HW 6 Even Solutions Math 3070-01

Required problems: Ch 4: 36, 60, 72bc

36.

    a. $P(X < 1500) = P(Z < 3) = \Phi(3) = .9987$; $P(X \geq 1000) = P(Z \geq -.33) = 1 - \Phi(-.33) = 1 - .3707 = .6293$.

    b. $P(1000 < X < 1500) = P(-.33 < Z < 3) = \Phi(3) - \Phi(-.33) = .9987 - .3707 = .6280$

    c. From the table, $\Phi(z) = .02 \Rightarrow z = -2.05 \Rightarrow x = 1050 - 2.05(150) = 742.5$ µm. The smallest 2% of droplets are those smaller than 742.5 µm in size.

    d. Let $Y =$ the number of droplets, out of 5, that exceed 1500 µm. Then Y is binomial, with $n = 5$ and $p =$ .0013 from **a**. So, $P(Y = 2) = \binom{5}{2}(.0013)^2(.9987)^3 \approx 1.68 \times 10^{-5}$.

60.

    a. $P(X \leq 100) = 1 - e^{-(100)(.01386)} = 1 - e^{-1.386} = .7499$.
        $P(X \leq 200) = 1 - e^{-(200)(.01386)} = 1 - e^{-2.772} = .9375$.
        $P(100 \leq X \leq 200) = P(X \leq 200) - P(X \leq 100) = .9375 - .7499 = .1876$.

    b. First, since X is exponential, $\mu = \dfrac{1}{\lambda} = \dfrac{1}{.01386} = 72.15$, $\sigma = 72.15$. Then
        $P(X > \mu + 2\sigma) = P(X > 72.15 + 2(72.15)) = P(X > 216.45) = 1 - (1 - e^{-.01386(216.45)}) = e^{-3} = .0498$.

    c. Remember the median is the solution to $F(x) = .5$. Use the formula for the exponential cdf and solve for x: $F(x) = 1 - e^{-.01386x} = .5 \Rightarrow e^{-.01386x} = .5 \Rightarrow -.01386x = \ln(.5) \Rightarrow x = -\dfrac{\ln(.5)}{.01386} = 50.01$ m.

72bc

    b. $P(X \leq 6) = 1 - e^{-(6/\beta)^\alpha} = 1 - e^{-(6/3)^2} = 1 - e^{-4} = .982$.

    c. $P(1.5 \leq X \leq 6) = (1 - e^{-(6/3)^2}) - (1 - e^{-(1.5/3)^2}) = e^{-.25} - e^{-4} = .760$.