45. $a \le X \le b$ means that $a \le x \le b$ for all x in the range of X. Hence $ap(x) \le xp(x) \le bp(x)$ for all x, and

$$\begin{aligned}\sum ap(x) &\le \sum xp(x) \le \sum bp(x)\\ a\sum p(x) &\le \sum xp(x) \le b\sum p(x)\\ a\cdot 1 &\le E(X) \le b\cdot 1\\ a &\le E(X) \le b\end{aligned}$$

Section 3.4

47.

a. $B(4;15,.7) = .001$.

b. $b(4;15,.7) = B(4;15,.7) - B(3;15,.7) = .001 - .000 = .001$.

c. Now $p = .3$ (multiple vehicles). $b(6;15,.3) = B(6;15,.3) - B(5;15,.3) = .869 - .722 = .147$.

d. $P(2 \le X \le 4) = B(4;15,.7) - B(1;15,.7) = .001$.

e. $P(2 \le X) = 1 - P(X \le 1) = 1 - B(1;15,.7) = 1 - .000 = 1$.

f. The information that 11 accidents involved multiple vehicles is redundant (since $n = 15$ and $x = 4$). So, this is actually identical to **b**, and the answer is .001.

49. Let X be the number of "seconds," so $X \sim \text{Bin}(6, .10)$.

a. $P(X=1) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{6}{1}(.1)^1(.9)^5 = .3543$.

b. $P(X \ge 2) = 1 - [P(X=0) + P(X=1)] = 1 - \left[\binom{6}{0}(.1)^0(.9)^6 + \binom{6}{1}(.1)^1(.9)^5\right] = 1 - [.5314 + .3543] =$ .1143.

c. Either 4 or 5 goblets must be selected.

Select 4 goblets with zero defects: $P(X=0) = \binom{4}{0}(.1)^0(.9)^4 = .6561$.

Select 4 goblets, one of which has a defect, and the 5th is good: $\left[\binom{4}{1}(.1)^1(.9)^3\right] \times .9 = .26244$

So, the desired probability is $.6561 + .26244 = .91854$.

51. Let X be the number of faxes, so $X \sim \text{Bin}(25, .25)$.

a. $E(X) = np = 25(.25) = 6.25$.

b. $V(X) = np(1-p) = 25(.25)(.75) = 4.6875$, so $SD(X) = 2.165$.

c. $P(X > 6.25 + 2(2.165)) = P(X > 10.58) = 1 - P(X \le 10.58) = 1 - P(X \le 10) = 1 - B(10;25,.25) = .030$.

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53. Let "success" = has at least one citation and define X = number of individuals with at least one citation. Then $X \sim \text{Bin}(n = 15, p = .4)$.

a. If at least 10 have no citations (failure), then at most 5 have had at least one (success): $P(X \le 5) = B(5;15,.40) = .403$.

b. Half of 15 is 7.5, so less than half means 7 or fewer: $P(X \le 7) = B(7;15,.40) = .787$.

c. $P(5 \le X \le 10) = P(X \le 10) - P(X \le 4) = .991 - .217 = .774$.

55. Let "success" correspond to a telephone that is submitted for service while under warranty and must be replaced. Then $p = P(\text{success}) = P(\text{replaced} \mid \text{submitted})\cdot P(\text{submitted}) = (.40)(.20) = .08$. Thus X, the number among the company's 10 phones that must be replaced, has a binomial distribution with $n = 10$ and $p = .08$, so $P(X = 2) = \binom{10}{2}(.08)^2(.92)^8 = .1478$.

57. Let X = the number of flashlights that work, and let event B = {battery has acceptable voltage}. Then $P(\text{flashlight works}) = P(\text{both batteries work}) = P(B)P(B) = (.9)(.9) = .81$. We have assumed here that the batteries' voltage levels are independent.
Finally, $X \sim \text{Bin}(10, .81)$, so $P(X \ge 9) = P(X = 9) + P(X = 10) = .285 + .122 = .407$.

59. In this example, $X \sim \text{Bin}(25, p)$ with p unknown.

a. $P(\text{rejecting claim when } p = .8) = P(X \le 15 \text{ when } p = .8) = B(15; 25, .8) = .017$.

b. $P(\underline{\text{not}} \text{ rejecting claim when p} = .7) = P(X > 15 \text{ when } p = .7) = 1 - P(X \le 15 \text{ when } p = .7) =$
$= 1 - B(15; 25, .7) = 1 - .189 = .811$.
For $p = .6$, this probability is $= 1 - B(15; 25, .6) = 1 - .575 = .425$.

c. The probability of rejecting the claim when $p = .8$ becomes $B(14; 25, .8) = .006$, smaller than in **a** above. However, the probabilities of **b** above increase to .902 and .586, respectively. So, by changing 15 to 14, we're making it less likely that we will reject the claim when it's true (p really is $\ge .8$), but more likely that we'll "fail" to reject the claim when it's false (p really is $< .8$).

61. If topic A is chosen, then $n = 2$. When $n = 2$, $P(\text{at least half received}) = P(X \ge 1) = 1 - P(X = 0) =$
$1-\binom{2}{0}(.9)^0(.1)^2 = .99$.

If topic B is chosen, then $n = 4$. When $n = 4$, $P(\text{at least half received}) = P(X \ge 2) = 1 - P(X \le 1) =$
$1-\left[\binom{4}{0}(.9)^0(.1)^4+\binom{4}{1}(.9)^1(.1)^3\right] = .9963$.
Thus topic B should be chosen if $p = .9$.

However, if $p = .5$, then the probabilities are .75 for A and .6875 for B (using the same method as above), so now A should be chosen.

63.

a. $b(x; n, 1-p) = \binom{n}{x}(1-p)^x(p)^{n-x} = \binom{n}{n-x}(p)^{n-x}(1-p)^x = b(n-x; n, p)$.

Conceptually, $P(x \text{ S's when } P(\text{S}) = 1-p) = P(n-x \text{ F's when } P(\text{F}) = p)$, since the two events are identical, but the labels S and F are arbitrary and so can be interchanged (if $P(\text{S})$ and $P(\text{F})$ are also interchanged), yielding $P(n-x \text{ S's when } P(\text{S}) = 1-p)$ as desired.

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b. Use the conceptual idea from **a:** $B(x; n, 1-p) = P(\underline{\text{at most}}\ x\ \text{S's when}\ P(\text{S}) = 1-p) =$
$P(\underline{\text{at least}}\ n\text{–}x\ \text{F's when}\ P(\text{F}) = p)$, since these are the same event
$= P(\text{at least}\ n\text{–}x\ \text{S's when}\ P(\text{S}) = p)$, since the S and F labels are arbitrary
$= 1 - P(\text{at most}\ n\text{–}x\text{–}1\ \text{S's when}\ P(\text{S}) = p) = 1 - B(n\text{–}x\text{–}1; n, p)$.

c. Whenever $p > .5$, $(1-p) < .5$ so probabilities involving X can be calculated using the results **a** and **b** in combination with tables giving probabilities only for $p \le .5$.

65.

a. Although there are three payment methods, we are only concerned with S = uses a debit card and F = does not use a debit card. Thus we can use the binomial distribution. So, if X = the number of customers who use a debit card, $X \sim \text{Bin}(n = 100, p = .2)$. From this,
$E(X) = np = 100(.2) = 20$, and $V(X) = npq = 100(.2)(1\text{–}.2) = 16$.

b. With S = doesn't pay with cash, $n = 100$ and $p = .7$, so $\mu = np = 100(.7) = 70$, and $V = 21$.

67. When $n = 20$ and $p = .5$, $\mu = 10$ and $\sigma = 2.236$, so $2\sigma = 4.472$ and $3\sigma = 6.708$.
The inequality $|X - 10| \ge 4.472$ is satisfied if either $X \le 5$ or $X \ge 15$, or
$P(|X - \mu| \ge 2\sigma) = P(X \le 5 \text{ or } X \ge 15) = .021 + .021 = .042$. The inequality $|X - 10| \ge 6.708$ is satisfied if either $X \le 3$ or $X \ge 17$, so $P(|X - \mu| \ge 3\sigma) = P(X \le 3 \text{ or } X \ge 17) = .001 + .001 = .002$.

Section 3.5

69. According to the problem description, X is hypergeometric with $n = 6$, $N = 12$, and $M = 7$.

a. $P(X = 4) = \dfrac{\binom{7}{4}\binom{5}{2}}{\binom{12}{6}} = \dfrac{350}{924} = .379$. $P(X \le 4) = 1 - P(X > 4) = 1 - [P(X = 5) + P(X = 6)] =$

$$1 - \left[\frac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}} + \frac{\binom{7}{6}\binom{5}{0}}{\binom{12}{6}}\right] = 1 - [.114 + .007] = 1 - .121 = .879.$$

b. $E(X) = n \cdot \dfrac{M}{N} = 6 \cdot \dfrac{7}{12} = 3.5$; $V(X) = \left(\dfrac{12-6}{12-1}\right)6\left(\dfrac{7}{12}\right)\left(1 - \dfrac{7}{12}\right) = 0.795$; $\sigma = 0.892$. So,
$P(X > \mu + \sigma) = P(X > 3.5 + 0.892) = P(X > 4.392) = P(X = 5 \text{ or } 6) = .121$ (from part **a**).

c. We can approximate the hypergeometric distribution with the binomial if the population size and the number of successes are large. Here, $n = 15$ and $M/N = 40/400 = .1$, so
$h(x;15, 40, 400) \approx b(x;15, .10)$. Using this approximation, $P(X \le 5) \approx B(5; 15, .10) = .998$ from the binomial tables. (This agrees with the exact answer to 3 decimal places.)

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75. Let X = the number of boxes that do not contain a prize until you find 2 prizes. Then $X \sim$ NB(2, .2).

a. [redacted] Then

$$P(X = x) = nb(x; 2, .2) = \binom{x+2-1}{2-1}(.2)^2(1-.2)^x = (x+1)(.2)^2(.8)^x.$$

b. $P(4 \text{ boxes purchased}) = P(2 \text{ boxes without prizes}) = P(X = 2) = nb(2; 2, .2) = (2+1)(.2)^2(.8)^2 = .0768.$

c. $P(\text{at most 4 boxes purchased}) = P(X \le 2) = \sum_{x=0}^{2} nb(x; 2, .8) = .04 + .064 + .0768 = .1808.$

d. $E(X) = \dfrac{r(1-p)}{p} = \dfrac{2(1-.2)}{.2} = 8$. The total number of boxes you expect to buy is $8 + 2 = 10$.

77. This is identical to an experiment in which a single family has children until exactly 6 females have been born (since $p = .5$ for each of the three families). So,
$p(x) = nb(x; 6, .5) = \binom{x+5}{5}(.5)^6(1-.5)^x = \binom{x+5}{5}(.5)^{6+x}$. Also, $E(X) = \dfrac{r(1-p)}{p} = \dfrac{6(1-.5)}{.5} = 6$; notice this is just $2 + 2 + 2$, the sum of the expected number of males born to each family.

Section 3.6

79. All these solutions are found using the cumulative Poisson table, $F(x; \mu) = F(x; 1)$.

a. $P(X \le 5) = F(5; 1) = .999.$

b. $P(X = 2) = \dfrac{e^{-1}1^2}{2!} = .184$. Or, $P(X = 2) = F(2; 1) - F(1; 1) = .920 - .736 = .184.$

c. $P(2 \le X \le 4) = P(X \le 4) - P(X \le 1) = F(4; 1) - F(1; 1) = .260.$

d. For X Poisson, $\sigma = \sqrt{\mu} = 1$, so $P(X > \mu + \sigma) = P(X > 2) = 1 - P(X \le 2) = 1 - F(2; 1) = 1 - .920 = .080.$

81. Let $X \sim \text{Poisson}(\mu = 20)$.

a. $P(X \le 10) = F(10; 20) = .011.$

b. $P(X > 20) = 1 - F(20; 20) = 1 - .559 = .441.$

c. $P(10 \le X \le 20) = F(20; 20) - F(9; 20) = .559 - .005 = .554;$
$P(10 < X < 20) = F(19; 20) - F(10; 20) = .470 - .011 = .459.$

d. $E(X) = \mu = 20$, so $\sigma = \sqrt{20} = 4.472$. Therefore, $P(\mu - 2\sigma < X < \mu + 2\sigma) =$
$P(20 - 8.944 < X < 20 + 8.944) = P(11.056 < X < 28.944) = P(X \le 28) - P(X \le 11) =$
$F(28; 20) - F(11; 20) = .966 - .021 = .945.$

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83. The exact distribution of X is binomial with $n = 1000$ and $p = 1/200$; we can approximate this distribution by the Poisson distribution with $\mu = np = 5$.

a. $P(5 \le X \le 8) = F(8; 5) - F(4; 5) = .492$.

b. $P(X \ge 8) = 1 - P(X \le 7) = 1 - F(7; 5) = 1 - .867 = .133$.

85.

a. $\mu = 8$ when $t = 1$, so $P(X = 6) = \dfrac{e^{-8}8^6}{6!} = .122$; $P(X \ge 6) = 1 - F(5; 8) = .809$; and $P(X \ge 10) = 1 - F(9; 8) = .283$.

b. $t = 90$ min $= 1.5$ hours, so $\mu = 12$; thus the expected number of arrivals is 12 and the standard deviation is $\sigma = \sqrt{12} = 3.464$.

c. $t = 2.5$ hours implies that $\mu = 20$. So, $P(X \ge 20) = 1 - F(19; 20) = .530$ and $P(X \le 10) = F(10; 20) = .011$.

87.

a. For a two hour period the parameter of the distribution is $\mu = \alpha t = (4)(2) = 8$, so $P(X = 10) = \dfrac{e^{-8}8^{10}}{10!} = .099$.

b. For a 30-minute period, $\alpha t = (4)(.5) = 2$, so $P(X = 0) = \dfrac{e^{-2}2^0}{0!} = .135$.

c. The expected value is simply $E(X) = \alpha t = 2$.

89. In this example, α = rate of occurrence = 1/(mean time between occurrences) = $1/.5 = 2$.

a. For a two-year period, $\mu = \alpha t = (2)(2) = 4$ loads.

b. Apply a Poisson model with $\mu = 4$: $P(X > 5) = 1 - P(X \le 5) = 1 - F(5; 4) = 1 - .785 = .215$.

c. For $\alpha = 2$ and the value of t unknown, P(no loads occur during the period of length t) = $P(X = 0) = \dfrac{e^{-2t}(2t)^0}{0!} = e^{-2t}$. Solve for t: $e^{-2t} \le .1 \Rightarrow -2t \le \ln(.1) \Rightarrow t \ge 1.1513$ years.

91.

a. For a quarter-acre (.25 acre) plot, the mean parameter is $\mu = (80)(.25) = 20$, so $P(X \le 16) = F(16; 20) = .221$.

b. The expected number of trees is $\alpha\cdot$(area) = 80 trees/acre (85,000 acres) = 6,800,000 trees.

c. The area of the circle is $\pi r^2 = \pi(.1)^2 = .01\pi = .031416$ square miles, which is equivalent to $.031416(640) = 20.106$ acres. Thus X has a Poisson distribution with parameter $\mu = \alpha(20.106) = 80(20.106) = 1608.5$. That is, the pmf of X is the function $p(x; 1608.5)$.

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