HW 3 Even Solutions

Required problems: Ch 2: 76, 80; Ch 3: 12, 18, 30, 38

76. Follow the same logic as in Exercise 75: If the probability of an event is p, and there are n independent "trials," the chance this event never occurs is $(1-p)^n$, while the chance of at least one occurrence is $1-(1-p)^n$. With $p = 1/9{,}000{,}000{,}000$ and $n = 1{,}000{,}000{,}000$, this calculates to $1 - .9048 = .0952$.

Note: The final answer should be 0.1052, not 0.0952.

80. Let A_i denote the event that component #i works ($i = 1, 2, 3, 4$). Based on the design of the system, the event "the system works" is $(A_1 \cup A_2) \cup (A_3 \cap A_4)$. We'll eventually need $P(A_1 \cup A_2)$, so work that out first: $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = (.9) + (.9) - (.9)(.9) = .99$. The third term uses independence of events. Also, $P(A_3 \cap A_4) = (.8)(.8) = .64$, again using independence.

Now use the addition rule and independence for the system:

$$\begin{aligned} P((A_1 \cup A_2) \cup (A_3 \cap A_4)) &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P((A_1 \cup A_2) \cap (A_3 \cap A_4)) \\ &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P(A_1 \cup A_2) \times P(A_3 \cap A_4) \\ &= (.99) + (.64) - (.99)(.64) = .9964 \end{aligned}$$

(You could also use deMorgan's law in a couple of places.)

12.

a. Since there are 50 seats, the flight will accommodate all ticketed passengers who show up as long as there are no more than 50. $P(Y \leq 50) = .05 + .10 + .12 + .14 + .25 + .17 = .83$.

b. This is the complement of part **a**: $P(Y > 50) = 1 - P(Y \leq 50) = 1 - .83 = .17$.

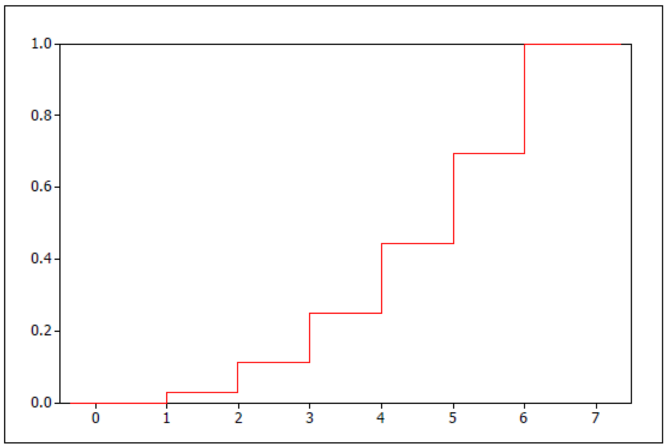
c. If you're the first standby passenger, you need no more than 49 people to show up (so that there's space left for you). $P(Y \leq 49) = .05 + .10 + .12 + .14 + .25 = .66$. On the other hand, if you're third on the standby list, you need no more than 47 people to show up (so that, even with the two standby passengers ahead of you, there's still room). $P(Y \leq 47) = .05 + .10 + .12 = .27$.

18.

a. $p(1) = P(M = 1) = P(\{(1,1)\}) = \frac{1}{36}$; $p(2) = P(M = 2) = P(\{(1,2)(2,1)(2,2)\}) = \frac{3}{36}$;
$p(3) = P(M = 3) = P(\{(1,3)(2,3)(3,1)(3,2)(3,3)\}) = \frac{5}{36}$. Continuing the pattern, $p(4) = \frac{7}{36}$, $p(5) = \frac{9}{36}$, and $p(6) = \frac{11}{36}$.

b. Using the values in **a**,

$$F(m) = \begin{cases} 0 & m < 1 \\ \frac{1}{36} & 1 \le m < 2 \\ \frac{4}{36} & 2 \le m < 3 \\ \frac{9}{36} & 3 \le m < 4 \\ \frac{16}{36} & 4 \le m < 5 \\ \frac{25}{36} & 5 \le m < 6 \\ 1 & m \ge 6 \end{cases}$$



30.

a. $E(Y) = \sum_{y=0}^{3} y \cdot p(y) = 0(.60) + 1(.25) + 2(.10) + 3(.05) = .60.$

b. $E(100Y^2) = \sum_{y=0}^{3} 100y^2 \cdot p(y) = 0(.60) + 100(.25) + 400(.10) + 900(.05) = \$110.$

38.

a. $E(X) = 1(.15) + 2(.35) + 3(.35) + 4(.15) = 2.5$. By linearity, $E(5 - X) = 5 - E(X) = 5 - 2.5 = 2.5$ as well.

b. Since $150/(5 - X)$ is not a linear function of X, we cannot use the results from **a**. Instead, we must create a new weighted average:
$E[150/(5 - X)] = [150/(5 - 1)](.15) + [150/(5 - 2)](.35) + [150/(5 - 3)](.35) + [150/(5 - 4)](.15) =$ 71.875. Since \$71.875 < \$75, they're better off in the long run charging a flat fee of \$75.