Poisson random variable with parameter $\mu_1 + \mu_2$. Therefore, the total number of errors, $X + Y$, also has a Poisson distribution, with parameter $\mu_1 + \mu_2$.

13.

a. $f(x,y) = f_X(x) \cdot f_Y(y) = \begin{cases} e^{-x-y} & x \ge 0, y \ge 0 \\ 0 & \text{otherwise} \end{cases}$

b. By independence, $P(X \le 1 \text{ and } Y \le 1) = P(X \le 1) \cdot P(Y \le 1) = (1 - e^{-1})(1 - e^{-1}) = .400$.

c. $P(X + Y \le 2) = \int_0^2 \int_0^{2-x} e^{-x-y}\,dy\,dx = \int_0^2 e^{-x}\left[1 - e^{-(2-x)}\right]dx = \int_0^2 (e^{-x} - e^{-2})dx = 1 - e^{-2} - 2e^{-2} = .594.$

d. $P(X + Y \le 1) = \int_0^1 e^{-x}\left[1 - e^{-(1-x)}\right]dx = 1 - 2e^{-1} = .264$,
so $P(1 \le X + Y \le 2) = P(X + Y \le 2) - P(X + Y \le 1) = .594 - .264 = .330$.

15.

a. Each X_i has cdf $F(x) = P(X_i \le x) = 1 - e^{-\lambda x}$. Using this, the cdf of Y is
$F(y) = P(Y \le y) = P(X_1 \le y \cup [X_2 \le y \cap X_3 \le y])$
$= P(X_1 \le y) + P(X_2 \le y \cap X_3 \le y) - P(X_1 \le y \cap [X_2 \le y \cap X_3 \le y])$
$= (1 - e^{-\lambda y}) + (1 - e^{-\lambda y})^2 - (1 - e^{-\lambda y})^3$ for $y > 0$.

The pdf of Y is $f(y) = F'(y) = \lambda e^{-\lambda y} + 2(1 - e^{-\lambda y})\left(\lambda e^{-\lambda y}\right) - 3(1 - e^{-\lambda y})^2\left(\lambda e^{-\lambda y}\right) = 4\lambda e^{-2\lambda y} - 3\lambda e^{-3\lambda y}$ for $y > 0$.

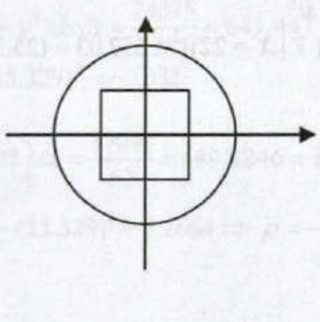
b. $E(Y) = \int_0^\infty y \cdot \left(4\lambda e^{-2\lambda y} - 3\lambda e^{-3\lambda y}\right)dy = 2\left(\frac{1}{2\lambda}\right) - \frac{1}{3\lambda} = \frac{2}{3\lambda}$.

17.

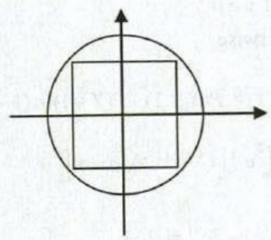
a. Let A denote the disk of radius $R/2$. Then $P((X,Y) \text{ lies in } A) = \iint_A f(x,y)\,dx\,dy$

$= \iint_A \frac{1}{\pi R^2}\,dx\,dy = \frac{1}{\pi R^2}\iint_A dx\,dy = \frac{\text{area of } A}{\pi R^2} = \frac{\pi (R/2)^2}{\pi R^2} = \frac{1}{4} = .25$. Notice that, since the joint pdf of X and Y is a constant (i.e., (X,Y) is uniform over the disk), it will be the case for any subset A that $P((X,Y)$ lies in $A) = \dfrac{\text{area of } A}{\pi R^2}$.

b. By the same ratio-of-areas idea, $P\left(-\frac{R}{2} \le X \le \frac{R}{2}, -\frac{R}{2} \le Y \le \frac{R}{2}\right) = \frac{R^2}{\pi R^2} = \frac{1}{\pi}$. This region is the square depicted in the graph below.



c. Similarly, $P\left(-\frac{R}{\sqrt{2}} \le X \le \frac{R}{\sqrt{2}}, -\frac{R}{\sqrt{2}} \le Y \le \frac{R}{\sqrt{2}}\right) = \frac{2R^2}{\pi R^2} = \frac{2}{\pi}$. This region is the slightly larger square depicted in the graph below, whose corners actually touch the circle.



d. $f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy = \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{1}{\pi R^2} dy = \frac{2\sqrt{R^2-x^2}}{\pi R^2}$ for $-R \le x \le R$.

Similarly, $f_Y(y) = \frac{2\sqrt{R^2-y^2}}{\pi R^2}$ for $-R \le y \le R$. X and Y are not independent, since the joint pdf is not the product of the marginal pdfs: $\frac{1}{\pi R^2} \ne \frac{2\sqrt{R^2-x^2}}{\pi R^2} \cdot \frac{2\sqrt{R^2-y^2}}{\pi R^2}$.

19. Throughout these solutions, $K = \frac{3}{380{,}000}$, as calculated in Exercise 9.

a. $f_{Y|X}(y\,|\,x) = \frac{f(x,y)}{f_X(x)} = \frac{K(x^2+y^2)}{10Kx^2+.05}$ for $20 \le y \le 30$.

$f_{X|Y}(x\,|\,y) = \frac{f(x,y)}{f_Y(y)} = \frac{K(x^2+y^2)}{10Ky^2+.05}$ for $20 \le x \le 30$.

b. $P(Y \ge 25\,|\,X=22) = \int_{25}^{30} f_{Y|X}(y\,|\,22)dy = \int_{25}^{30} \frac{K((22)^2+y^2)}{10K(22)^2+.05}dy = .5559$.

$P(Y \ge 25) = \int_{25}^{30} f_Y(y)dy = \int_{25}^{30}(10Ky^2+.05)dy = .75$. So, given that the right tire pressure is 22 psi, it's much less likely that the left tire pressure is at least 25 psi.

c. $E(Y\,|\,X=22) = \int_{-\infty}^{\infty} y \cdot f_{Y|X}(y\,|\,22)dy = \int_{20}^{30} y \cdot \frac{K((22)^2+y^2)}{10K(22)^2+.05}dy = 25.373$ psi.

$E(Y^2\,|\,X=22) = \int_{20}^{30} y^2 \cdot \frac{k((22)^2+y^2)}{10k(22)^2+.05}dy = 652.03 \Rightarrow$

$V(Y\,|\,X=22) = E(Y^2\,|\,X=22) - [E(Y\,|\,X=22)]^2 = 652.03 - (25.373)^2 = 8.24 \Rightarrow$

$SD(Y\,|\,X=22) = 2.87$ psi.

21.

a. $f_{X_3|X_1,X_2}(x_3 \mid x_1,x_2) = \dfrac{f(x_1,x_2,x_3)}{f_{X_1,X_2}(x_1,x_2)}$, where $f_{X_1,X_2}(x_1,x_2) =$ the marginal joint pdf of X_1 and X_2, i.e.

$f_{X_1,X_2}(x_1,x_2) = \int_{-\infty}^{\infty} f(x_1,x_2,x_3)\,dx_3$.

b. $f_{X_2,X_3|X_1}(x_2,x_3 \mid x_1) = \dfrac{f(x_1,x_2,x_3)}{f_{X_1}(x_1)}$, where $f_{X_1}(x_1) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1,x_2,x_3)\,dx_2\,dx_3$, the marginal pdf of X_1.

Section 5.2

23. $E(X_1 - X_2) = \sum_{x_1=0}^{4}\sum_{x_2=0}^{3}(x_1 - x_2)\cdot p(x_1,x_2) = (0-0)(.08) + (0-1)(.07) + \ldots + (4-3)(.06) = .15.$

Note: It can be shown that $E(X_1 - X_2)$ always equals $E(X_1) - E(X_2)$, so in this case we could also work out the means of X_1 and X_2 from their marginal distributions: $E(X_1) = 1.70$ and $E(X_2) = 1.55$, so $E(X_1 - X_2) = E(X_1) - E(X_2) = 1.70 - 1.55 = .15$.

25. The expected value of X, being uniform on $[L - A, L + A]$, is simply the midpoint of the interval, L. Since Y has the same distribution, $E(Y) = L$ as well. Finally, since X and Y are independent, $E(\text{area}) = E(XY) = E(X)\cdot E(Y) = L\cdot L = L^2$.

27. The amount of time Annie waits for Alvie, if Annie arrives first, is $Y - X$; similarly, the time Alvie waits for Annie is $X - Y$. Either way, the amount of time the first person waits for the second person is $h(X, Y) = |X - Y|$. Since X and Y are independent, their joint pdf is given by $f_X(x)\cdot f_Y(y) = (3x^2)(2y) = 6x^2y$. From these, the expected waiting time is

$E[h(X,Y)] = \int_0^1\int_0^1 |x-y|\cdot f(x,y)\,dx\,dy = \int_0^1\int_0^1 |x-y|\cdot 6x^2y\,dx\,dy$

$= \int_0^1\int_0^x (x-y)\cdot 6x^2y\,dy\,dx + \int_0^1\int_x^1 (x-y)\cdot 6x^2y\,dy\,dx = \dfrac{1}{6} + \dfrac{1}{12} = \dfrac{1}{4}$ hour, or 15 minutes.

29. $\text{Cov}(X,Y) = -\dfrac{2}{75}$ and $\mu_X = \mu_Y = \dfrac{2}{5}$.

$E(X^2) = \int_0^1 x^2\cdot f_X(x)\,dx = 12\int_0^1 x^3(1 - x^2dx) = \dfrac{12}{60} = \dfrac{1}{5}$, so $V(X) = \dfrac{1}{5} - \left(\dfrac{2}{5}\right)^2 = \dfrac{1}{25}$.

Similarly, $V(Y) = \dfrac{1}{25}$, so $\rho_{X,Y} = \dfrac{-\frac{2}{75}}{\sqrt{\frac{1}{25}}\cdot\sqrt{\frac{1}{25}}} = -\dfrac{50}{75} = -\dfrac{2}{3}$.

31.

a. $E(X) = \int_{20}^{30} x f_X(x)\,dx = \int_{20}^{30} x\left[10Kx^2 + .05\right]dx = \dfrac{1925}{76} = 25.329 = E(Y)$,

$E(XY) = \int_{20}^{30}\int_{20}^{30} xy\cdot K(x^2 + y^2)\,dx\,dy = \dfrac{24375}{38} = 641.447 \Rightarrow$

$\text{Cov}(X, Y) = 641.447 - (25.329)^2 = -.1082$.

b. $E(X^2) = \int_{20}^{30} x^2\left[10Kx^2 + .05\right]dx = \dfrac{37040}{57} = 649.8246 = E(Y^2) \Rightarrow$

$V(X) = V(Y) = 649.8246 - (25.329)^2 = 8.2664 \Rightarrow \rho = \dfrac{-.1082}{\sqrt{(8.2664)(8.2664)}} = -.0131.$

39. X is a binomial random variable with $n = 15$ and $p = .8$. The values of X, then $X/n = X/15$ along with the corresponding probabilities $b(x; 15, .8)$ are displayed in the accompanying pmf table.

x	0	1	2	3	4	5	6	7	8	9	10
$x/15$	0	.067	.133	.2	.267	.333	.4	.467	.533	.6	.667
$p(x/15)$	.000	.000	.000	.000	.000	.000	.001	.003	.014	.043	.103

x	11	12	13	14	15
$x/15$	.733	.8	.867	.933	1
$p(x/15)$	.188	.250	.231	.132	.035

41. The tables below delineate all 16 possible (x_1, x_2) pairs, their probabilities, the value of $\bar{x}$ for that pair, and the value of r for that pair. Probabilities are calculated using the independence of X_1 and X_2.

(x_1, x_2)	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4
probability	.16	.12	.08	.04	.12	.09	.06	.03
$\bar{x}$	1	1.5	2	2.5	1.5	2	2.5	3
r	0	1	2	3	1	0	1	2

(x_1, x_2)	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
probability	.08	.06	.04	.02	.04	.03	.02	.01
$\bar{x}$	2	2.5	3	3.5	2.5	3	3.5	4
r	2	1	0	1	3	2	1	2

a. Collecting the $\bar{x}$ values from the table above yields the pmf table below.

$\bar{x}$	1	1.5	2	2.5	3	3.5	4
$p(\bar{x})$	.16	.24	.25	.20	.10	.04	.01

b. $P(\bar{X} \le 2.5) = .16 + .24 + .25 + .20 = .85$.

c. Collecting the r values from the table above yields the pmf table below.

r	0	1	2	3
$p(r)$	.30	.40	.22	.08

d. With $n = 4$, there are numerous ways to get a sample average of at most 1.5, since $\bar{X} \le 1.5$ iff the sum of the X_i is at most 6. Listing out all options, $P(\bar{X} \le 1.5) = P(1,1,1,1) + P(2,1,1,1) + \ldots + P(1,1,1,2) + P(1,1,2,2) + \ldots + P(2,2,1,1) + P(3,1,1,1) + \ldots + P(1,1,1,3)$
$= (.4)^4 + 4(.4)^3(.3) + 6(.4)^2(.3)^2 + 4(.4)^2(.2)^2 = .2400$.

43. The statistic of interest is the fourth spread, or the difference between the medians of the upper and lower halves of the data. The population distribution is uniform with $A = 8$ and $B = 10$. Use a computer to generate samples of sizes $n = 5$, 10, 20, and 30 from a uniform distribution with $A = 8$ and $B = 10$. Keep the number of replications the same (say 500, for example). For each replication, compute the upper and lower fourth, then compute the difference. Plot the sampling distributions on separate histograms for $n = 5$, 10, 20, and 30.

b. The sports report begins 260 minutes after he begins grading papers.

$$P(T_0 > 260) = P\left(Z > \frac{260-240}{37.95}\right) = P(Z > .53) = .2981.$$

51. Individual times are given by $X \sim N(10, 2)$. For day 1, $n = 5$, and so

$$P(\bar{X} \le 11) = P\left(Z \le \frac{11-10}{2/\sqrt{5}}\right) = P(Z \le 1.12) = .8686.$$

For day 2, $n = 6$, and so

$$P(\bar{X} \le 11) = P(\bar{X} \le 11) = P\left(Z \le \frac{11-10}{2/\sqrt{6}}\right) = P(Z \le 1.22) = .8888.$$

Finally, assuming the results of the two days are independent (which seems reasonable), the probability the sample average is at most 11 min on both days is (.8686)(.8888) = .7720.

53.

a. With the values provided,

$$P(\bar{X} \ge 51) = P\left(Z \ge \frac{51-50}{1.2/\sqrt{9}}\right) = P(Z \ge 2.5) = 1 - .9938 = .0062.$$

b. Replace $n = 9$ by $n = 40$, and

$$P(\bar{X} \ge 51) = P\left(Z \ge \frac{51-50}{1.2/\sqrt{40}}\right) = P(Z \ge 5.27) \approx 0.$$

55.

a. With Y = # of tickets, Y has approximately a normal distribution with $\mu = 50$ and $\sigma = \sqrt{\mu} = 7.071$. So, using a continuity correction from [35, 70] to [34.5, 70.5],

$$P(35 \le Y \le 70) \approx P\left(\frac{34.5-50}{7.071} \le Z \le \frac{70.5-50}{7.071}\right) = P(-2.19 \le Z \le 2.90) = .9838.$$

b. Now $\mu = 5(50) = 250$, so $\sigma = \sqrt{250} = 15.811$.

Using a continuity correction from [225, 275] to [224.5, 275.5], $P(225 \le Y \le 275) \approx$

$$P\left(\frac{224.5-250}{15.811} \le Z \le \frac{275.5-250}{15.811}\right) = P(-1.61 \le Z \le 1.61) = .8926.$$

c. Using software, part (a) = $\sum_{y=35}^{70} \frac{e^{-50}50^y}{y!} = .9862$ and part (b) = $\sum_{y=225}^{275} \frac{e^{-250}250^y}{y!} = .8934$. Both of the approximations in (a) and (b) are correct to 2 decimal places.

57. With the parameters provided, $E(X) = \alpha\beta = 100$ and $V(X) = \alpha\beta^2 = 200$. Using a normal approximation,

$$P(X \le 125) \approx P\left(Z \le \frac{125-100}{\sqrt{200}}\right) = P(Z \le 1.77) = .9616.$$