Lifeisgood: Learning Invariant Features via In-Label Swapping for Generalizing Out-of-Distribution in Machine Fault Diagnosis

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To facilitate reading the paper, abbreviations, general rule of superscripts and subscripts, and important symbols (all sorted primarily in alphabet order) are listed as follows:

Nomenclature

Abbreviations

1D-CNN one dimensional convolutional neural network

CaSN causal representation of sufficiency and necessity

CausIRL_MMD causality based invariant representation learning

CondCAD conditional contrastive adversarial domain

CWRU Case Western Reserve University

DG domain generalization

EPdM Ecole Polytechnique de Montr´eal

ERM empirical risk minimization

IB_IRM information bottleneck invariant risk minimization

iDAG invariant directed acyclic graph

IIB invariant information bottleneck

IRM invariant risk minimization

KAIST Korea Advanced Institute of Science and Technology

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MDGCML multisource domain-class gradient coordination meta-learning

RDM risk distribution matching

RIDG rational invariance for domain generalization

S0-1 loss swapping 0-1 loss

SAGM sharpness-aware gradient matching

SCE loss swapping cross-entropy loss

SCEDUB swapping cross-entropy difference upper bound

SCIRM sparsity constraint invariant risk Minimization

SelfReg self-supervised contrastive regularization

T-SNE distributed stochastic neighbor embedding

UBFC University of Bourgogne Franche-Comt ́e

VNE von Neumann entropy

VREx variance of risk extrapolation

w.r.t with respect to

General Rule of Superscripts and Subscripts (without otherwise specified)

V^2 variable V is squared

V^c variable V is associated with the label c

V^j variable V is indexed by j where $j = 1, 2, 3, ...$

V_i^j variable V is indexed by or associated with both i and j

V^* variable V is the optimal value

V^δ variable V is associated with the swapping set δ

$V^{\mathcal{U}(lb,ub)}$ variable V has a power of $\mathcal{U}$ that is drawn uniformly at random from the range $[lw, up]$ where lb and ub is the lower bound and upper bound respectively

V_i variable V is indexed by i where $i = 1, 2, 3, ...$

V_n variable V is associated with the n-th data instance X_n

$V_n(e_i)$ variable V is associated with the n-th data instance X_n from i-th data domain e_i

$V_{\{0,1\}}$ variable V is associated with the 0-1 loss

$V_{\{i,j\}}$ variable V is associated with a feature pair (Z_i, Z_j)

V_{CE} variable V is associated with the cross-entropy loss

V_{erm} variable V is associated with empirical risk minimization

V_{irm} variable V is associated with invariant risk minimization

$V_{n_i(\delta)}$ variable V is associated with swapping set δ and n_i-th data instance X_{n_i} where $i = 1, 2, 3...$ such that X_{n_1}, X_{n_2}, etc., denote different data instances

V_{te} variable V is associated with the testing domain distribution

V_{tr} variable V is associated with the training domain distribution

$V_{y_{n_i}}$ variable V the n_i-th data instance and its label y_{n_i} where $i = 1, 2, 3, ...$

Symbols

$:=$ defined as

$[C]$ label index set

$[E]$ domain index set

$[M]$ data index set

$[N]$ sample index set

& logic "AND"

$\overset{\star}{\mathcal{L}}{}^{\delta}_{\{0,1\}}$ Bayes optimal risks on top of invariant features for the S0-1 loss $\mathcal{L}^{\delta}_{\{0,1\}}$

$\overset{\star}{\mathcal{L}}{}^{\delta^\star}_{CE}$ Bayes optimal risks on top of invariant features for the SCE loss $\mathcal{L}^{\delta^\star}_{CE}$

α descending flag

$\bar{\mathcal{F}}$ a non-empty subset of the model hypothesis class $\mathcal{F}$ such that $\forall \bar{f} \in \bar{\mathcal{F}}$, the featurizer $\bar{h}_\theta$ of $\bar{f}$ generates invariant features

β Keeping rate

δ instance of Δ

$\Delta := \mathcal{P}([U])$ meta-swapping set, defined as the power set of $[U]$

$\delta^\star$ optimal swapping set

ϵ acceptable margin

$\hat{\mathcal{L}}^{\delta^\star}_{CE}$ practical SCE loss

$\hat{P}_{te}$ empirical meta-testing distribution

$\hat{P}_{tr}$	empirical meta-training distribution
κ	weight decay
λ	balancing weight
λ_{IRM}	balancing weight of IRM
$\mathbb{I}$	indicator function
$\mathbb{R}^M$	real value set of dimension M, the data space
$\mathbb{R}^U$	real value set of dimension U, the feature space
$\mathbb{R}_+$	positive real values
$\mathbb{V}$	variance
$\mathbf{X}$	collection of data
$\mathcal{A}(r)$	variable only dependent on the Hellinger distance constraint
$\mathcal{B}(r)$	distribution set determined by the Hillinger distance constraint r
$\mathcal{C}(r)$	variable determined by the Hillinger distance constraint r
$\mathcal{D}(r, N, \sigma, \rho)$	variable only depending on the Hillinger distance constraint r, the sample size N, the confidence parameter σ, and the maximum loss difference between features with the different labels ρ
$\mathcal{F}$	model hypothesis class
$\mathcal{L}_{\{0,1\}}$	0-1 loss
$\mathcal{L}^{\delta}_{\{0,1\}}$	S0-1 loss with the swapping set δ
$\mathcal{L}_{CE}$	cross-entropy loss
$\mathcal{P}_+$	pair sets of same labels
$\mathcal{P}_-$	pair sets of different labels
$\mathcal{Z}$	feature space for theoretical proofs
μ	reference probability measure
$\nabla_{\{\omega,\theta\}}\hat{\mathcal{L}}^{\delta^\star}_{CE}$	gradients of the featurizer and classifier
$\odot$	point-wise product
Ω	parameter set of the classifier
ω	parameter instance of the classifier
ϕ	function that is, non-negative, convex, differentiable at 0, and $\phi'(0) \leq 0$

$\psi(\epsilon)$	variable only determined by the acceptable margin ϵ
ρ	maximum loss difference between the features with the different labels
σ	confidence parameter
Θ	parameter set of the featurizer
θ	parameter instance of the featurizer
$\tilde{\mathcal{L}}^{\delta}_{\{0,1\}}$	Bayes optimal risks based on all features of training domains for the S0-1 loss $\mathcal{L}^{\delta}_{\{0,1\}}$
$\tilde{\mathcal{L}}^{\delta^\star}_{CE}$	Bayes optimal risks based on all features of training domains for the S0-1 loss $\mathcal{L}^{\delta^\star}_{CE}$
$\tilde{G}^{c}_{\{i,\tilde{j}\}}$	same labeled features with a randomized index order from $G^{c}_{\{i,j\}}$
$\tilde{Z}_{n_1(\delta)}$	swapped feature of the feature Z_{n_1} based on the swapping set δ
Υ	union of index sets where the entries of the features are not identical
υ	element of the union of index sets Υ
a_i	classification accuracy of a trial
A_t	average accuracy at task level
$A_{(t,alg)}$	average accuracy of a method alg at task level
A_{alg}	average accuracy of a method alg at dataset level
Avg	average accuracy at dataset level
B	a batch for computing a loss
C^2_N	number of combining 2 elements from a set of N elements
$Count$	count of surpassing ERM
$Count_{all}$	total count of surpassing ERM
d_t	standard deviation at task level
$d_{(t,alg)}$	standard deviation of a method alg at task level
e_i	the i-th domain instance
f	learning model
$Freq_{erm}$	frequency of surpassing ERM
g	featurizer

$G_i^c = \left(G^c_{\{i,j\}}, \tilde{G}^c_{\{i,\tilde{j}\}}\right)$ pair-wise cosine similarity between two collections of same labled features corresonding to the label c

g_ω linear classifier with parameter ω

$G^c_{\{i,j\}} = \{Z_j, y_j\}_{[J(c)]}$ grouping features by labels where $[J(c)]$ is the index set of the same labeled features with the label as c

H Hellinger distance

h featurizer

h_θ featurizer with parameter θ

l_n instance-wise loss corresponding to data instance X_n and label instance y_n

M number M

N number N

n_i element of the sample index set $[N]$

N_t number of trials on the DG task

N_{all} total number of comparisons on all datasets

$P_1, P_2 \in \mathcal{B}(r)$ P_1, P_2 are the elements of a set of distributions constraint by the Hellinger distance constraint r

p_n^c the probability of predicting the feature Z_n or the data instance X_n to the label $c \in [C]$, which is also c-th entry of p_n

P_{te} meta-testing distribution

P_{tr} meta-training distribution

R risk function

S training step

$S_N^2(\delta)_{\{0,1\}}$ S0-1 difference between a pair of the swapped feature and the original feature, where N, 2, δ, $\{0, 1\}$ indicate sample size, square operation, swapping set, 0-1 loss, respectively

w_υ υ-th entry of the linear classifier weight $W_{y_{n_1}}$

$W_{y_{n_1}}$ class-wise weight vector of the linear classifier g_ω corresponding to the label y_{n_1}

X_n the n-th data instance of $\mathbf{X}$

x_n^M the M-th entry of X_n

Y	collection of labels
Z_n	the feature with the input as X_n
$Z_n(e_i)$	the n-th feature given by the n-th data instance of the i-th domain
$z_{n_1}^u(e_1)$	the u-th entry of a feature Z with the data instance idnex n_1 and the domain index e_1
$l_{\{0,1\}_n}$	instance-wise 0-1 loss
l_{CEn}	instance-wise cross-entropy loss
$n_1(\delta)$	instance index n_1 that is associate with the swapping set δ
$\mathcal{L}_{CE}^{\delta^\star}$	SCE loss with the optimal swapping set $\delta^\star$
B_0	a batch at training