HW 5 Rubric Math 3070-01

Required problems: Ch 4: 2, 4, 10, 14abc, 24

2. $f(x) = \frac{1}{10}$ for $-5 \le x \le 5$ and $= 0$ otherwise

a. $P(X < 0) = \int_{-5}^{0} \frac{1}{10}\,dx = .5$.

b. $P(-2.5 < X < 2.5) = \int_{-2.5}^{2.5} \frac{1}{10}\,dx = .5$.

c. $P(-2 \le X \le 3) = \int_{-2}^{3} \frac{1}{10}\,dx = .5$.

d. $P(k < X < k+4) = \int_{k}^{k+4} \frac{1}{10}\,dx = \frac{1}{10}x\Big]_{k}^{k+4} = \frac{1}{10}[(k+4) - k] = .4$.

4.

a. $\int_{-\infty}^{\infty} f(x;\theta)dx = \int_{0}^{\infty} \frac{x}{\theta^2} e^{-x^2/2\theta^2}\,dx = -e^{-x^2/2\theta^2}\Big]_{0}^{\infty} = 0 - (-1) = 1$

b. $P(X \le 200) = \int_{-\infty}^{200} f(x;\theta)dx = \int_{0}^{200} \frac{x}{\theta^2} e^{-x^2/2\theta^2}\,dx = -e^{-x^2/2\theta^2}\Big]_{0}^{200} \approx -.1353 + 1 = .8647$.

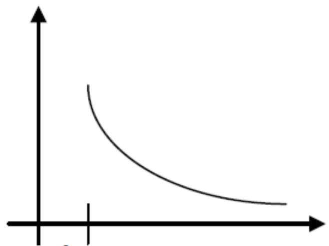
$P(X < 200) = P(X \le 200) \approx .8647$, since X is continuous.
$P(X \ge 200) = 1 - P(X < 200) \approx .1353$.

c. $P(100 \le X \le 200) = \int_{100}^{200} f(x;\theta)dx = -e^{-x^2/20{,}000}\Big]_{100}^{200} \approx .4712$.

d. For $x > 0$, $P(X \le x) = \int_{-\infty}^{x} f(y;\theta)dy = \int_{0}^{x} \frac{y}{\theta^2} e^{-y^2/2\theta^2}\,dx = -e^{-y^2/2\theta^2}\Big]_{0}^{x} = 1 - e^{-x^2/2\theta^2}$.

10.

a. The pdf is a decreasing function of x, beginning at $x = \theta$.



b. $\int_{-\infty}^{\infty} f(x;k,\theta)\,dx = \int_{\theta}^{\infty} \frac{k\theta^k}{x^{k+1}}dx = k\theta^k \int_{\theta}^{\infty} x^{-k-1}\,dx = \theta^k \cdot (-x^{-k})\Big]_{\theta}^{\infty} = 0 - \theta^k \cdot (-\theta^{-k}) = 1$.

c. $P(X \le b) = \int_{\theta}^{b} \frac{k\theta^k}{x^{k+1}}dx = -\frac{\theta^k}{x^k}\Big]_{\theta}^{b} = 1 - \left(\frac{\theta}{b}\right)^k$.

d. $P(a \le X \le b) = \int_{a}^{b} \frac{k\theta^k}{x^{k+1}}dx = -\frac{\theta^k}{x^k}\Big]_{a}^{b} = \left(\frac{\theta}{a}\right)^k - \left(\frac{\theta}{b}\right)^k$.

14.

a. If X is uniformly distributed on the interval from A to B, then $E(X) = \int_{A}^{B} x \cdot \frac{1}{B-A}dx = \frac{A+B}{2}$, the midpoint of the interval. Also, $E(X^2) = \frac{A^2 + AB + B^2}{3}$, from which $V(X) = E(X^2) - [E(X)]^2 = \ldots = \frac{(B-A)^2}{12}$.

With $A = 7.5$ and $B = 20$, $E(X) = 13.75$ and $V(X) = 13.02$.

b. From Example 4.6, the complete cdf is $F(x) = \begin{cases} 0 & x < 7.5 \\ \frac{x-7.5}{12.5} & 7.5 \le x < 20 \\ 1 & 20 \le x \end{cases}$.

c. $P(X \le 10) = F(10) = .200$; $P(10 \le X \le 15) = F(15) - F(10) = .4$.

24.

a. $E(X) = \int_\theta^\infty x \cdot \frac{k\theta^k}{x^{k+1}} dx = k\theta^k \int_\theta^\infty \frac{1}{x^k} dx = \frac{k\theta^k x^{-k+1}}{-k+1}\Big]_\theta^\infty = \frac{k\theta}{k-1}$.

b. If we attempt to substitute $k = 1$ into the previous answer, we get an undefined expression. More precisely, $\lim_{k \to 1^+} E(X) = \infty$.

c. $E(X^2) = k\theta^k \int_\theta^\infty \frac{1}{x^{k-1}} dx = \frac{k\theta^2}{k-2}$, so $V(X) = \left(\frac{k\theta^2}{k-2}\right) - \left(\frac{k\theta}{k-1}\right)^2 = \frac{k\theta^2}{(k-2)(k-1)^2}$.

d. Using the expression above, $V(X) = \infty$ since $E(X^2) = \infty$ if $k = 2$.

e. $E(X^n) = k\theta^k \int_\theta^\infty x^{n-(k+1)} dx$, which will be finite iff $n - (k+1) < -1$, i.e. if $n < k$.