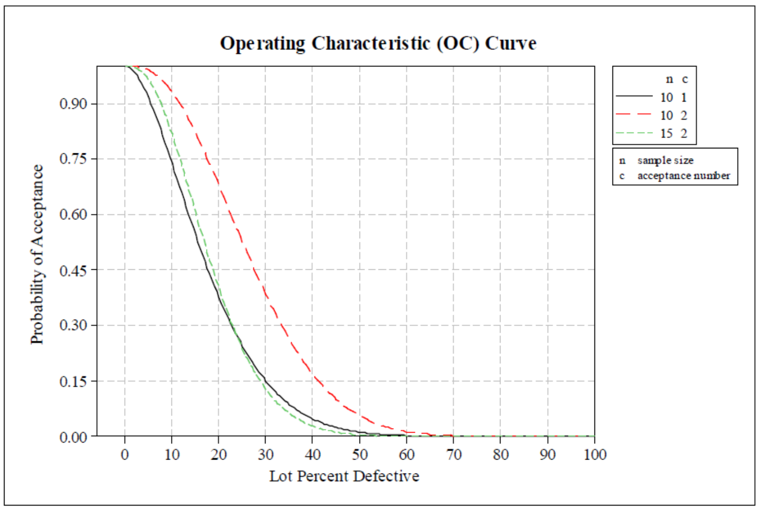
d. $E(X) = np = (25)(.05) = 1.25$, $\sigma_X = \sqrt{np(1-p)} = \sqrt{25(.05)(.95)} = 1.09$.

e. With $n = 50$, $P(X = 0) = \binom{50}{0}(.05)^0(.95)^{50} = (.95)^{50} = .077$.

58. Let p denote the actual proportion of defectives in the batch, and X denote the number of defectives in the sample.

a. If the actual proportion of defectives is p, then $X \sim \text{Bin}(10, p)$, and the batch is accepted iff $X \le 2$. Using the binomial formula, $P(X \le 2) = \binom{10}{0}p^0(1-p)^{10} + \binom{10}{1}p^1(1-p)^9 + \binom{10}{2}p^2(1-p)^8 =$ $[(1-p)^2 + 10p(1-p) + 45p^2](1-p)^8$. Values for this expression are tabulated below.

p:	.01	.05	.10	.20	.25
$P(X \le 2)$:	.9999	.9885	.9298	.6778	.5256



62.

a. $np(1 - p) = 0$ if either $p = 0$ (whence every trial is a failure, so there is no variability in X) or if $p = 1$ (whence every trial is a success and again there is no variability in X).

b. $\frac{d}{dp}[np(1-p)] = n[(1)(1-p) + p(-1)] = n[1 - 2p] = 0 \Rightarrow p = .5$, which is easily seen to correspond to a maximum value of $V(X)$.

68.

a. There are 18 items (people) total, 8 of which are "successes" (first-time examinees). Among these 18 items, 6 have been randomly assigned to this particular examiner. So, the random variable X is hypergeometric, with $N = 18$, $M = 8$, and $n = 6$.

b. $P(X = 2) = \dfrac{\binom{8}{2}\binom{18-8}{6-2}}{\binom{18}{6}} = \dfrac{\binom{8}{2}\binom{10}{4}}{\binom{18}{6}} = \dfrac{(28)(210)}{18564} = .3167.$

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2) = \dfrac{\binom{8}{0}\binom{10}{6}}{\binom{18}{6}} + \dfrac{\binom{8}{1}\binom{10}{5}}{\binom{18}{6}} + .3167 =$

$.0113 + .1086 + .3167 = .4366.$
$P(X \ge 2) = 1 - P(X \le 1) = 1 - [P(X = 0) + P(X = 1)] = 1 - [.0113 + .1086] = .8801.$

c. $E(X) = n \cdot \frac{M}{N} = 6 \cdot \frac{8}{18} = 2.67$; $V(X) = \left(\frac{18-6}{18-1}\right) \cdot 6\left(\frac{8}{18}\right)\left(1 - \frac{8}{18}\right) = 1.04575$; $\sigma = 1.023$.

72.

a. There are $N = 11$ candidates, $M = 4$ in the "top four" (obviously), and $n = 6$ selected for the first day's interviews. So, the probability x of the "top four" are interviewed on the first day equals $h(x; 6, 4, 11) =$
$\dfrac{\binom{4}{x}\binom{7}{6-x}}{\binom{11}{6}}$.

b. With X = the number of "top four" interview candidates on the first day, $E(X) = n \cdot \frac{M}{N} = 6 \cdot \frac{4}{11} = 2.18$.

80. Solutions are found using the cumulative Poisson table, $F(x; \mu) = F(x; 4)$.

a. $P(X \le 4) = F(4; 4) = .629$, while $P(X < 4) = P(X \le 3) = F(3; 4) = .434$.

110. The number of grasshoppers within a circular region of radius R follows a Poisson distribution with $\mu = \alpha \cdot$ area $= \alpha\pi R^2$.

P(at least one) = 1 – P(none) = 1 – $\dfrac{e^{-\alpha\pi R^2}(\alpha\pi R^2)^0}{0!} = 1 - e^{-\alpha\pi R^2} = .99 \Rightarrow e^{-\alpha\pi R^2} = .01 \Rightarrow R^2 = \dfrac{-\ln(.01)}{\alpha\pi} =$ $.7329 \Rightarrow R = .8561$ yards.