

1 Introduction

Blabla . Here \sim Vila. \wedge Encore.

2 Assoc

Blablabla

Print *objects*.

```
Inductive objects : Set :=
  letter : letters → objects | up_0 : objects → objects → objects
```

Print *nodes*.

```
Inductive nodes : objects → Set :=
  self : ∀ A : objects, A
| at_left : ∀ A : objects, A → ∀ B : objects, A ∧0 B
| at_right : ∀ A B : objects, B → A ∧0 B
```

Print *lt_right*.

```
Inductive lt_right : ∀ A : objects, A → A → Set :=
  lt_right_cons1 : ∀ (B : objects) (z : B) (C : objects),
    self (C ∧0 B) <r at_right C z
| lt_right_cons2 : ∀ (B C : objects) (x y : B),
  x <r y → at_left x C <r at_left y C
| lt_right_cons3 : ∀ (B C : objects) (x y : B),
  x <r y → at_right C x <r at_right C y
```

Print *arrows*.

```
Inductive arrows : objects → objects → Set :=
  unitt : ∀ A : objects, arrows A A
| bracket_left : ∀ A B C : objects,
  arrows (A ∧0 B ∧0 C) ((A ∧0 B) ∧0 C)
| up_1 : ∀ A B A0 B0 : objects,
  arrows A B → arrows A0 B0 → arrows (A ∧0 A0) (B ∧0 B0)
| com : ∀ A B C : objects, arrows A B → arrows B C → arrows A C
```

Print *arrows_assoc*.

Inductive *arrows_assoc* : *objects* → *objects* → **Set** :=
 | *unitt_assoc* : ∀ *A* : *objects*, *arrows_assoc* *A* *A*
 | *bracket_left_assoc* : ∀ *A B C* : *objects*,
 arrows_assoc (*A* ∧₀ *B* ∧₀ *C*) ((*A* ∧₀ *B*) ∧₀ *C*)
 | *bracket_right_assoc* : ∀ *A B C* : *objects*,
 arrows_assoc ((*A* ∧₀ *B*) ∧₀ *C*) (*A* ∧₀ *B* ∧₀ *C*)
 | *up_1_assoc* : ∀ *A B A0 B0* : *objects*,
 arrows_assoc *A B* →
 arrows_assoc *A0 B0* → *arrows_assoc* (*A* ∧₀ *A0*) (*B* ∧₀ *B0*)
 | *com_assoc* : ∀ *A B C* : *objects*,
 arrows_assoc *A B* → *arrows_assoc* *B C* → *arrows_assoc* *A C*

About *arrows_coerce*.

arrows_coerce : ∀ *A B* : *objects*, *arrows* *A B* → *arrows_assoc* *A B*

Print *same*.

Inductive *same* : ∀ *A B* : *objects*, *arrows* *A B* → *arrows* *A B* → **Set** :=
 | *same_refl* : ∀ (*A B* : *objects*) (*f* : *arrows* *A B*), *f* ~_s *f*
 | *same_trans* : ∀ (*A B* : *objects*) (*f g h* : *arrows* *A B*),
 f ~_s *g* → *g* ~_s *h* → *f* ~_s *h*
 | *same_sym* : ∀ (*A B* : *objects*) (*f g* : *arrows* *A B*), *f* ~_s *g* → *g* ~_s *f*
 | *same_cong_com* : ∀ (*A B C* : *objects*) (*f f0* : *arrows* *A B*)
 (*g g0* : *arrows* *B C*),
 f ~_s *f0* → *g* ~_s *g0* → (*g* <_o *f*) ~_s (*g0* <_o *f0*)
 | *same_cong_up_1* : ∀ (*A B A0 B0* : *objects*) (*f f0* : *arrows* *A B*)
 (*g g0* : *arrows* *A0 B0*),
 f ~_s *f0* → *g* ~_s *g0* → (*f* ∧₁ *g*) ~_s (*f0* ∧₁ *g0*)
 | *same_cat_left* : ∀ (*A B* : *objects*) (*f* : *arrows* *A B*),
 (*unitt* *B* <_o *f*) ~_s *f*
 | *same_cat_right* : ∀ (*A B* : *objects*) (*f* : *arrows* *A B*),
 (*f* <_o *unitt* *A*) ~_s *f*
 | *same_cat_assoc* : ∀ (*A B C D* : *objects*) (*f* : *arrows* *A B*)
 (*g* : *arrows* *B C*) (*h* : *arrows* *C D*),
 (*h* <_o *g* <_o *f*) ~_s ((*h* <_o *g*) <_o *f*)
 | *same_bif_up_unit* : ∀ *A B* : *objects*,
 (*unitt* *A* ∧₁ *unitt* *B*) ~_s *unitt* (*A* ∧₀ *B*)
 | *same_bif_up_com* : ∀ (*A B C A0 B0 C0* : *objects*)
 (*f* : *arrows* *A B*) (*g* : *arrows* *B C*)
 (*f0* : *arrows* *A0 B0*) (*g0* : *arrows* *B0 C0*),
 ((*g* <_o *f*) ∧₁ (*g0* <_o *f0*)) ~_s (*g* ∧₁ *g0* <_o *f* ∧₁ *f0*)
 | *same_bracket_left_5* : ∀ *A B C D* : *objects*,

$$\begin{aligned}
& (bracket_left (A \wedge 0 B) C D < o \\
& \quad bracket_left A B (C \wedge 0 D)) \sim s \\
& (bracket_left A B C \wedge 1 unitt D < o \\
& \quad bracket_left A (B \wedge 0 C) D < o \\
& \quad unitt A \wedge 1 bracket_left B C D)
\end{aligned}$$

Voila la fin